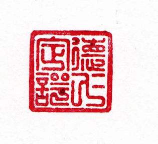
Variations on the phases of $\mathcal{N} = 2$

Jean-Pierre Derendinger

AEC, ITP, University of Bern

Ignatios Antoniadis Day

Planck 2022, Ecole Polytechnique, Paris, June 3rd., 2022



UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Ignatios, my timeline

- 1984, fall, in Stanford (?) (after Green-Schwarz, before the heterotic string . . .)
- 1990–1991, IA: large extra dimensions in strings . . .
  (long before the 1998 papers . . .)
- 1991: threshold corrections from superstrings

  effective supergravity by Ferrara, Kounnas, Zwirner, JPD
  confirmed in string theory by IA, Gava and Narain

  A great PASCOS conference at Northeastern, Boston, 1991
- 1995: The APT (Partouche, Taylor) model 1995
  IA: talk at CERN
- 1999: work with IA and Kounnas
  Temperature instabilities in $\mathcal{N} = 4$ strings

Ignatios, my timeline

- Starting ∼2006: a series of works and papers on $\mathcal{N} = 2$ global and local.

  With Petropoulos, Siampos, Tartaglino-Mazzuchelli, Farakos, Jiang, Maillard, Ambrosetti, Tziveloglou, Jacot, Markou

- *And, until 2020: six years at the Albert Einstein Center, University of Bern, where some of the $\mathcal{N} = 2$ researches have been performed.*

Over all these years, plenty of funny, interesting, intelligent hours with IA and friends (many from Greece)

This could well be what attracted me to work on $\mathcal{N} = 2$, which is now my subject . . .

IA can be very persuasive

$\mathcal{N} = 2$ with Ignatios et al.

- Nonlinear $\mathcal{N} = 2$ supersymmetry, effective actions and moduli stabilization (+ Maillard)
  Nonlinear supersymmetry, brane-bulk interactions and super-Higgs without gravity (+ Ambrosetti, Tziveloglou)

- The hypermultiplet with Heisenberg isometry in $\mathcal{N} = 2$ global and local supersymmetry (+ Ambrosetti, Tziveloglou)
  Heisenberg symmetry and hypermultiplet manifolds / Isometries, gaugings and $\mathcal{N} = 2$ supergravity decoupling (+ Petropoulos, Siampos)

- Nonlinear $\mathcal{N} = 2$ global supersymmetry (+ Markou)
  All partial breakings in $\mathcal{N} = 2$ supergravity with a single hypermultiplet (+ Petropoulos, Siampos)
  Magnetic deformation of super-Maxwell theory in supergravity (+ Jiang, Tartaglino-Mazzucchelli)

- $\mathcal{N} = 2$ supersymmetry breaking at two different scales (+ Jacot)

- New Fayet-Iliopoulos terms in $\mathcal{N} = 2$ supergravity (+ Farakos, Tartaglino-Mazzucchelli)

$\mathcal{N} = 2$: phases

Global supersymmetry: $\mathcal{N} = 2$ (unbroken)

- $\mathcal{N} = 0$, single scale (FI terms, Maxwell, ...).
- $\mathcal{N} = 1$, partial breaking, Maxwell, APT model.
- $\mathcal{N} = 1$, hypermultiplet dual to single-tensor theory, ADM model.
- $\mathcal{N} = 1$, hypermultiplet and *two* Maxwell. (Partouche, Pioline)
- $\mathcal{N} = 0$, two scales.

Supergravity: $\mathcal{N} = 2$, supergravity, Minkowski or Anti-de Sitter

- $\mathcal{N} = 0$, single scale (dS and .... ?).
- $\mathcal{N} = 1$, partial breaking to Minkowski:
  Minimal: one Maxwell, one hypermultiplet (FGP model) unique !
  Non-minimal: in principle possible, examples ?
- $\mathcal{N} = 1$, partial breaking to AdS: possible in general (one less condition).
- $\mathcal{N} = 0$: two (parametric) scales in Minkowski, FGP model.

Partial breaking

Partial breaking of susy had a poor start:

Two no-go claims, for global and local supersymmetry

- Global: simply wrong. Superalgebra based, ill-defined Noether (super)charges, an irrelevant notion of vacuum energy
  (classical and quantum mechanics, QFT, only know about energy *differences*)
  Disproved at the current algebra level (before $\int d^3x$ to charges)
  (Polchinski + Hugues, Liu)

- Local: the no-go claim is correct (Cecotti, Girardello, Porrati, 84-86)
  But too strong restrictions (use of e.-m. duality not fully understood in 86)

Few really cared, phenomenology likes chiral fermion representations and then $\mathcal{N} = 1$ or 0.

But string compactifications / branes / fluxes care.

Breaking susy: spontaneous ?

- Gauge theories: spontaneous $\iff$ vacuum degeneracy
  Induced by the ground state: scalar vev's (orbit of $\langle \phi_G \rangle$)

  $\delta\,\phi_G = C\alpha + \text{linear}$ connects equivalent vacua, ϕ_G massless

  The *scalar potential* is gauge invariant.

- Supersymmetry: the *action* is invariant.

- Then for susy: one or several goldstinos in the ground state:

  $\boldsymbol{\delta\,\psi_G = C\epsilon + \text{linear}}$ (ψ_G massless $C\epsilon$: fermion shift)

  Nonlinear realizations
  obtained by deformations of the linear theory (invariant dynamics, same degrees of freedom (for off-shell representations)

  Or with less components (constrained multiplets):
  Volkov-Akulov, DBI, ...

Partial breaking of $\mathcal{N} = 2$ supergravity

- The FGP model: (Ferrara, Girardello, Porrati, 1996)
  Field content: supergravity, a Maxwell multiplet, a single hypermultiplet on $SO(4,1)/SO(4)$. Minkowski ground state with $\mathcal{N} = 1$.

- No other example worked out with one hypermultiplet.

- *Seems* relatively common with several hypermultiplets, but explicit examples hard to find (*any published example ?*)

  (Needed: quaternion-Kähler metrics in $4n_H \geq 8$ dimensions ??)
  (Louis, Smyth, Triendl, 2009-2010)

- We decided to explicitly find and to classify all $\mathcal{N} = 2$ supergravity theories with a single hypermultiplet admitting vacua with $\mathcal{N} = 1$ supersymmetry in Minkowski space-time ...

- ... we found that the FGP model is indeed unique
  (Antoniadis, JPD, Petropoulos, Siampos, 2018)

Partial breaking, supergravity with one hypermultiplet

- Hypermultiplets of supergravity live on quaternion-Kähler spaces ($4n_H$ real dimensions). (Bagger, Witten)
- $n_H = 1$: Weyl self-dual. Generic metrics are known with
  - one isometry (Przanowski-Tod, PT) or
  - two commuting isometries (Calderbank-Pedersen, CP).

  Depend on solutions of differential equations, Toda for PT.
- $n_H > 1$: explicit metrics missing.
- Whenever CP coordinates exist, partial breaking is impossible.
- The $SO(4,1)/SO(4)$ FGP model with commuting translation symmetries has PT coordinates but does not admit CP coordinates. There is no other case: the FGP model is unique, with one hypermultiplet.
- In contradiction with the published claim of CP.

Calderbank-Pedersen

Coordinates: ρ, η, ψ, ϕ, two commuting shift isometries $\delta\psi = c$, $\delta\phi = d$.

For $F(\rho, \eta)$ such that $\dfrac{\partial^2 F}{\partial\rho^2} + \dfrac{\partial^2 F}{\partial\eta^2} = \dfrac{3F}{4\rho^2}$ quaternion-Kähler metric:

$$ds^2 = \frac{4\rho^2(F_\rho^2 + F_\eta^2) - F^2}{4F^2}\, d\ell^2 + \frac{((F - 2\rho F_\rho)\alpha - 2\rho F_\eta \beta)^2 + ((F + 2\rho F_\rho)\beta - 2\rho F_\eta \alpha)^2}{F^2(4\rho^2(F_\rho^2 + F_\eta^2) - F^2)}$$

$$\alpha = \sqrt{\rho}\, d\varphi \qquad \beta = (d\psi + \eta d\varphi)/\sqrt{\rho} \qquad d\ell^2 = \rho^{-2}(d\rho^2 + d\eta^2)$$

Calderbank-Pedersen: (arXiv:math/0105253)

"Any selfdual Einstein metric of nonzero scalar curvature with two linearly independent commuting Killing fields arises locally in this way (i.e., in a neighbourhood of any point, it is of the form (1.1) up to a constant multiple)."

Global $\mathcal{N} = 2$, partial breaking, single-tensor multiplet

The APT model: (Antoniadis, Partouche, Taylor, 1996)

- A class of $\mathcal{N} = 2$ theories broken into $\mathcal{N} = 1$ with a single Maxwell multiplet in global susy. Depends on a holomorphic function $F(X)$, non canonical $F_{XXX} \neq 0$. Goldstino partner of the massless photon field. A chiral multiplet with mass $\sim \langle F_{XXX} \rangle$
  (APT model: Antoniadis, Partouche, Taylor, 1996)

The ADM model: (Antoniadis, JPD, Markou, 2017!)

- A class of $\mathcal{N} = 2$ theories broken into $\mathcal{N} = 1$ with a single single-tensor multiplet (dual to a hypermultiplet) in global susy. Depends on a holomorphic function $W(\Phi)$, non canonical $W_{\Phi\Phi} \neq 0$. Goldstino partner of the $B_{\mu\nu}$ gauge field. A chiral multiplet with mass $\sim \langle W_{\Phi\Phi} \rangle$
  (ADM model: Antoniadis, JPD, Markou, 2017 !)

ADM model

Use a single-tensor $\mathcal{N} = 2$ multiplet

- Hypermultiplet with a (translational) isometry
  - $\Rightarrow$ dualize the axionic scalar to $B_{\mu\nu}$
  - $\Rightarrow$ single-tensor multiplet with $B_{\mu\nu}$ (gauge field, 1_B on-shell)
- An off-shell representation, $8_B + 8_F$
  (bosons: $B_{\mu\nu}$, a real $SU(2)$ triplet of propagating scalars, a complex auxiliary scalar)
- $\mathcal{N} = 1$ superfields: L (real linear) or $\overline{D}_{\dot{\alpha}}L$ (chiral spinor), Φ (chiral)
- Compare with Maxwell $\mathcal{N} = 1$ superfields: W_α and X.

- $16_B + 16_F$ version: $\mathcal{N} = 1$ superfields χ_α, Φ and Y:
  χ_α as prepotential of $L = D\chi - \overline{D}\overline{\chi}$
- Compare with Maxwell $W_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V_2$ and $X = \frac{1}{2}\overline{DD}V_1$.

Single-tensor partial breaking

Analogy with Maxwell (and APT): (Note change of chirality)

$\overline{D}_{\dot\alpha} L$	$\Longleftrightarrow$	W_α	Φ	$\Longleftrightarrow$	X	chiral
$W(\Phi)$	$\Longleftrightarrow$	$F_X(X)$	F(X) prepotential			
$\widetilde{M}^2\, W$	$\Longleftrightarrow$	$M^2\, F_X$	magnetic prepotential / FI term			

In both cases, the deformation parameter $\widetilde{M}^2$ or M^2 CANNOT be produced by the shift of an auxiliary field: this would destroy the partial breaking. An intrinsic deformation of the linear representation.

Not a “pure" spontaneous breaking induced by scalar vev’s. It is induced by the deformation.

(Scalars actually used to protect the unbroken susy)

ADM lagrangian

$$\mathcal{L} = \int d^2\theta \left[\frac{i}{2} W_\Phi \,(\overline{D}L)(\overline{D}L) - \frac{i}{4} W \,\overline{DD}\,\overline{\Phi} + \widetilde{m}^2\Phi + \widetilde{M}^2\, W\right] + \text{h.c.}$$

$$= i\int d^2\theta d^2\overline{\theta}\left[-L^2(W_\Phi - \overline{W}_{\overline{\Phi}}) + \overline{\Phi}W - \Phi\overline{W}\right] \qquad \Longleftarrow \text{Laplace}$$

$$+\int d^2\theta\left[\widetilde{m}^2\Phi + \widetilde{M}^2\, W\right] + \text{h.c.}$$

The second supersymmetry is deformed: Goldstino: $\overline{D}_{\dot\alpha} L|_{\theta=0}$

$$\delta^*\, L \;=\; \delta^*_{nl}\, L - \frac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi}) \qquad \delta^*_{nl}\, L \;=\; \sqrt{2}\,\widetilde{M}^2\,(\overline{\theta\eta} + \theta\eta)$$

$\delta^*\,\Phi$ unchanged $\qquad \delta^*_{nl}\,\overline{D}_{\dot\alpha} L \;=\; -\sqrt{2}\,\widetilde{M}^2\,\overline{\eta}_{\dot\alpha}$

L: massless ($B_{\mu\nu}$ gauge field) Φ: mass $\sim \langle W_{\Phi\Phi}\rangle$

$\langle W_{\Phi\Phi}\rangle \to \infty$: constrained single-tensor multiplet (Bagger-Galperin), next . . .

DBI – dilaton, the problem

- Classes of superstring compactifications with $\mathcal{N} = 1$ have a universal sector with $\mathcal{N} = 2$ properties.
- Bulk has $\mathcal{N} = 2$ supersymmetry. Dilaton in the universal hypermultiplet, or dual version with tensor(s).
- Gauge fields located on D–branes, with Dirac-Born-Infeld (DBI) lagrangian coupled to the dilaton (and SUSY partners).
  Breaking 1/2 SUSY.
- Expect: linear $\mathcal{N} = 1$, gauge fields with nonlinear second SUSY.
  Partial breaking of linear supersymmetry $\mathcal{N} = 2 \implies \mathcal{N} = 1$
  Goldstino in $\mathcal{N} = 1$ Maxwell multiplet.
- *How does the dilaton from a hypermultiplet enter Maxwell kinetic terms ?*
  “Factorization theorem of (linear) $\mathcal{N} = 2$":
  Maxwell fields do not interact with hypermultiplet fields.

DBI – dilaton: kinetic lagrangians

$\mathcal{N} = 2$ Maxwell theory:

$$\mathcal{L}_{Max.} = \frac{1}{4}\int d^2\theta \left[\mathcal{F}''(X)WW - \frac{1}{2}\mathcal{F}'(X)\overline{DD}\,\overline{X}\right] + \text{c.c.} + \mathcal{L}_{F.I.}$$

$\mathcal{F}(X)$: holomorphic prepotential

Canonical: $F(X) = X^2/2$

Single-tensor kinetic term:

$$\mathcal{L}_{s.-t.} = \int d^2\theta d^2\overline{\theta}\,\mathcal{H}(L, \Phi, \overline{\Phi})$$

$$\left(\frac{\partial^2}{\partial L^2} + 2\frac{\partial^2}{\partial\Phi\partial\overline{\Phi}}\right)\mathcal{H} = 0$$

(Lindström, Roček, 1983)

DBI – dilaton: Chern-Simons interaction

$B \wedge F$ interaction:

$$\mathcal{L}_{CS} = -g \int d^2\theta d^2\overline{\theta} \left[LV_2 + (\Phi + \overline{\Phi})V_1 \right]$$

$$\mathcal{L}_{CS,\chi} = g \int d^2\theta \left[\chi^\alpha W_\alpha + \tfrac{1}{2}\Phi X \right] + g \int d^2\overline{\theta} \left[-\overline{\chi}_{\dot\alpha} \overline{W}^{\dot\alpha} + \tfrac{1}{2}\overline{\Phi X} \right]$$

$L = D^\alpha \chi_\alpha - \overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}$, gauge invariance: $\delta\chi_\alpha = -\frac{i}{4}\overline{D}\overline{D}D_\alpha\Lambda$

$L = \frac{1}{2}\theta\sigma^\mu\overline{\theta}\,\epsilon_{\mu\nu\rho\sigma}\partial^{[\nu}b^{\rho\sigma]} + \dots$ $\qquad 4_B + 4_F$

$\chi_\alpha = \dots - \frac{1}{4}\theta_\alpha C + \frac{1}{2}(\theta\sigma^\mu\overline{\sigma}^\nu)_\alpha\, b_{\mu\nu} + \dots$ $\qquad 8_B + 8_F$

The contraint, $\mathcal{N}=2 \to \mathcal{N}=1$, DBI theory

- $\mathcal{N}=1$: Martin Roček showed in 1978 that the Volkov-Akulov theory follows from the constraint $\Phi^2 = 0$ applied to a chiral $\mathcal{N}=1$ superfield Φ, provided that in Φ $\langle f \rangle \neq 0$ (source of supersymmetry breaking).

  The Goldstino is ψ_α in Φ.

- Extended to partial breaking $\mathcal{N}=2 \longrightarrow \mathcal{N}=1$ by Roček and Tseytlin, Bagger and Galperin, ...

- Consider a $\mathcal{N}=2$ Maxwell multiplet with $\mathcal{N}=1$ superfields W_α and X. Impose the constraint:

$$WW - \frac{1}{2} X \overline{DDX} = \frac{1}{\kappa} X$$

  Left-hand side: $\mathcal{N}=2$ invariance for solutions.
  Deform $\delta^* W$ to match the variation of the right-hand side.
  The Goldstino is the gaugino in W.

$\mathcal{N} = 2$ DBI

Solution:

$$X(WW) = \kappa W^2 - \kappa^3 \overline{DD} \left[\frac{W^2 \overline{W^2}}{1 + A + \sqrt{1 + 2A + B^2}} \right]$$

where

$$A = \frac{\kappa^2}{2}(DD\, W^2 + \overline{DD}\, \overline{W}^2) = A^*, \quad B = \frac{\kappa^2}{2}(DD\, W^2 - \overline{DD}\, \overline{W}^2) = -B^*$$

The constraint deforms the second SUSY variation of W_α:

$$\delta^*_{deformed} W_\alpha = \sqrt{2}\, i \left[\frac{1}{2\kappa}\eta_\alpha + \frac{1}{4}\eta_\alpha \overline{DD}\, \overline{X} + i(\sigma^\mu \overline{\eta})_\alpha\, \partial_\mu X \right]$$

Goldstino chiral superfield, $\overline{D}_{\dot\alpha} \Lambda_\alpha = 0$:

$$\Lambda_\alpha = -\frac{\sqrt{2} i W_\alpha}{1 + \frac{\kappa}{2}\overline{DDX}} \qquad \delta^* \Lambda_\alpha = \frac{1}{\kappa}\eta_\alpha + 2i\kappa(\Lambda \sigma^\mu \overline{\eta})\partial_\mu \Lambda_\alpha$$

DBI – dilaton lagrangian

Lagrangian for the coupled Maxwell DBI – single-tensor system

$$\mathcal{L} = g \int d^2\theta \left[\frac{1}{2} \Phi X(WW) + \chi^\alpha W_\alpha - \frac{i}{2\kappa} Y \right] + \text{c.c.}$$

supplemented by the kinetic term of the single-tensor multiplet.

- Describes a massive $\mathcal{N} = 1$ vector multiplet (because of the $b \wedge F$ interaction).
- The Goldstino has been "Higgsed" in the massive Dirac gaugino.
- This super-Higgs mechanism is made possible by the four-form field. . . .

Maxwell – dilaton interaction induced by (susy) $b \wedge F$

Terms in the bosonic action

A (semi-positive) scalar potential:

$$V(C, \mathrm{Re}\,\Phi) = \frac{2g\,\mathrm{Re}\,\Phi - \xi_1}{8\kappa}\left[\sqrt{1 + \frac{2g^2C^2}{(2g\,\mathrm{Re}\,\Phi - \xi_1)^2}} - 1\right]$$

Minimum at $C = 0$, for $\mathrm{Re}\,\Phi$ arbitrary (flat direction at $C = 0$), and with $V = 0$: linear $\mathcal{N} = 1$, nonlinear second susy.

Antisymmetric tensor terms:

$$g\epsilon^{\mu\nu\rho\sigma}\left(\frac{\kappa}{4}\,\mathrm{Im}\,\Phi F_{\mu\nu}F_{\rho\sigma} - \frac{1}{4}b_{\mu\nu}F_{\rho\sigma} + \frac{1}{24\kappa}C_{\mu\nu\rho\sigma}\right)$$

include a linear term in the four form-field.
(As in ten dimensions, RR–brane coupling $\sim \sum_k C_k \wedge e^F$).

Essential for $\mathcal{N} = 2$, not consistent alone: more terms from other sources (tadpole cancellation) required.

Last

My best wishes to Ignatios,

for many more ideas, intelligence, lucidity,

friendly and intense collaborations . . .

and, above all, a sweet life

And many thanks, it's already almost forty years