New methods for studying the Electroweak phase transition

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Phase transitions in the early universe

Why care about phase transitions?

Baryon asymmetry problem $\rightarrow$ Electroweak baryogenesis

Gravitational waves $\rightarrow$ Pushing field theory to its limit!

Outline

Sources of theoretical uncertainties

Consistent methods to handle (tree-level) radiative barriers

Importance of radiative corrections for the nucleation rate

Talk based on the papers

2205.08815 by AE, Philipp Schicho and Tuomas V.I. Tenkanen

2205.07241 by AE, Oliver Gould and Johan Löfgren

2205.05145 by AE

Problems at high temperatures

Slow convergence

$T^2 \gg m^2 \implies$ Large Logs

$m^2_{\text{eff}} = (m^2 + aT^2) \ll m^2$

$\rightarrow$ Need 2-loop thermal masses

Huge uncertainty for GW spectrum (2104.04399)

High-temperature effective theory

Integrate out $E \sim T$ modes

$\rightarrow$ Logs made small by matching at $\mu \approx T$

$\rightarrow$ Unambiguous resummations

$\rightarrow$ Regular (T=0) field theory in 3d

No more thermal integrals!

`DRalgo` : Automatic matching to two loops (2205.08815)

Matches to the effective theory

$\rightarrow$ Two-loop thermal masses

$\rightarrow$ Two-loop Debye masses

$\rightarrow$ One-loop thermal couplings

$\rightarrow$ Two-loop effective potential

Phase transitions

3d calculation: $V_{\text{1-loop}} \sim -\frac{m^3_{\text{eff}}}{12\pi}$

Nucleation rate: $\Gamma \sim e^{-S_3/T}$

$\rightarrow$ Get S_3 from the effective theory

$\rightarrow$ Same procedure; better accuracy!

First-order phase transitions for radiative barriers

Start with the effective theory

Scalar potential $V(\phi) = \frac{1}{2}m_3^2\phi^2 + \frac{1}{4}\lambda_3\phi^4$ ($\lambda_3 = T\lambda_{4d}$ to LO)

Integrating out vector-bosons generates a barrier: $\delta V(\phi) = -\frac{1}{16\pi}g_3^3\phi^3$

Only consistent if $\frac{m_H^2}{m_A^2} \sim \frac{\lambda_3}{g_3^2} \equiv x \ll 1$

After a field-rescaling: $V_{\text{LO}}(\phi) = \frac{1}{2}y\phi^2 - \frac{1}{16\pi}\phi^3 + \frac{1}{4}x\phi^4$, $y \equiv \frac{m_3^2}{g_3^4} \sim x^{-1}$

Different minima: $\phi_{\text{s}} = 0, \phi_{\text{b}} \sim x^{-1} \neq 0$

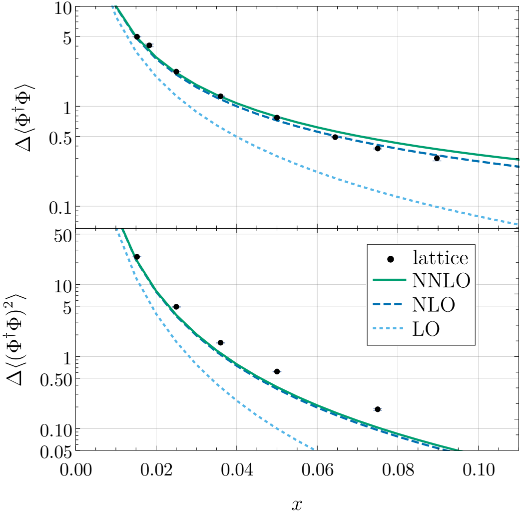
Critical temperature (mass)

Minima coincide when $\Delta V(x, y_c) \equiv V_{\text{LO}}(\phi_{\text{b}}) - V_{\text{LO}}(\phi_{\text{s}}) = 0 \implies$ Critical mass y_c

Consistent expansion: $\phi_b = \phi_{\text{LO}} + x\phi_{\text{NLO}} + \ldots \implies$ Gauge invariance

Critical mass: $y_c = y_{\text{LO}} + xy_{\text{NLO}} + \ldots \implies$ Exact RG-invariance at every order

Comparison with Lattice (new lattice results taken from 2205.07238)



Condensates to NNLO

$$\Delta\left\langle \Phi^\dagger\Phi \right\rangle = \frac{1+\frac{51}{2}x+13\sqrt{2}x^{3/2}}{2(8\pi x)^2}$$

$$\Delta\left\langle (\Phi^\dagger\Phi)^2 \right\rangle = \frac{1+51x+14\sqrt{2}x^{3/2}}{4(8\pi x)^4}.$$

Latent heat: $L \approx 4\times\Delta\left\langle \Phi^\dagger\Phi \right\rangle$

NLO corresponds to 2-loops

$\rightarrow$ 2-loop corrections are essential

Radiative corrections to the nucleation rate

Nucleation rate

$$\Gamma = \underbrace{\Gamma_{\text{stat}}}_{\text{Boltzmann factor}} \times \underbrace{\Gamma_{\text{dyn}}}_{\text{Damping}} \quad (2201.07331)$$

Effective action: $\Gamma_{\text{stat}} = e^{-S_{\text{eff}}}$

To leading order $S_{\text{eff}} = S_3 + S_{\text{NLO}} + \dots$

Function determinant gives S_{NLO}

1-Loop correction

$S_{\text{NLO}} = \frac{1}{2}\sum_i \text{Tr}\log\left[-\nabla^2 + M_i^2[\phi_B]\right]$

$S_{\text{NLO}} \sim R^3$ and $S_3 \sim R^2$ for large bubbles

$\Longrightarrow$ Corrections to the bounce are important

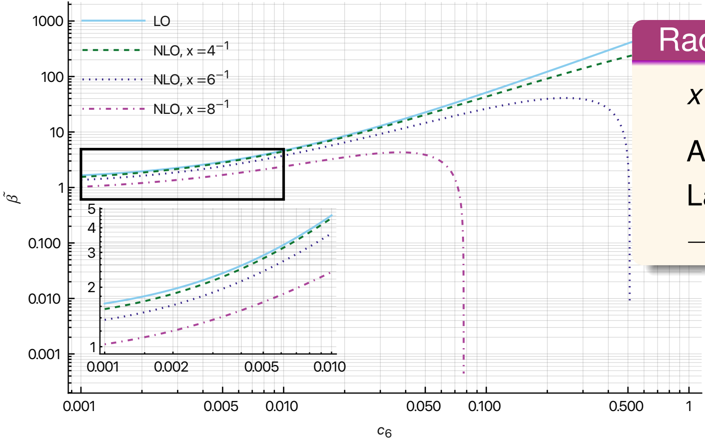
Example: Dimension-6 operator

$V(\phi) = \frac{1}{2}m_3^2\phi^2 - \frac{1}{4}\lambda_3\phi^4 + \frac{1}{32}c_6\phi^6 \; (c_6 = T^2 c_{6,4d})$

$S_{\text{eff}} = S_{\text{LO}} + S_{\text{NLO}} + \dots$

$\beta_N/H_N \sim \tilde{\beta} \to$ Observable in the effective 3d theory

Results for $\tilde{\beta}$ (2205.05145)



Radiative corrections can be large

$$x = \frac{\lambda_3}{g_3^2} \approx \frac{\lambda}{g^2}, \quad c_6 = T^2 c_{6,4d}$$

Absolute upper bound $c_6 \lesssim x^3$

Large corrections for small quartics

$\rightarrow$ Corrections propagate to GWs

Summary

The Electroweak phase transition is a hot topic

$\rightarrow$Uncertainties for common methods span orders of magnitude

$\rightarrow$ High-temperature effective theory key to reduce RG-scale dependence

$\rightarrow$ Consistent perturbative expansion key to remove gauge dependence

$\rightarrow$ Going beyond the bounce-action key for accurate predictions

Thank You

Backup slides

DRalgo example: Standard-Model with nF fermion families

Effective Couplings: Lb, Lf $\sim \log \mu / T$ (matching scale $\mu \sim T$)

Out[∘]=

$$\left\{\mathrm{gw3d}^2 \to \frac{\mathrm{gw}^4\, T\,(43\,\mathrm{Lb} - 8\,\mathrm{Lf}\,\mathrm{nF} + 4)}{96\,\pi^2} + \mathrm{gw}^2\, T,\ \mathrm{gY3d}^2 \to \mathrm{gY}^2\, T - \frac{\mathrm{gY}^4\, T\,(3\,\mathrm{Lb} + 40\,\mathrm{Lf}\,\mathrm{nF})}{288\,\pi^2},\ \mathrm{gs3d}^2 \to \frac{\mathrm{gs}^4\, T\,(33\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{nF} + 3)}{48\,\pi^2} + \mathrm{gs}^2\, T,\right.$$

$$\left.\lambda 1\mathrm{H3d} \to \frac{T\left(24\,\lambda 1\mathrm{H}\left(3\,\mathrm{gw}^2\,\mathrm{Lb} + \mathrm{gY}^2\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{yt}^2\right) + (2 - 3\,\mathrm{Lb})\left(3\,\mathrm{gw}^4 + 2\,\mathrm{gw}^2\,\mathrm{gY}^2 + \mathrm{gY}^4\right) + 256\,\pi^2\,\lambda 1\mathrm{H} - 192\,\lambda 1\mathrm{H}^2\,\mathrm{Lb} + 48\,\mathrm{Lf}\,\mathrm{yt}^4\right)}{256\,\pi^2}\right\}$$

One-loop scalar masses

Out[∘]=

$$\left\{\mathrm{m23d} \to \frac{1}{16}\, T^2\left(3\,\mathrm{gw}^2 + \mathrm{gY}^2 + 8\,\lambda 1\mathrm{H} + 4\,\mathrm{yt}^2\right) + \mathrm{m2}\right\}$$

Two-loop Debye masses

Out[∘]=

$$\left\{\mu\mathrm{sqSU2} \to \frac{\mathrm{gw}^2\left(T^2\left(\mathrm{gw}^2\left(86\,\mathrm{Lb}\,(2\,\mathrm{nF} + 5) - 32\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + (44 - 80\,\mathrm{Lf})\,\mathrm{nF} + 207\right) - 3\left(6\left(8\,\mathrm{gs}^2\,\mathrm{nF} - 4\,\lambda 1\mathrm{H} + \mathrm{yt}^2\right) + \mathrm{gY}^2\,(4\,\mathrm{nF} - 3)\right)\right) + 144\,\mathrm{m2}\right)}{1152\,\pi^2},\right.$$

$$\mu\mathrm{sqSU3} \to \frac{\mathrm{gs}^2\, T^2\left(4\,\mathrm{gs}^2\,(33\,\mathrm{Lb}\,(\mathrm{nF} + 3) + \mathrm{nF}\,(-4\,\mathrm{Lf}\,(\mathrm{nF} + 3) + 4\,\mathrm{nF} + 3) + 45) - 27\,\mathrm{gw}^2\,\mathrm{nF} - 11\,\mathrm{gY}^2\,\mathrm{nF} - 36\,\mathrm{yt}^2\right)}{576\,\pi^2},$$

$$\left.\mu\mathrm{sqU1} \to -\frac{\mathrm{gY}^2\left(T^2\left(18\left(88\,\mathrm{gs}^2\,\mathrm{nF} - 36\,\lambda 1\mathrm{H} + 33\,\mathrm{yt}^2\right) + 81\,\mathrm{gw}^2\,(4\,\mathrm{nF} - 3) + \mathrm{gY}^2\left(6\,\mathrm{Lb}\,(10\,\mathrm{nF} + 3) + 800\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + 60\,(4\,\mathrm{Lf} + 17)\,\mathrm{nF} - 45\right)\right) - 1296\,\mathrm{m2}\right)}{10\,368\,\pi^2}\right\}$$

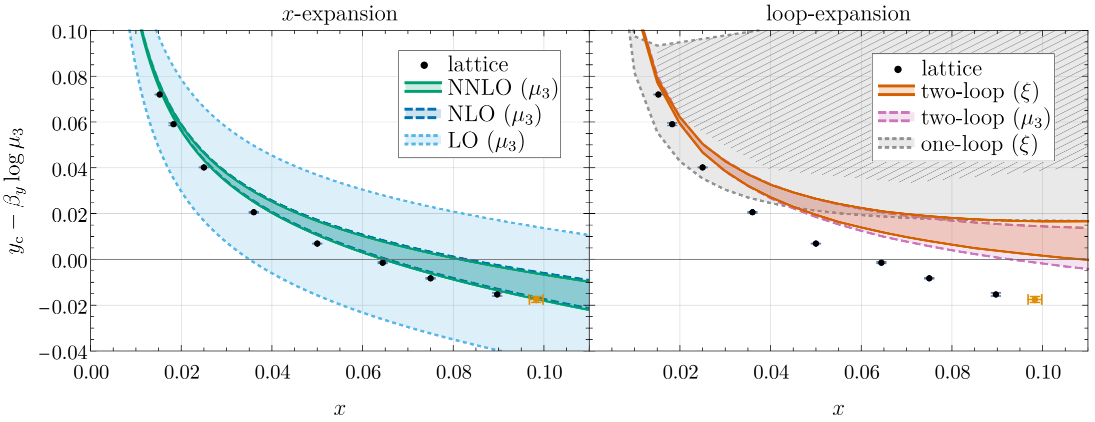
From 3d observables to the latent heat

Observables in the 3d to two-loops (2205.07241)

$$\Delta \left\langle \Phi^\dagger \Phi \right\rangle \equiv \partial_y \Delta V(x, y_c), \quad \Delta \left\langle (\Phi^\dagger \Phi)^2 \right\rangle \equiv \partial_x \Delta V(x, y_c)$$

$$\Delta \left\langle \Phi^\dagger \Phi \right\rangle = \frac{1 + \frac{51}{2} x + 13\sqrt{2} x^{3/2}}{2(8\pi x)^2}, \quad \Delta \left\langle (\Phi^\dagger \Phi)^2 \right\rangle = \frac{1 + 51 x + 14\sqrt{2} x^{3/2}}{4(8\pi x)^4}$$

Comparison with other methods



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$\Gamma_{\rm stat} = e^{-S_{\rm eff}}$. To leading order $\Gamma = e^{-S_3}$

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Radiative effects can be large:

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$\rightarrow$ Dimensionless variables

$(x = \frac{\lambda_3}{g_3^2}, \quad y = \frac{m_3^2}{\lambda_3^2})$

Observables

Nucleation mass/temperature: $S_{\rm eff}(x, y_N) = 126$

$\beta_N/H_N = \frac{d}{d\log T}S_{\rm eff}(x, y_N) \approx \frac{d}{d\log T}y \times \nabla_y S_{\rm eff}(x, y_N) \implies \tilde{\beta} \equiv \nabla_y S_{\rm eff}(x, y_N)$