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String loop corrections and de Sitter vacua

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based on works with:
I. Antoniadis, Y. Chen 1803.08941, 1909.10525
I. Antoniadis, O. Lacombe 2007.10362, 2109.03243
Pramod Shukla 2203.03362

On the occasion of the retirement of

Prof. Ignatios Antoniadis

Reminiscences and Landmarks

▲ *Late 1980's at CERN:* acquaintance with Ignatios's work

▲ 1988: First collaboration

$SO(6) \times SO(4)$ *inspired from 4-d superstring.*

▲ c.1995 : *Ecole Polytechnique*

▲ c.2012 -2015 : *CERN*

F-theory models, Mordell-Weil $U(1)$*'s etc*

▲ 2018–: *LPTHE, Paris*

Moduli Stabilisation and String Inflation...

Lausanne 1988

Motivation

▲ **String Derived Effective Field Theories (EFT)** ▲

Basic problems that must be solved before considered as viable candidates for particel physics models

▲ **Compactifications** characterised by **large numbers** of **massless scalar fields** (moduli)

▲ Find CY compactification inducing an EFT with $V_{\text{min}} > 0$ and positive masses-squared for all moduli fields ⇒

⇒ *Moduli Stabilisation* ⇐

▲ Look for possible **Inflaton** candidates among *moduli*

▲ at **String Theory** level: ▲

▲ # CY of **Compactifications** and # fluxes ⇒
Enormous number of String Vacua

⇓

String Landscape

▲ **Long standing Question** ▲

▲ Are there any de Sitter vacua in the **Landscape**?

... Even if the answer is Yes... we know that they are...

⇒ *Certainly Scarce* ⇐

$\mathcal{F}$ocus of present $\mathcal{T}$alk

Context: type $\mathcal{IIB}$ theory:

- A solution to the Moduli Stabilisation problem

- Find a *de Sitter* vacuum in String Theory

... based only on **perturbative** correction

- If yes:

- Examine cosmological implications such as inflation.

★ Type II-B/F-theory

★ **Moduli** Space (*notation*)

▲ Graviton, dilaton and Kalb-Ramond (KR)-field

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

▲ Scalar, 2- and 4-index fields (*p-form potentials*)

$$\mathbf{C_0}, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

1. ▲ $C_0,\ \phi \to$ *combined to* **axion-dilaton** *modulus:*

$$S = C_0 + i\, e^{-\phi} \equiv C_0 + \frac{i}{g_s}$$

2. z_a : *Complex Structure (CS)* moduli (*shape*)
3. T_i : Kähler moduli (*size*)

Type II-B effective Supergravity

Basic ‘ingredients’:

Superpotential $\mathcal{W}$ *and Kähler potential* $\mathcal{K}$

▲ The **Superpotential** $\mathcal{W}$ ▲

▲ *Field strengths:*

$$F_p := d\,C_{p-1},\;\; H_3 := d\,B_2,\;\; \Rightarrow \mathbf{G_3} := F_3 - S\,H_3$$

▲ *Holomorphic* $(3,0)$*-form :* $\Omega(z_a)$

Flux-induced superpotential (*G. V. W. hep-th/9906070*):

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a)$$

▲ $\mathcal{W}$-Flatness conditions:

$$\mathcal{D}_{z_a}\mathcal{W} = 0, \quad \mathcal{D}_S\mathcal{W} = 0\;:$$

$\Rightarrow$ z_a *and* S **stabilised** $\Leftarrow$

but!

▲ Kähler moduli $\notin \mathcal{W}_0 \;\Rightarrow$ remain unfixed! ▲

▲ The Kähler potential ▲

$$\mathcal{K}_0 = -\sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(i \int \Omega \wedge \bar{\Omega}) \cdot$$

▲ The classical scalar potential ▲

$$V = e^{\mathcal{K}} (\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2) \equiv 0$$

$\Downarrow$

Kähler moduli completely **undetermined**!

due to flatness conditions and the no-scale structure

$\Downarrow$

Engineering the appropriate geometric set up and compute:

Kähler moduli-dependent QUANTUM corrections

The Kähler potential $\mathcal{K}$

and

$\mathcal{PERTURBATIVE}$

String Loop Corrections

▲ α'^3 **Corrections** ▲

Imply redefinition of 4-d dilaton (*Becker et al, hep-th:0204254*)

$$\begin{aligned} e^{-2\phi_4} &= e^{-2\phi_{10}}(\mathcal{V}+\xi) \\ &= e^{-\frac{1}{2}\phi_{10}}\,(\hat{\mathcal{V}}+\hat{\xi})\ \ \text{(Einstein frame)} \end{aligned}$$

$\mathcal{V}(t^k)$ *6d-volume*, with t^k =ImT^k:

$$\begin{aligned} \hat{\mathcal{V}} &= \frac{1}{3!}\kappa_{ijk}\hat{t}^i\hat{t}^j\hat{t}^k \\ t^k &= -\mathrm{Im}(T^k) \equiv \hat{t}^k\, g_s^{1/2} \\ \xi &= -\frac{\zeta(3)}{4(2\pi)^3}\chi \equiv \hat{\xi}\, g_s^{3/2} \end{aligned}$$

Hence, $\hat{\xi}$ incorporated into Kähler potential through the **shift** $(\hat{\mathcal{V}}+\hat{\xi})$. *(to simplify, from now on we write* $\mathcal{V}$*.)*

▲▲

D7-branes and Logarithmic corrections

Two ingredients needed for log*-corrections:*

▲▲

$\mathcal{A}$) Intersecting **D7-brane configuration**:

D7s	Minkowski				Compact Dimensions					
	0	1	2	3	4	5	6	7	8	9
$D7_a$		*	*	*	*	*	*	*		
$D7_b$		*	*	*	*	*			*	*
$D7_c$		*	*	*			*	*	*	*

$\mathcal{B}$) **Higher derivative couplings in curvature**

(generated by multigraviton scattering)

(see hep-th/9704145; 9707013; 9707018)

Leading correction term in type II-B action:

proportional to the fourth power of curvature:

$$\boxed{\propto R^4}$$

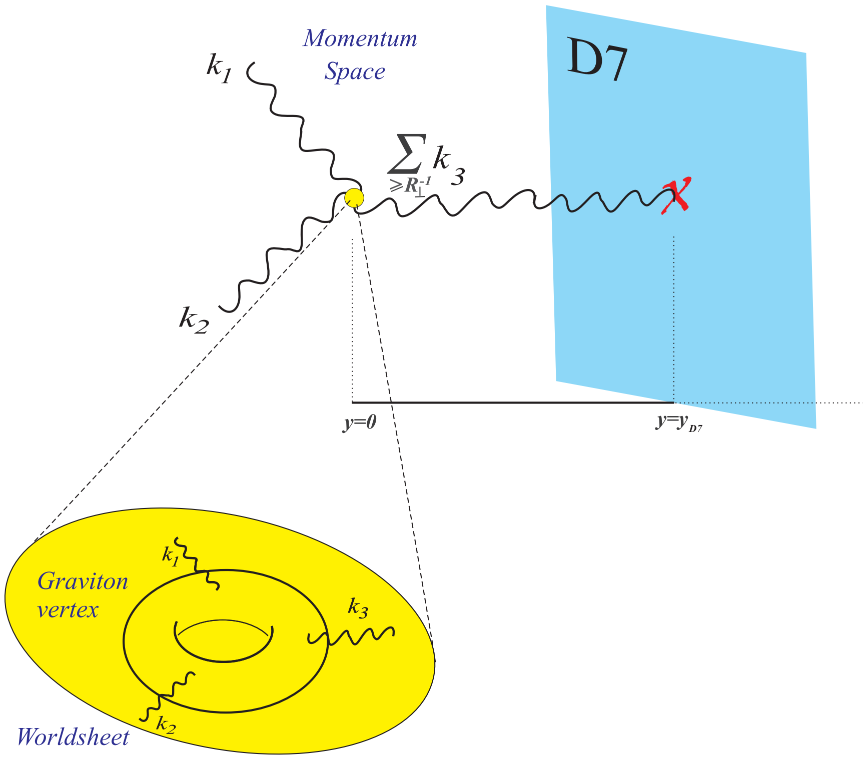
After reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) R^4 induces a **novel** Einstein-Hilbert term $\mathcal{R}_{(4)} \propto$ by the Euler characteristic χ:

$$\propto \quad \underbrace{\chi \int_{M_4} (\zeta(2) - \zeta(3) e^{-2\phi}) \mathcal{R}_{(4)}}_{induced\ \mathcal{EH}\ term},$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

▲▲ New $\mathcal{EH}$-term localised at points with $\chi \neq 0$▲▲

KK-exchange between graviton vertex and a $D7$-brane



Corrections

$$\propto \zeta(2)\chi \int_{M_4} (1 + \sum_{i=1,2,3} e^{2\phi}\mathcal{T}_i \log(R_\perp^i)\mathcal{R}_{(4)} \ ,$$

▲ $\mathcal{T}_i$: D7-brane tension ($= g_s T_0$)

▲ $R_\perp^i$: D7-transverse 2-dimension

Kähler potential :

$\mathcal{K} = -\log(-i(S - \bar{S})) - 2\log(\mathcal{V} + \xi/2 + \eta \log \mathcal{V}) + K_{cs}$

$$\eta = -\frac{1}{2} g_s T_0 \xi \ \ ; \ \ \xi = -\frac{\chi}{4} \times \begin{cases} \frac{\pi^2}{3} g_s^2 & \text{for orbifolds} \\ \zeta(3) & \text{for smooth CY} \end{cases} \tag{1}$$

Kähler moduli

STABILISATION

within a concrete Global Model:

(*GKL & Pramod Shukla* **2203.03362**)

Kreuzer-Skarke (KS) in hep-th/0002240 introduced toric methods to construct Calabi-Yau manifolds in terms of

Reflexive Polyhedra

...exploring the KS dataset ...⇒

Explicit CY_3 Manifold

$$h^{1,1} = 3,\ h^{2,1} = 115,\ \chi = -224$$

$$\hat{\xi} = -\frac{\chi\,\zeta[3]}{2(2\pi)^3\,{g_s}^{3/2}} = \frac{14\,\zeta[3]}{\pi^3\,{g_s}^{3/2}}, \qquad \frac{\hat{\xi}}{\hat{\eta}} = -\frac{\zeta[3]}{\zeta[2]}\frac{1}{{g_s}^2}$$

Assuming a basis of smooth divisors D_1, D_2, D_3, the case under consideration gives intersection polynomial with only one non-zero intersection:

$$I_3 = 2D_1D_2D_3$$

with Kähler form $J = 2\sum_{k=1}^{3} t^k D_k$, and 6d-volume:

$$\mathcal{V} = 2\,t^1t^2t^3 = \frac{1}{\sqrt{2}}\sqrt{\tau_1\tau_2\tau_3}$$

($t^i \to$ 2-cycle, $\tau_i \to$ 4-cycle moduli, subject to $\tau_i = 2\,t^jt^k$)

▲ Kähler potential including α' and loop corrections:

$$\mathcal{K}(T_i, S, z_a) = -\log\{-i(S-\bar{S})\} - 2\ln\mathcal{U} + K_{cs}(z_a) \qquad (2)$$

$$\mathcal{U}(T_i, S) = \mathcal{V} + \frac{\hat{\xi}}{2} + \mathcal{U}_1 \qquad (3)$$

with $\mathcal{U}_1(S, T^i)$ incorporating any loop corrections.

Computation of V_F requires inverse Kähler metric (basis S, T^i, z^a)

$$\mathcal{K}^{A\bar{B}} = \begin{pmatrix} \tilde{\mathcal{P}}_1 & k_\alpha \tilde{\mathcal{P}}_2 & \mathcal{O} \\ k_\alpha \tilde{\mathcal{P}}_2 & k_\alpha\, k_\beta\, \tilde{\mathcal{P}}_3 - k_{\alpha\beta}\, \tilde{\mathcal{P}}_4 & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & K_{cs}^{i\bar{j}} \end{pmatrix} \qquad (4)$$

$\Rightarrow$ *block-diagonal for $K_{cs}^{i\bar{j}}$ but S and T_i ($\mathcal{V}$) mix :* $\tilde{\mathcal{P}}_I = \tilde{\mathcal{P}}_I(\mathcal{V}, S)$.

Master formula for F-term potential $(generic\, \mathcal{U}_1)$

$$\Downarrow$$

$$\boxed{V_{\alpha'+\log} = e^{\mathcal{K}}\left(\frac{3\mathcal{V}}{2\mathcal{U}^2}\left(1+\frac{\partial\mathcal{U}_1}{\partial\mathcal{V}}\right)^2\frac{4\mathcal{V}^2+\mathcal{V}\hat{\xi}+4\hat{\xi}^2}{\mathcal{V}-\hat{\xi}}-3\right)|W_0|^2}$$

For α' and log corrections $\mathcal{U}_1 = -\hat{\eta} + \hat{\eta}\log\mathcal{V}$:

$$V_{\alpha'+\log} = 12 g_s e^{K_{cs}}|W_0|^2\,\underbrace{\hat{\xi}\frac{\mathcal{V}^2+7\hat{\xi}\mathcal{V}+\hat{\xi}^2}{\left(\mathcal{V}-\hat{\xi}\right)\left(2\mathcal{V}+\hat{\xi}\right)^4}}_{\alpha'^3-corrections}$$

$$-\frac{3\kappa}{2}|W_0|^2\underbrace{\frac{2\hat{\eta}-\hat{\eta}\log\mathcal{V}}{2\mathcal{V}^3}}_{logarithmic}+\cdots$$

Large Volume Limit

$$V_F \approx C\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta}\log(\mathcal{V})}{\mathcal{V}^3}$$

Properties

- Minimum exists for $\hat{\eta} < 0$.
- Stabilisation at **large** volume:

$$\mathcal{V}_{\min} = e^{\frac{7}{3} + \frac{\hat{\xi}}{2|\hat{\eta}|}} \sim e^{\frac{1}{g_s^2}}$$

- For F-term potential, AdS-minimum

$$(V_F)_{\min} \propto \frac{\eta}{\mathcal{V}^3} < 0$$

▲ **Uplift** to dS occurs through $\mathcal{D}$**-terms** (*see Lüst et al hep-th/0609211*) associated with universal $U(1)$'s of D7-stacks:

$$V_{\mathcal{D}} = \frac{g_{D7_i}^2}{2}\left(Q_i \partial_{T_i} K + \sum_j q_j |\Phi_j|^2\right)^2 , \quad \frac{1}{{g_{D7_i}}^2} = \mathrm{Re} T_i + \cdots$$

Minimising the total potential:

$$V_{\mathrm{eff}} = V_F + V_{\mathcal{D}}$$

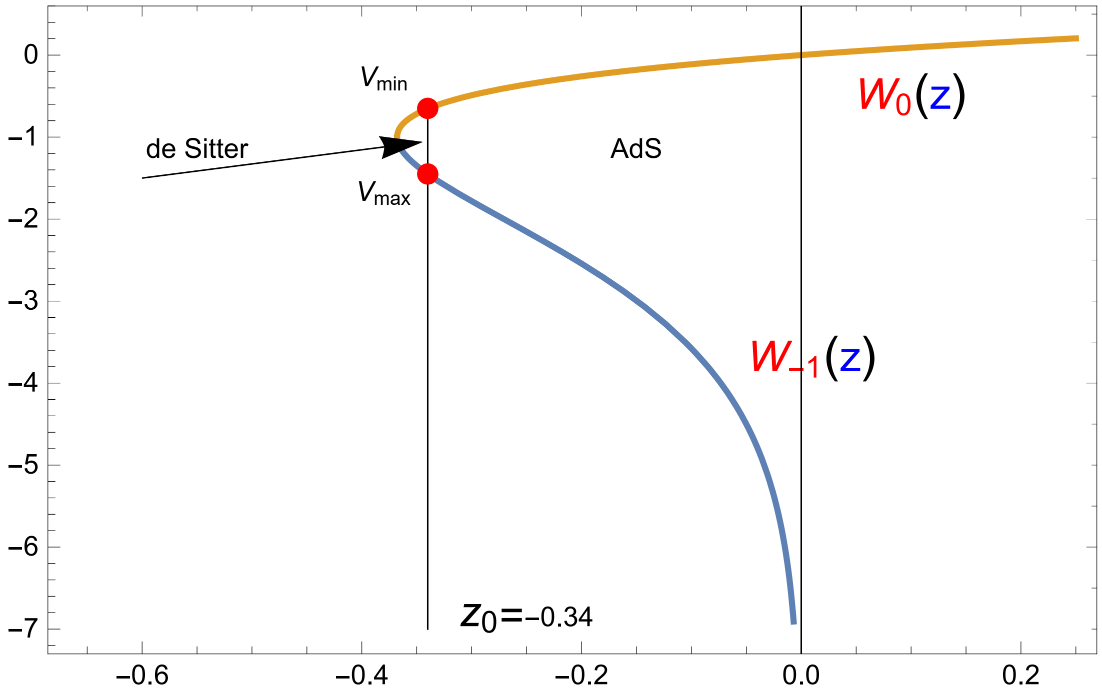
$\Rightarrow$ a minimum and a maximum defined by the double-valued **Lambert** W-function ($we^w = z$):

$$\boxed{\mathcal{V}_{\min} = \frac{\hat{n}}{d}\mathbf{W}_{\mathbf{0}/-\mathbf{1}}\left(\frac{d}{\hat{n}} e^{\frac{7}{3} - \frac{\hat{\xi}}{2\hat{n}}}\right)}$$

▲ de Sitter vacua ▲

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

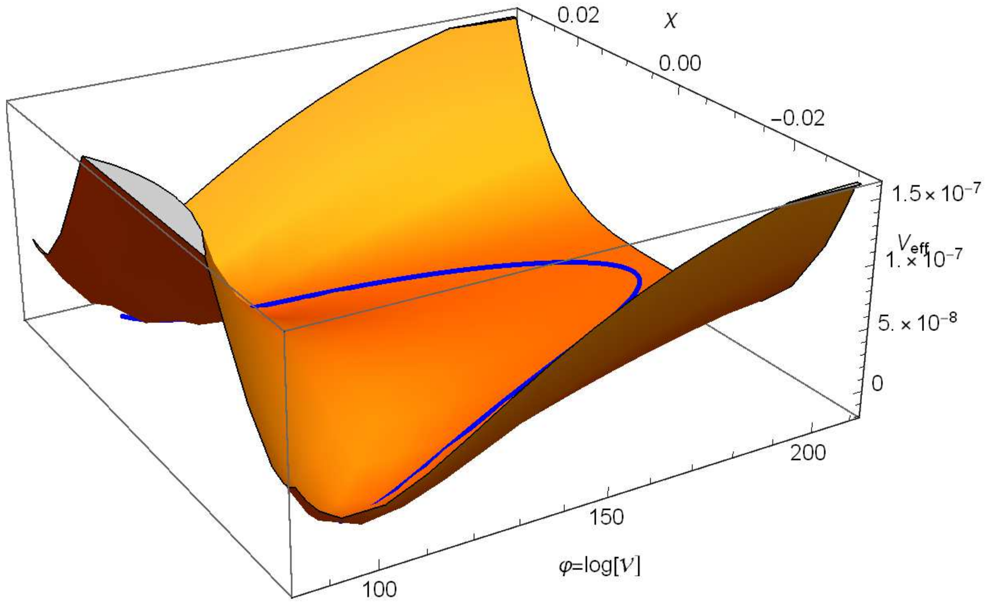
$$V_{\rm eff}^{min} = \frac{c}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0]$$



This dramatically constrains acceptable string vacua (fluxes etc)

Inflation

Hybrid scenario with open string states at D7-brane intersections playing the role of waterfall fields, (Antoniadis, Lacombe, GKL 2109.03243): $V_\chi \sim m^2(\mathcal{V})\chi^2 + \lambda(\mathcal{V})\chi^4$



▲ Blue curve: waterfall field χ trajectory (inflaton:$\phi \propto \log \mathcal{V}$)

★ Conclusions ★

★ *IIB/F-theory:*

• Stabilisation of Kähler Moduli possible with

Perturbative Corrections only:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \hat{\xi}/2 + \hat{\eta}\ln\mathcal{V}\right) + \cdots$$

Origin of log-corrections:

Induced Einstein-Hilbert terms from R^4-couplings in 10-d theory.

This $\mathcal{EH}$-term $\exists$ in $4d$ only!

$\Downarrow$

★ *induced* $\mathcal{EH}$–term ... indispensable element for a:

$4d$ de Sitter Universe

★ Hybrid Inflation with $\phi \sim \log\mathcal{V}$ role of the inflaton

★ $I\gamma\nu\alpha\tau\iota\epsilon$:

Thank you for being my friend and collaborator ★

APPENDIX

The analysis of the divisor topologies using *cohomCalg* shows that divisors are of $K3$ and SD types and can be represented by the following Hodge diamonds:

$$K3 \equiv \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}, \qquad \mathrm{SD} \equiv \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 27 & & 184 & & 27 \\ & 0 & & 0 & \\ & & 1 & & \end{array}.$$

Cancellation of all D7-charges

Introduce N_a D7-branes wrapped around divisors D_a and orientifold images D'_a (*0811.2936*)

$$\sum_k N_k\left([D_k + D'_k]\right) = 8[O7]$$

D7-branes and O7-planes also give rise to D3-tadpoles which receive contributions also from background 3-form fluxes

Assuming simple case:
$D7$-tadpoles are cancelled by placing 4 $D7 + D7'$-branes on top of $O7$-plane:

$$N_{D3} + \frac{1}{2} N_{\text{flux}} + N_{\text{gauge}} = \frac{1}{4}\left(O3 + \chi(O7)\right)$$

Example

Specific brane setting involving 2 stacks of $D7$-branes wrapping the divisors D_1, D_6 in the basis,

$$8[O7] = 4\left([D_1 + D_1']\right) + 4\left([D_6 + D_6']\right)$$

$D3$ tadpole condition

$$N_{D3} + \frac{1}{2} N_{flux} + N_{gauge} = 12$$

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