2010.00317

bottom-up EFT for $\mu \leftrightarrow e$

Sacha Davidson (+...)
IN2P3/CNRS, France

1. this is a talk about BSM chez les leptons (puzzling m_ν/flav. diag. anomalies...)
   LFV = flavour-changing contact interactions of charged leptons (FCNC for charged leptons)

2. LFV should occur : no symmetries forbid it, if add m_ν to SM.
   ... but we have not seen it (not know rates)

3. ν expts and LFV give complementary info about leptonic NP
   ...so...by measuring LFV rates, what can one learn about leptonic NP?
   Ideally: reconstruct the NP Lagrangian. But how close can we really get?

What we know/what we can learn

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$ (SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(16\to?)}$ (Mu2e,COMET)
		$10^{-(18\to?)}$ (PRISM/PRIME/ENIGMA)
$K^+ \to \pi^+ \bar{\mu} e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
...		
$B^+ \to \bar{\mu}\nu$	$< 1.0 \times 10^{-6}$ (Belle)	$\sim 10^{-7}$ (BelleII)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to \ell\{\pi, \rho, \phi, K, ...\}$	$\lesssim \text{few} \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to ...$		
$h \to \tau^\pm \ell^\mp$	$< 1.5, 2.2 \times 10^{-3}$ (ATLAS/CMS)	
$h \to \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ (ATLAS)	

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

highlights of that table (for this talk)

1. restrictive bounds on three $\mu \to e$ processes, with

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \lesssim 10^{-12} \Rightarrow \Lambda_{\rm NP} \gtrsim 10^3 \langle v \rangle \sim 100 \text{ TeV}$$

   upcoming expts aim for BR $\sim 10^{-16} \Rightarrow \Lambda_{\rm NP} \sim \text{PeV}$.
   promising for discovery of LFV?

2. bounds on a multitude of $\tau \to \{e, \mu\}$ processes

$$\frac{\Gamma(\tau \to 3e)}{\Gamma(\tau \to e\bar{\nu}\nu)} \times .2 \lesssim \text{few} \times 10^{-8} \Rightarrow \Lambda_{\rm NP} \gtrsim 55 \langle v \rangle \gtrsim 10 \text{ TeV}$$

   BelleII will improve sensitivities to $\sim 10^{-9}$.
   promising for identification of LFV NP: complementary observables allow to constrain most/all SMEFT coefficients. Provided NP not to heavy.

3. (bds on lepton and quark FC interactions: independent info)

4. (heavy particle LFV decays: independent info)

Outline

1. 3 processes at low energy ($\sim m_\mu$): $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu A \to eA$

2. parametrise in EFT$\Rightarrow$ 2 questions:

   (a) will we see $\mu \to e$ flavour change if its there?
   (3 processes $\approx$ 12 constraints vs $\sim$ 90 $\mu \to e$ operators with $\leq$ 4 legs below m_W)

   (b) what can we learn if we see it?

3. (assume NP heavy) use EFT to include SM loops between $\Lambda_{expt} \to \Lambda_{NP}$

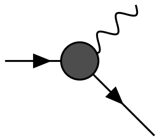
4. find that:

   (a) (almost) all the 90 coefficients contribute to at least one of the processes, suppressed at most by 10^{-3}.

   (b) a recipe to study this: use observable-motivated basis for the constrainable subspace ($\perp$ to "flat directions")

parametrising $\mu \to e\gamma$ and $\mu \to e\bar{e}e$

Two dipole operators constrained by $\mu \to e\gamma$:

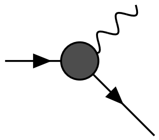


$$
\begin{aligned}
\delta\mathcal{L}_{meg} &= \frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,L}\overline{\mu_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,R}\overline{\mu_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta} \right) \\
BR(\mu \to e\gamma) &= 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.2 \times 10^{-13} \\
&\Rightarrow |C_{D,X}| \lesssim 10^{-8}
\end{aligned}
$$

MEG expt, PSI

parametrising $\mu \to e\gamma$ and $\mu \to e\bar{e}e$

Two dipole operators constrained by $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = \frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,L}\overline{\mu_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,R}\overline{\mu_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta} \right)$$

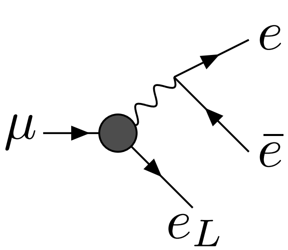
$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.2 \times 10^{-13}$$

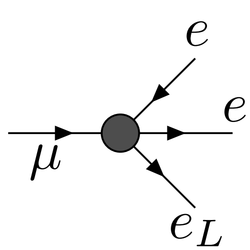
$$\Rightarrow |C_{D,X}| \lesssim 10^{-8}$$

MEG expt, PSI

$\mu \to e_L\bar{e}e$ **:** add $2\sqrt{2}G_F\Big[C_{V,LL}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_L e) + C_{V,LR}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_R e) + C_{S,RR}(\bar{e}P_R\mu)(\bar{e}P_R e)\Big]$

(e relativistic $\approx$ chiral, neglect interference between e_L, e_R)





$$BR = \frac{|C_{S,RR}|^2}{8} + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,R}|^2$$
$$+2|C_{V,LL} + 4eC_{D,R}|^2 + |C_{V,LR} + 4eC_{D,R}|^2 + \{L \leftrightarrow R\}$$
$$\leq 10^{-12} \Rightarrow C_{SXX}, C_{VXY} \lesssim 10^{-6}$$

SINDRUM, PSI

$\Rightarrow \mu \to e\gamma + \mu \to e\bar{e}e$ give 8 contraints
distinguish operators via angular correlations in final state

OkadaOkomuraShimizu

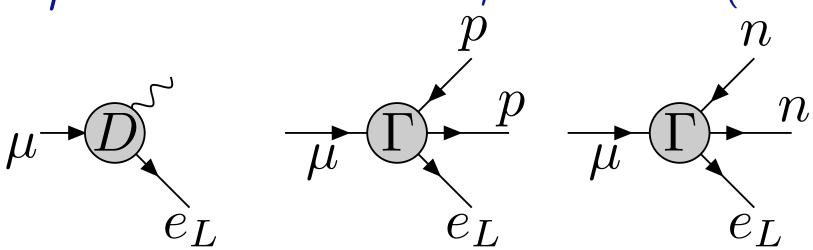
$\mu A \to eA$: sensitive to $\mu \to e$ on quarks

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit

$\mu A \to eA$: sensitive to $\mu \to e$ on quarks

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV: μ interacts with $\vec{E}$, nucleons (via $\widetilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

$(E_e \approx m_\mu$ so $e_L/e_R)$



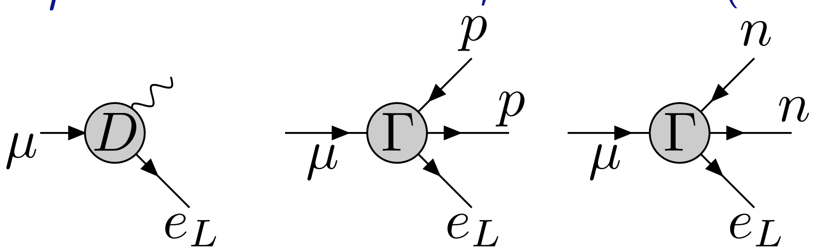
$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$

$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\mu A \to eA$: sensitive to $\mu \to e$ on quarks

- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV: μ interacts with $\vec{E}$, nucleons (via $\tilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

$(E_e \approx m_\mu$ so $e_L/e_R)$



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V, A, T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

KitanoKoikeOkada

$$BR_{SI} \sim \quad Z^2|V^p_A\tilde{C}^p_V + S^p_A\tilde{C}^p_S + V^n_A\tilde{C}^n_V + S^b_A\tilde{C}^n_S + D_A C_D|^2$$

S^N_A, V^N_A = integral over nucleus A of N distribution$\times$lepton wavefns...

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over nucleons $\propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

CiriglianoDavidsonKuno
HoferichterMenendezNoel

so Lagrangian at exptal scale (for precisely measured $\mu \to e_L$ observables)

$$\delta\mathcal{L} = 2\sqrt{2}G_F\Big[C_{DR}(m_\mu \overline{e}\sigma \cdot F P_R \mu) + C_{SRR}(\overline{e}P_R\mu)(\overline{e}P_R e) + C_{VLR}(\overline{e}\gamma^\alpha P_L \mu)(\overline{e}\gamma_\alpha P_R e)$$
$$+C_{VLL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_L e) + C_{Al}\mathcal{O}_{Al} + C_{Au\perp}\mathcal{O}_{Au\perp}\Big]$$

(past expts used Titanium (≈Aluminium) and Gold)

What are $\mathcal{O}_{Al}$, $\mathcal{O}_{Au\perp}$?

so Lagrangian at exptal scale (for precisely measured $\mu \to e_L$ observables)

$$\delta\mathcal{L} = 2\sqrt{2}G_F\Big[C_{DR}(m_\mu \overline{e}\sigma\cdot F P_R\mu) + C_{SRR}(\overline{e}P_R\mu)(\overline{e}P_R e) + C_{VLR}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_R e)$$
$$+C_{VLL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_L e) + C_{Al}\mathcal{O}_{Al} + C_{Au\perp}\mathcal{O}_{Au\perp}\Big]$$

(past expts used Titanium (≈Aluminium) and Gold)
What are $\mathcal{O}_{Al}$, $\mathcal{O}_{Au\perp}$?
Four $2l2$-nucleon operators can contribute to SpinIndep $\mu A \to eA$:

$$\delta\mathcal{L}_{SI} = 2\sqrt{2}G_F\Big[\tilde{C}^p_{SR}\overline{e}P_R\mu\overline{p}p + \tilde{C}^n_{SR}\overline{e}P_R\mu\overline{n}n + \tilde{C}^p_{VL}\overline{e}\gamma^\alpha P_L\mu\overline{p}\gamma_\alpha p + \tilde{C}^n_{VL}\overline{e}\gamma^\alpha P_L\mu\overline{n}\gamma_\alpha n\Big]$$

but a target is sensitive to a linear combo.(≈direction in coeff space), determined by "overlap integrals",eg

$$\mathcal{O}_{Al} \equiv \frac{1}{4}(\overline{e}P_R\mu\overline{p}p + \overline{e}P_R\mu\overline{n}n + \overline{e}\gamma^\alpha P_L\mu\overline{p}\gamma_\alpha p + \overline{e}\gamma^\alpha P_L\mu\overline{n}\gamma_\alpha n)$$

$\mathcal{O}_{Au\perp}$ = combo of ops probed by Au, not Al.

($L \leftrightarrow R$ not identical in SMEFT, but not worry)

take observable-motivated basis to Λ_{NP}?

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$

solving RGEs gives $\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}(m_\mu, m_W)$, $\Rightarrow$ define $\vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$ such that:

$$C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$$

($L \leftrightarrow R$ not identical in SMEFT, but not worry)

take observable-motivated basis to Λ_{NP}?

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$

solving RGEs gives $\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}(m_\mu, m_W)$, $\Rightarrow$ define $\vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$such that:

$C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$

$$
\begin{aligned}
C_{D,X}(m_\mu) \quad = \quad & C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\
& -\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\
& +16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\
& -8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\ \mathrm{GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right) f_{TD} \\
& +16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\ \mathrm{GeV}}\left(\sum_{u,c} 4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)
\end{aligned}
$$

all coeffs on right side $C(m_W)$ (basis vectors rotate and change length with scale)

$\lambda = \alpha_s(m_W)/\alpha_s(2\mathrm{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

($L \leftrightarrow R$ not identical in SMEFT, but not worry)

take observable- motivated basis to Λ_{NP} (if is a bad idea, SVP tell me why?)

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
solving RGEs gives $\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}(m_\mu, m_W)$, $\Rightarrow$ define $\vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$such that:
$C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$, Λ = scale of RGEs)

2.for $\mu \to e_L \overline{e_L} e_L$, define

$$C_{V,LL}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to 3e_L}(m_\mu, \Lambda)$$

$$BR(\mu \to e\bar{e}e) = 2|\vec{C}(\Lambda) \cdot (\vec{v}_{\mu\to 3e_L} + 4e\vec{v}_{\mu\to e\gamma})|^2 + \ldots$$

etc for $\mu \to e_L \overline{e_R} e_R$, and $\mu \to e_L \overline{e_R} e_L$.

3.for, eg, $\mu Al \to eAl$, define $\vec{v}_{\mu Al\to eAl}(\Lambda)$ to pick out correct quark operators:

$$BR(\mu Al \to eAl) = \#|\vec{C}(\Lambda) \cdot (\#\vec{v}_{\mu\to e\gamma} + \#\vec{v}_{\mu Al\to eAl})|^2$$

and a different $\vec{v}_{\mu Au\to e_L Au}(\Lambda)$ for Gold, etc.

obtain a scale-dependent basis for the experimentally constrainable subspace; the "flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

Should be the finite-eigenvalue subspace of the correlation matrix.

what to do with this basis?

check a few things

1. Do the basis vectors stay orthogonal? =*Do* $\mu\to e\gamma, \mu\to e\bar{e}e$ *and* $\mu A\to eA$ *give complementary information about NP models?*

   (a) Yes, to $\mathcal{O}(10^{-3})$ in running 2 GeV$\to m_W$

   (b) But changing EFTs can give overlaps (diff. low-E operators can match to same high-E operator $\leftrightarrow$ measure same thing)

   ex1: at m_W, all low-E vector 4f operators match to penguins $C_{HE}^{e\mu}, C_{HL}^{e\mu}$.

   ex2: in matching at 2 GeV:

   $\langle p|\bar{u}u|p\rangle = \langle n|\bar{d}d|n\rangle$ (isospin ?)

   but also: $\langle p|\bar{d}d|p\rangle \simeq \langle p|\bar{u}u|p\rangle \simeq \langle n|\bar{d}d|n\rangle \simeq \langle n|\bar{u}u|n\rangle$

2. the basis vectors change length...by $\mathcal{O}(1)$ factors, so ok

   eg importance of dipole for $\mu\to e\bar{e}e$ grows with scale

Wanted to use EFT to take exptal info to models... so:

1. *match to models, and explore what we can learn*

   (not need to run RGEs at each point in model space)

   are some regions of 6-d space inaccessible to some models?

2. make plots of the excluded region in 6-d space ?
   $\Leftrightarrow$ illustrate the reach and complementarity of experiments

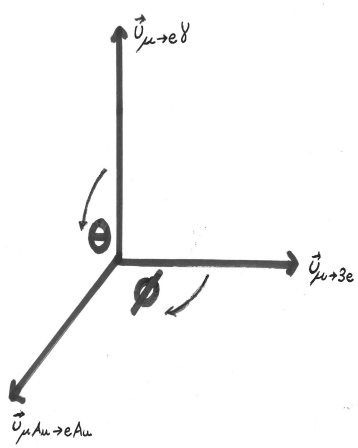
3. ... ?(why don't people already do this?)

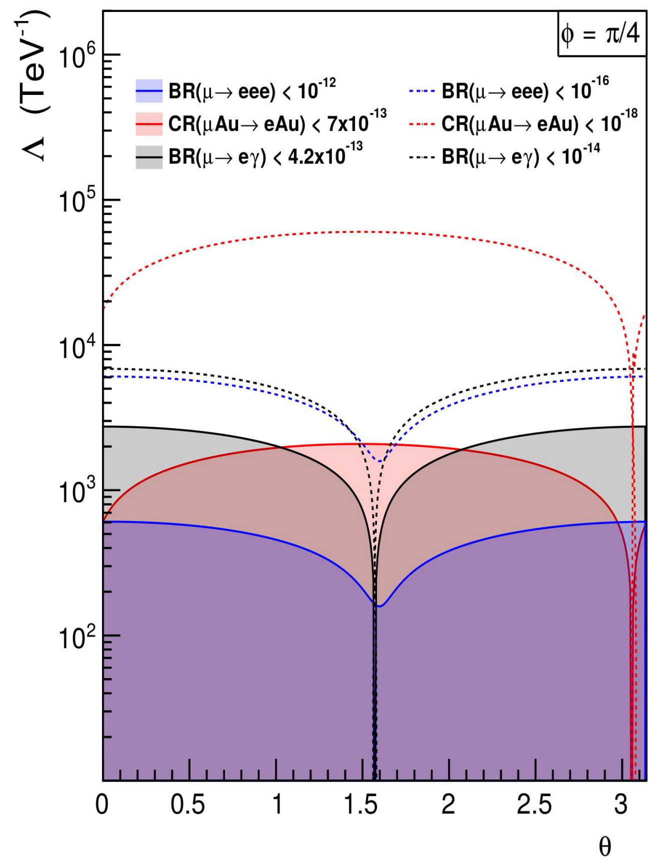
Plotting complementarity and reach of $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$

Restrict to 3-d space of coefficients of $\vec{v}_{\mu\to e_L\gamma}, \vec{v}_{\mu\to 3e_L}, \vec{v}_{\mu Au\to e_L Au} (= z, x, y)$.
Current *allowed* region an ellipse around origin... write instead:

$$\vec{C} \cdot \vec{v}_{\mu\to e_L\gamma} \equiv |\vec{v}_{\mu\to e_L\gamma}| \frac{v^2 \cos\theta}{\Lambda_{NP}^2}$$

$\Rightarrow \Lambda_{NP} \to \infty$ allowed





see 2204.00564

Summary

$\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ have experimental sensitivity to only a few operators at low energy, so:

1. worth to include RGEs at leading order, because allow to mix almost every coefficient (in chiral basis) into the testables ones
   $\Rightarrow$ almost every $\mu \to e$ interaction below m_W (otherwise flav. diag., ≤ 4 legs) contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$ and/or $\mu \to e\bar{e}e$ and/or $\mu A \to eA$
   (possible exceptions :$\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

2. most directions in coefficient space are untestable ("flat")
   (not an EFT-problem, its a consequence of searching for NP under the lamppost, affects model studies in same way.)

3. no physics in a basis choice; one should choose a convenient basis for the calculation: a convenient basis for comparing models to $\mu \to e$ flavour-changing observables can be constructed from the observables. (It should span the same subspace as the eigenvectors of the correlation matrix with finite non-zero eigenvalues.)

BackUp

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon +BowmanChengLiMatis

Operator basis $m_\tau \to m_W$: ~ 90 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad \textit{dim } 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad \textit{dim } 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

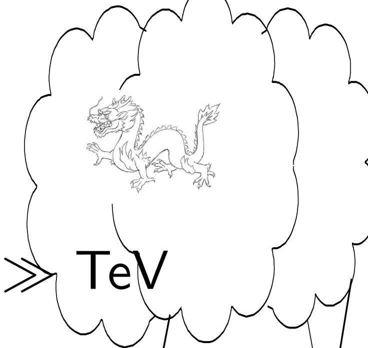
$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \textit{dim } 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

...zzz...but ~ 90 coeffs!

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

EFT for Heavy LFV Physics...



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$\mathcal{L}_{SM}$ $+\mathcal{L}(\text{SM invar. operators, dim6})$

$m_W \sim m_h \sim m_t$

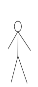
$\{\gamma, g, f\}$

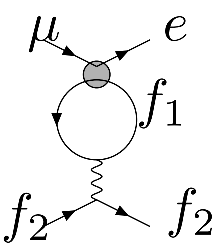
$\mathcal{L}_{QED\times QCD}$ $+\mathcal{L}(3\rightarrow 4 \text{ legged QCDxQED invar. ops})$ $\sim$ 90 of them!

2 GeV $\sim m_c, m_b, m_\tau$

$\mathcal{L}(n, p, \pi, \gamma, e, \mu)$ $+\mathcal{L}(\text{3or4 legged QED invar. ops})$

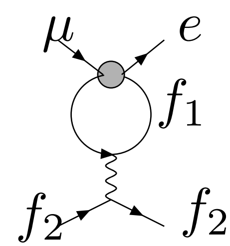
data $(\mu\rightarrow e\gamma, \mu\rightarrow e\bar{e}e, \mu A\rightarrow eA)$

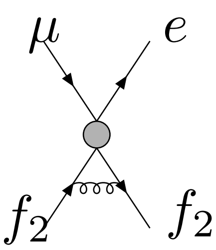


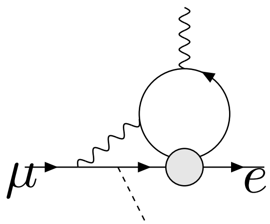


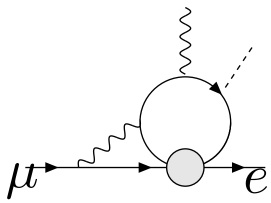
Including RGEs

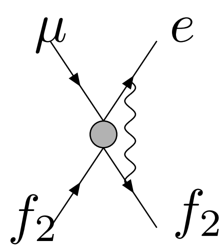
eg below m_W: 1-loop QED + QCD (+2-loop QED V→D)

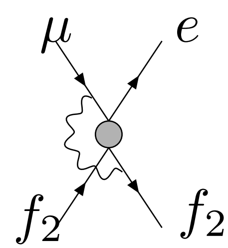


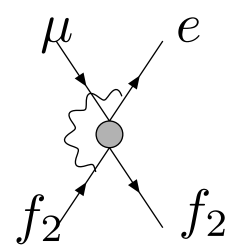












$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

QCD: not mix ops, should resum ⇒ multiplicative renorm S,T ops: diagonal **D**

QED: *does* mix ops, but $\alpha_{em} \ll$, solve perturbatively

$$\vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G} \quad , \quad \boldsymbol{G} = \boldsymbol{D}\left(\mathbf{1} - \frac{\alpha_e}{4\pi} \mathbf{\Gamma} + \frac{\alpha_e^2}{32\pi^2} \mathbf{\Gamma}\mathbf{\Gamma} + ...\right.$$

And models may *not* generate at tree level operators expts probe...

ex: $\mu A \to eA$ **in a model giving tensor** $C_T^{uu}(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u)$ **at weak scale**

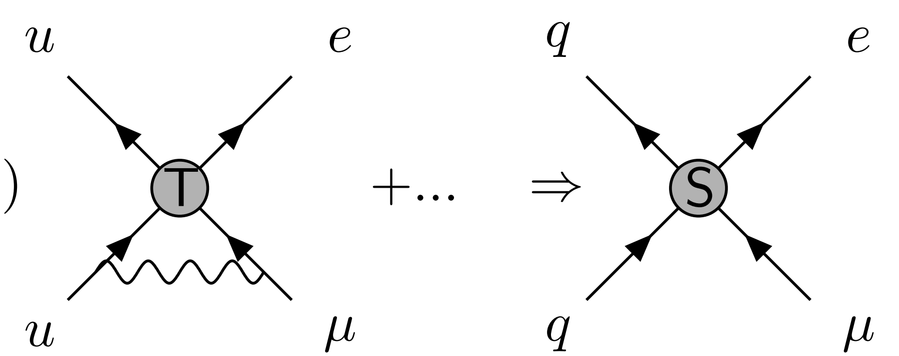
1: forget RGEs Match to nucleons $N \in \{n,p\}$ as $\widetilde{C}_T^{NN} \simeq \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4}C_T^{uu}$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T^{uu}|^2}$$

nuclear matrix elements: EngelRTO, KlosMGS

2: include RGEs

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$



$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$

$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$

Then match to nucleons: $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained

Quantifying which targets give independent information (on nucleons)

1. neglect Dipole (better sensitivity of $\mu \to e\gamma$ (MEGII) and $\mu \to e\bar{e}e$(Mu3e).
   remain to determine: $\vec{C} \equiv (\widetilde{C}^{pp}_{VR}, \widetilde{C}^{pp}_{SL}, \widetilde{C}^{nn}_{VR}, \widetilde{C}^{nn}_{SL})$

2. recall that
   $$BR_{SI}(A\mu \to Ae) \propto \left|\vec{C} \cdot \vec{v}_A\right|^2$$
   where target vector for nucleus A
   $$\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}\right)$$

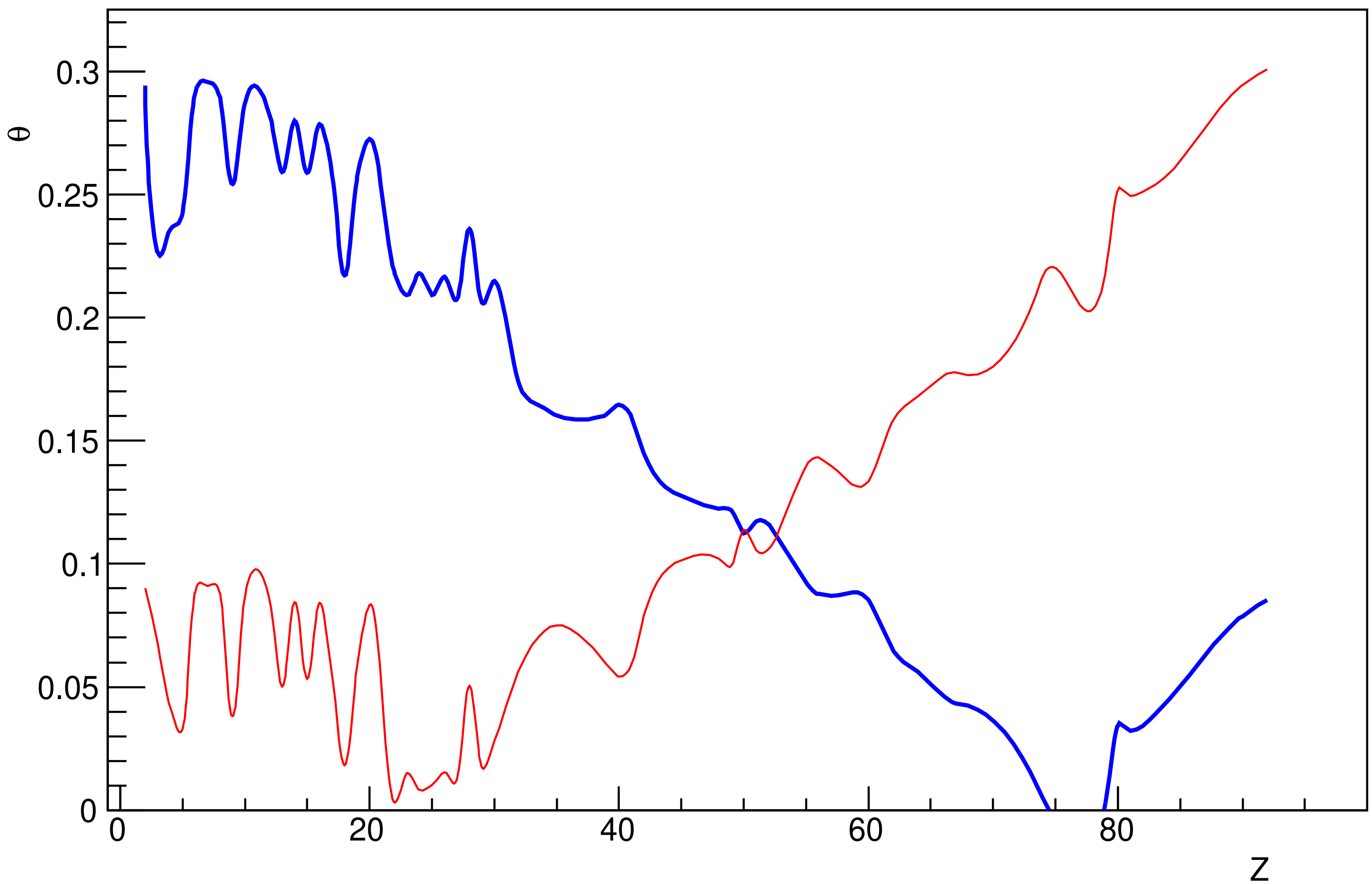
3. So first experimental search (eg on Aluminium) probes projection of $\vec{C}$ of $\vec{v}_{Al}$
   ... next target needs to have component $\perp$ to Aluminium!
   $\Leftrightarrow$ plot misalignment angle θ between target vectors

4. how big does θ need to be?
   overlap integrals have theory uncertainty: $\Delta\theta \left\{ \begin{array}{cc} \text{nuclear} & \sim 5\% (KKO) \\ NLO\ \chi\text{PT} & \sim 10\%(?) \end{array} \right.$
   Both vectors uncertain by $\Delta\theta$; need misaligned by $2\Delta\theta \approx 10 \to 20\%$

Current data+ theory uncertainty $\sim 10\%$**: two targets give** $\Delta\theta > 0.2$

$$\boldsymbol{BR(\mu Au \to eAu) \leq 7 \times 10^{-13}} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

$$\hat{v}_{Al} \approx \frac{1}{2}(1,1,1,1)$$

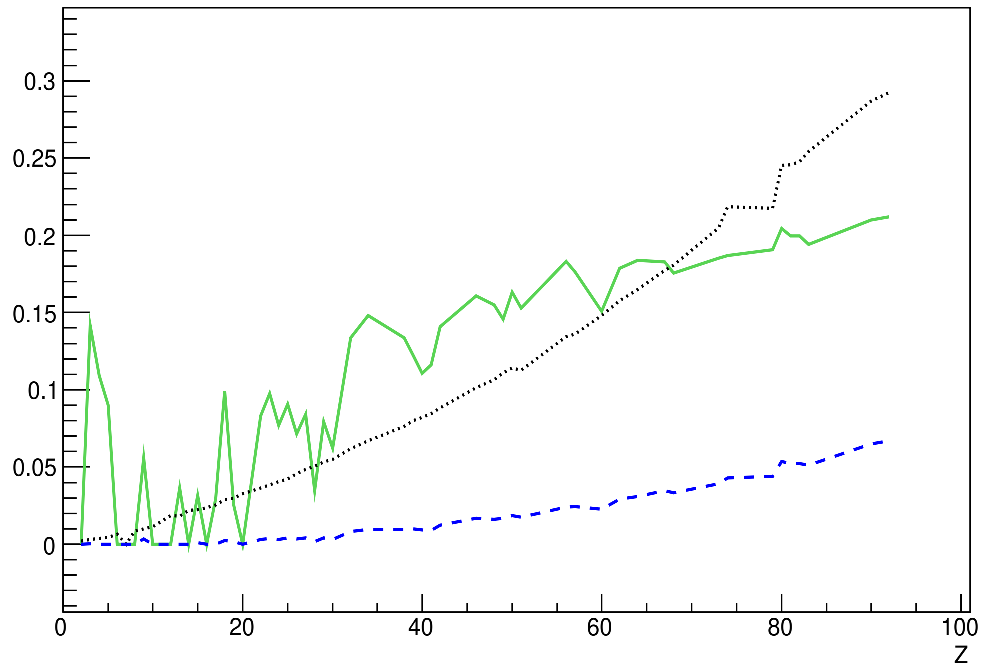
(recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$

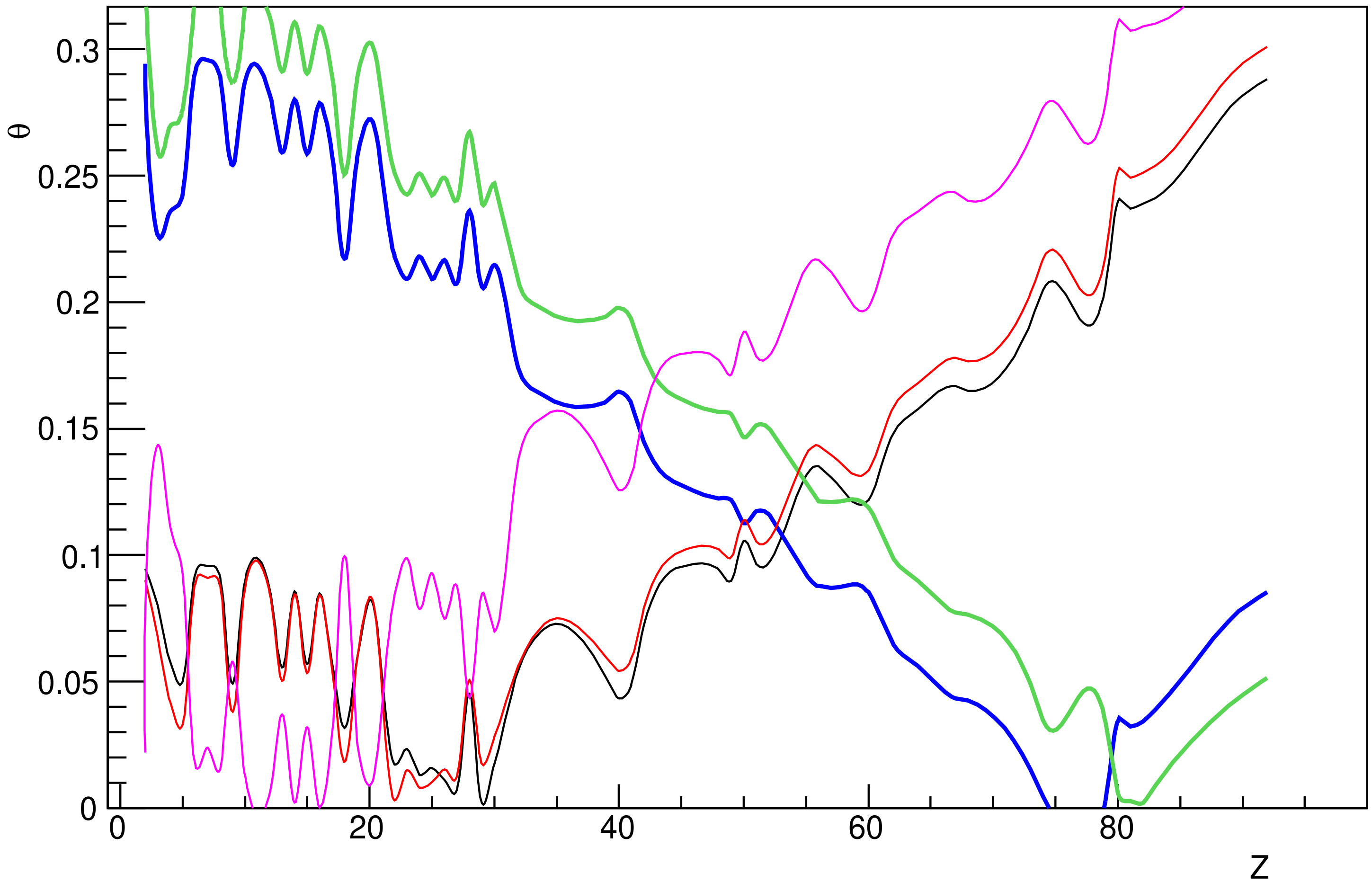


probe 3 combinations of SI coeffs

All current data...

$BR(\mu Au \to eAu) \leq 7 \times 10^{-13}$ $(Au : Z = 79)$

$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12}$ $(Ti : Z = 22)$



$BR(\mu Pb \to ePb) \leq 4.6 \times 10^{-11}$

$BR(\mu S \to eS) \leq 7 \times 10^{-11}$ S = Sulpher, Z = 16

$BR(\mu Cu \to eCu) \leq 1.6 \times 10^{-8}$ Cu = Copper, Z = 29

But what happens when match nucleons to quarks?

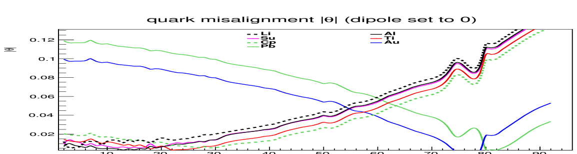
By measuring $\mu A \to eA$ on different targets, could determine coefficients of LFV ops with vector and scalar currents of n or p.

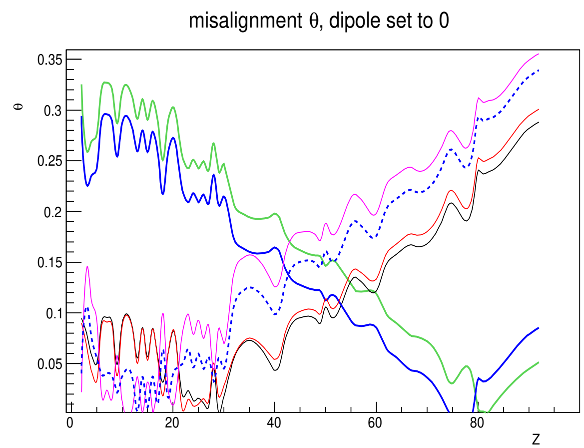
Match to quarks: $(\Gamma_O \in \{I, \gamma_5, \gamma^\alpha, \gamma^\beta\gamma_5, \sigma^{\alpha\beta}\})$

$$\begin{aligned}
\langle N(P_f)|\bar{q}(x)\Gamma_O q(x)|N(P_i)\rangle &= G_O^{N,q}\langle N|\bar{N}(x)\Gamma_O N(x)|N\rangle \\
&= G_O^{N,q}\overline{u_N}(P_f)\Gamma_O u_N(P_i)e^{-i(P_f-P_i)x}
\end{aligned}$$

But for scalar ops, $G_S^{p,u} = G_S^{n,d} \simeq G_S^{p,d} \simeq G_S^{n,u}$

so need great precision to differentiate LFV ops with scalar currents of u or d :(

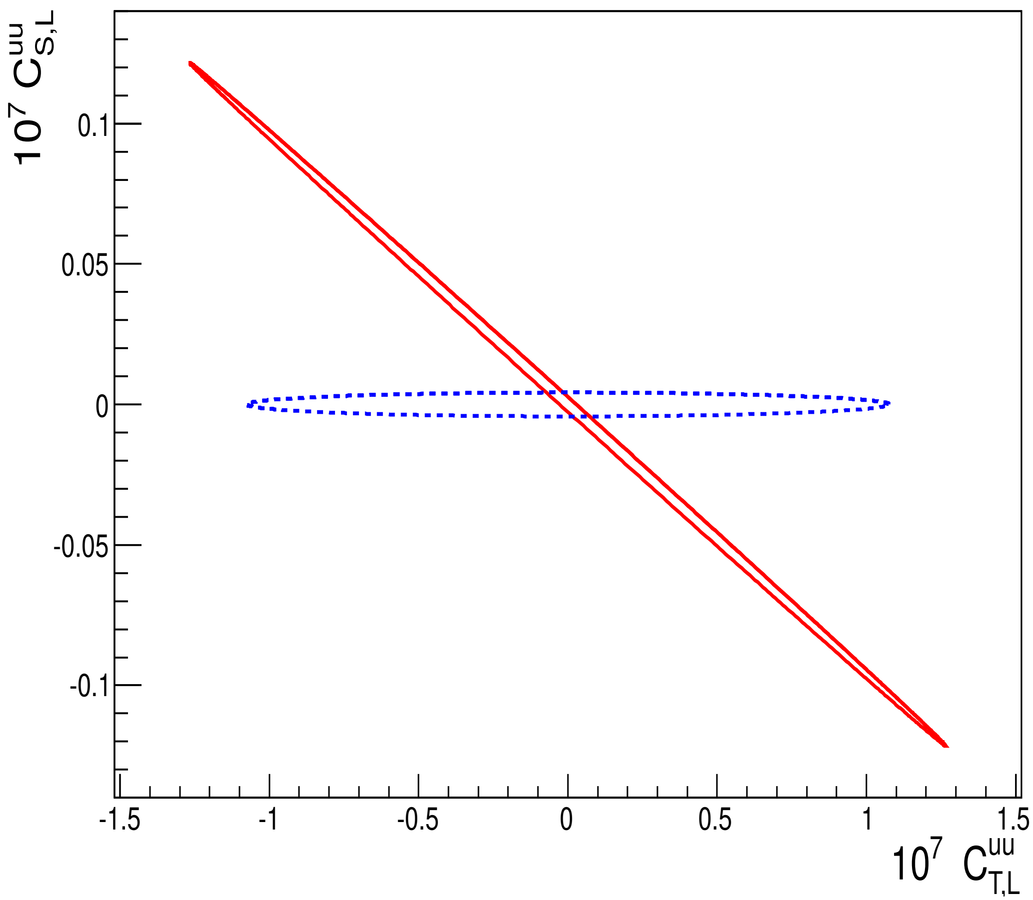




sensitivity vs constraint

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$, and :

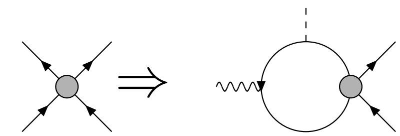
$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y\mu)(\overline{u}u)$$



C_T^{uu}, C_S^{uu} constrained to live inside blue (red) ellipse at exptal scale (at m_W): sensitivity to C_S^{uu} = cut ellipse @ $C_T^{uu} = 0$; constraint = live in projection of ellipse onto C_S^{uu} axis.

“Accidental cancellations” and “naturalness” in EFT

(“accidental” cancellations occur; in 1-loop RGEs give, for coeff.s at $\sim m_W$ of
$\mathcal{O}_{D,L} = m_\mu(\overline{e}\sigma \cdot F P_L \mu)$, $\mathcal{O}^{\tau\tau}_{T,LL} = (\overline{e}\sigma P_L \mu)(\overline{\tau}\sigma P_L \tau)$:



$$BR(\mu \to e\gamma) \approx |0.938 C_{D,L} + 0.981 C^{\tau\tau}_{T,LL} + ...|^2$$)

If imagine that NP knows about all the SM parameters, but not about the scale at which you do expts, could argue that RG-stable cancellations in EFT can be “natural”.

(caveat: NP does know about all the mass scales in the theory, which often determine the scales in the logs...)

So if resum RGs, cancellations among coeff.s with same anom dim are ok?
If not resum, can allow cancellations among all coeff.s who multiply same log?

Interest of this argument, is that forbidding “unnatural” calcellations transforms a single exptal bound into many bounds...but unnatural cancellations occur, see green parenthese: dipole is tree, tensor is log-enhanced loop.