Bubble wall dynamics at the electroweak phase transition

Andrea Guiggiani

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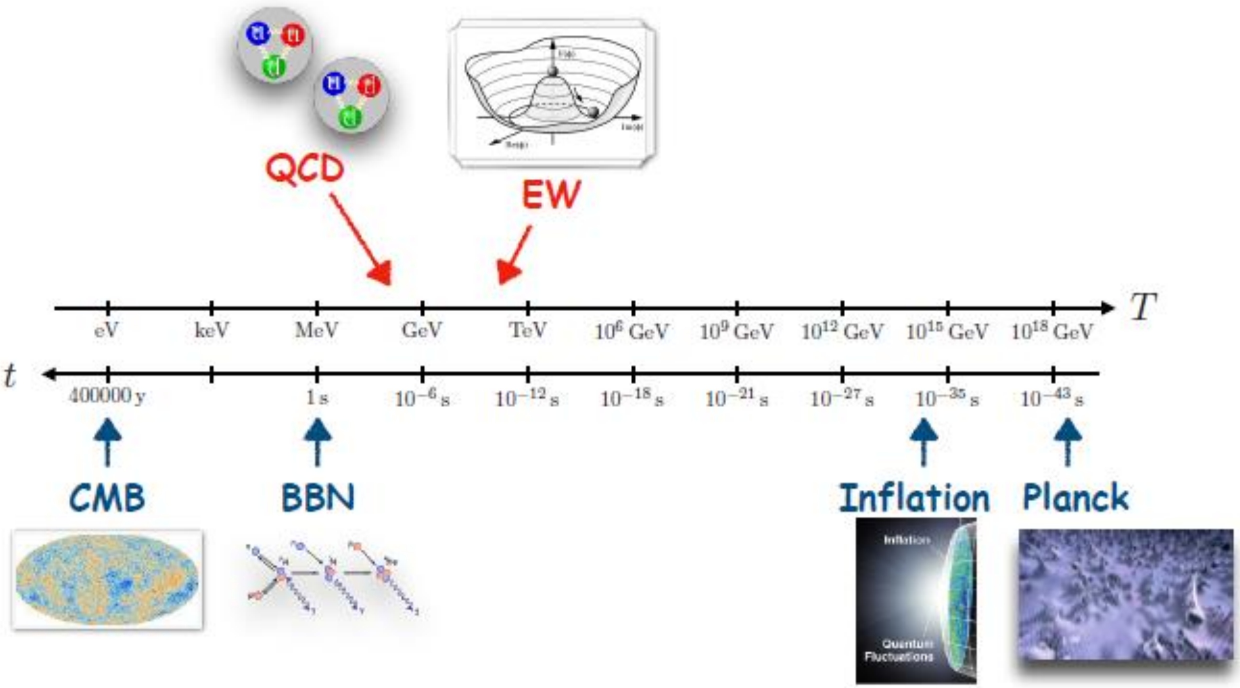
Based on: De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico [2201.08220]

Phase transitions in the SM

Phase trasitions are important events in the evolution of the Universe

- ➢ the SM predicts two of them (QCD confinement EW symmetry breaking)



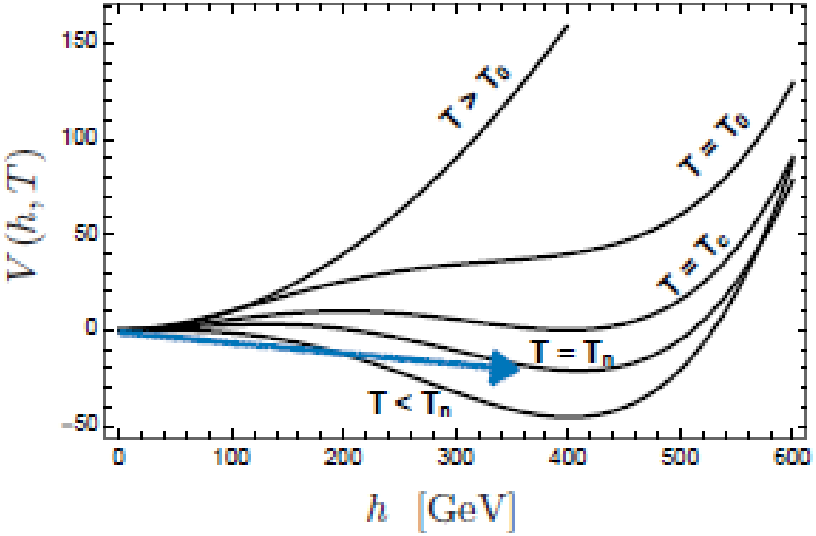
In the SM the QCD and EW PhTs are **extremely weak**

No distinctive experimental signatures and breaking of thermal equilibrium

First-order EWPhT

Several extensions of the SM predict a first-order EWPhT

- two **minima** are separated by a **barrier** *(two phases may coexist)*
- the field tunnels from false to true minimum at $T = T_n < T_c$
- the transition proceeds through bubble nucleation

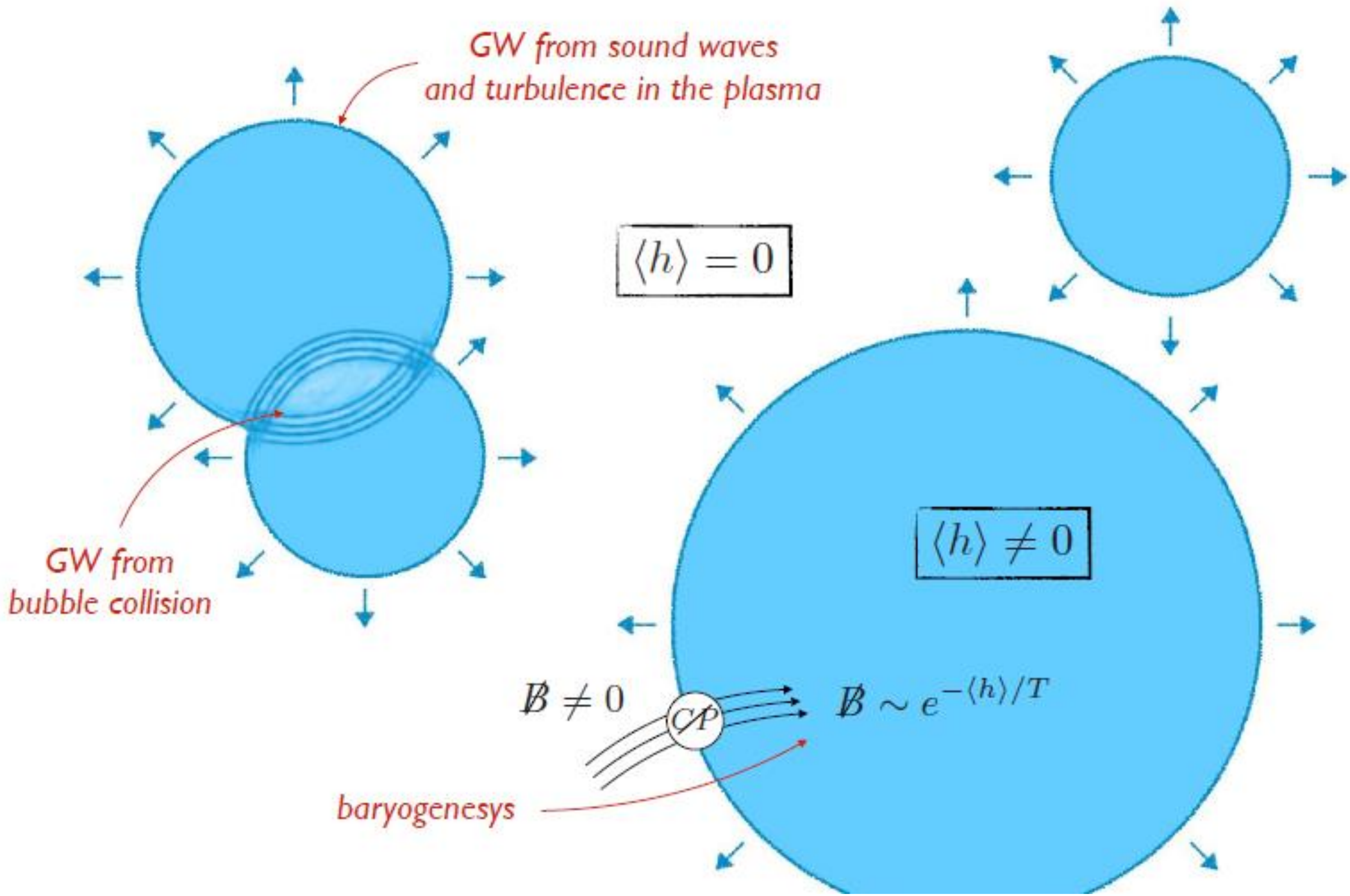


➢ interesting experimental signatures (*GW*)

➢ possible explanation of baryogenesis

Bubble nucleation

Bubble dynamics can produce **gravitational waves** and **baryogenesys**



Key features of a first-order phase transition

- the nucleation temperature T_n
- the strength α
- the (inverse) time duration of the transition β/H
- the speed of the bubble wall v_w
- the thickness of the bubble wall L_w

The parameters T_n, α, β/H are quite easy to compute
instead, v_w, L_w are much more challenging

Gravitational waves and the efficiency of the EW-baryogenesis crucially depend on them

EWBG thought to be efficient for slow moving walls. Recent results showed efficiency also
For fast moving walls

GWs are maximised for fast-moving walls

The dynamics of a bubble wall

Dynamics of a "simple" system

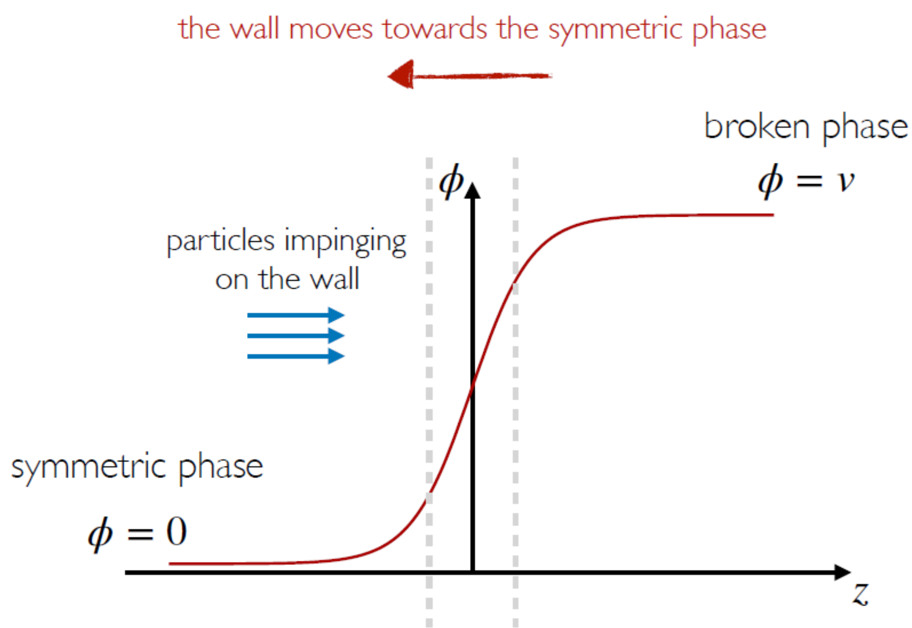
Scalar field + plasma

the wall front can reach a terminal velocity v_w if
the **pressure inside the bubble** balances the **friction of the plasma**

otherwise the bubbles never stop accelerating (run-away regime)

We assume **planar walls** and **steady state** (terminal velocity v_w reached)

Origin of the friction



- Friction arise from momentum transfer between bubble wall and particles
- Bubble wall movement brings the plasma out of equilibrium
- Fluctuations generate dissipation (Friction)

$$\phi' \Box \ \phi + \frac{dV_T}{dz} + \sum_i N_i \frac{dm_i^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E} \delta f_i(p,z) = 0$$

The effective Boltzmann equation

Assumptions on the plasma

- High temperature, weakly coupled plasma
- Higgs varying scale $L_w \gg q^{-1}$ inverse of momentum transfer in the plasma
- Only $2 \to 2$ processes in the plasma are considered
- Plasma made of two different species
  - *Top quark (main contributions)*
  - *All the other SM particles (background, assumed to be in equilibrium)*

Effective Boltzmann equation

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right) f \equiv \mathcal{L}[f] = -\mathcal{C}[f]$$

Collision operator

Goal: solve the Boltzmann equation to obtain the friction

Previous approaches to the solution

To deal with the collision term previous approaches made assumptions on the **shape** of the **perturbation**

- Fluid approximation [1]
- Extended fluid approximation [2]
- New formalism [3]

} Old formalism

[1] Moore, Prokopec, 1995
[2] Dorsch, Huber, Konstandin, 2021
[3] Laurent, Cline, 2020

- Neglection of the $\partial_{p_z}\delta f$ term
- Boltzmann equation integrated with a set of weights leading to a set of **differential equations**

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➢ Neglection of the $\partial_{p_z}\delta f$ term

➢ Boltzmann equation integrated with a set of weights leading to a set of **differential equations**

Recently a new method has been proposed

- Expansion using a polynomial basis [4]

[4] Laurent, Cline, 2022

Full solution to the Boltzmann equation

We propose an algorithm to solve the Boltzmann equation numerically **without** relying on any **ansatz** on the **shape** of δf

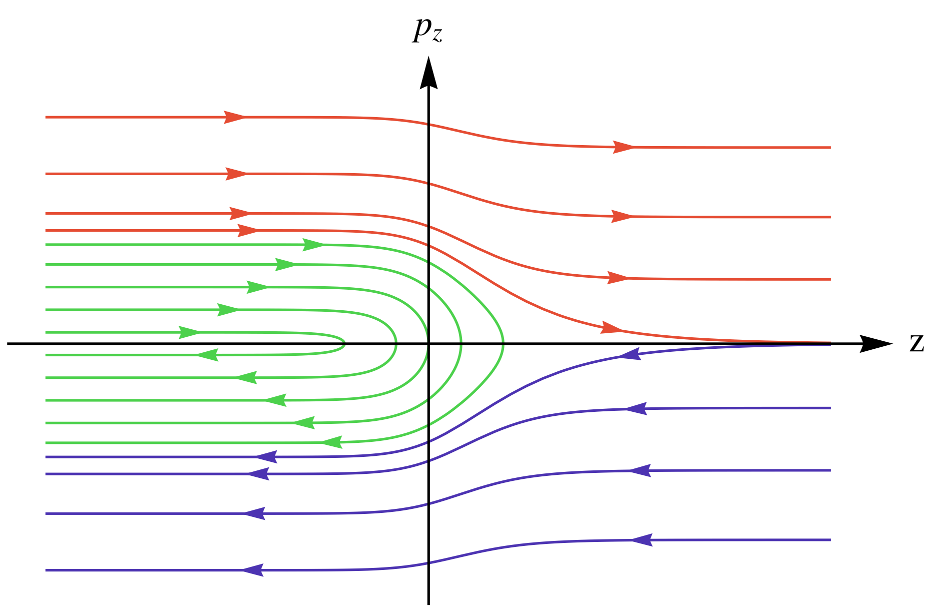
Key features

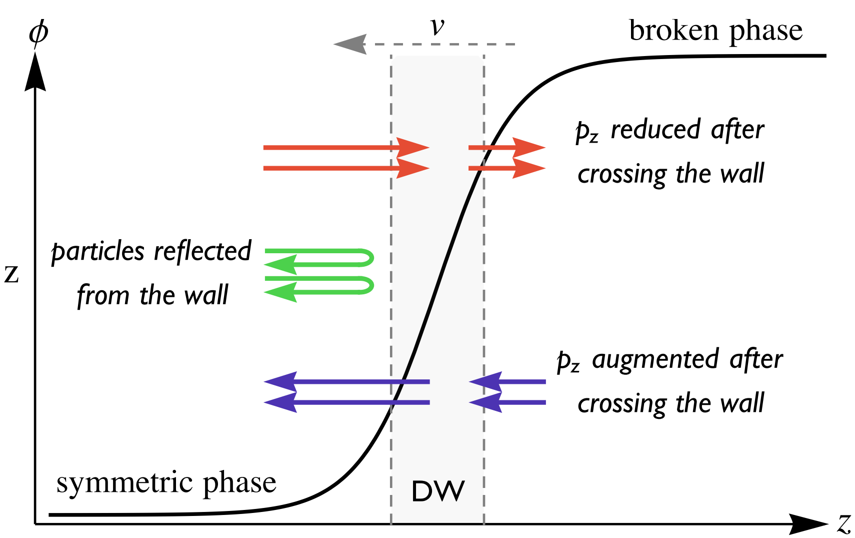
- Inclusion of **all terms** in the Boltzmann equation
- New approach to deal with collision integrals
- **Iterative** routine where convergence is achieved in **few steps**

Structure of the Liouville operator

Liouville operator is a derivative along flow paths

$$\mathcal{L}[f] = \left(\frac{p_z}{E}\partial_z - \frac{\left(m^2(z)\right)'}{2E}\partial_{p_z}\right) f \quad \longrightarrow \quad \frac{p_z}{E}\frac{df}{dz}$$





$E, p_\perp$ and $c = \sqrt{p_z^2 + m^2(z)}$ are **conserved** along the flow paths

Structure of the collision integrals

Linearized collision operator yields two terms

$$\overline{\mathcal{C}}[\delta f_i] = \mathcal{Q}\frac{\delta f}{f_v'(p)} + \left(\langle \delta f(k) \rangle - \langle \delta f(p') \rangle - \langle \delta f(k') \rangle\right)$$

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Kernels for processes

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- Reduce the integrations involving δf
- Performed once, increasing timing performances
- Depends on Lorentz invariants

General structure of the Boltzmann equation

$$\frac{d}{dz}\delta f - \frac{Q}{p_z}\frac{\delta f}{f_v'} = \mathcal{S}$$

$$f_v = \frac{1}{e^{\gamma(E - v\,p_z)} + 1}$$

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Iterative procedure

- Initial guess on the solution
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$$\frac{d}{dz}\delta f_n - \frac{Q}{p_z}\frac{\delta f_n}{f_v'} = \mathcal{S}_n \qquad \mathcal{S}_n = \frac{(m^2)'}{2p_z}\partial_{p_z} f_v + \left(\langle \delta f_{n-1}(k)\rangle - \langle \delta f_{n-1}(p')\rangle - \langle \delta f_{n-1}(k')\rangle\right)$$

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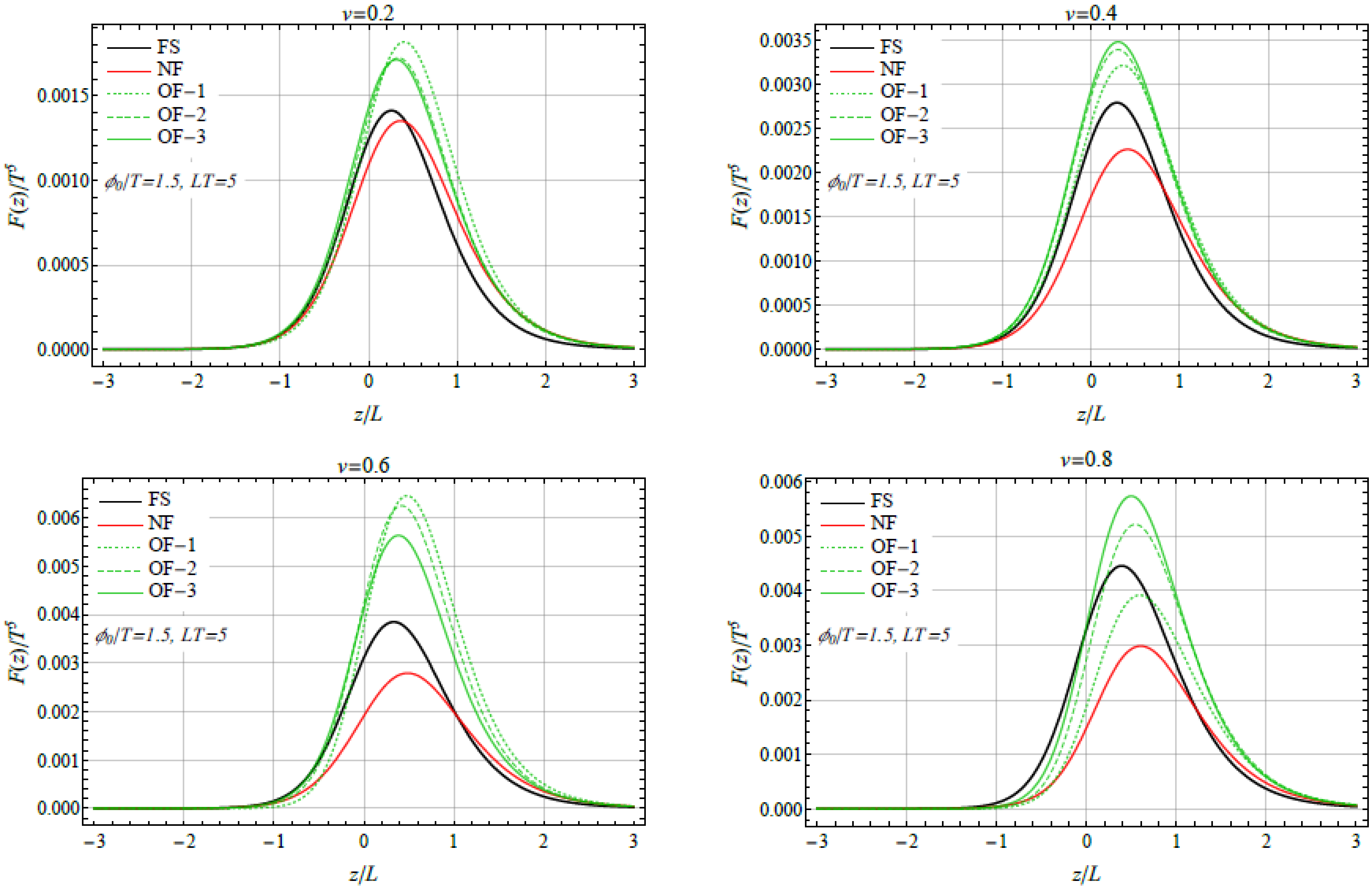
- Stop when $\sim 1\%$ convergence is reached

De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico, 2022

Friction results



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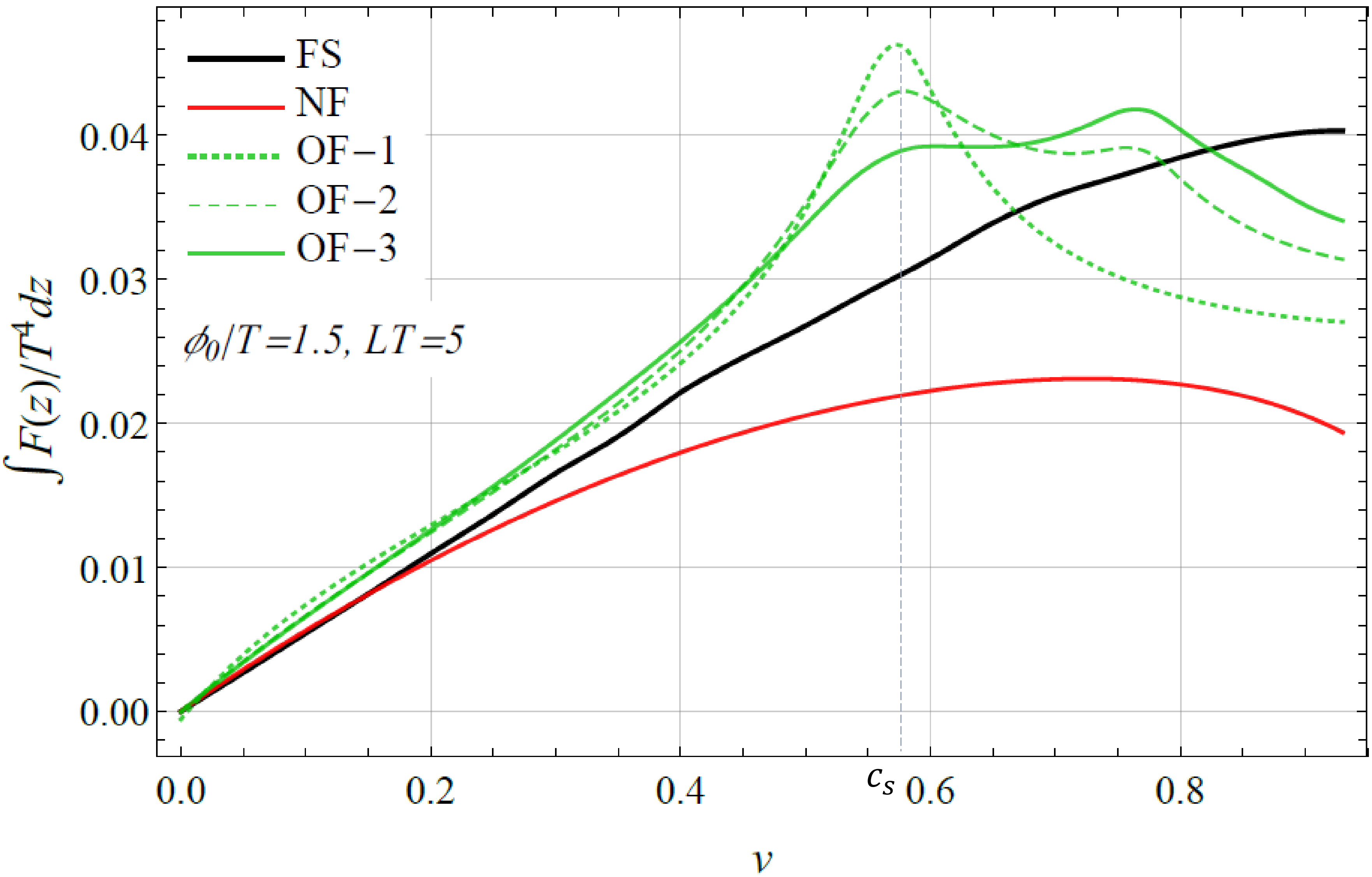
Integrated friction results



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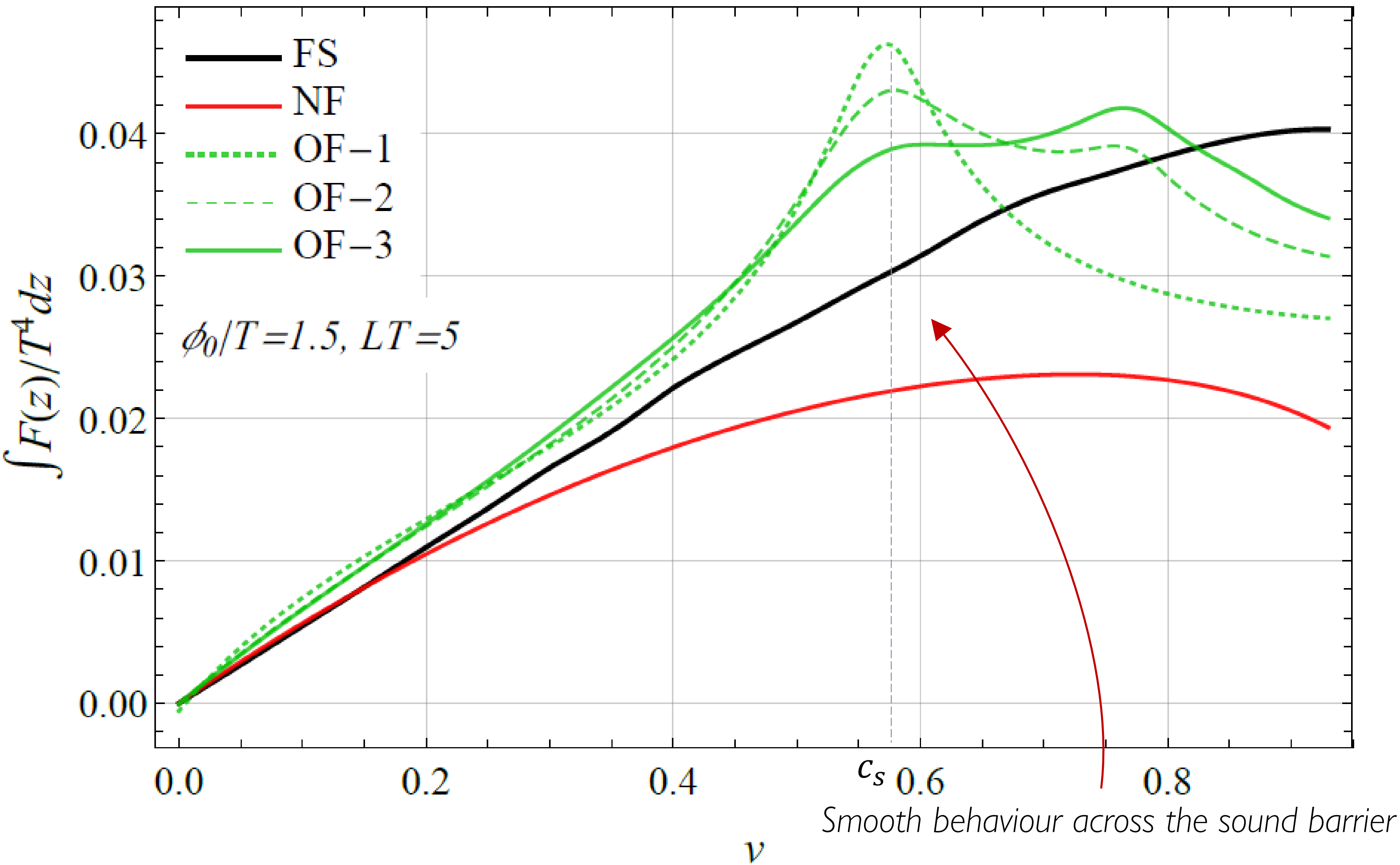

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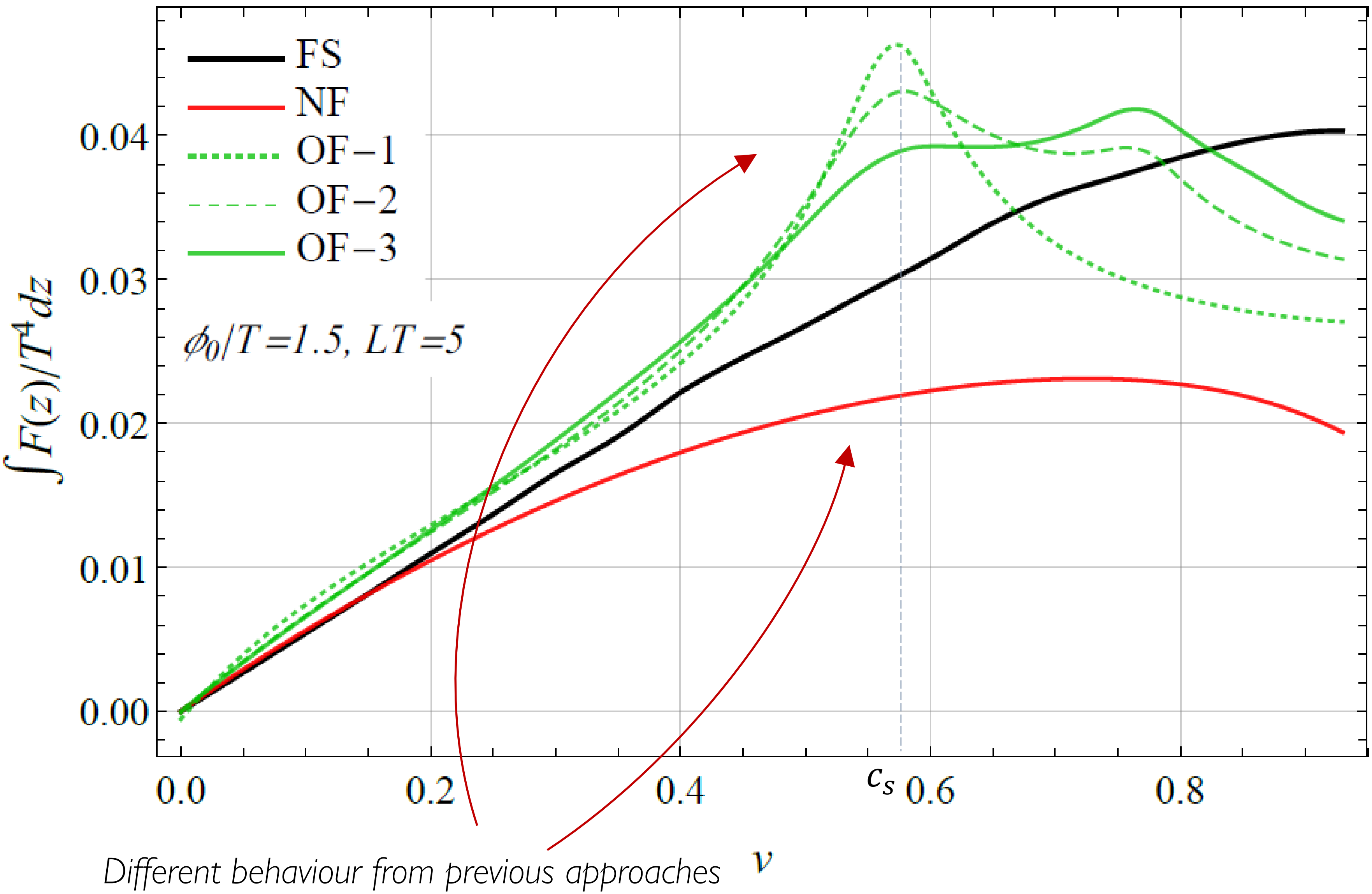

Conclusions and outlook

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- Inclusion of the background
- Inclusion of $1 \to 2$ and $2 \to 1$ processes

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Future perspectives:

- Inclusion of the W, Z bosons
- Inclusion of the background → Can be partially done as in [Cline, Laurent 2022]
- Inclusion of $1 \to 2$ and $2 \to 1$ processes