Gravitational wave induced baryon acoustic oscillations

arXiv: 2107.10283

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Christian Döring

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In collaboration with:

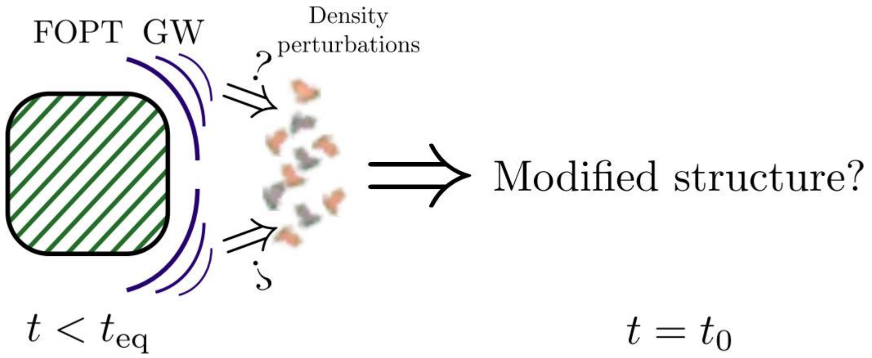
Salvador Centelles Chuliá

Manfred Lindner

Björn Malte Schäfer

Matthias Bartelmann

Question: Can GWs from FOPTs impact structure formation?
If so, can we infer bounds on the FOPT parameters from SF?



Overview

- Short review of cosmological first order phase transitions (FOPT) and gravitational waves (GWs)
- Short review on structure formation (SF)
- Physical idea
- Methods
- Results
- Summary

Short intro to FOPTs

Particle (scalar) model

$$\mathcal{L} \supset \partial_\mu \phi \partial^\mu \phi^* - V^{(0)}(\phi)$$

$$V_{\rm eff}(\phi, T) = V^{(0)}(\phi) + V^{(1)}(\phi) + V_T^{(1)}(\phi)$$

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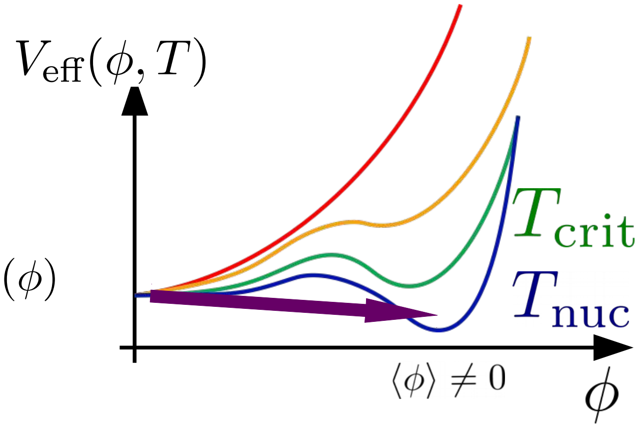

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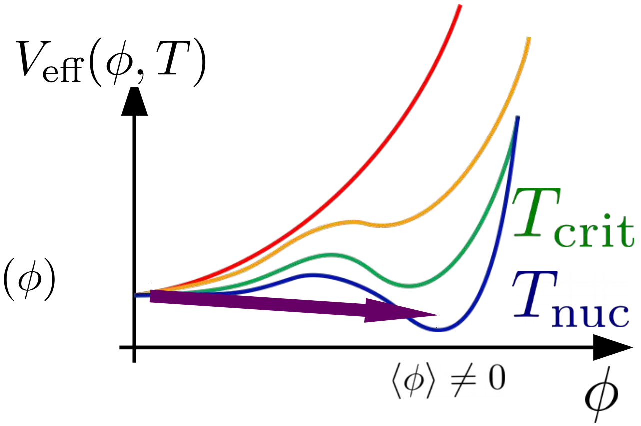

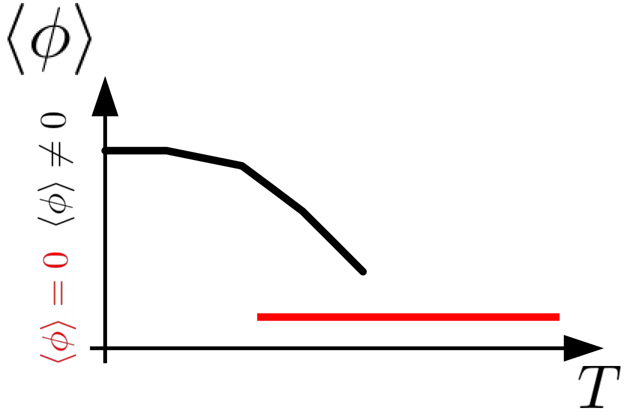

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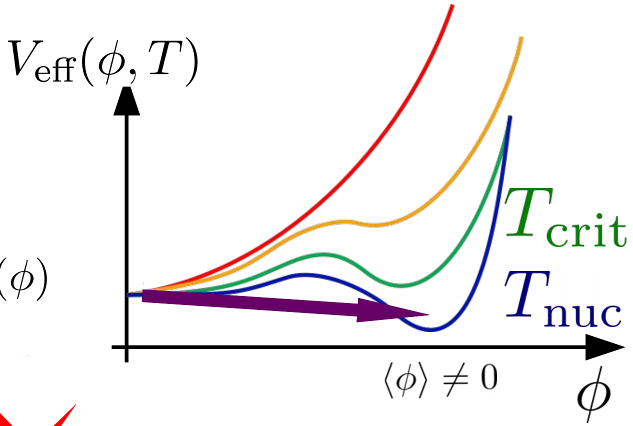

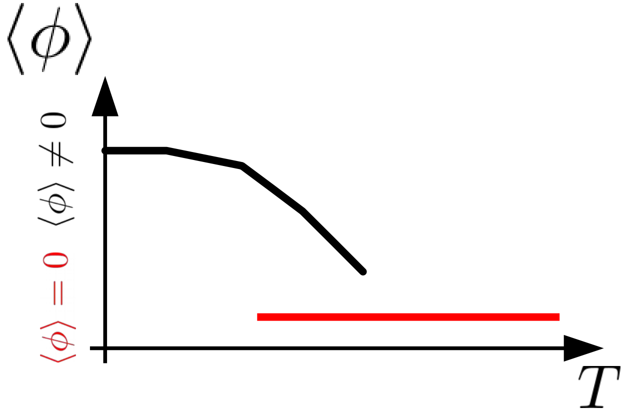

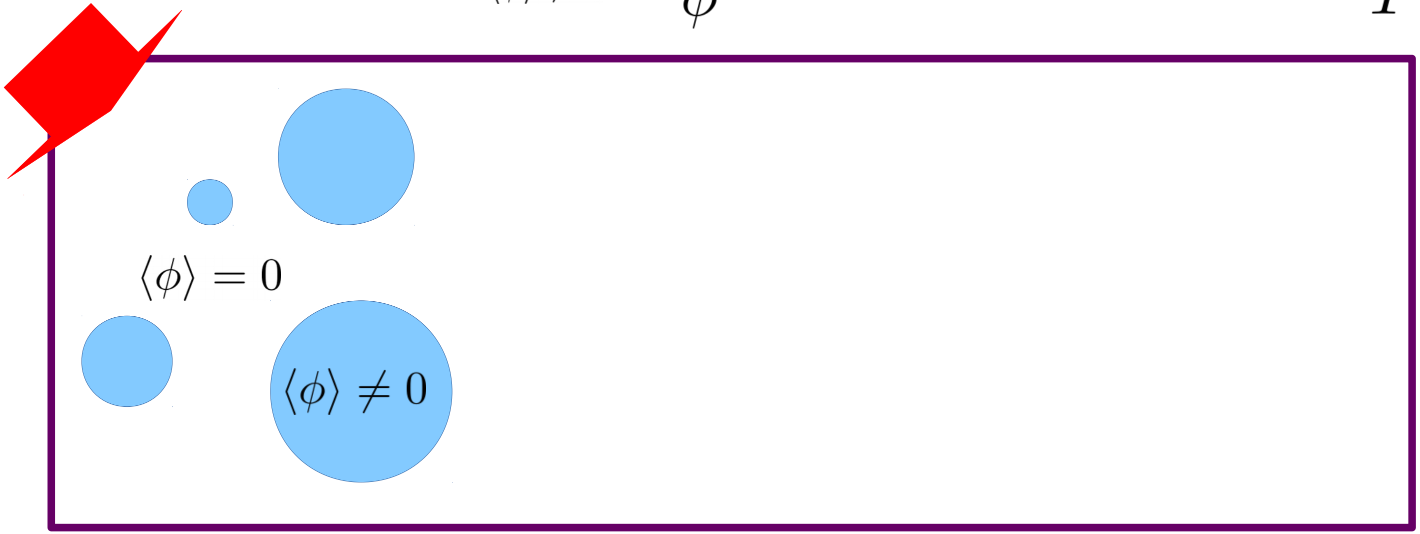

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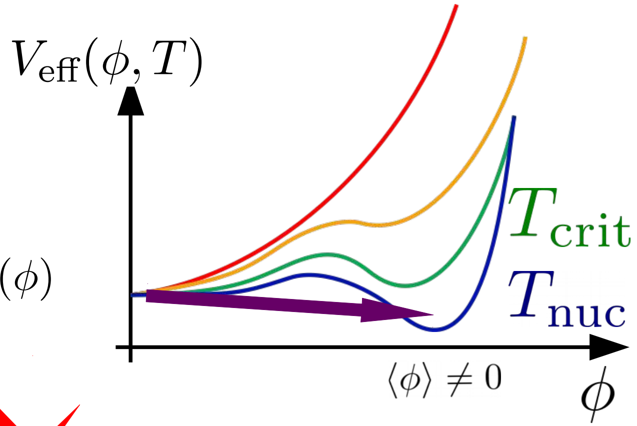

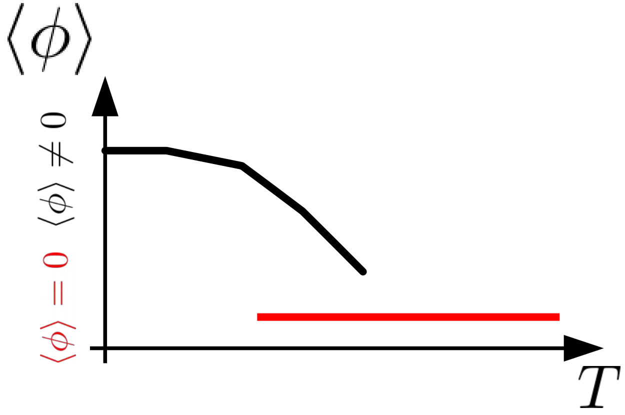

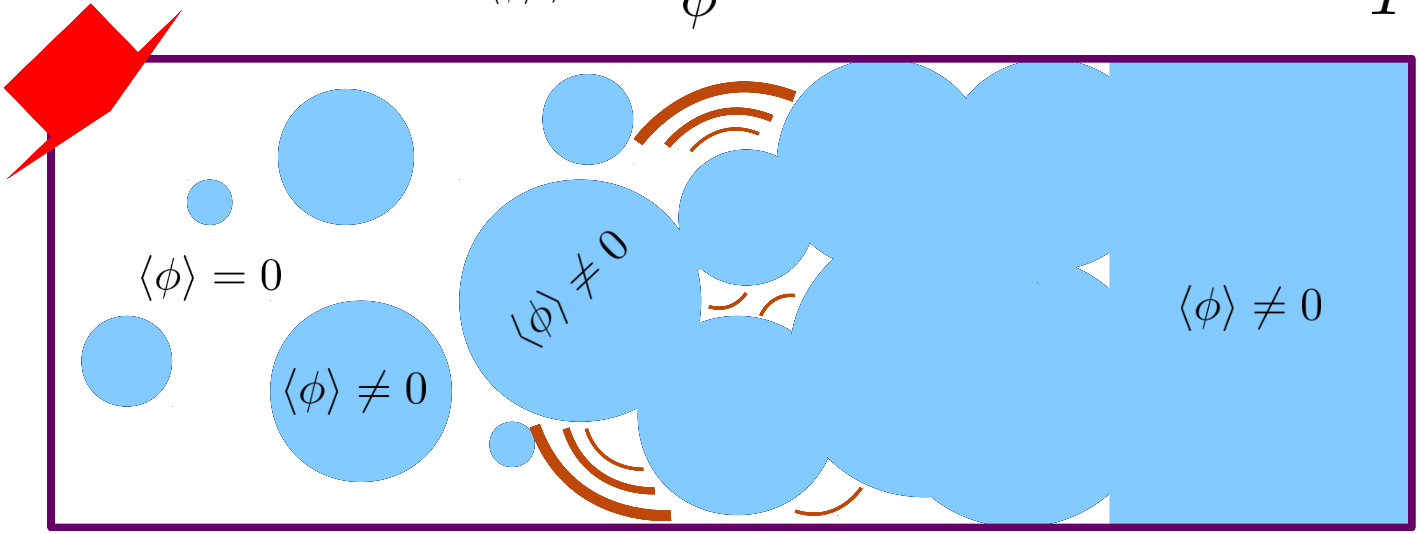

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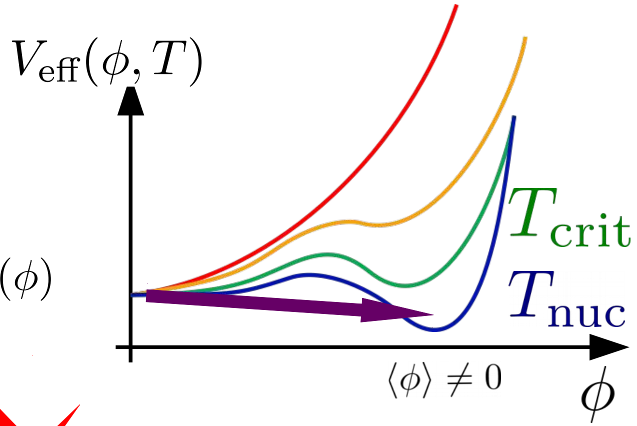
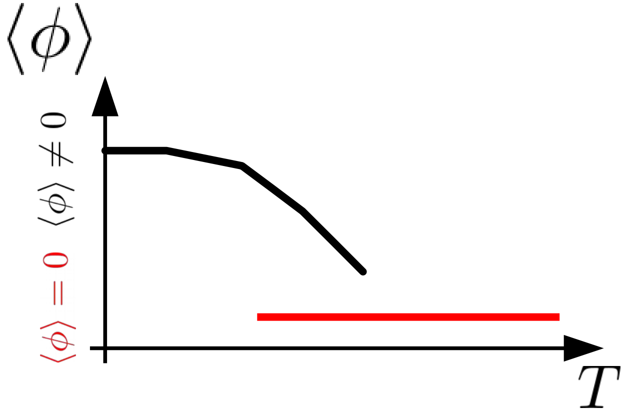
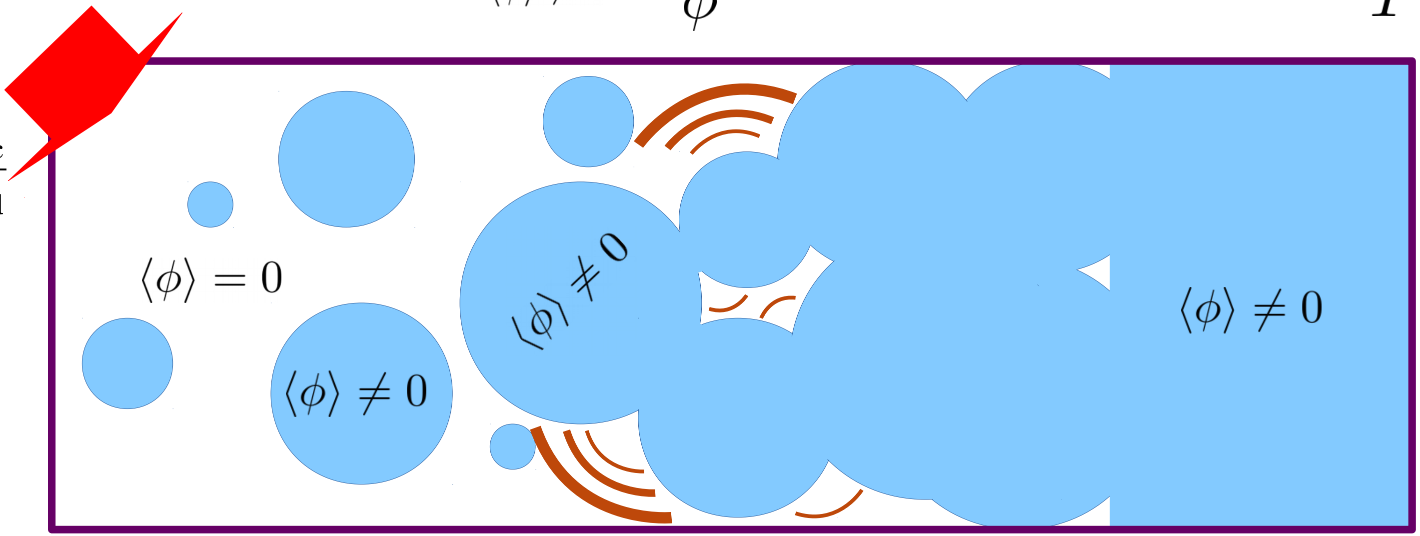




- Strength $\alpha := \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}}$
- Duration β^{-1}
- Scale/Temperature T_{nuc}/T_*



[Review: C. Caprini, D. Figueroa, Class.Quant.Grav. 35 (2018) 16, 163001, arXiv:1801.04268]

Bubble nucleation in supercooled water

Nucleation in old phase

Source:
https://www.youtube.com/watch?v=_9N-Y2CyYhM

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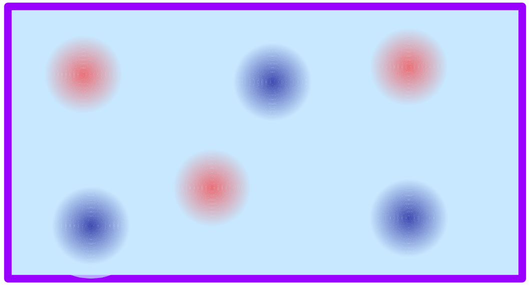

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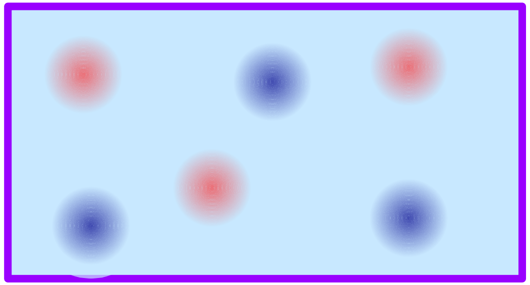

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Perturbed equations:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$
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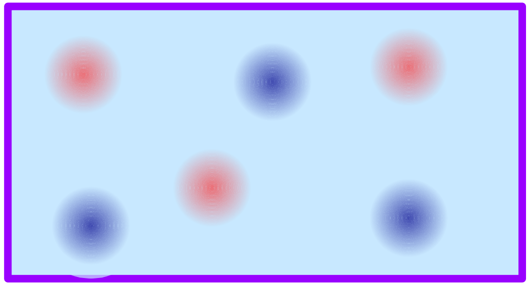

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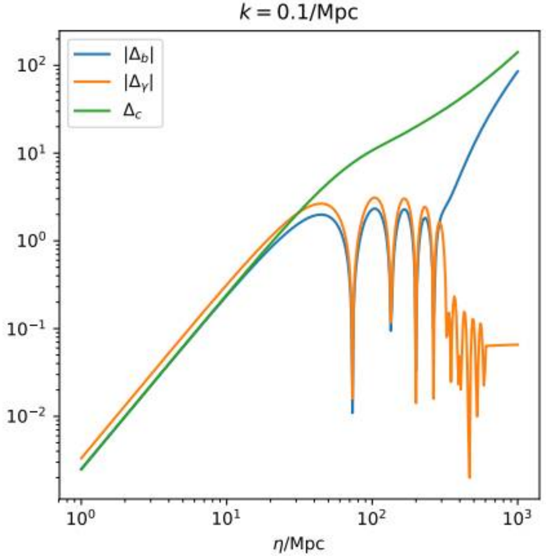

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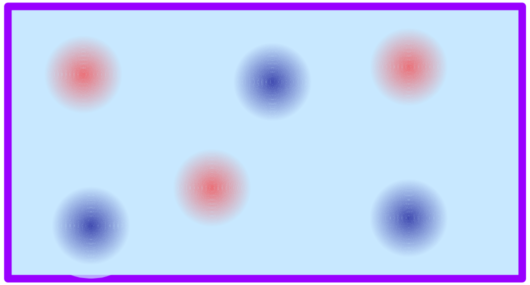

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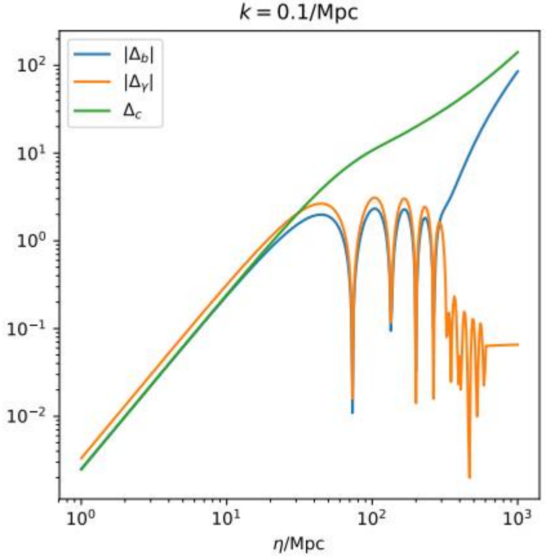

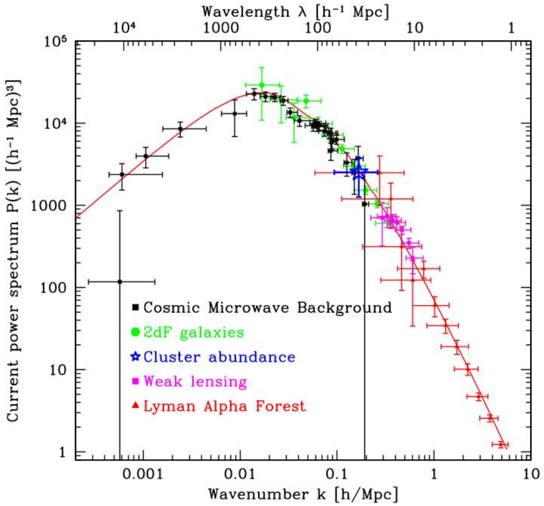

[M. Tegmark, M. Zaldarriaga, 2002, Phys. Rev. D, 66, 103508]

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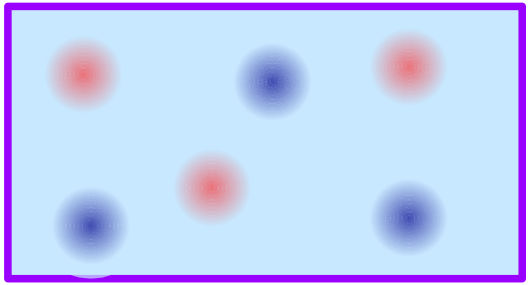

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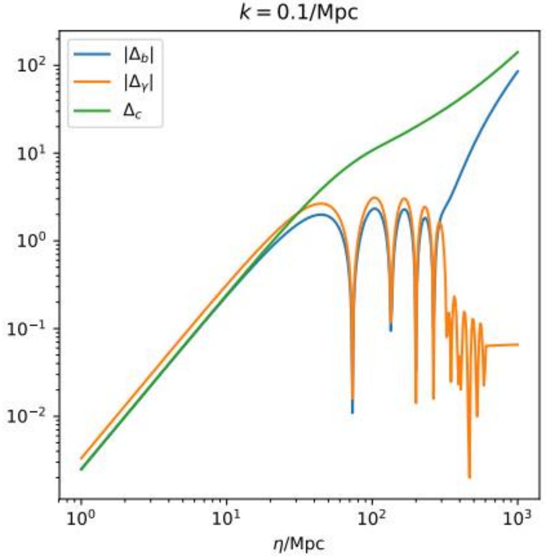

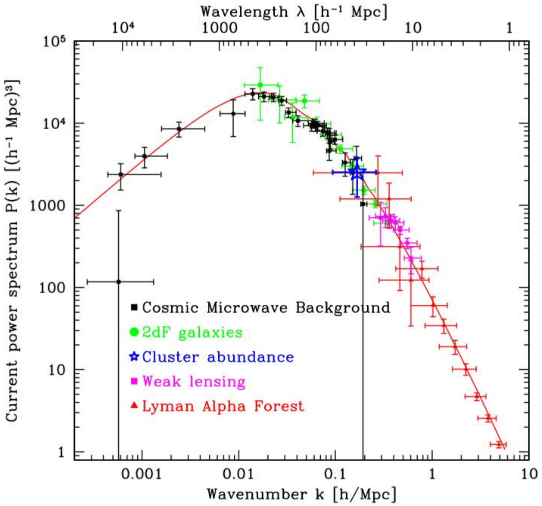

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=> supression of small scales

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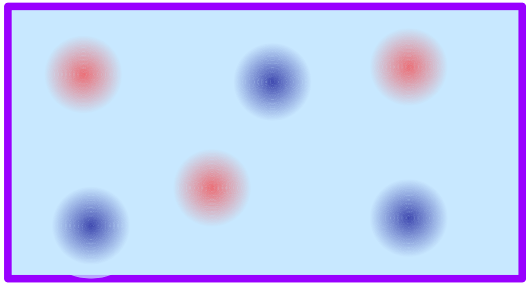

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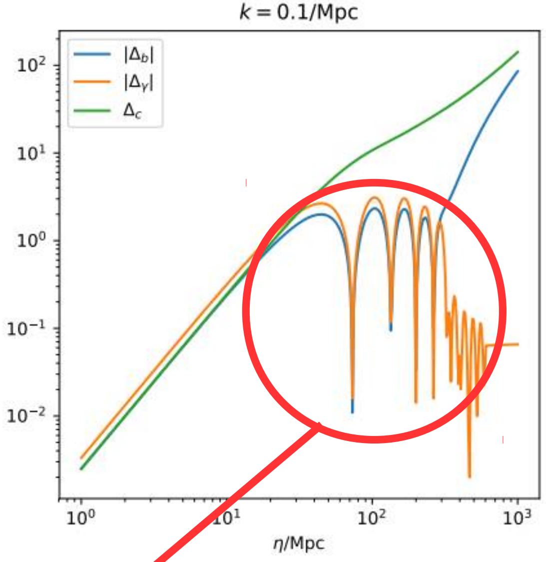
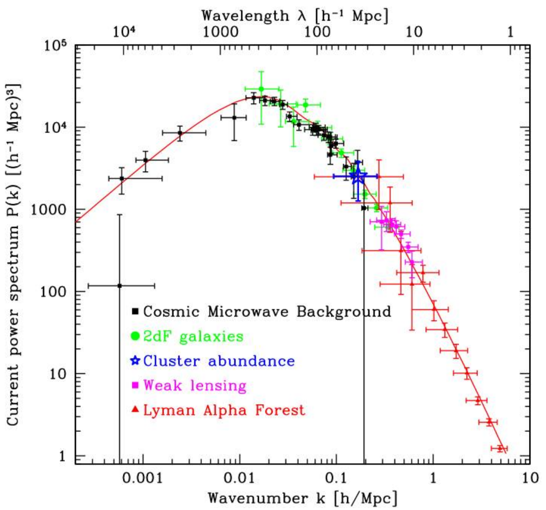


Baryon acoustic oscillations (BAOs)

$$\ddot{\delta}_\gamma + c_s^2 \frac{k^2}{a^2}\delta = \frac{4}{3} 4\pi G \left(\rho_d^{(0)}\delta_d + \rho_b^{(0)}\delta_b\right)$$

wiggles on MP spectrum



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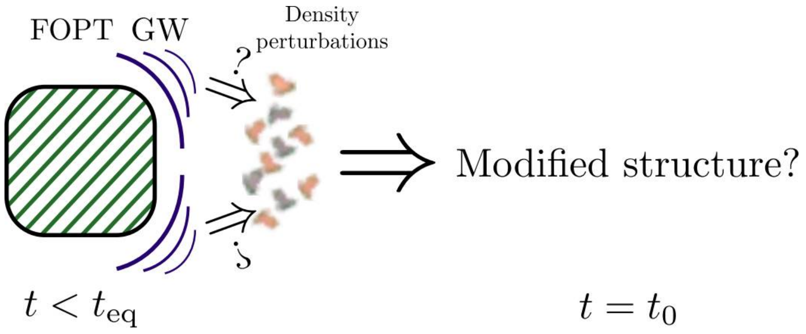

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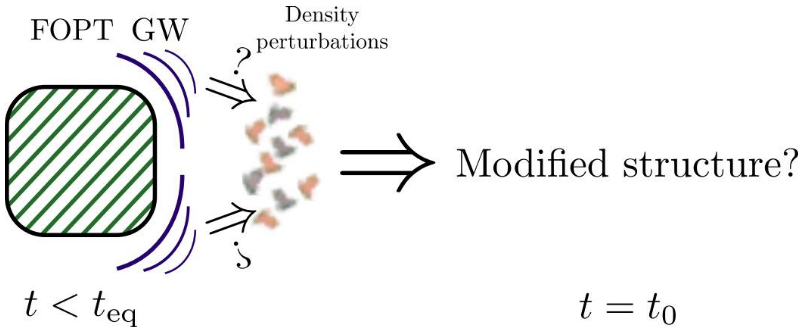

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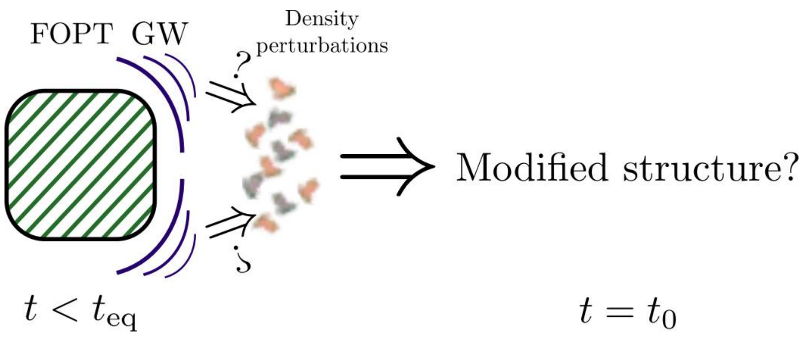

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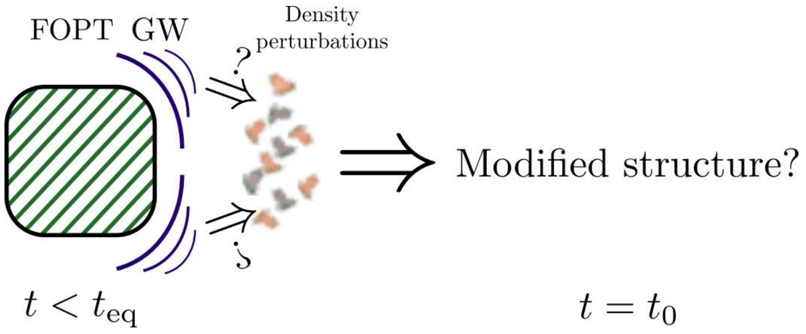

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Similar calculation Matter dom. & superhorizon

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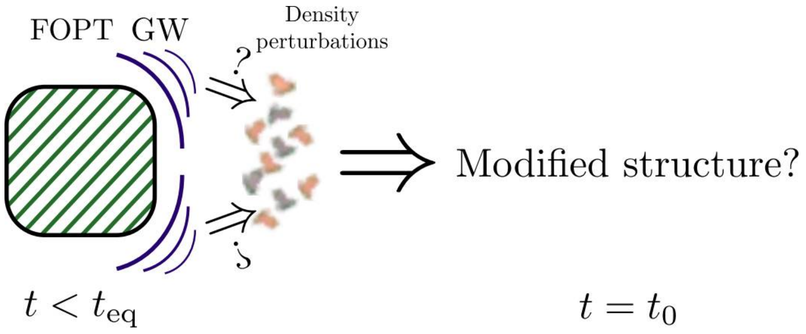

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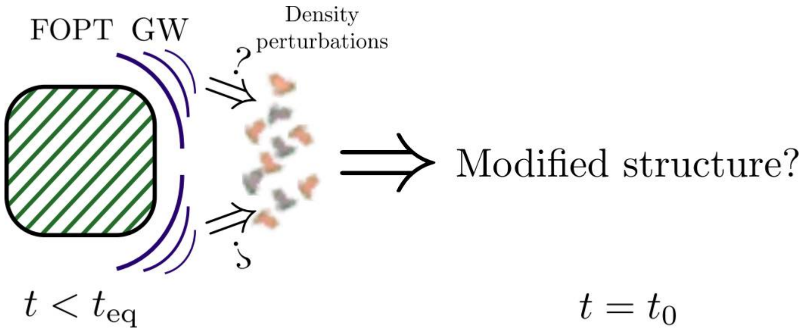

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- Takes place on sub-horizon scales
- FOPT completes within a Hubble time

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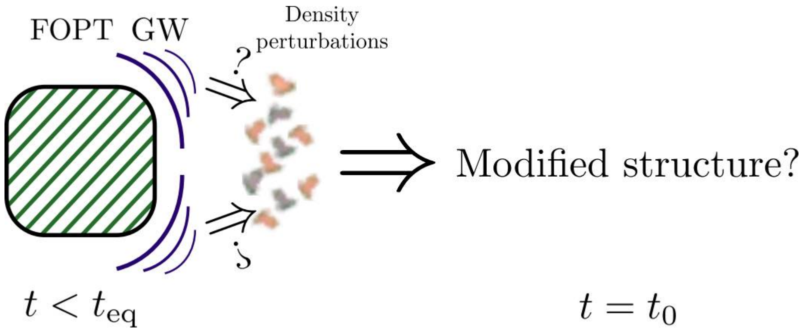

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General observations

- Takes place on sub-horizon scales $k \gtrsim a_* H_*$
- May complete within a Hubble time $\beta^{-1} \lesssim H_*^{-1}$

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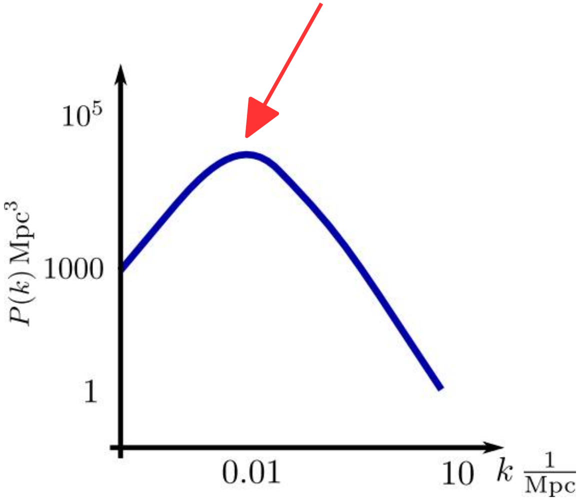
scale of MR equality



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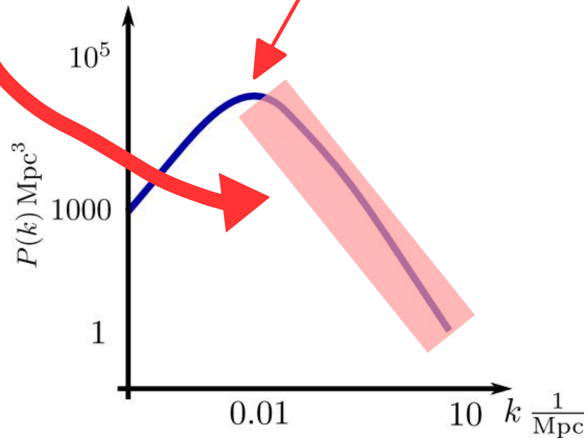

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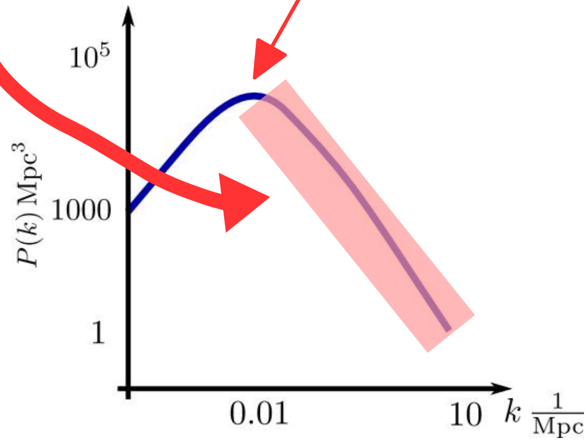

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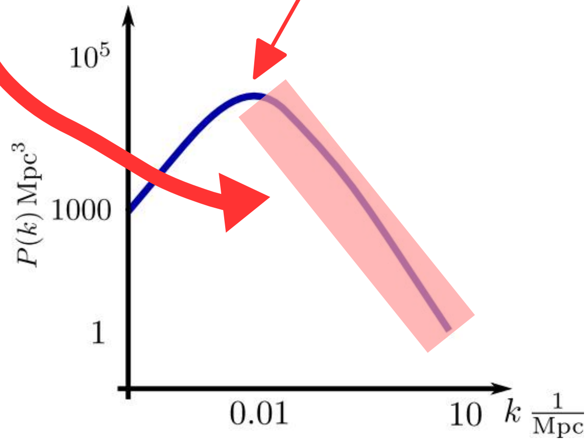

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Peak wave number of GW energy density

$$k_{\mathrm{peak}} = 2\pi \frac{\beta}{H_*} H_* a_*$$

GW induced density perturbations

Technicalities:

Step 1: Perturbation theory in 1+3 framework

$$\Delta_a : \frac{a}{\rho} \mathrm{D}_a \rho \quad Z_a := a \mathrm{D}_a \Theta \quad \sigma_{ab} = a^2 \dot{h}_{\alpha\beta}$$

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Step 4: Interpretation

Second order baryon acoustic oscillations driven by GW from FOPT

Impact on linear MP spectrum

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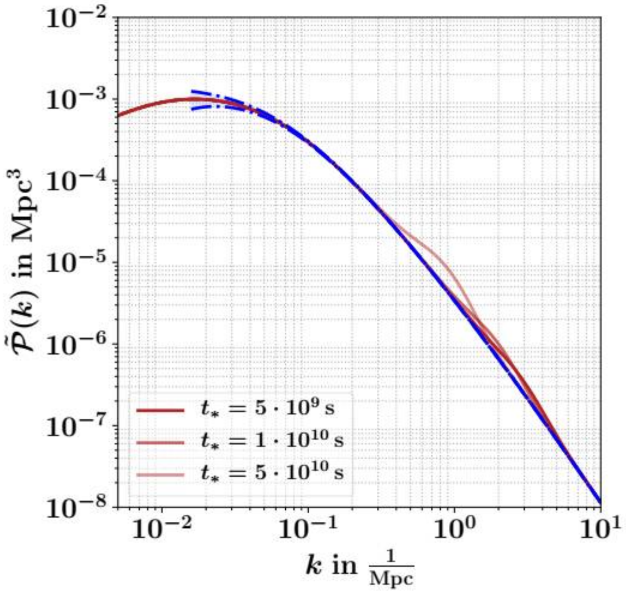
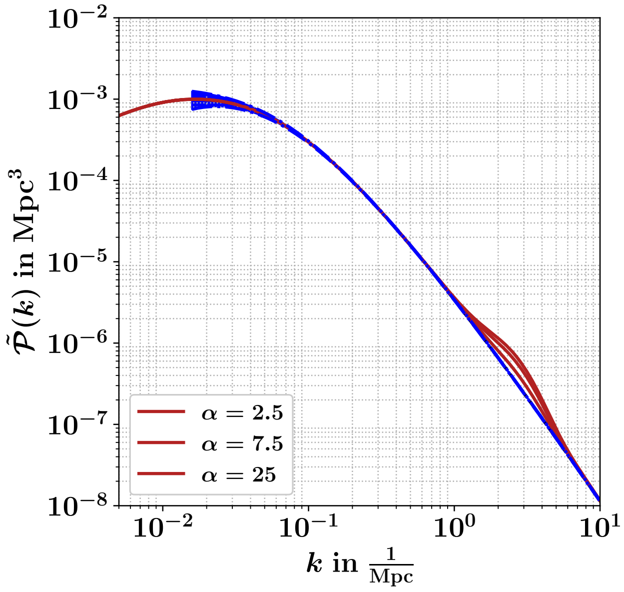
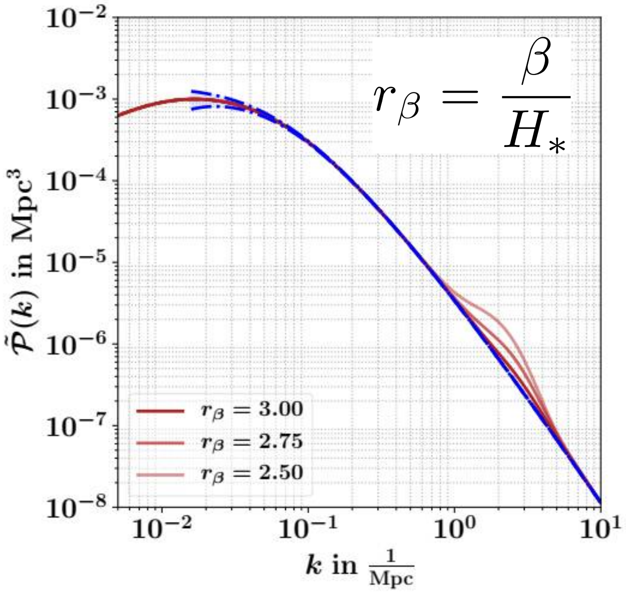


Changing scale



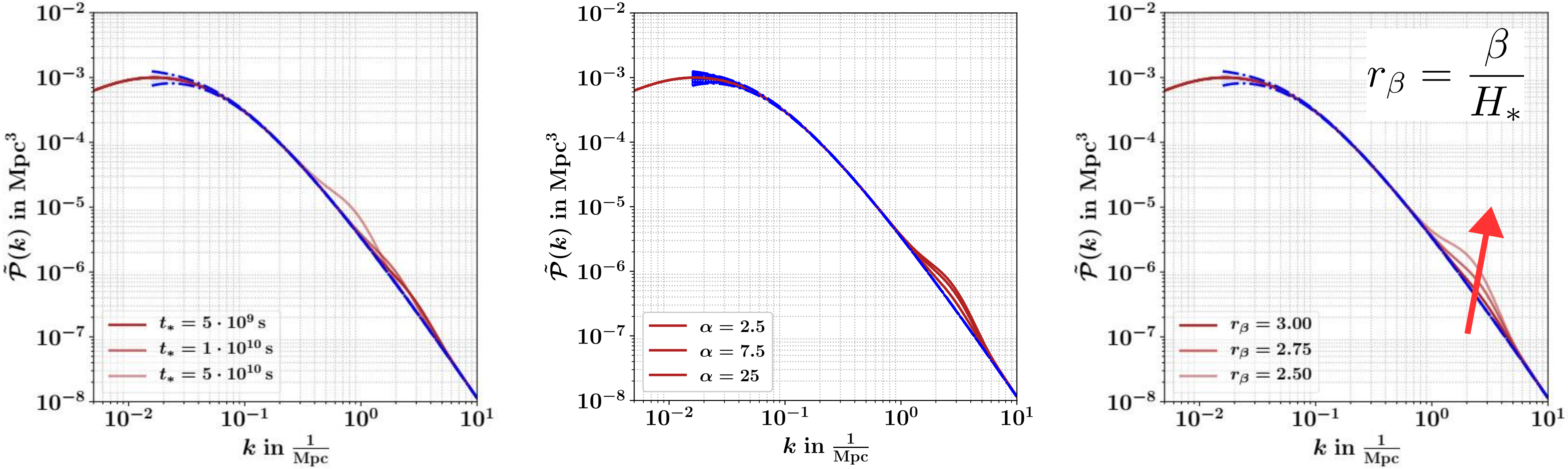
Changing strength



Changing duration

Blue line: Cosmic Variance Bound

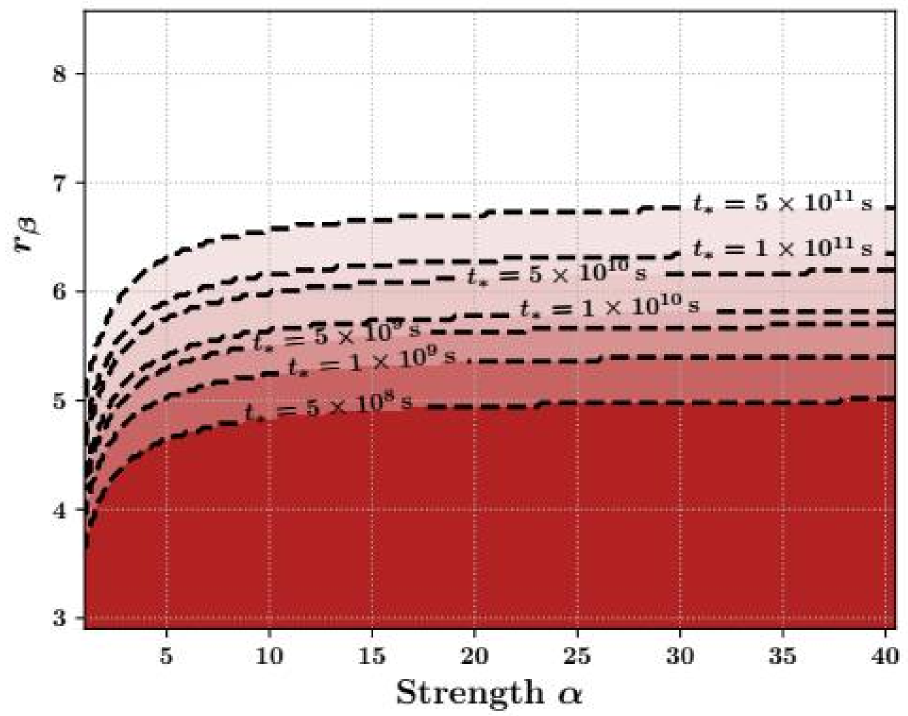
Impact on linear MP spectrum

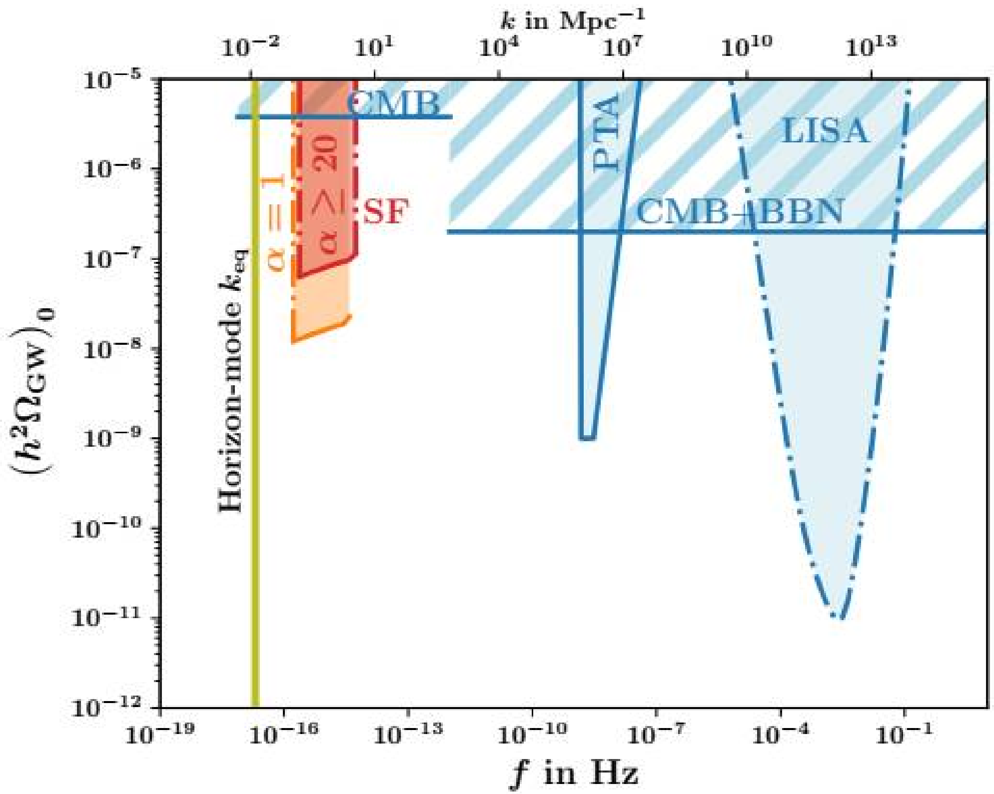


Changing scale | Changing strength | Changing duration

Blue line: Cosmic Variance Bound

Limits from cosmic variance





Particle models that can achive this: e.g. conformal models

Summary

Max-Planck-Institut für Kernphysik Heidelberg

- GWs from FOPTs can seed density perturbations at second order
- Effect is bound to the scale at which the FOPT occurs → late FOPTs
- Only very strong and long FOPTs can have significant impact
- Cosmic variance bound leads to new limit on very small GW frequencies

Backup slides

1+3 Decomposition

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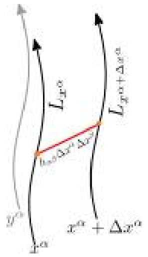
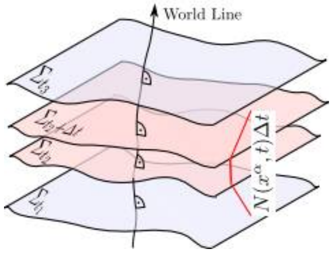
Spacetime decomposition:

$$u^a = \frac{\mathrm{d}x^a}{\mathrm{d}\tau} \qquad h_{ab} := g_{ab} + u_a u_b$$



1+3 approach



3+1 approach

Stewart & Walker Lemma:

$$S^{(1)} \to S^{(1)} + \epsilon \mathfrak{L}_\xi S^{(0)}$$

Gauge invariant if zero

J. M. Stewart, M. Walker, Proc. R. Soc. Lond. A 341 no. 49, (1974)

Motion of test particle

volume expansion

$$\nabla_b u_a = \sigma_{ab} + \omega_{ab} + \frac{1}{3}\Theta h_{ab} - A_a u_b$$

shear vorticity acceleration

density perturbation

$$\Delta_a : \frac{a}{\rho}\mathrm{D}_a \rho$$

volume gradient

$$Z_a := a\mathrm{D}_a \Theta$$

$$a\mathrm{D}_b \Delta_a = \frac{1}{3}\Delta h_{ab} + \Delta_{\langle ab \rangle} + \Delta_{[ab]}$$

P. K. S. Dunsby, M. Bruni, G.F.R. Ellis, Class. Quant. Grav. 14 (1997) 1215-1222

Evolution equations

$$\dot{\Delta}_{\langle a\rangle} = \frac{p}{\rho}\Theta\Delta_a - \left(1+\frac{p}{\rho}\right)Z_a + a\frac{\Theta}{\rho}\left(\dot{q}_{\langle a\rangle} + \frac{4}{3}\Theta q_a\right) - \frac{a}{\rho}{}^b{}_a q_b + a\frac{\Theta}{\rho}{}^b\pi_{ab}$$

$$- \left(\sigma^b{}_a + \omega^b{}_a\right)\Delta_b - \frac{a}{\rho}{}_a\left(2A^b q_b + \sigma^{bc}\pi_{bc}\right) + a\frac{\Theta}{\rho}\left(\sigma_{ab}+\omega_{ab}\right)q^b + a\frac{\Theta}{\rho}\pi_{ab}A^b$$

$$+ \frac{1}{\rho}\left({}^b q_b + 2A^b q_b + \sigma^{bc}\pi_{bc}\right)\left(\Delta_a - aA_a\right)$$

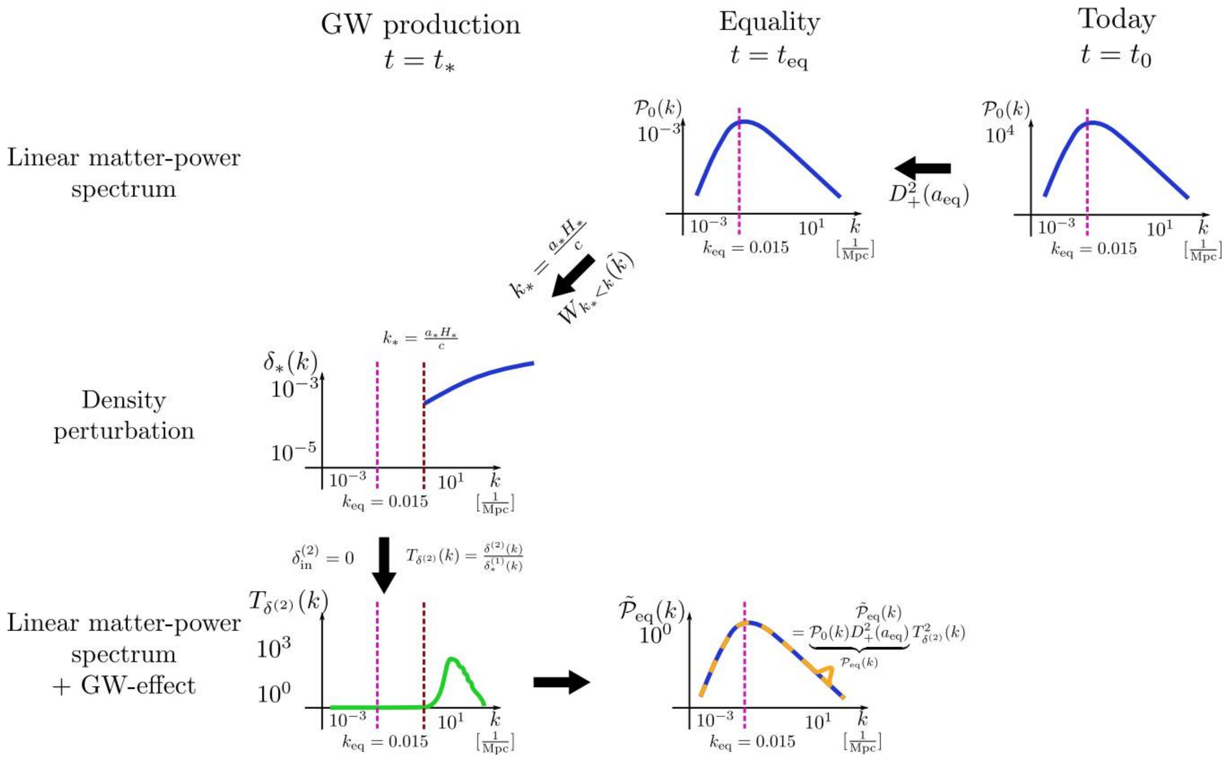
$$\dot{Z}_{\langle a\rangle} = -\frac{2}{3}\Theta Z_a - \frac{1}{2}\kappa\rho\Delta_a - \frac{3}{2}\kappa a_a p - a\left[\frac{1}{3}\Theta^2 + \frac{1}{2}\kappa(\rho+3p) - \Lambda\right]A_a + a^b_a A_b$$

$$- \left(\sigma^b{}_a + \omega^b{}_a\right)Z_b - 2a_a\left(\sigma^2 - \omega^2\right) + 2aA^b_a A_b$$

$$- a\left[2\left(\sigma^2-\omega^2\right) -^b A_b - A^b A_b\right]A_a$$

Transferfunction

Estimating the linear density perturbation from the linear MP spectrum:



Examples for late PTs

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- I. Wasserman: Phys. Rev. Lett, 57:2234-2236, 1986
- A. Patwardhan, G. Fuller: Phys. Rev. D, 90(6):063009, 2014
- Xiao-chun Luo, D. Schramm: Astrophys. J., 421:393-399, 1994