Phase Transitions from Stars

Planck 2022

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Technische Universität München

Based on 2105.13354 and 2106.11320
Reuven Balkin, Javi Serra, Konstantin Springmann, SS and Andreas Weiler

01.06.2022

Phase transition from stars

Analogy: supercooled water

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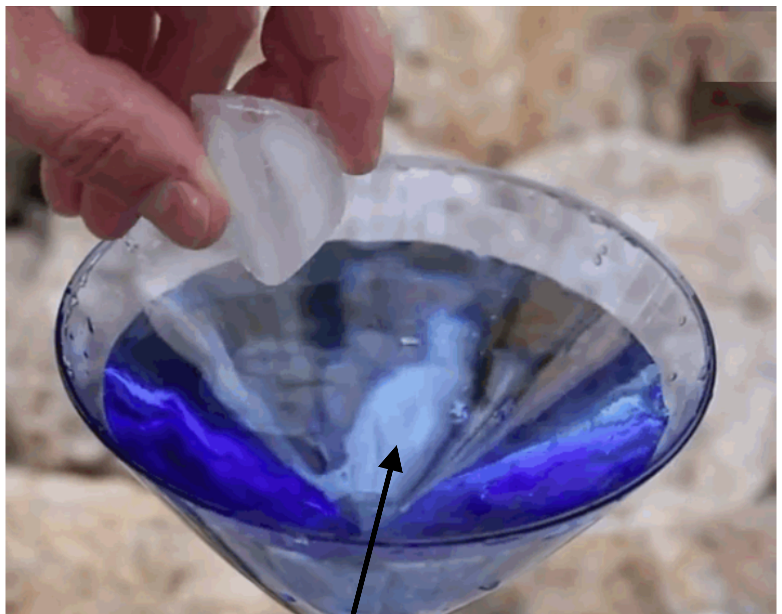
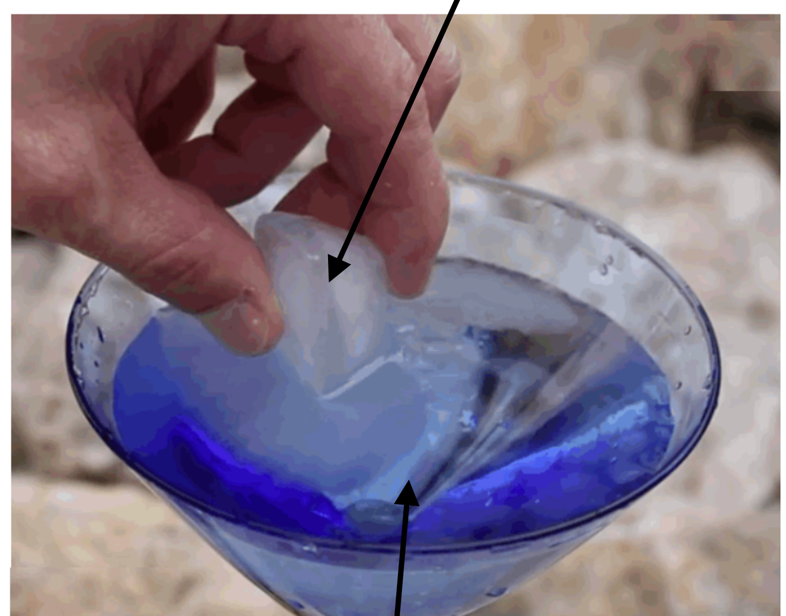


Universe in meta-stable ground state

star triggering the phase transition



phase transition

Universe in lower energy final state

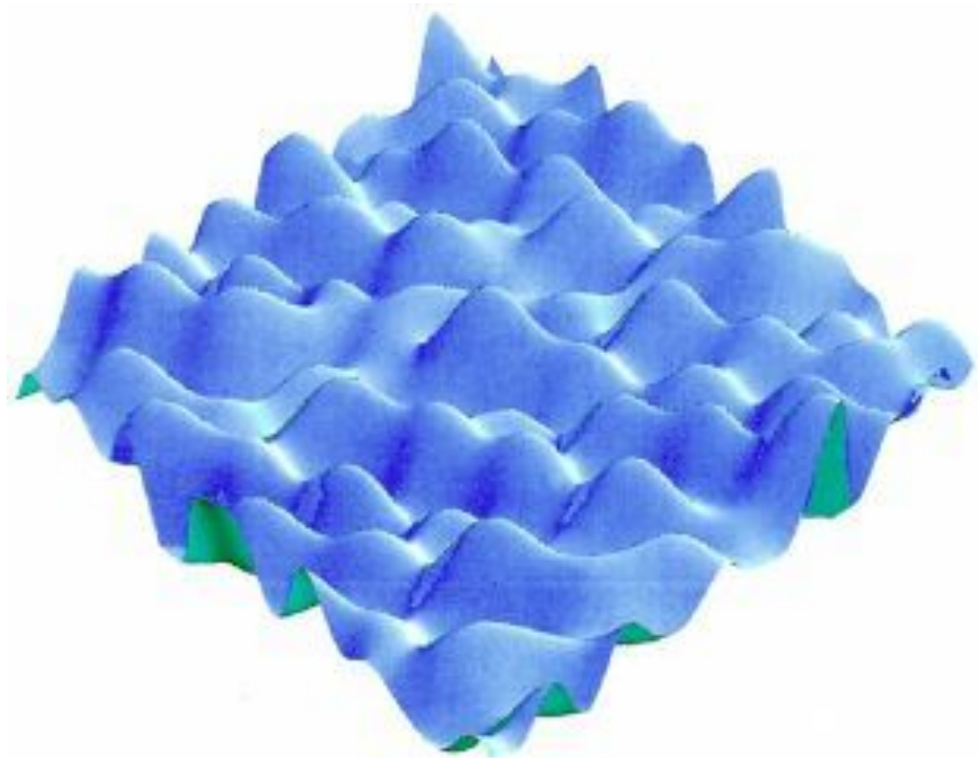
Landscapes

Theories with multiple vacua are very common in BSM

String Theory

Cosmological Constant Problem

Electroweak Hierarchy Problem



Experimental evidence of a vacuum different than ours would be revolutionary!

Scalar potential at finite density

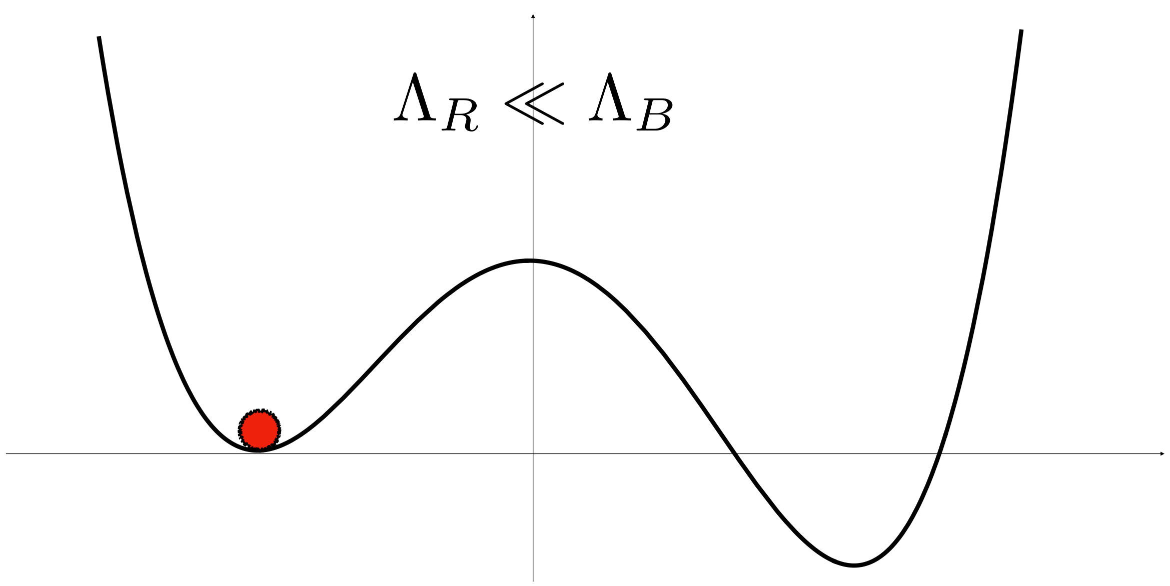
Simple potential à la Coleman

$$V(\phi) = -\Lambda_R^4 \frac{\phi}{f} + \Lambda_B^4 \left(\frac{\phi^2}{f^2} - 1 \right)^2$$

$$\delta^2 \equiv 1 - \frac{\Lambda_R^4}{\Lambda_B^4}$$

$$\delta \sim 1$$

Deep minimum



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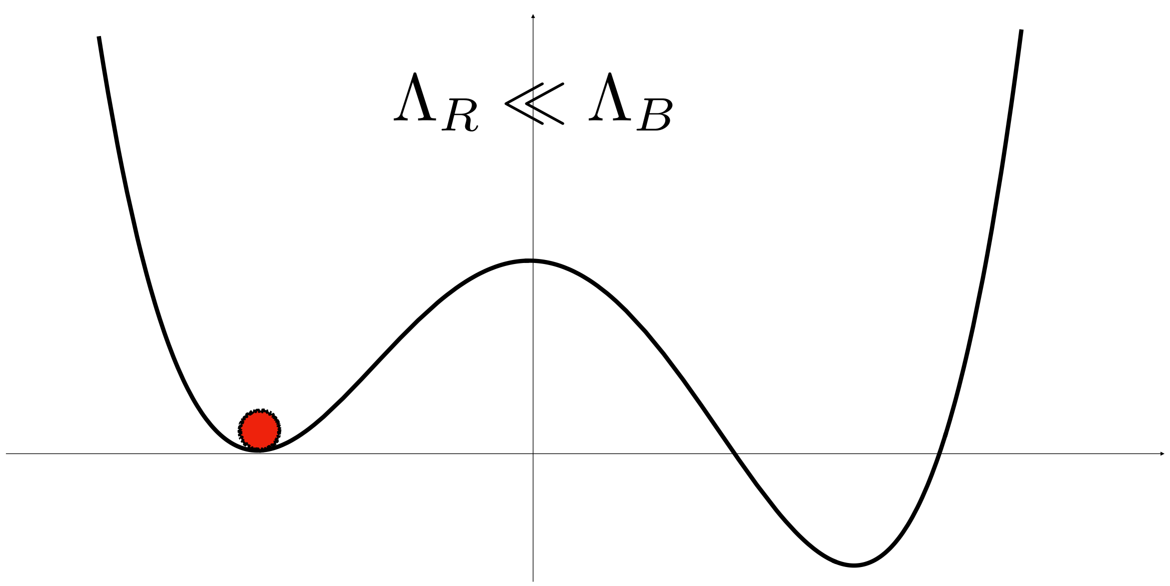

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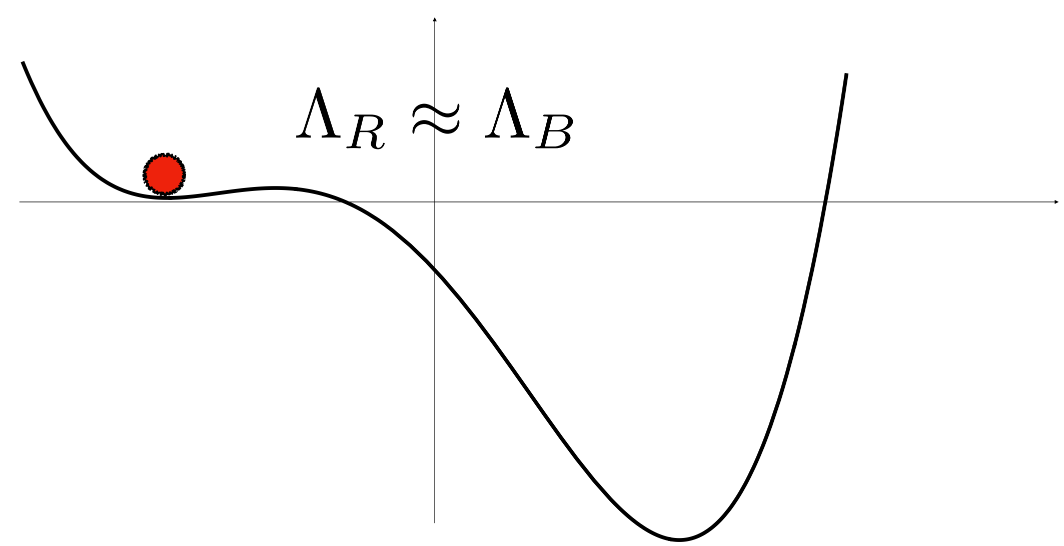

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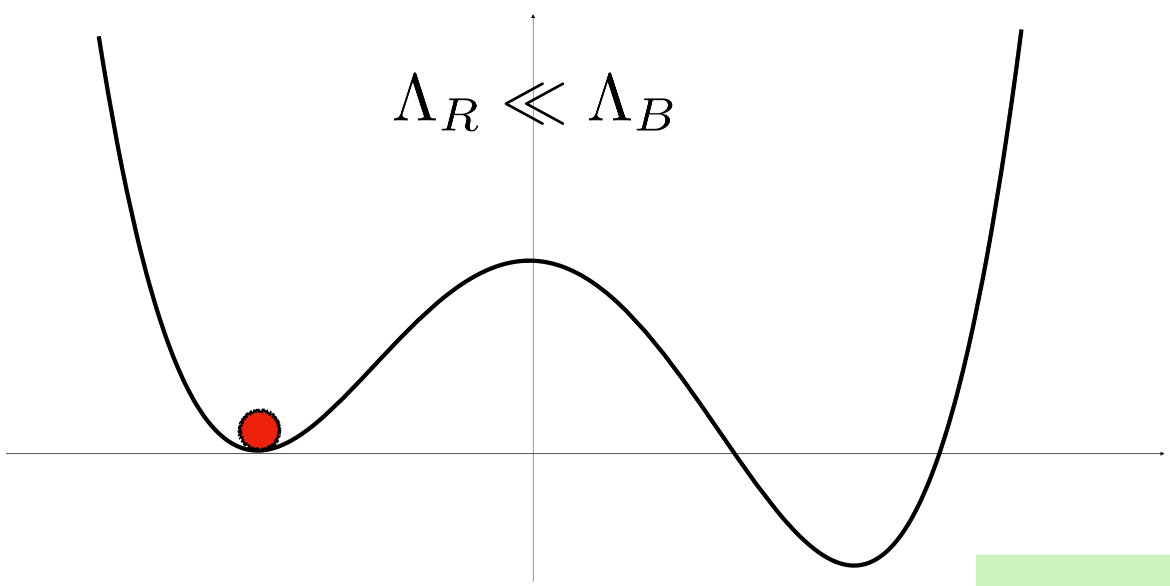

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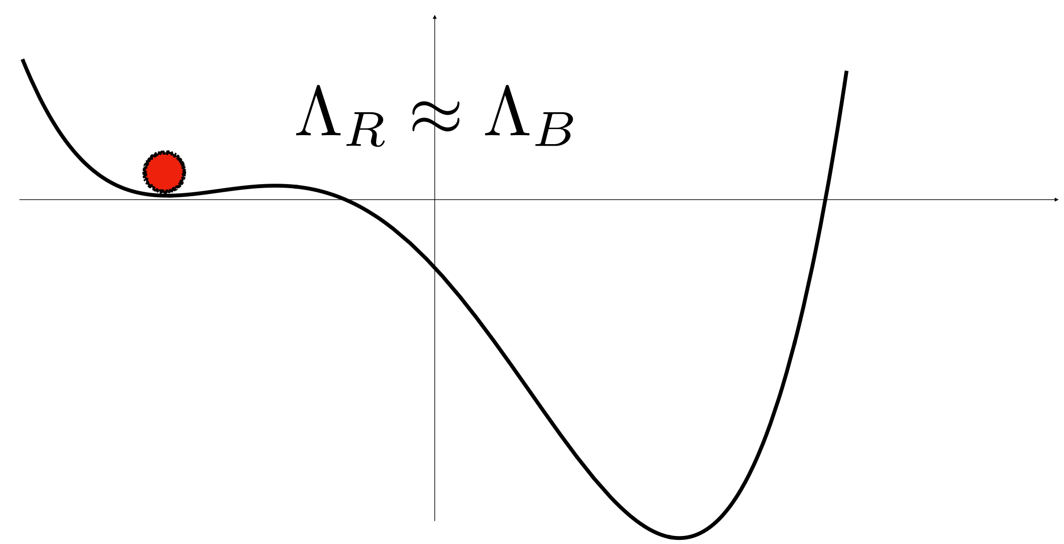

$$\Delta\Lambda \sim -\Lambda_R^4$$

Density dependent potential

(Hook, Huang '19)

Nucleon number density: $$n_N = \langle \bar{N}\gamma^0 N\rangle \approx \langle \bar{N}N\rangle$$

NR-limit

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Simple example:

$$V(\phi) + f(\phi)\bar{N}N \xrightarrow{\text{NR - nucleon background density}} V(\phi) + f(\phi) n_N$$

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Density dependent potenial

Star: spherically symmetric dense object: $n_N = n_N(r)$

Position dependent potential!

Density dependent potential

Motivated and predictive class of models:

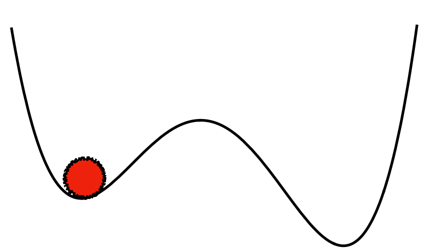
$$\Lambda_B^4 \to \Lambda_B^4(n_N) < \Lambda_B^4$$

Metastable minimum disappears at critical density:

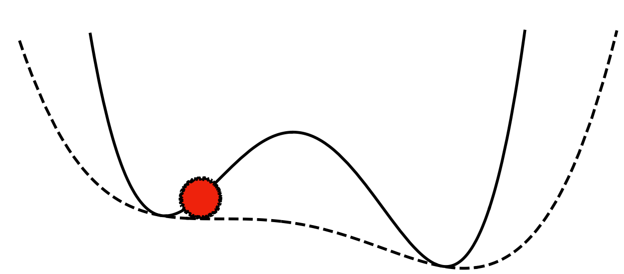
$$\Lambda_B^4(n_c) = \Lambda_R^4$$

Deep minimum:

$$n_N = 0$$



$$n_N = n_c$$



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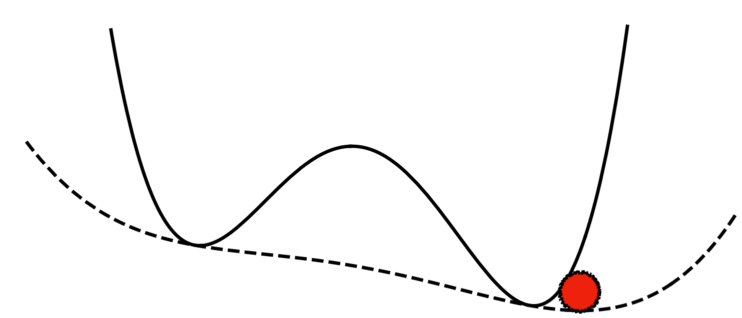

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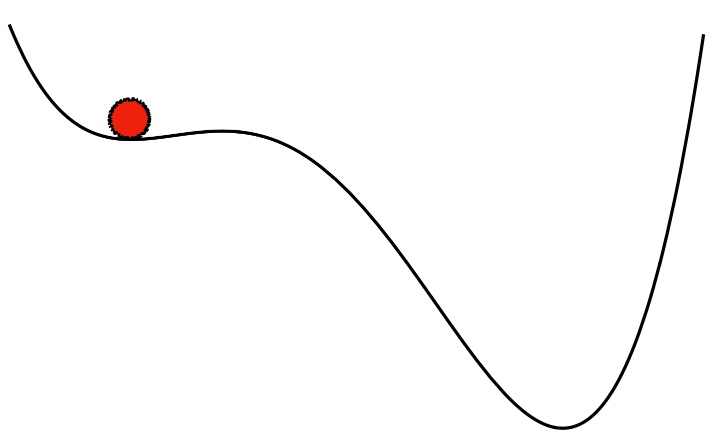

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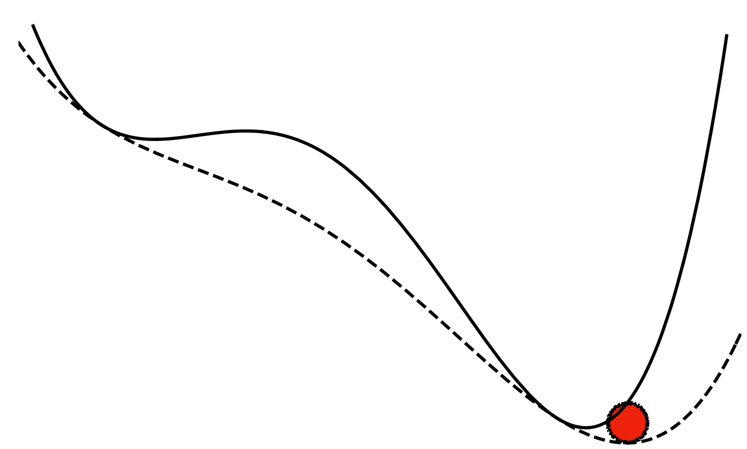

Bubble Dynamics

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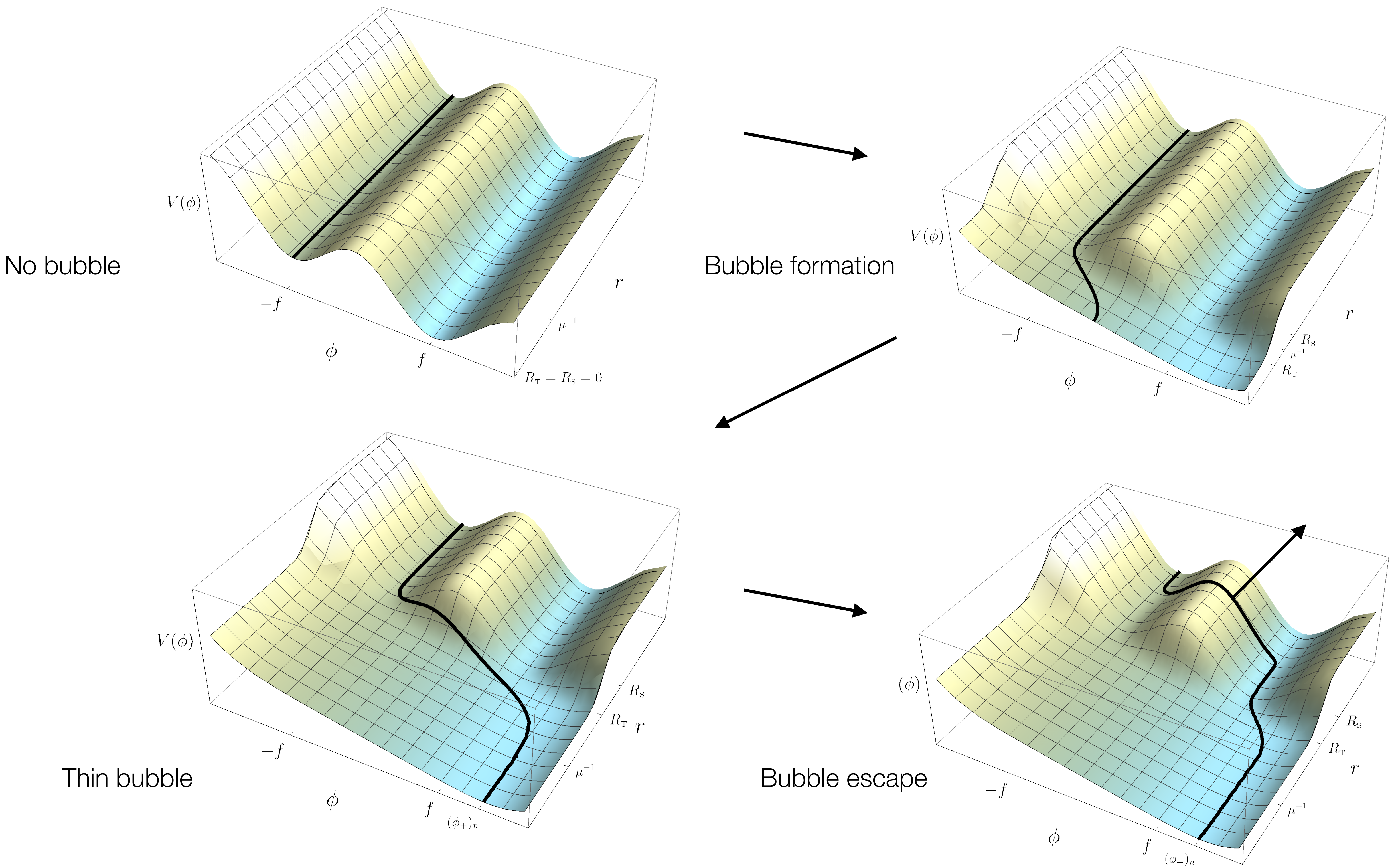

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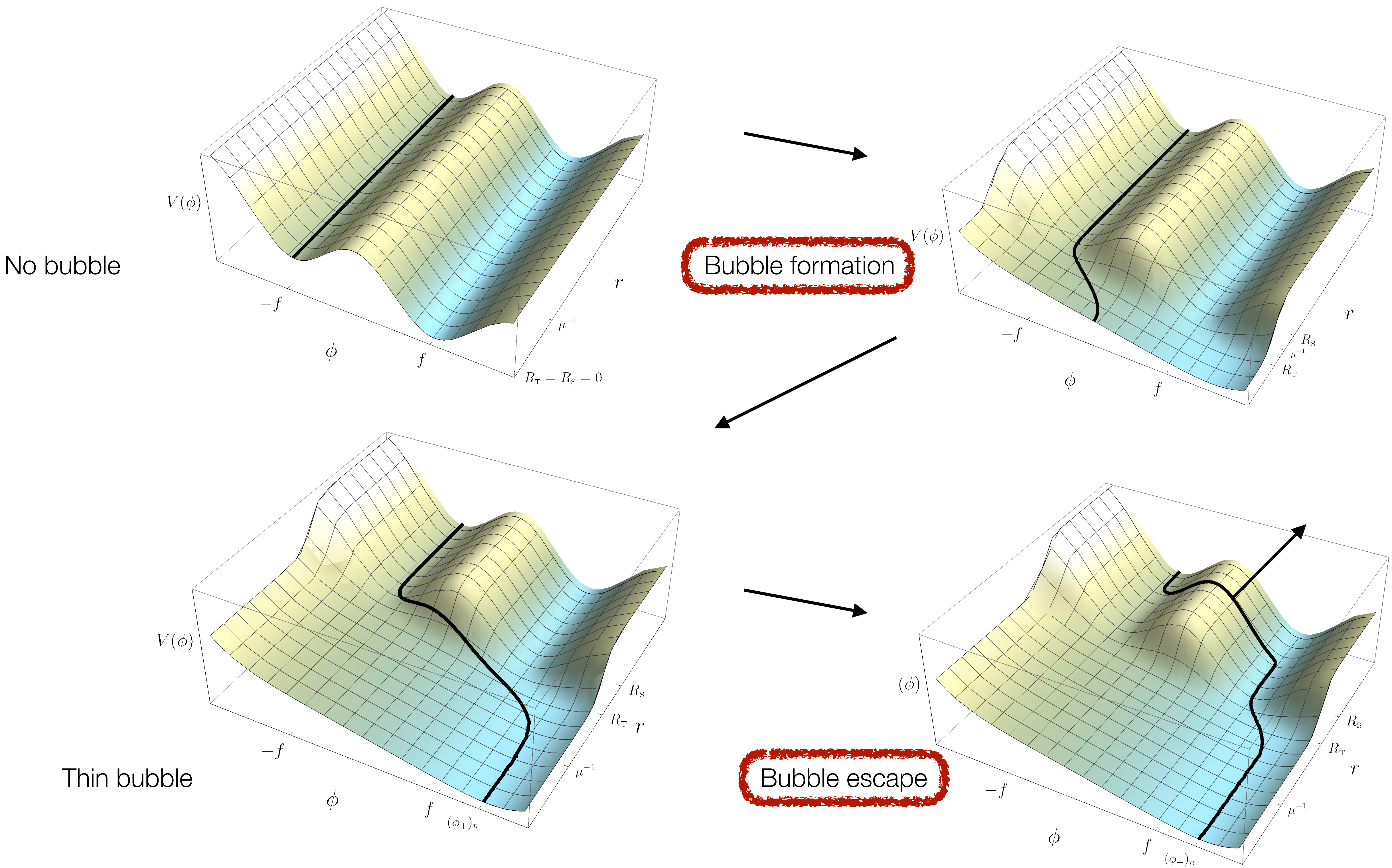
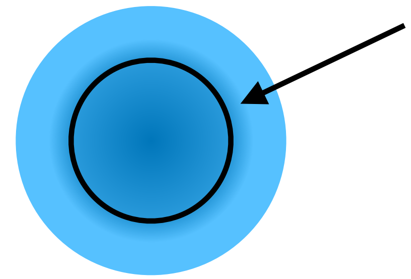
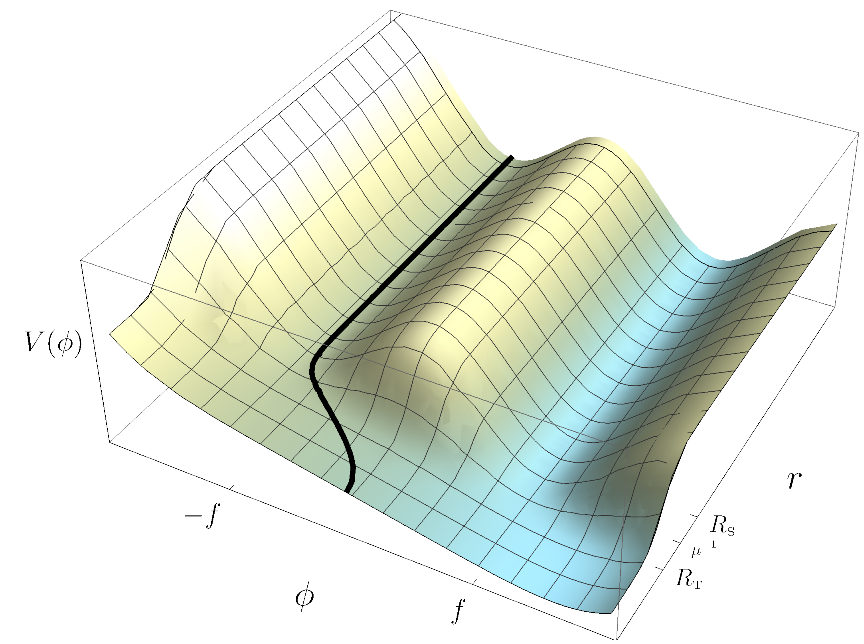


Bubble Formation

Dense enough

$n_N > n_c$ **for** $r < R_T$





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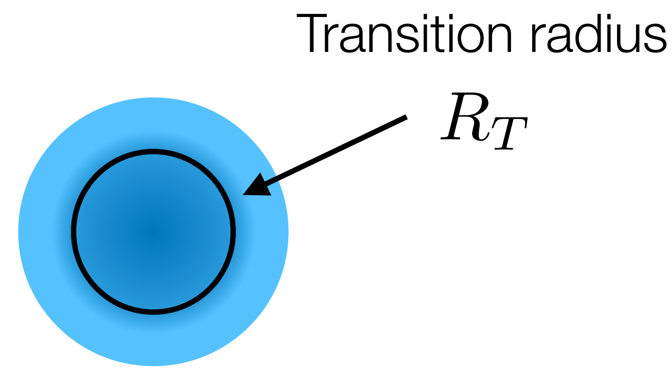

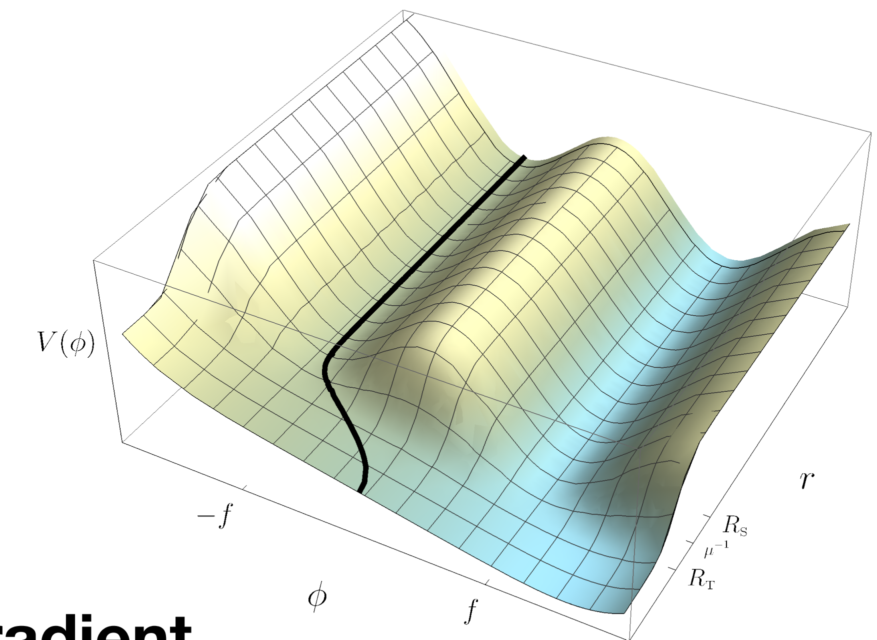

Energy from vacuum = Energy in gradient

$$(\phi')^2 \sim \Delta\Lambda$$

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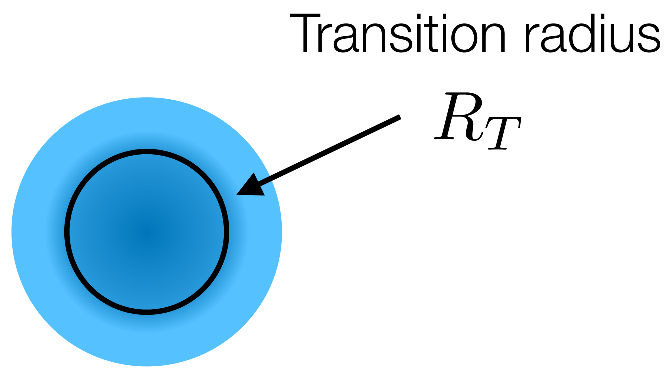

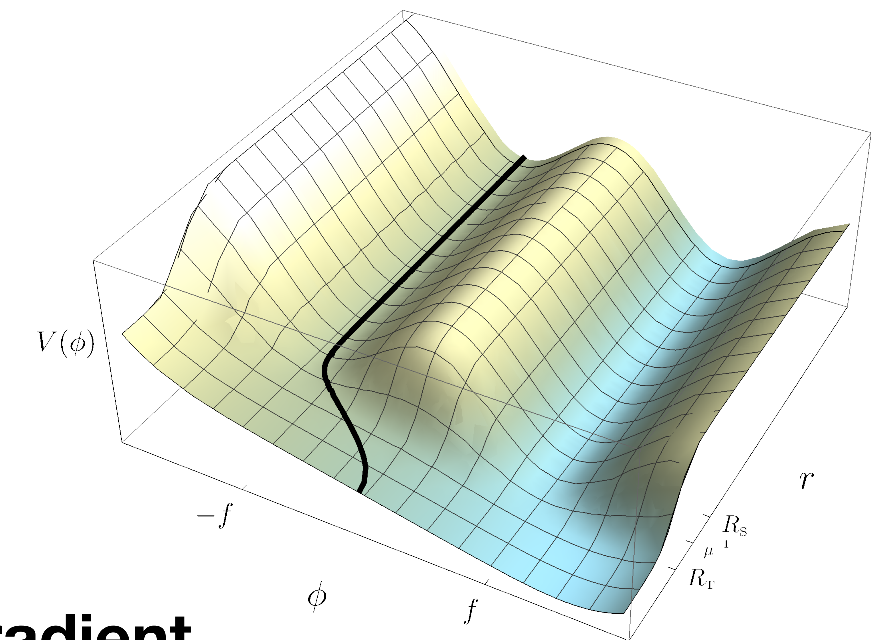

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Bubble fully formed when $\phi \sim f$

Space ΔR **that bubble needs to fully form** $\dfrac{f^2}{\Delta R^2} \sim \Lambda_R^4$

↓

Formation Condition $R_T \gtrsim \dfrac{f}{\Lambda_R^2}$

Bubble Escape

As core of star grows, bubble becomes thinner

Dynamics of thin wall bubbles can easily be understood:

$$E(R) = -\frac{4\pi}{3}R^3\epsilon + 4\pi R^2\sigma(R)$$

$$\epsilon = -\Delta\Lambda \sim \Lambda_R^4$$

Main novelty: Position dependent barriers

Radius dependent tension

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Radius dependent tension

Thin bubble EOM $$\sigma\ddot{R} = \epsilon - \frac{2\sigma}{R} - \sigma'$$

Additional contracting force

Escape condition: $$\epsilon \gtrsim \sigma' \sim \frac{\sigma(R_s) - \sigma(R_T)}{R_s - R_T}$$

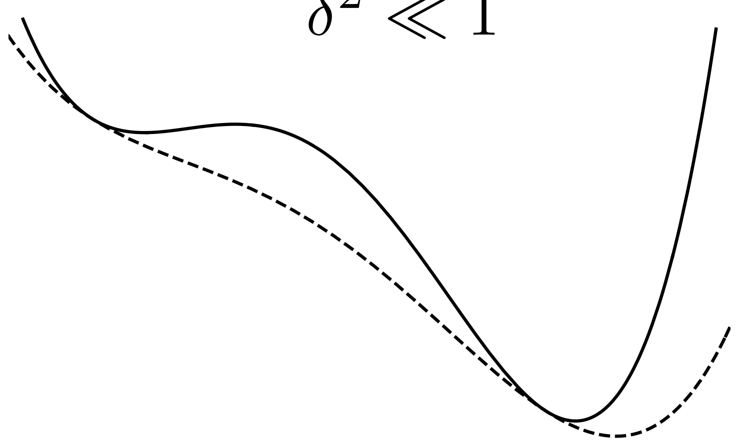
With stellar radius R_s

Bubble Escape

Escape condition: $$\epsilon \gtrsim \sigma' \sim \frac{\sigma(R_s) - \sigma(R_T)}{R_S - R_T}$$

Shallow

$\delta^2 \ll 1$



Tension is gradient dominated

$$\sigma(R_S) \approx \sigma(R_T)$$

Every bubble that forms also escapes

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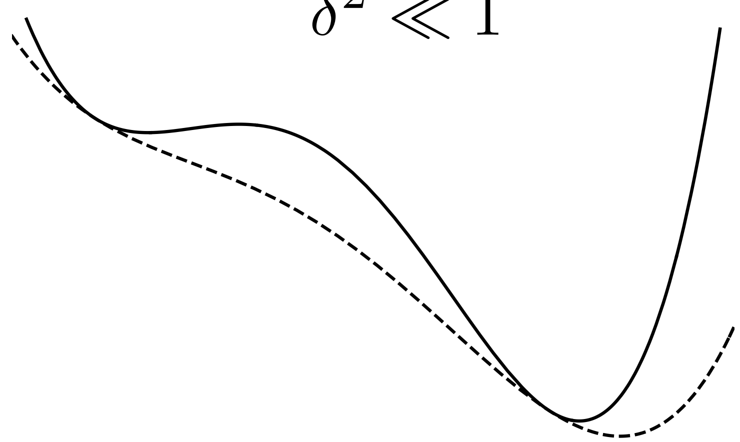

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Deep

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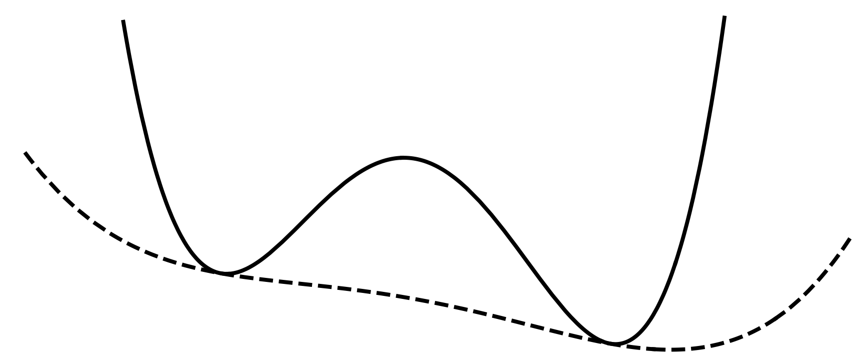


$$\sigma(R_S) \sim \Lambda_B^2/f \gg \sigma(R_T)$$

Escape condition:

$$R_S - R_T \gtrsim \frac{f\Lambda_B^2}{\Lambda_R^4}$$

Summary:
Conditions for Phase Transition

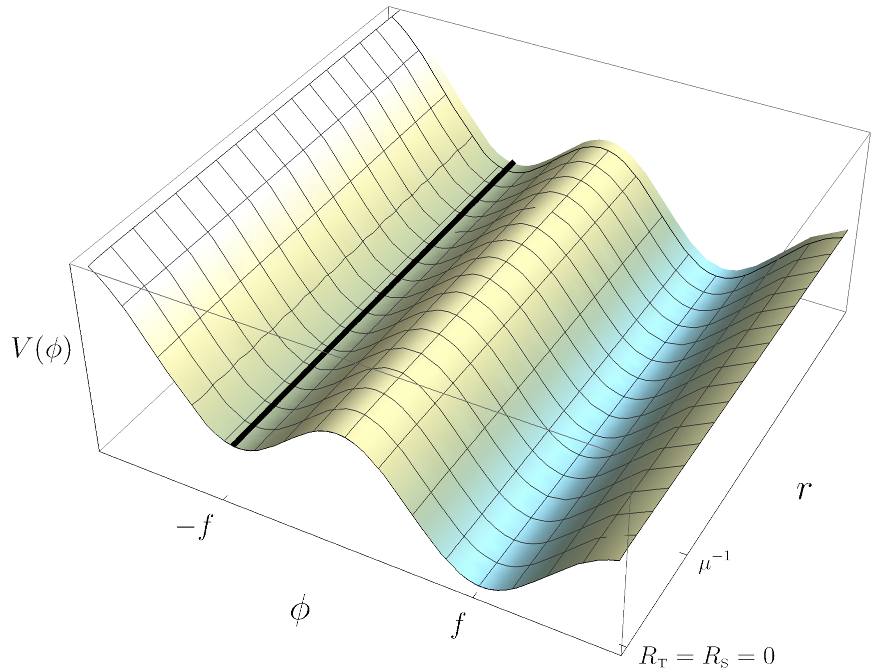
Dense enough: $n > n_c$

Large enough:

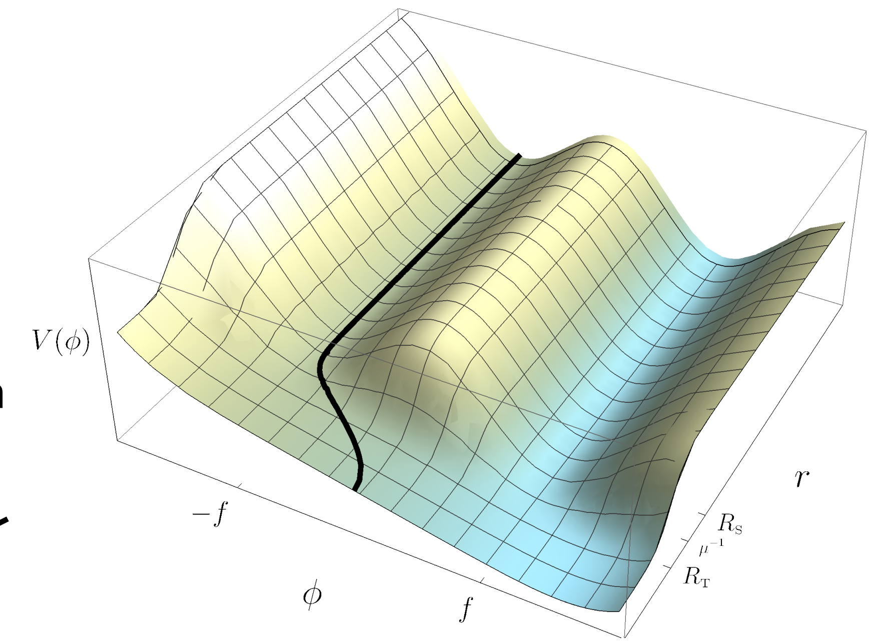
Shallow		Deep
$R_T \gtrsim \frac{f}{\Lambda_R^2}$	**Formation**	$R_T \gtrsim \frac{f}{\Lambda_R^2}$
$R_S - R_T \gtrsim \frac{f}{\Lambda_R^2}\delta^2$	**Escape**	$R_S - R_T \gtrsim \frac{f\Lambda_B^2}{\Lambda_R^4}$

Bubble dynamics - full numerical solution

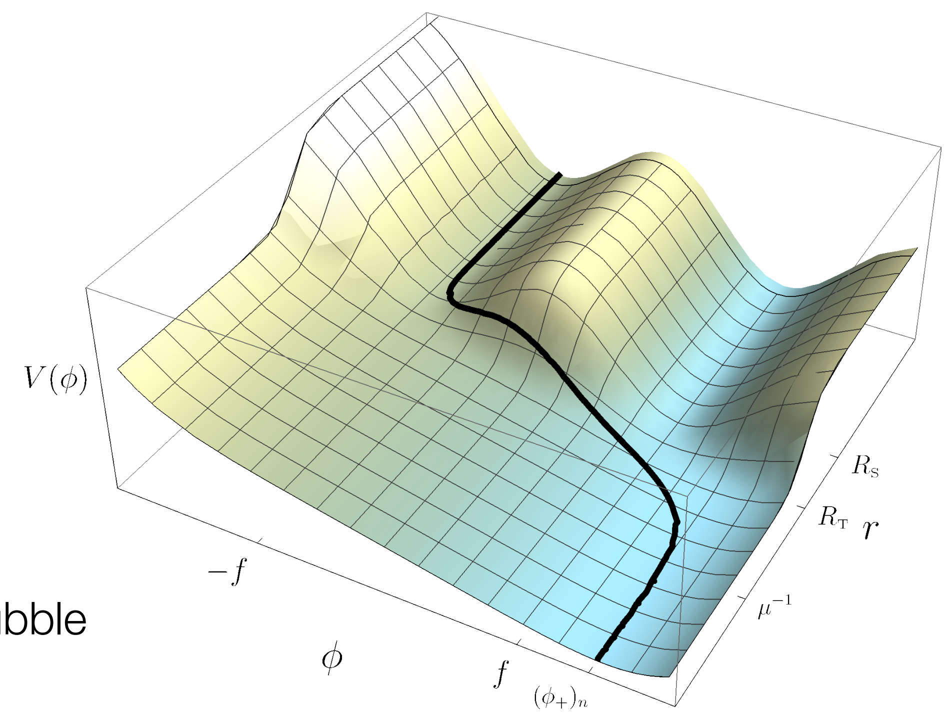
No bubble



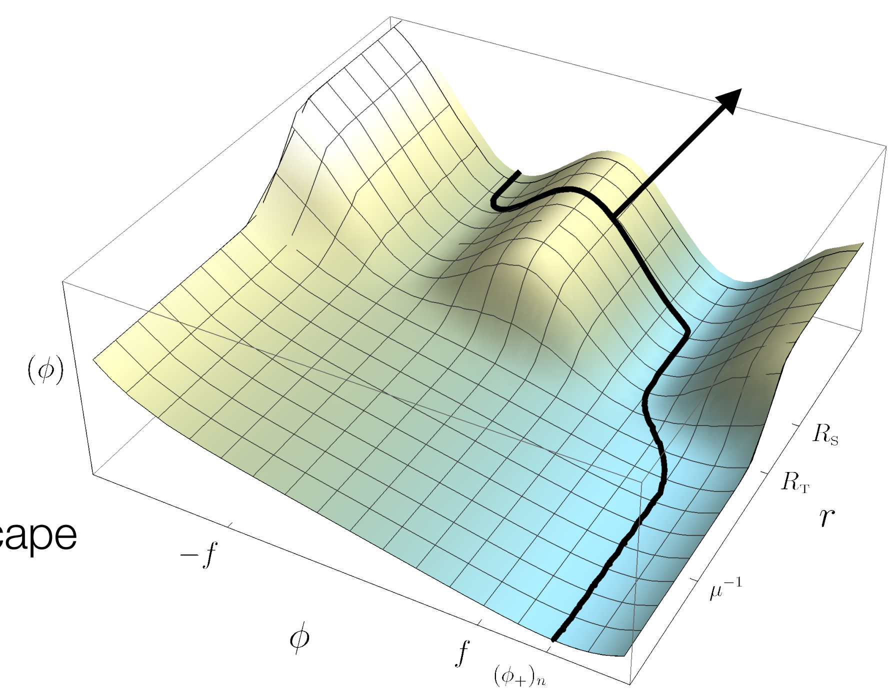
Bubble formation



Thin bubble



Bubble escape



Are such phase transitions cosmologically viable?

Density induced phase transitions

Happens at very low redshift $z \lesssim 20$ **when first compact stars form**

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Strong bounds on change of Cosmological Constant from early vs late Hubble measurements

Constraints on early dark energy recently studied in context of Hubble tension:

Karwal, Kamionkowski '16

$$\Delta\Lambda \lesssim 10^2 \Lambda_0 \quad \textbf{at} \quad z \sim 10$$

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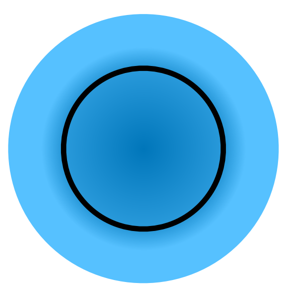


Density induced phase transitions have:

Formation condition

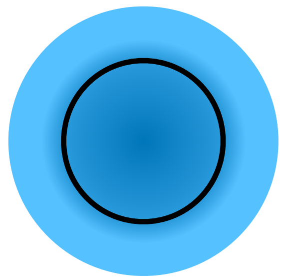
Typical NS radius

Radius of largest known stars

$$-\Delta\Lambda \sim \Lambda_R^4 \gtrsim \left(\frac{f}{R_S}\right)^2 \approx 10^{16}\Lambda_0 \left(\frac{f}{10\mathrm{TeV}}\right)^2 \left(\frac{10\mathrm{km}}{R_s}\right)^2 \approx \Lambda_0 \left(\frac{f}{10\mathrm{TeV}}\right)^2 \left(\frac{10^9\mathrm{km}}{R_s}\right)^2$$

Bound generically violated, only largest stars could lead to allowed PT

Density induced phase transitions



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Bound generically violated, only largest stars could lead to allowed PT

Strong constraints on models with metastable ground state!

Phase transitions from EM backgrounds

$$g_{\phi\gamma\gamma}\frac{\phi}{f}F^{\mu\nu}\tilde{F}_{\mu\nu}$$

Vacuum Dipole as model for magnetic NS:

$$\vec{E}\cdot\vec{B}\sim\left(\frac{B_s^2 R_s^6 \Omega_s}{4r^5}\right)\Theta(r-R_S)$$

$$\downarrow$$

$$\Lambda_R^4(\vec{E}\cdot\vec{B}) > \Lambda_R^4$$

Bubbles can form! (Conditions similar to before)

$$\downarrow$$

Gives bounds on metastable theories with coupling to photons (e.g. Technicolored Relaxion)

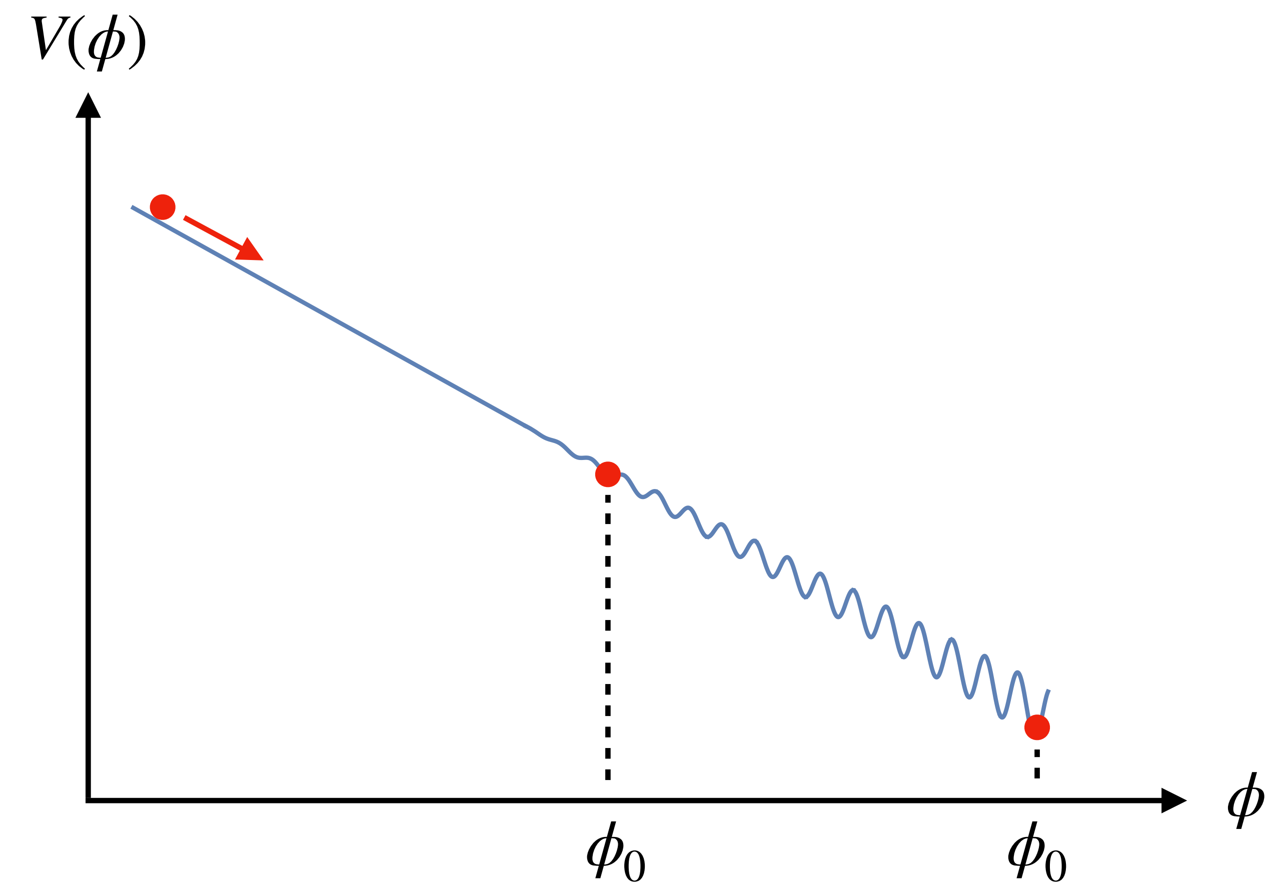
Relaxion

Solving the Hierarchy Problem Dynamically: The Relaxion

(Graham, Kaplan, Rajendran ’15)

$$V(\phi) = -\Lambda_R^4 \frac{\phi}{f} + \Lambda_B^4(\phi)\cos\left(\frac{\phi}{f}\right)$$

+ couplings with Higgs



How does this solve the hierarchy problem?

Stopping point must be determined by small Higgs vev!

(e.g. Higgs dependent backreaction potential)

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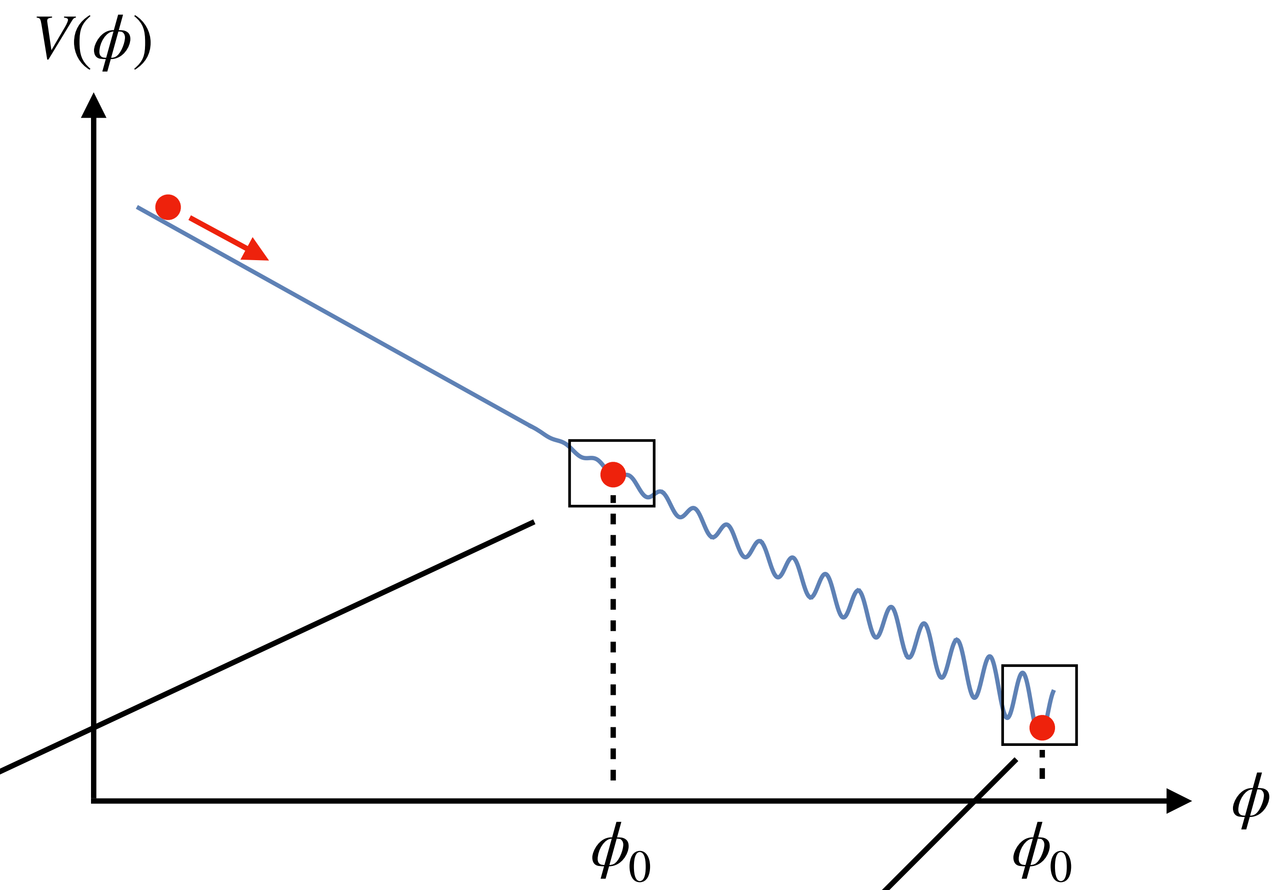
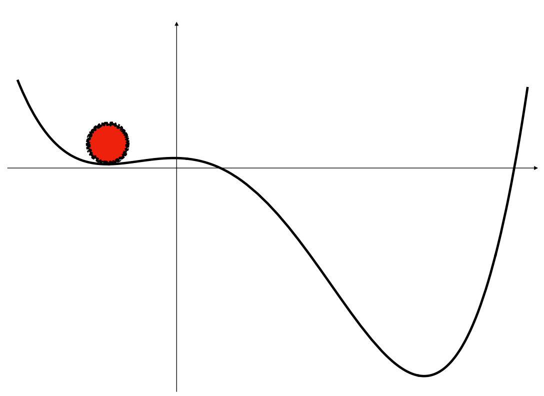
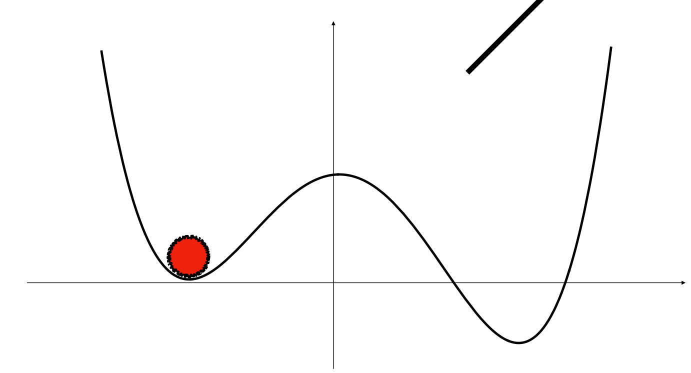




First minima are **shallow**

Banerjee et al. 2020



Further down: **deep**

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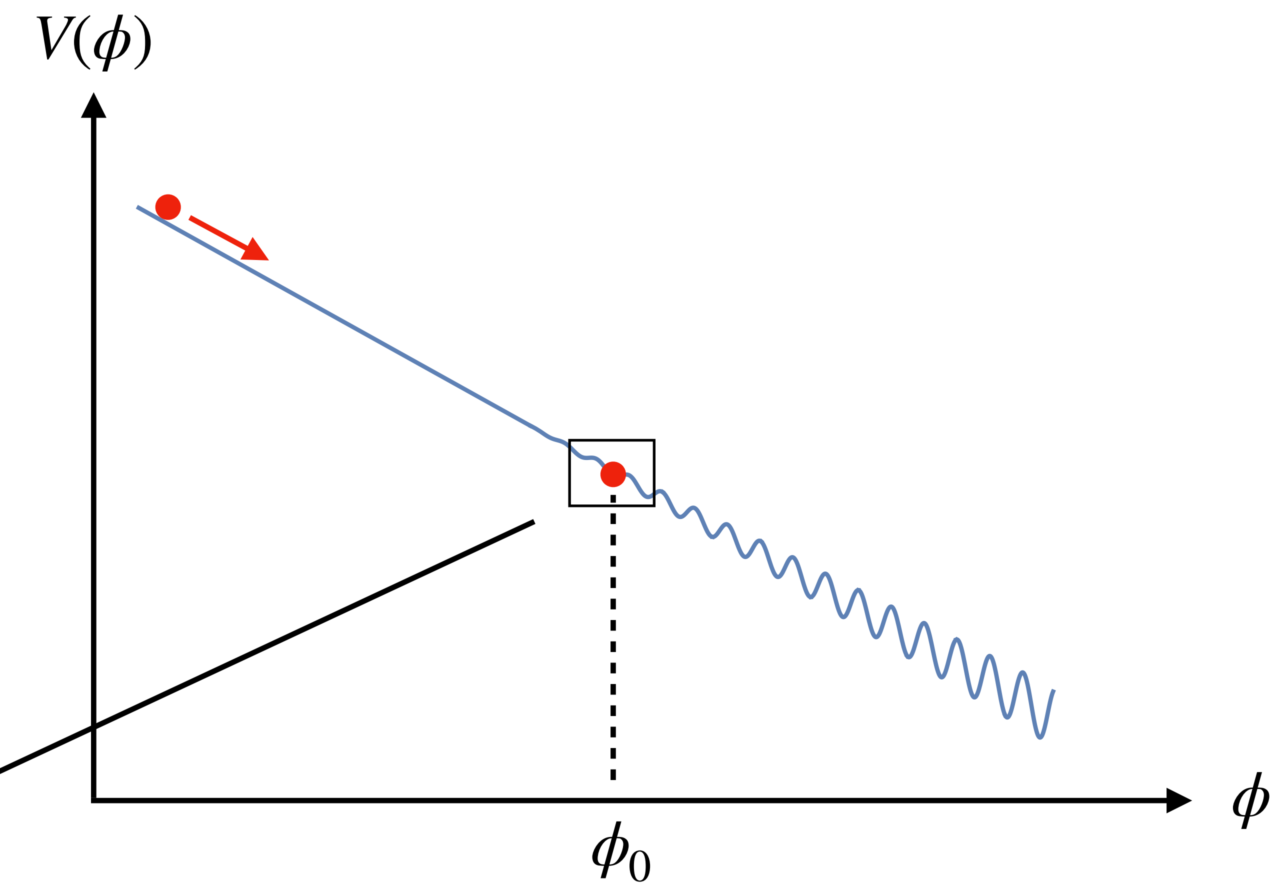

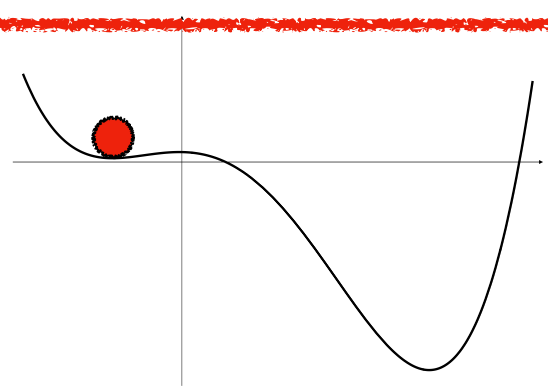

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$$\delta^2_{l=1} \sim \frac{\Lambda^4_{\mathrm{QCD},C}}{v^2 M^2} \quad \text{with cutoff} \quad M > \mathrm{TeV}$$

The QCD Relaxion

QCD axion like barriers: $\Lambda_B^4 = \Lambda_{\rm QCD}^4 \frac{h}{v} \sim m_q \langle \bar{q} q \rangle$

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At finite density $\Big\downarrow$ $\langle \bar{q} q \rangle \to \langle \bar{q} q \rangle (n)$

$$\Lambda_B^4(n_N) = \Lambda_B^4 \left(1 - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right)$$

The QCD Relaxion

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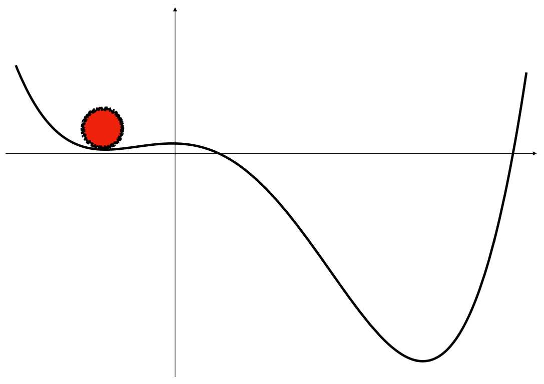

Dense enough:

$$n_N \gtrsim \frac{1}{M^2} \frac{\pi \Lambda_{QCD}^8}{\sigma_N v^2}$$

Large enough:

$$R_S \gtrsim \frac{f}{\Lambda_{QCD}^2}$$

The QCD Relaxion

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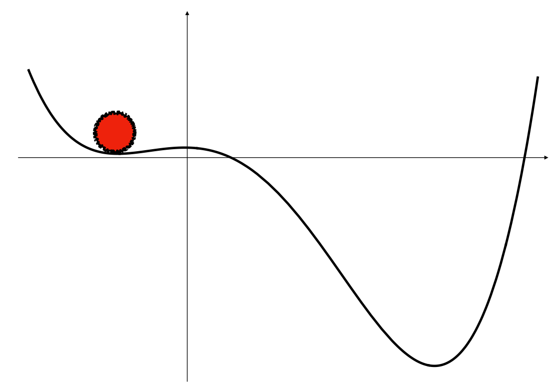

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Easily fulfilled for Neutron Stars and White Dwarfs for $f \lesssim M_p$ **and** $M \gtrsim \mathrm{TeV}$

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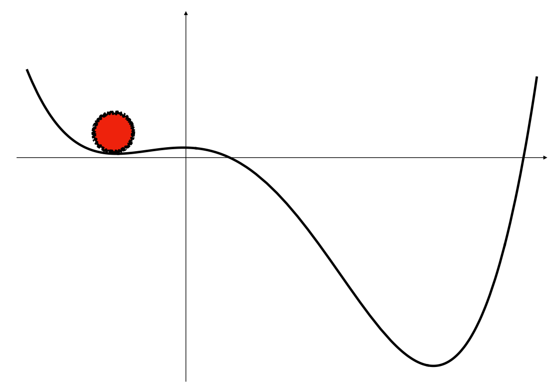

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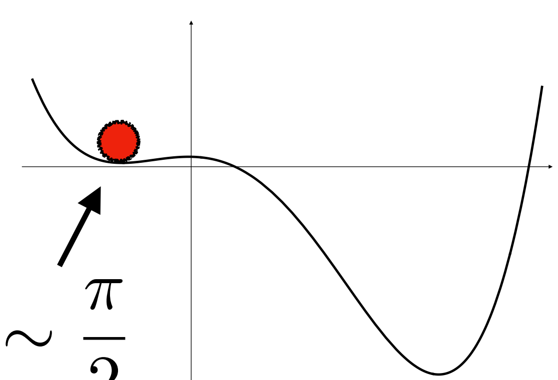

At shallow minimum $\theta_{\mathrm{QCD}} \sim \frac{\pi}{2}$

The Non - QCD Relaxion

$$\Lambda_B^4 = \Lambda_C^4 \frac{h^2}{v^2}$$

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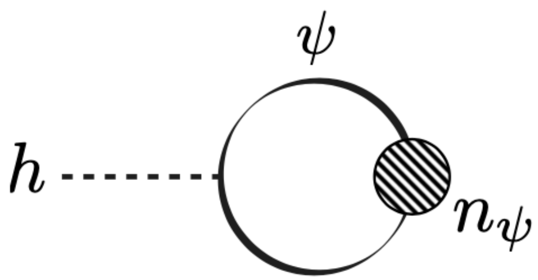
At finite density $\quad h \to h(n)$



$$\Lambda_B^4(n) = \Lambda_B^4 \left(1 - \frac{\sigma_N n_N}{m_h^2 v^2}\right)$$

The Non - QCD Relaxion

$$\Lambda_B^4 = \Lambda_C^4 \frac{h^2}{v^2}$$

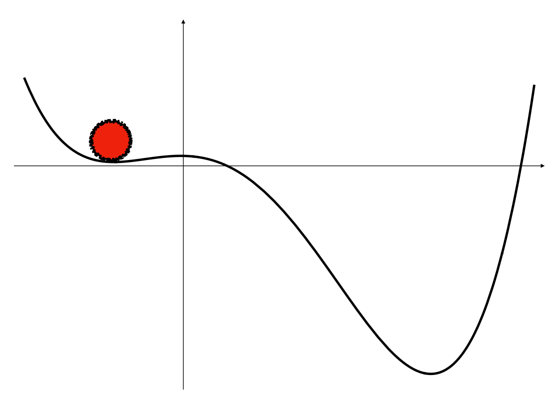
At finite density $h \to h(n)$

$$\Lambda_B^4(n) = \Lambda_B^4 \left(1 - \frac{\sigma_N n_N}{m_h^2 v^2}\right)$$

Dense enough:

$$n_N \gtrsim \frac{\Lambda_C^4 v^2}{M^2 \sigma_N}$$

First minimum

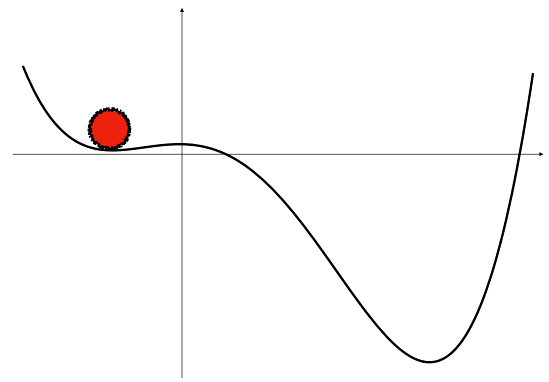


Large enough:

$$R_s \gtrsim \frac{f}{\Lambda_C^2}$$

The Non - QCD Relaxion

First minimum



Dense enough:

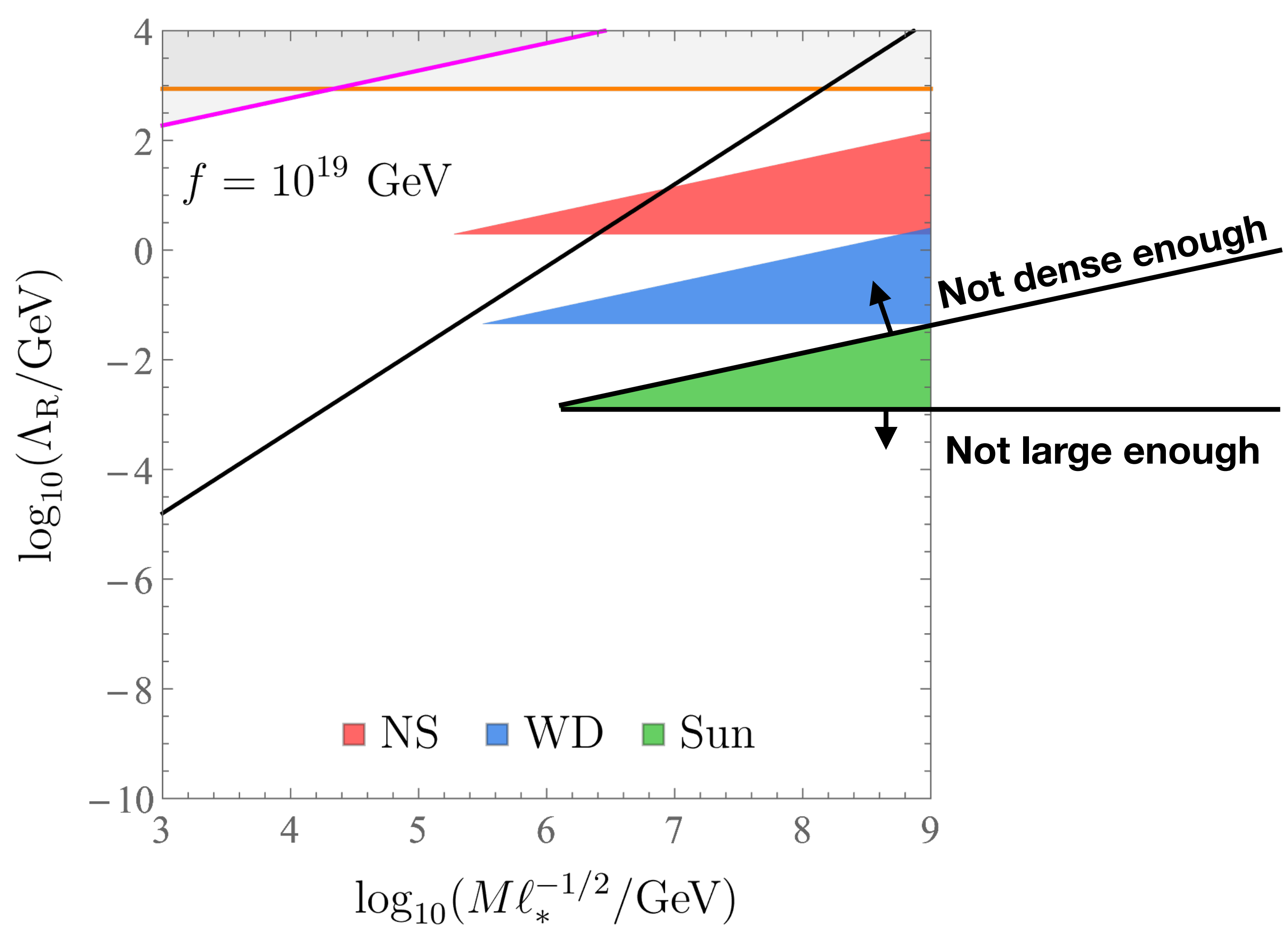
$$n_N \gtrsim \frac{\Lambda_C^4 v^2}{M^2 \sigma_N}$$

Large enough:

$$R_s \gtrsim \frac{f}{\Lambda_C^2}$$

If conditions are satisfied: change in the CC too large!

$$\Lambda_R \sim \Lambda_C$$



The Non - QCD Relaxion

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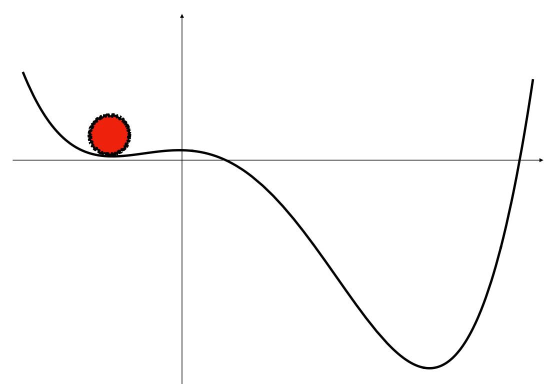

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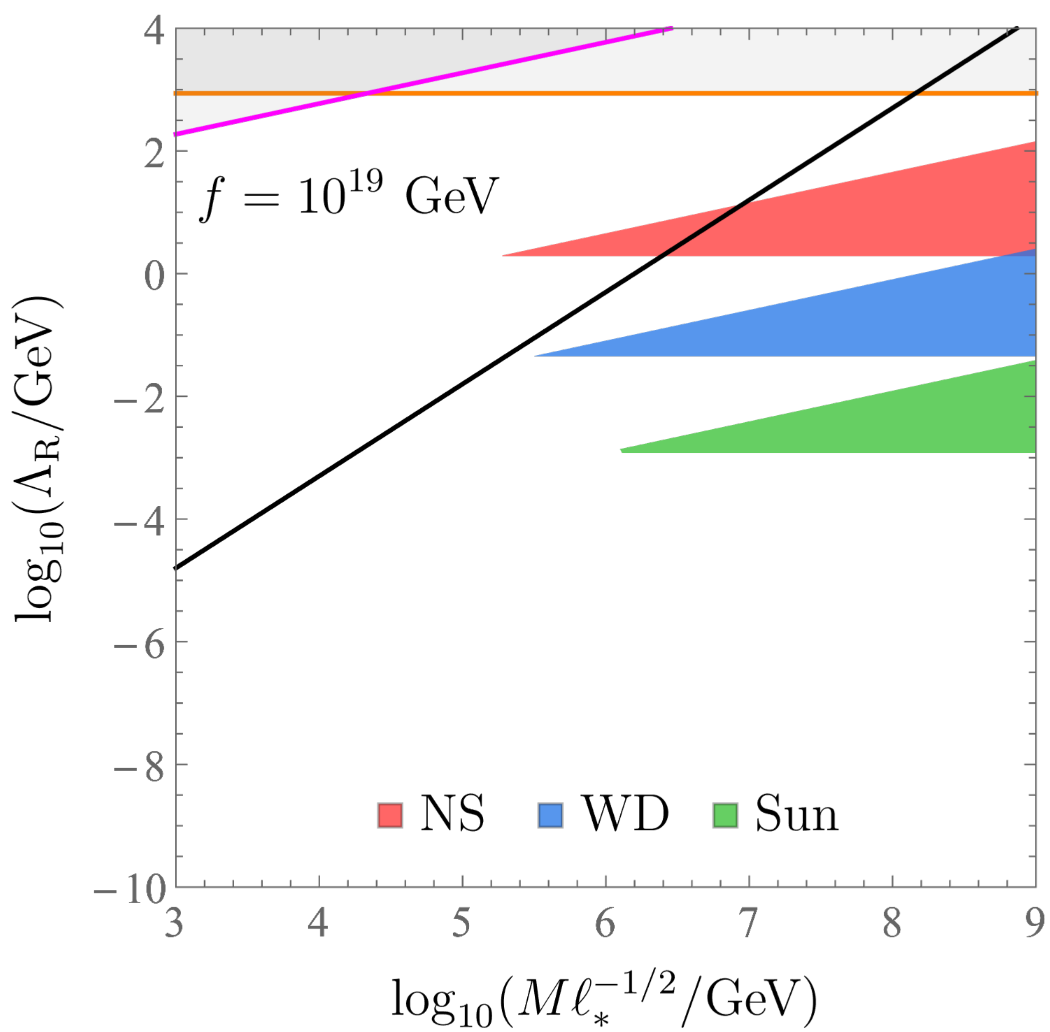
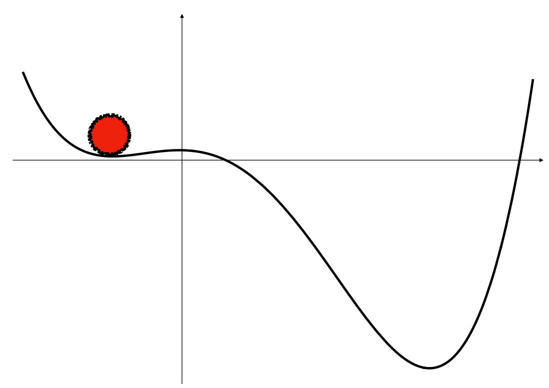


The Non - QCD Relaxion

First minimum



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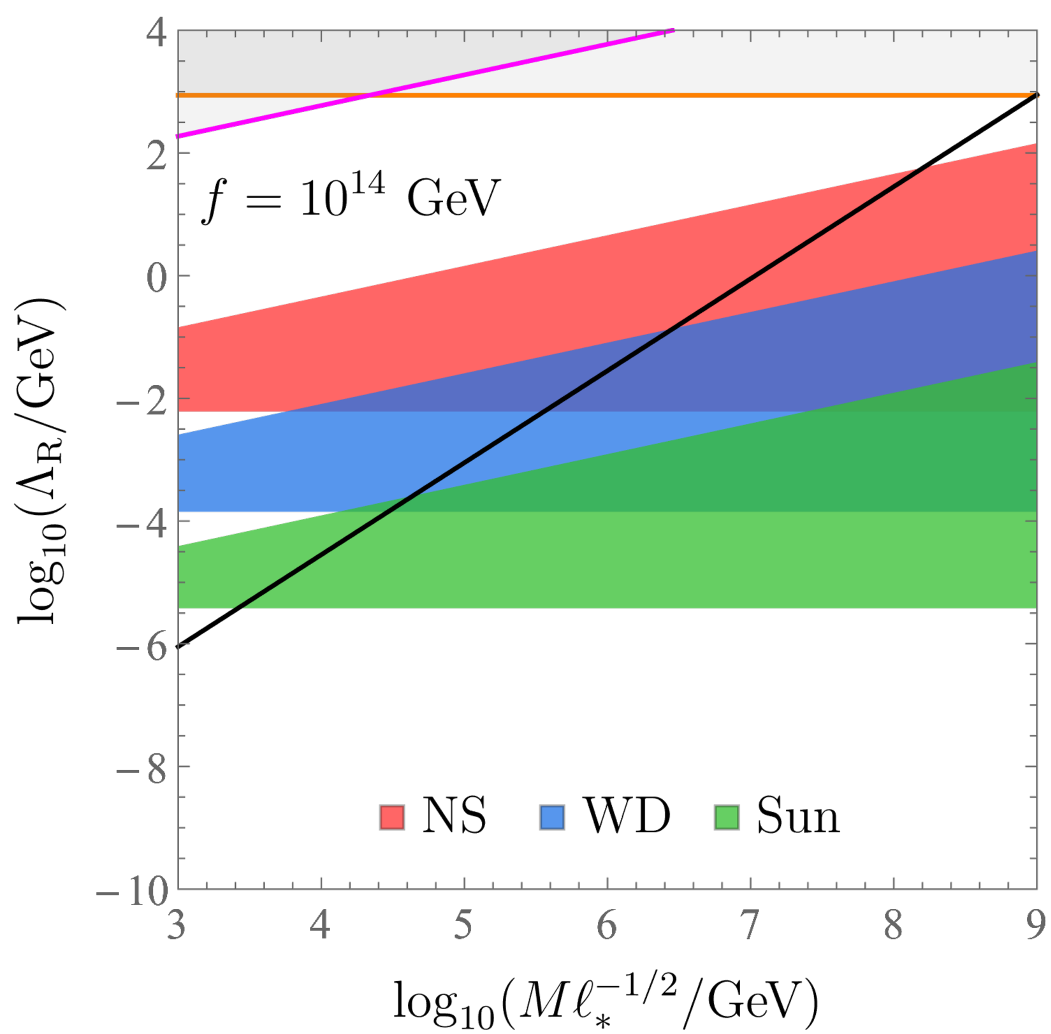
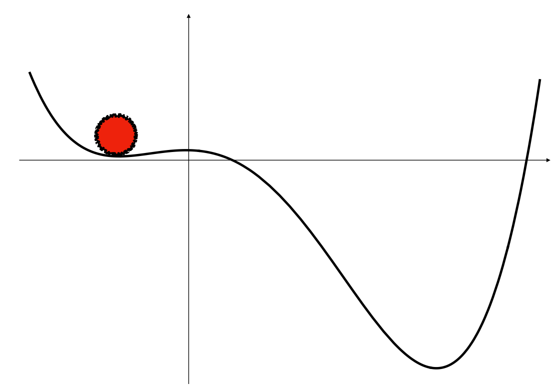


The Non - QCD Relaxion

First minimum



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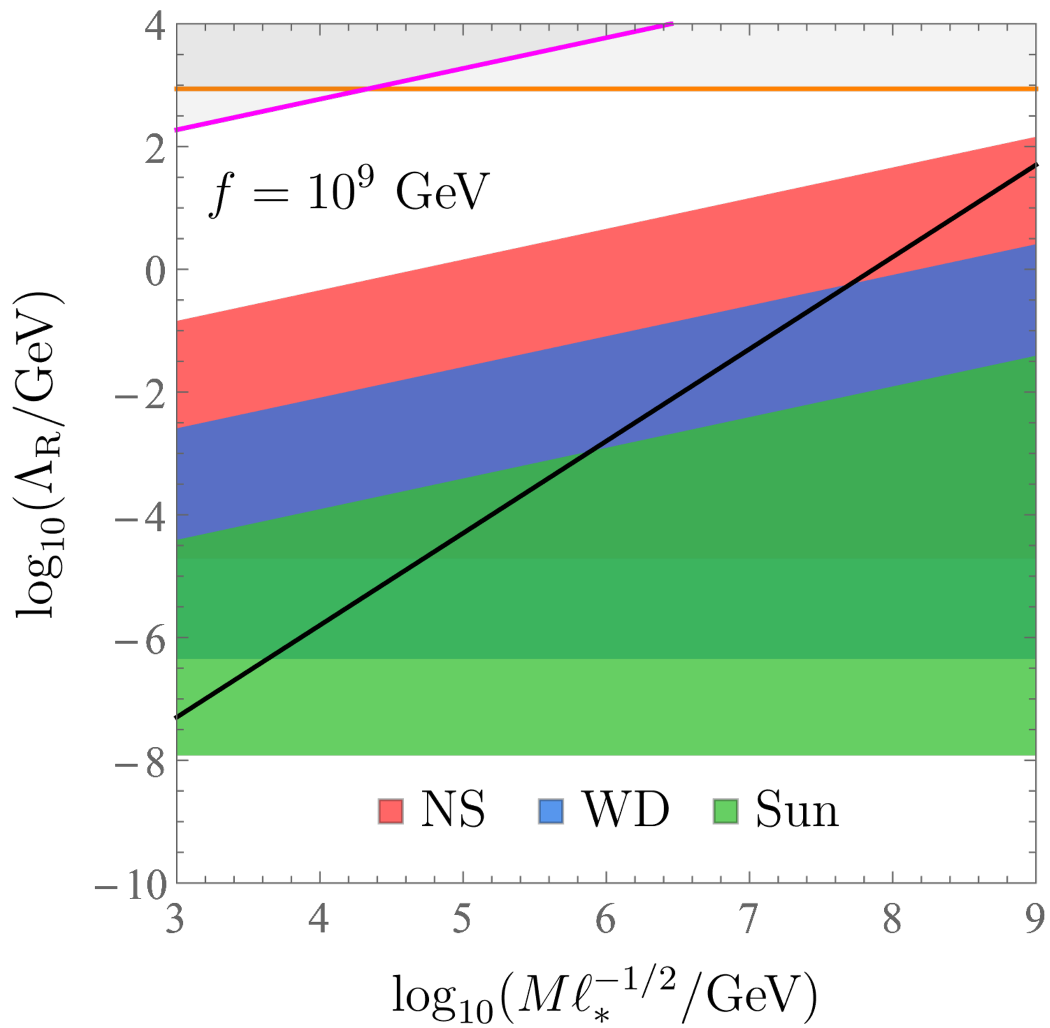


Summary

Density can destabilise the vacuum

This leads to **confined bubbles** or **phase transitions**
(Hook, Huang ’19)

Bounds on late time phase transitions → **New bounds on relaxion models**

Landscapes sensitive to SM densities are constrained from seeded phase transitions

Such seeded instabilities can still happen today!
(cf. arxiv: 0011262 and 1205.6260)
Can this be seen?

In Collaboration with
Reuven Balkin, Javi Serra, Konstantin Springmann and Andreas Weiler

Backup Slides

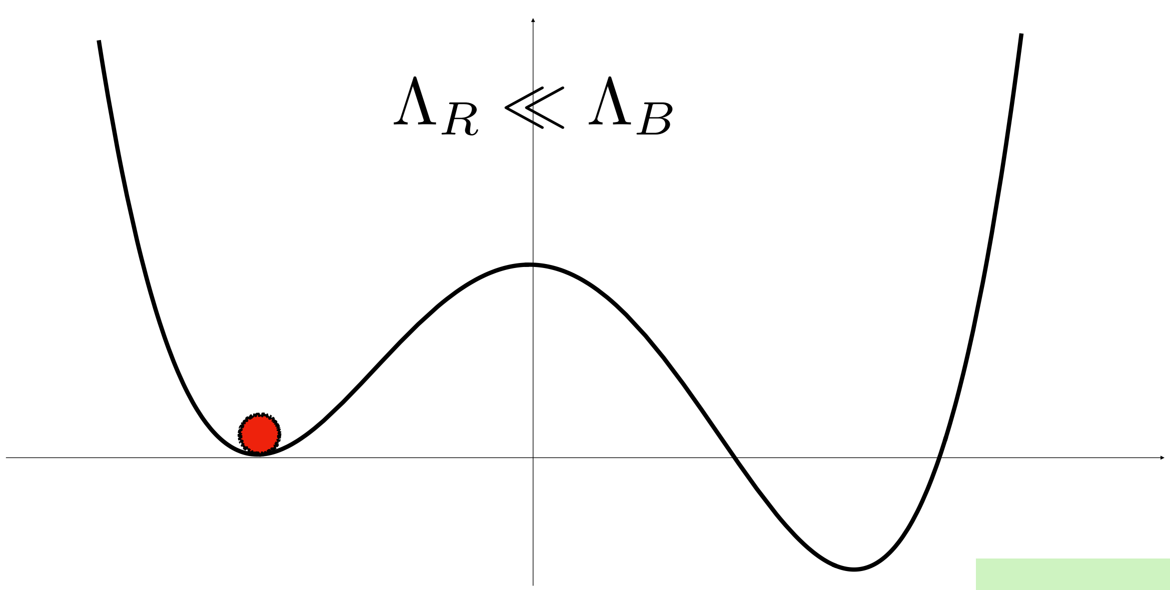
Simple potential à la Coleman

$$V(\phi) = -\Lambda_R^4 \frac{\phi}{f} + \Lambda_B^4 \left(\frac{\phi^2}{f^2} - 1 \right)^2$$

$$\delta^2 \equiv 1 - \frac{\Lambda_R^4}{\Lambda_B^4}$$

$\delta \sim 1$

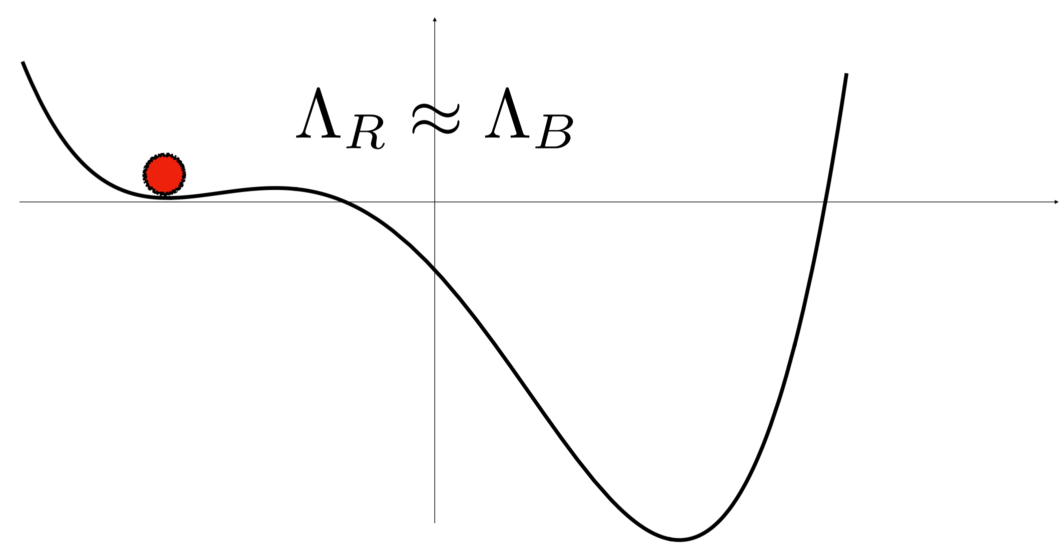
Deep minimum



$m_\phi^2 \sim \Lambda_B^4/f^2$

$\delta \ll 1$

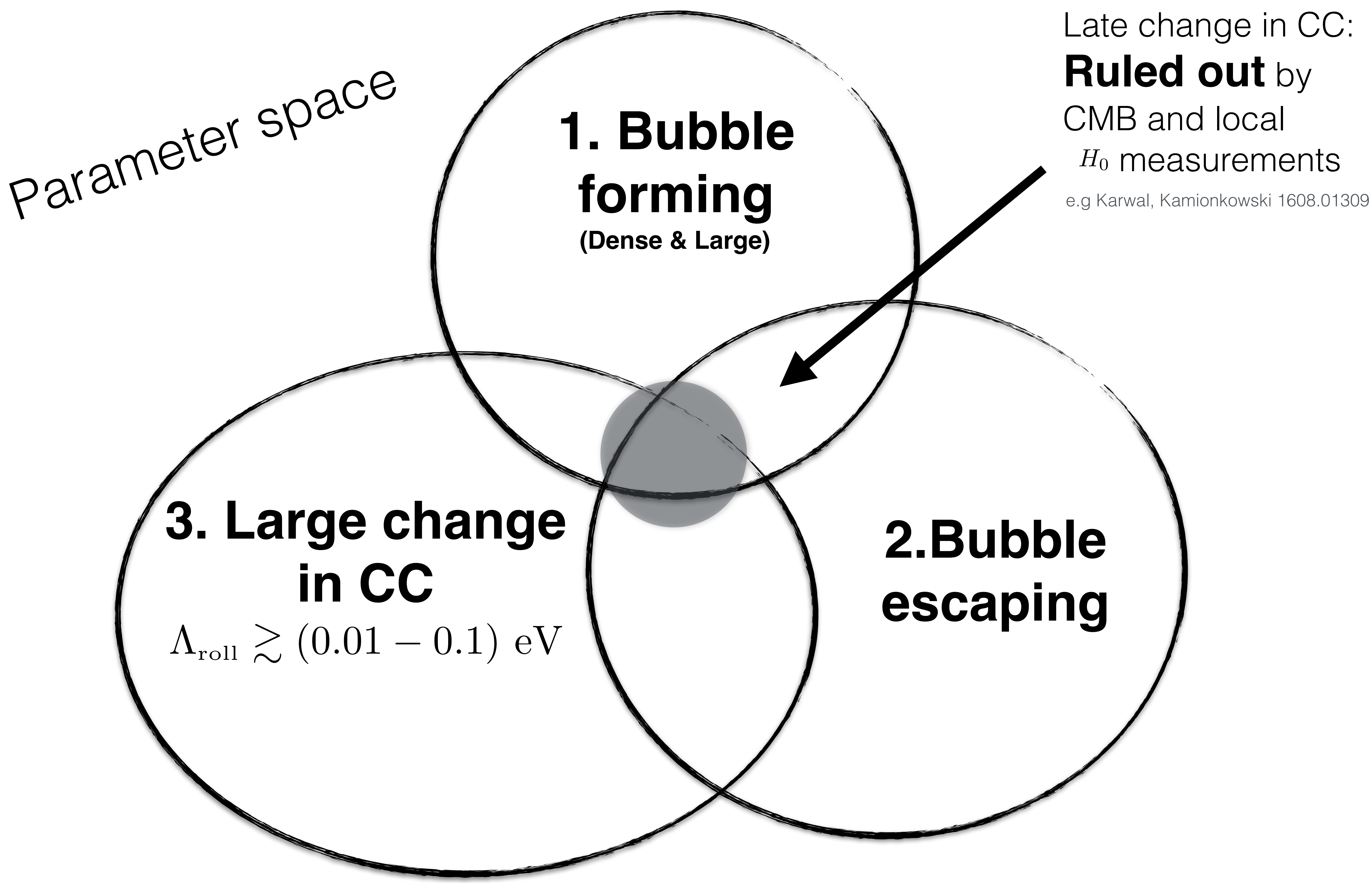
Shallow minimum



$m_\phi^2 \sim \delta \Lambda_B^4/f^2$

$$\boxed{\Delta\Lambda \sim -\Lambda_R^4}$$

Backup - parameter space exclusion



Backup - Time scales

Time scale of field reaction:

$$t_\phi \sim \frac{f}{\Lambda_r^2}$$

Time scale of object formation:

$$T_o$$

$t_\phi \ll T_o$ **: Adiabatic, field evolves in quasi-static background**

$t_\phi \gg T_o$ **: Sudden, field only sees final density configuration**

If final density distribution allows for classical rolling:

Only small difference due to kinetic energy

($\mathcal{O}(1)$ factors)

If final density does not allow for rolling, but intermediate does

Big difference!

Backup - Gravity

$$F_{grav} = -\frac{1}{M_p^2}\frac{M_{star}M_{bubble}}{r_b^2} = -\frac{1}{M_p^2}\frac{\left(\frac{4\pi}{3}r_b^3\rho\right)\left(4\pi\sigma r_b^2\right)}{r_b^2}$$

Using NDA $R_{star}^2 \sim \frac{M_p^2}{\rho}$

we find O(1) change in eom:

Inside: $$\sigma\,\ddot{r} = \epsilon - \frac{2\sigma}{r}\left(1 + \frac{2\pi}{3}\frac{r_b^2}{R_{star}^2}\right) - \frac{d\sigma}{dr}$$ $r_b < R_{star}$

Outside: $$\sigma\,\ddot{r} = \epsilon - \frac{2\sigma}{r}\left(1 + \frac{2\pi}{3}\frac{R_{star}}{r_b}\right) - \frac{d\sigma}{dr}$$ $r_b > R_{star}$

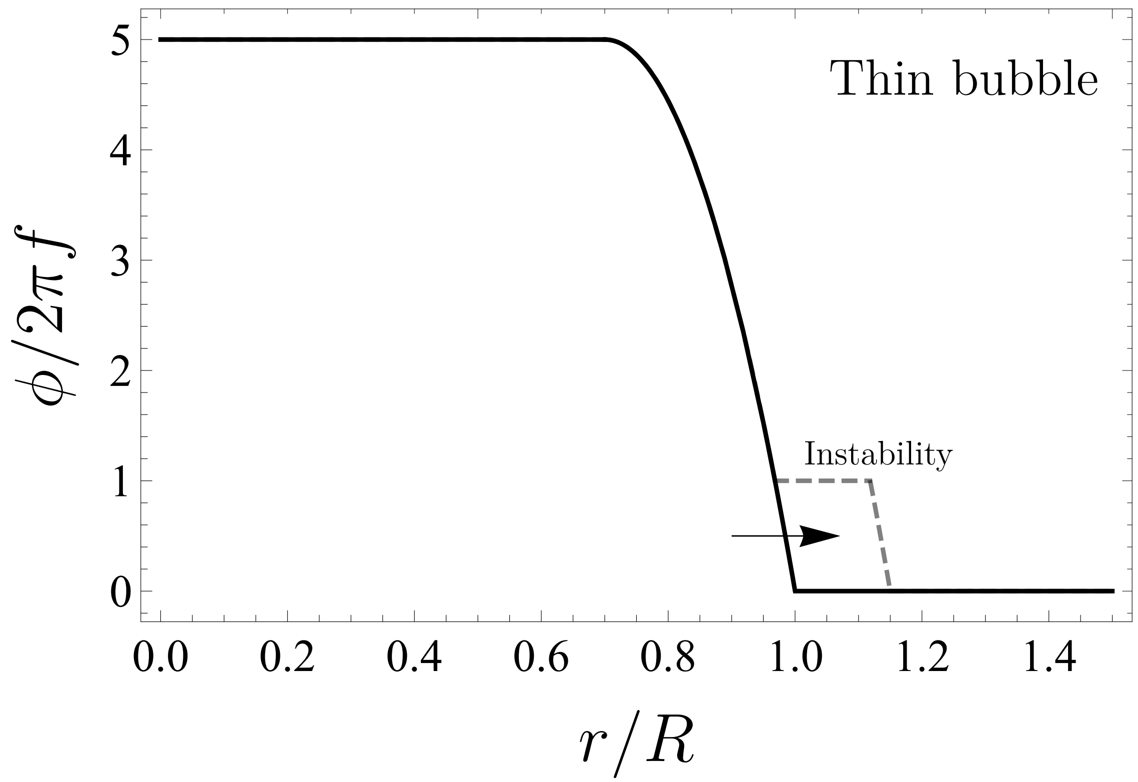
Backup - Bubbles connecting multiple vacua

$$\Delta\phi = 2\pi f N$$

Split bubble into sub-bubbles with $\Delta\phi = 2\pi f$

Tension of each sub-bubble in enhanced by $\sqrt{N}$

Furthermost sub-bubble can split off more easy



Backup - Coupled system NS - scalar

$$p' + \phi'\left(\frac{\mathrm{d}V}{\mathrm{d}\phi}\right) = -\frac{(\epsilon + p)\,e^{\sigma}}{2r}\left[1 - e^{-\sigma} + \kappa r^2\left(p + \frac{e^{-\sigma}}{2}(\phi')^2\right)\right],$$

$$\sigma' = \kappa r e^{\sigma}\left[\epsilon + \frac{e^{-\sigma}}{2}\,(\phi')^2\right] - \frac{e^{\sigma} - 1}{r},$$

$$\phi'' + \frac{2}{r}\left[\frac{1 + e^{\sigma}}{2} + \frac{\kappa r^2 e^{\sigma}}{4}\,(p - \epsilon)\right]\phi' = e^{\sigma}\frac{\mathrm{d}V}{\mathrm{d}\phi}.$$

Outlook - Phase transition from EM fields

$$g_{\phi\gamma\gamma}\frac{\phi}{f}F^{\mu\nu}\tilde{F}_{\mu\nu}$$

Vacuum Dipole as model for magnetic NS:

$$\vec{E}\cdot\vec{B}\sim\left(\frac{B_s^2R_s^6\Omega_s}{4r^5}\right)\Theta(r-R_S)$$

$$\downarrow$$

$$\Lambda_R^4(\vec{E}\cdot\vec{B})>\Lambda_R^4$$

Bubbles can form!

$$\downarrow$$

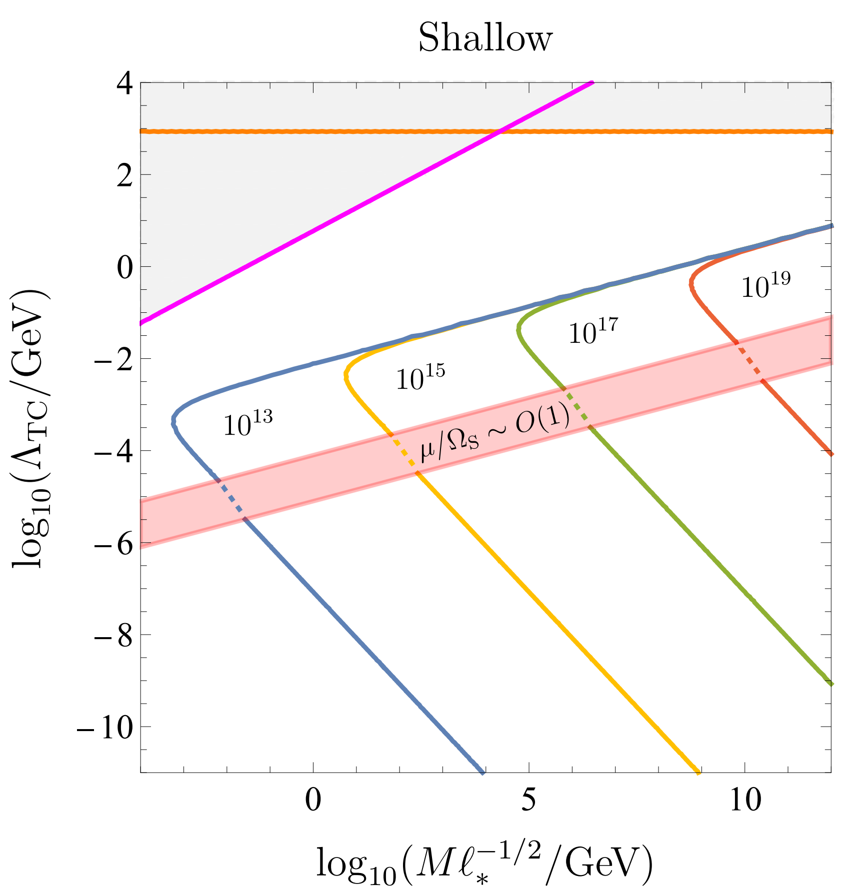
Gives bounds on metastable theories with coupling to photons (e.g. Technicolored Relaxion)

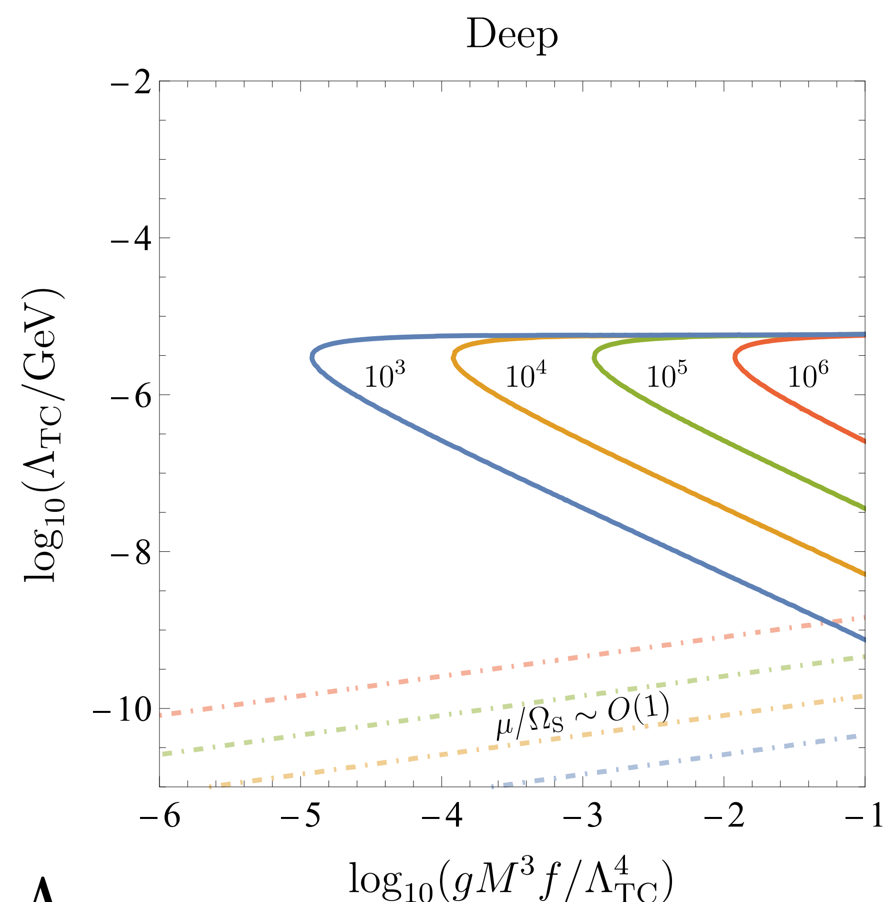
Backup - Phase transition from EM fields

Analysis very similar to matter density

Escape condition

$$R_T^{EM} - R_s \gtrsim \frac{f\Lambda_B^2}{\Lambda_R^4}$$





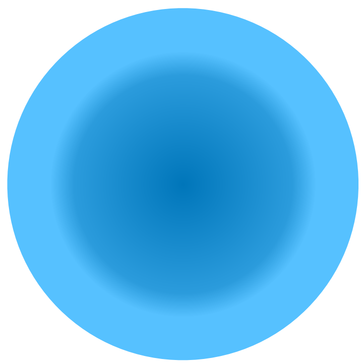
$$\Lambda_{TC} \sim \Lambda_B$$

Technicolored Relaxion

Outlook - Phase transitions from dark compact objects

Dark ‘neutron stars’ can seed PT (e.g. in non-QCD relaxion)

Fermi pressure vs Gravity



$$n_\psi \sim m_\psi^3$$

$$R_s \sim \frac{M_p}{m_\psi^2}$$

$$\frac{\Lambda_B(n_\psi)^4}{\Lambda_B^4} \sim 1 - \frac{\tilde{\sigma} n_\psi}{\Lambda_C^4}$$

Dense enough: $\Lambda_B(n_\psi)^4 \lesssim \Lambda_R$

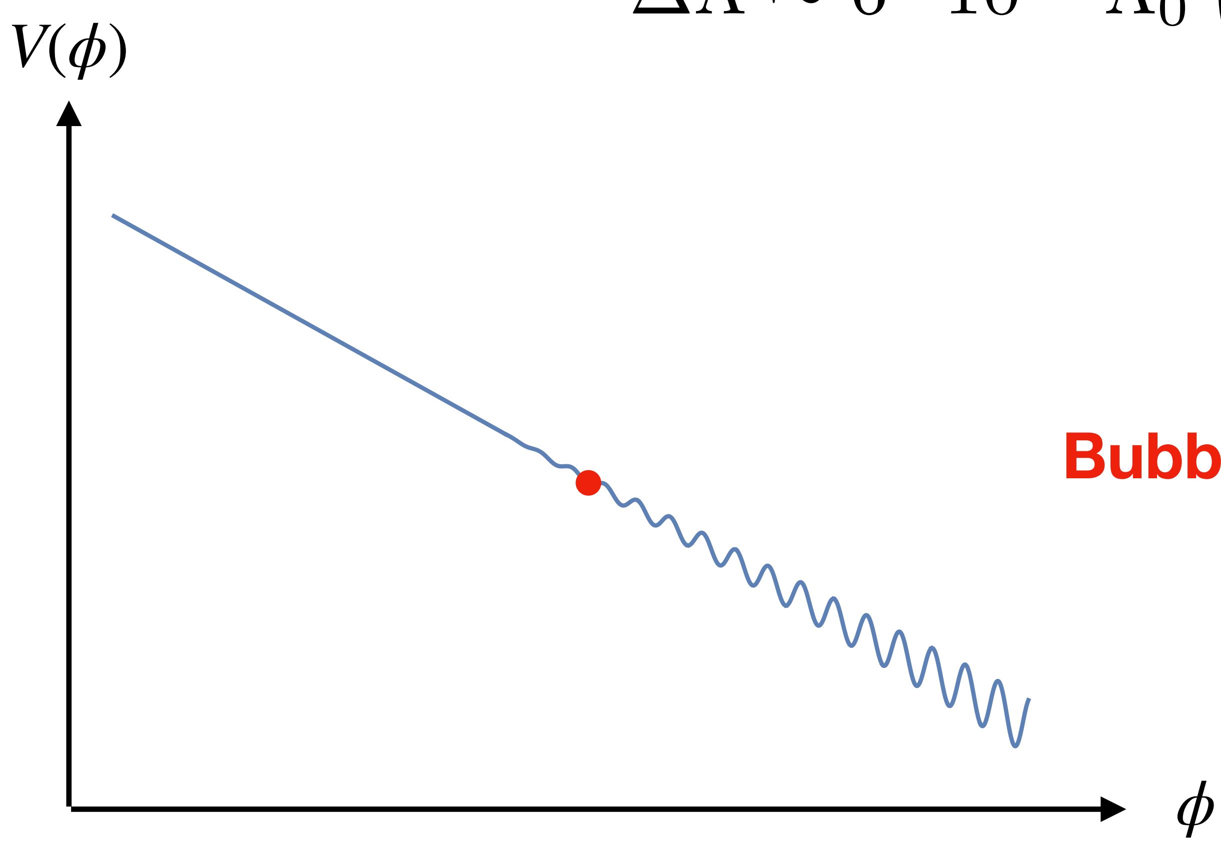
Large enough: $m_\psi \lesssim \Lambda_C \sqrt{\frac{M_p}{f}}$

$$-\Delta\Lambda \sim 6 \cdot 10^{-3} \Lambda_0 \left(\frac{m_\psi}{10\mathrm{keV}}\right)^4 \left(\frac{f}{10\mathrm{TeV}}\right)^2$$

Outlook - Phase transitions from dark compact objects

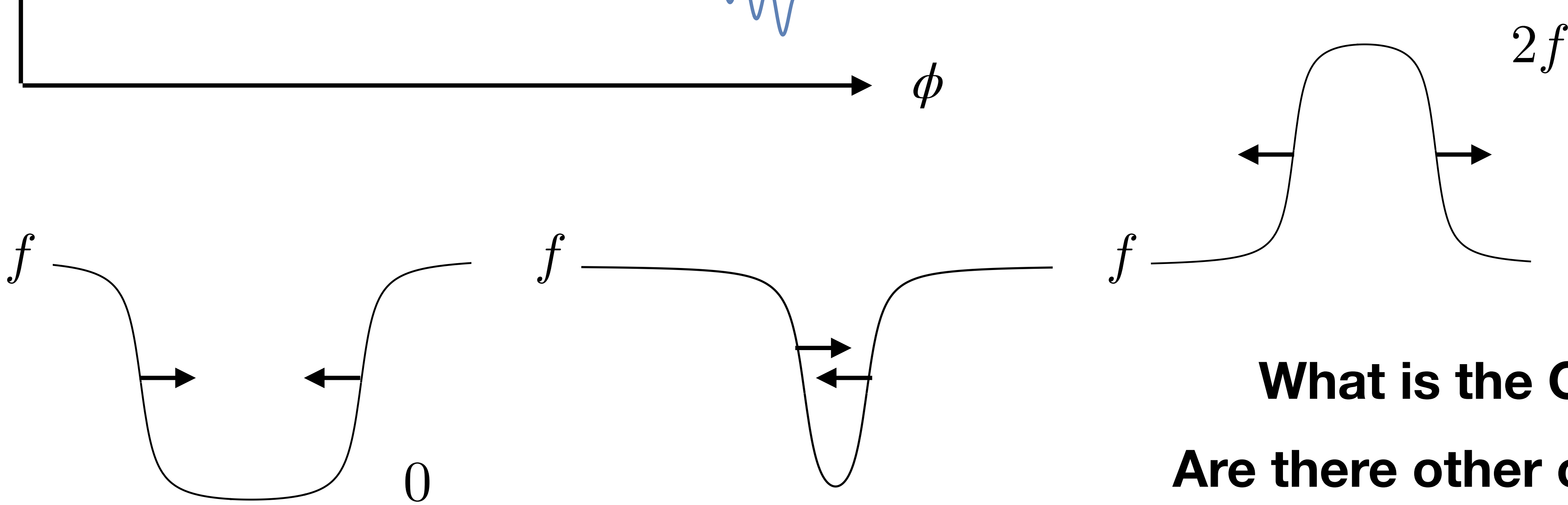
Ongoing work

$$-\Delta\Lambda \sim 6 \cdot 10^{-3} \Lambda_0 \left(\frac{m_\psi}{10\mathrm{keV}}\right)^4 \left(\frac{f}{10\mathrm{TeV}}\right)^2$$



First bubbles at star formation

Bubble collisions trigger subsequent bubbles



What is the GW signal?

Are there other observables?