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Axion decay and Chiral anomalies

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ÉCOLE POLYTECHNIQUE

Outline of talk

- Motivation
- Toy model for axion decay
- Nother current of chiral gauge and global axial symmtery
    - At classical level
    - At quantum level
- Generalization of toy model for SM gauge symmetry
- Conclusion

Motivation

Axions are blind to anomalies

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- This paper claims that in chiral gauge theories, axion decay to 2 gauge boson is completely independent to axial anomaly.
- They computed Jacobian for anomaly computation, and computed axion decay using diagramtic approach for type-II THDM.

Motivation

The Anomalous Case of Axion EFTs and Massive Chiral Gauge Fields

Quentin Bonnefoy,[a] Luca Di Luzio,[a] Christophe Grojean,[a,b] Ayan Paul[a,b] and Alejo N. Rossia[a,b]

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- They used the traditional method of anomaly matching by weakly gauging the axial symmetry.
- They found, in chiral gauge theory, axion decay have contribution from two operators

$$\mathcal{C}\, aF\tilde{F} = \mathcal{A}_A aF\tilde{F} + \underbrace{\mathcal{C}_1(\partial_\mu a(x) - X_\mu)\left(A_\nu - \frac{\partial\varphi(x)}{f}\right)\tilde{F}^{\mu\nu}}$$

Gauge and axial symmetry invariant

(In vector gauge theory, this operator is not gauge invariant)

Basic Information

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L \imath \not{D}_L \psi_L + \bar{\psi}_R \imath \not{D}_R \psi_R - (M\bar{\psi}_L\psi_R + h.c.)$$

Axial and gauge current

$$\begin{aligned} J^\mu_A &= \frac{1}{2}\overline{\psi}\gamma^\mu\gamma_5\psi \\ J^\mu_G &= -\overline{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi \end{aligned}$$

$$2\alpha = q_L + q_R \quad 2\beta = q_L - q_R$$

The divergence of current at classical level

$$\begin{aligned} \partial_\mu J^\mu_G &= 2\beta M g\overline{\psi}\gamma_5\psi \\ \partial_\mu J^\mu_A &= M g\overline{\psi}\gamma_5\psi \end{aligned}$$

In vector gauge theory (β=0), Gauge current is conserved.

Toy model for axion

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L \iota \not{D}_L \psi_L + \bar{\psi}_R \iota \not{D}_R \psi_R + |D_{\phi_1}\phi_1|^2 + |D_{\phi_2}\phi_1|^2 - (y\phi_1\bar{\psi}_L\psi_R + h.c.)$$
$$- \lambda_1(f_1^2 - |\phi_1|^2)^2 - \lambda_2(f_2^2 - |\phi_2|^2)^2$$

Gauge tranformation →

$$\psi_L \to e^{igq_L\theta(x)}\psi_L \qquad \psi_R \to e^{igq_R\theta(x)}\psi_R$$
$$\phi_i \to e^{igq_{\phi_i}\theta(x)}\phi_i, \quad q_{\phi_1} = q_L - q_R = 2\beta$$

Axial transformation →

$$\psi_L \to e^{ia_L\theta}\psi_L, \quad \psi_R \to e^{-ia_L\theta}\psi_R,$$
$$\phi_i \to e^{ia_{\phi_i}\theta}\phi_i \quad a_L = -a_R \quad a_{\phi_1} = 2a_L$$

The axial and gauge current, using $\phi_i = f_i e^{ia_i(x)/f_i}/\sqrt{2}$

$$J_A^\mu = a_L\bar{\psi}\gamma^\mu\gamma_5\psi + a_{\phi_1}f_1\partial_\mu a_1(x) + a_{\phi_2}f_2\partial_\mu a_2(x) - gA^\mu(q_{\phi_1}a_{\phi_1}f_1^2 + q_{\phi_2}a_{\phi_2}f_2^2)$$
$$J_G^\mu = -g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi + q_{\phi_1}gf_1\partial^\mu a_1(x) + q_{\phi_2}gf_2\partial^\mu a_2(x) - g^2A^\mu(q_{\phi_1}^2f_1^2 + q_{\phi_2}^2f_2^2)$$

Divergence of both current at classical level is zero. →

$$\partial_\mu J_G^\mu = 0$$
$$\partial_\mu J_A^\mu = 0$$

Toy model for axion

In $(a,\ \varphi)$ basis

Physical axion $a(x) = ca_1(x) - sa_2(x)$

Would be Goldstone boson $\varphi(x) = sa_1(x) + ca_2(x)$

Gauge tranformation → $a(x) \to a(x)$ $\quad \varphi(x) \to \varphi(x) + f\theta_g(x)$

Axial transformation → $a(x) \to a(x) + f\theta$ $\quad \varphi(x) \to \varphi(x)$

The current in $(a,\ \varphi)$ basis

$$\begin{aligned} J^\mu_A &= a_L\overline{\psi}\gamma^\mu\gamma_5\psi + f\partial_\mu a(x) \\ J^\mu_G &= -g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi + gf\partial^\mu\varphi(x) - g^2A^\mu f^2 \end{aligned}$$

What is problem?

- Axion decay to two gauge boson operator in this toy model

$$\mathcal{L}_{eff} \propto \frac{g^2}{16\pi^2}\frac{a(x)}{f}F\tilde{F}\left(\alpha^2 + \frac{1}{3}\beta^2\right)$$

- And anomaly associate with U(1) global axial current is

Jacobian

$$\mathcal{A} = \theta\frac{g^2}{16\pi^2}F\tilde{F}(\alpha^2 + \beta^2)$$

Direct calculation of Anomaly using one-loop three point function

$$J^\mu_A = a_L\bar{\psi}\gamma^\mu\gamma_5\psi + f\partial_\mu a(x) = J^{\mu\psi}_A + J^{\mu a}_A$$

$$J^\mu_G = \underbrace{-g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi}_{J^{\mu\psi}_G} + \underbrace{gf\partial^\mu\varphi(x)}_{J^{\mu\varphi}_G} - g^2A^\mu f^2$$

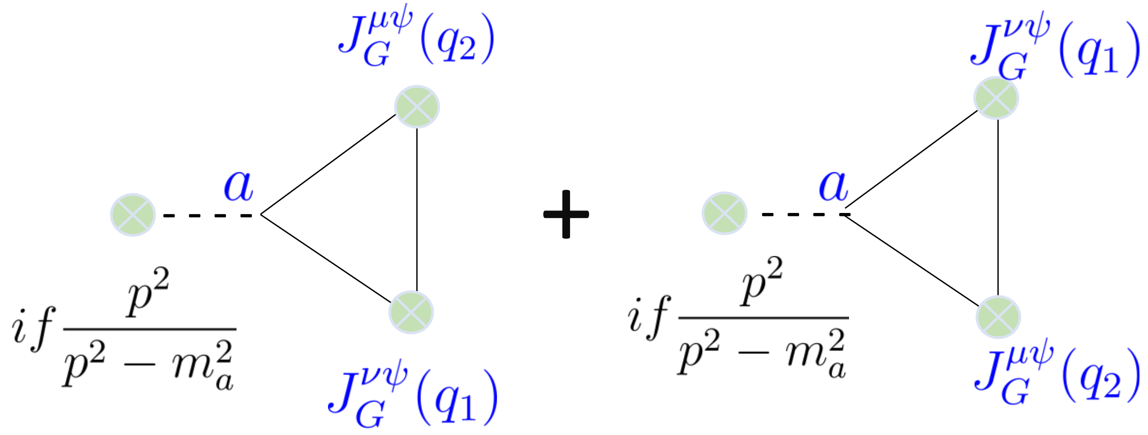
$$\begin{aligned}
\partial_\rho\langle J^\rho_G J^\mu_G J^\nu_G\rangle &= \partial_\rho\langle J^{\rho\psi}_G J^{\mu\psi}_G J^{\nu\psi}_G\rangle + \partial_\rho\langle J^{\rho\varphi}_G J^{\mu\psi}_G J^{\nu\psi}_G\rangle \\
\partial_\rho\langle J^\rho_A J^\mu_G J^\nu_G\rangle &= \partial_\rho\langle J^{\rho\psi}_A J^{\mu\psi}_G J^{\nu\psi}_G\rangle + \partial_\rho\langle J^{\rho a}_A J^{\mu\psi}_G J^{\nu\psi}_G\rangle \\
\partial_\mu\langle J^\rho_A J^\mu_G J^\nu_G\rangle &= \partial_\mu\langle J^{\rho\psi}_A J^{\mu\psi}_G J^{\nu\psi}_G\rangle + \partial_\mu\langle J^{\rho\psi}_A J^{\mu\varphi}_G J^{\nu\psi}_G\rangle \\
\partial_\nu\langle J^\rho_A J^\mu_G J^\nu_G\rangle &= \partial_\nu\langle J^{\rho\psi}_A J^{\mu\psi}_G J^{\nu\psi}_G\rangle + \partial_\nu\langle J^{\rho\psi}_A J^{\mu\psi}_G J^{\nu\varphi}_G\rangle
\end{aligned}$$

→ This will give us gauge and axial anomaly.

$$\begin{aligned}
\partial_\rho\langle J^{\rho a}_G J^{\mu a}_G J^{\nu a}_G\rangle &= 0 \\
\partial_\rho\langle J^{\rho\psi}_G J^{\mu\psi}_G J^{\nu a}_G\rangle &= 0 \\
\partial_\rho\langle J^{\rho a}_G J^{\mu\psi}_G J^{\nu a}_G\rangle &= 0
\end{aligned}
\qquad
\begin{aligned}
\partial_\rho\langle J^{\rho\psi}_G J^{\mu a}_G J^{\nu\psi}_G\rangle &= 0, \\
\partial_\rho\langle J^{\rho a}_G J^{\mu a}_G J^{\nu\psi}_G\rangle &= 0, \\
\partial_\rho\langle J^{\rho\psi}_G J^{\mu a}_G J^{\nu a}_G\rangle &= 0,
\end{aligned}$$

Direct calculation of Anomaly using one-loop three point function

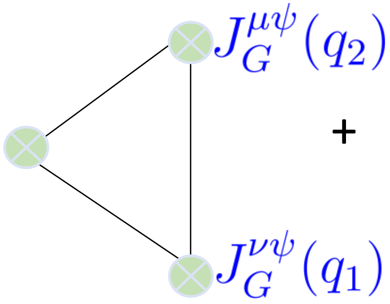
$$\partial_\rho \langle J_A^{\rho a} J_G^{\mu\psi} J_G^{\nu\psi} \rangle =$$



$$= \; if \left(1 + \frac{m_a^2}{p^2 - m_a^2}\right) \mathcal{M}_{\mu\nu}(a \to Z \; Z)$$

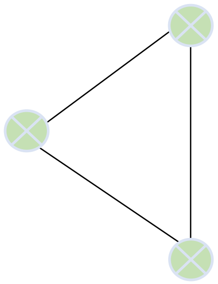
Direct calculation of Anomaly using one-loop three point function

$$\partial_\rho \langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = \quad ip.J_A^{\psi}$$



+ cross diagram

$$\partial_\rho \langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = \Bigg($$



$$+ \text{ cross diagram} \Bigg)_{m=0} - \; if\mathcal{M}_{\mu\nu}(a \to Z\ Z)$$

Linearly divergent integral

- If we shift 1st diagram loop momentum k→ k + m and 2nd diagram loop momentum k → k+ n. It will cancel the linearly divergent term but total contribution will be proportional to momentum ambiguity parameter c_1, c_2
- Where we parametrize m-n=$c_1 q_1 + c_2 q_2$

Direct calculation of Anomaly using one-loop three point function

$$\partial_\rho\langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle = -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta - if\mathcal{M}_{\mu\nu}(a \to Z\ Z)$$

$$\partial_\rho\langle J_A^{\rho a} J_G^{\mu\psi} J_G^{\nu\psi}\rangle = if\left(1+\frac{m_a^2}{p^2-m_a^2}\right)\mathcal{M}_{\mu\nu}(a \to Z\ Z)$$

❖ The divergence of full axial current

$$\partial_\rho\langle J_A^{\rho} J_G^{\mu} J_G^{\nu}\rangle = -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

where (c_1, c_2) are momentum ambiguity parameter.

Direct calculation of Anomaly using one-loop three point function

Similarly for gauge-gauge-gauge current

$$\partial_\rho \langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -q_{\phi_1}(c_1 - c_2)\frac{g^2 3(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta - if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z)$$
$$\partial_\rho \langle J_G^{\rho\varphi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z)$$

❖ The divergence of full chiral gauge current

$$\partial_\rho \langle J_G^{\rho} J_G^{\mu} J_G^{\nu} \rangle = -q_{\phi_1}(c_1 - c_2)\frac{3g^2(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta$$

Quantum correction in divergence of gauge & axial current

Add massless fermion will cancel it

$$\overline{\psi}_s \not{D} \psi_s$$

$$\partial_\rho \langle J^\rho_G J^\mu_G J^\nu_G \rangle = -q_{\phi_1}(c_1 - c_2)\frac{3g^3(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta$$

$$\partial_\rho \langle J^\rho_A J^\mu_G J^\nu_G \rangle = -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2 + \beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha (q_2)_\beta$$

Related to axial symmetry breaking

$$\partial_\mu \langle J^\rho_A(p) J^\mu_G(q_2) J^\nu_G(q_1) \rangle = a_{\phi_1}(c_1 - 2)\frac{g^2(\alpha^2 + \beta^2)}{16\pi^2}\epsilon^{\nu\rho\alpha\beta}(q_1)_\alpha (q_2)_\beta$$

$$\partial_\nu \langle J^\rho_A(p) J^\mu_G(q_2) J^\nu_G(q_1) \rangle = a_{\phi_1}(c_2 + 2)\frac{g^2(\alpha^2 + \beta^2)}{16\pi^2}\epsilon^{\nu\rho\alpha\beta}(q_1)_\alpha (q_2)_\beta$$

Related to gauge symmetry breaking

❖ Gauge invariance demands $c_1 = -c_2 = 2$

Result

The divergence of gauge and Axial current at quantum level

Using $c_1 = -c_2 = 2$ and $m \neq 0$

Divergence of axial current

$$\begin{aligned} \partial_\rho J_A^{\rho\psi} &= \mathcal{A} - if\mathcal{M}_{\mu\nu}(a \to Z\ Z) \\ \partial_\rho J_A^{\rho a} &= if\mathcal{M}_{\mu\nu}(a \to Z\ Z) \\ \partial_\rho J_A^{\rho} &= \mathcal{A} \end{aligned}$$

Divergence of gauge current

$$\begin{aligned} \partial_\rho J_G^{\rho\psi} &= \mathcal{A}_g - if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z) \\ \partial_\rho J_G^{\rho\varphi} &= if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z) \\ \partial_\rho J_G^{\rho} &= \mathcal{A}_g \end{aligned}$$

where $\mathcal{A} \propto (\alpha^2 + \beta^2)$

$$\mathcal{M}_{\mu\nu}(a \to Z\ Z) \propto (\alpha^2 + \frac{\beta^2}{3})$$

$$\mathcal{A}_g \propto 6\beta(\alpha^2 + \frac{\beta^2}{3})$$

$$\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z) \propto 2\beta(\alpha^2 + \frac{\beta^2}{3})$$

Using $c_1 = -c2 = 2$ amd m=0

$$\begin{aligned} \partial_\rho J_A^{\rho\psi} &= \mathcal{A} \\ \partial_\rho J_A^{\rho a} &= 0 \\ \partial_\rho J_A^{\rho} &= \mathcal{A} \end{aligned}$$

$$\begin{aligned} \partial_\rho J_G^{\rho\psi} &= \mathcal{A}_g \\ \partial_\rho J_G^{\rho\varphi} &= 0 \\ \partial_\rho J_G^{\rho} &= \mathcal{A}_g \end{aligned}$$

Conclusion

- Axion decay contribution exactly cancel in full current. So axion decay is independent of anomaly.

- But this is true for both Chiral and vector gauge theories. In vector gauge theories, axion decay accidently looks same as axial anomaly.

- Axion decay is exactly zero in exact massless limit but anomalies are mass independent.

- It looks like $^1/_3\ factor$ in axion decay is related to gauge current divergence of scalar current due to simultaneously breaking of both gauge and axial symmetry (But still in progress to proof it)

Generalization of toy model for SM gauge symmetry

Effective Lagrangian for Axion decay

Where axion couple to fermion through Yukawa terms

- We can absorb axion coupling in Yukawa term by rotating fermion fields. Let's consider a general rotation

$$\psi' \to \exp\left(-(\chi_V + \chi_A\gamma_5)\frac{\imath a(x)}{f}\right)\psi, \qquad \bar{\psi}' \to \bar{\psi}\exp\left((\chi_V - \chi_A\gamma_5)\frac{\imath a(x)}{f}\right)$$

where $\chi_V = \chi_R + \chi_L$ and $\chi_V = \chi_R - \chi_L$

- This rotation will lead to two term in the Lagrangian which will give axion decay operator

$$\mathcal{L}_a \;\propto\; \frac{g^2}{16\pi^2 f}a(x)F\tilde{F}\left(\chi_A(\alpha^2+\beta^2) - \chi_V\,2\alpha\beta\right) \longrightarrow \text{Jacobian}$$

$$+\;\frac{\partial_\mu a(x)}{f}(\chi_V\bar{\psi}\gamma^\mu\psi + \chi_A\bar{\psi}\gamma^\mu\gamma_5\psi) \longrightarrow \text{Fermionic Kinetic term}$$

$$-\frac{a(x)}{f}\left(\chi_V\partial_\mu J_V^{\mu\psi} + \chi_A\partial_\mu J_A^{\mu\psi}\right)$$

Effective Lagrangian for Axion decay

$$-\frac{a(x)}{f}\left(\chi_V \partial_\mu J_V^{\mu\psi} + \chi_A \partial_\mu J_A^{\mu\psi}\right)$$

m = 0:

$$\frac{g^2}{16\pi^2 f} a(x) F\tilde{F}\left(-\chi_A(\alpha^2+\beta^2) + \chi_V\, 2\alpha\beta\right)$$

m → ∞:

$$\frac{g^2}{16\pi^2 f} a(x) F\tilde{F}\left(\chi_A(-\frac{2\beta^2}{3}) + \chi_V\, 2\alpha\beta\right)$$

- The effective operator for axion decay (with massive fermion)

$$\mathcal{L}_{agg} \propto \frac{g^2}{16\pi^2 f} a(x) F\tilde{F} \sum_f \left(\chi_A^f(\alpha^2+\beta^2) - \chi_V^f\, 2\alpha\beta\right) + \frac{g^2}{16\pi^2 f} a(x) F\tilde{F} \sum_f \left(\chi_A^f(-\frac{2\beta^2}{3}) + \chi_V^f\, 2\alpha\beta\right)$$

$$\propto \frac{g^2}{16\pi^2 f} a(x) F\tilde{F} \sum_f \left(\chi_A^f(\alpha^2 + \frac{1}{3}\beta^2)\right)$$

- The effective operator for axion decay (with massless fermion)

$$\mathcal{L}_{agg} = 0$$

Effective Lagrangian for Axion decay in SM gauge theory

$$
\begin{aligned}
\mathcal{L}_{agg}^{SM} \;\propto\;& N_c \frac{g_s^2}{16\pi^2 f} a(x) G^a \tilde{G}^a + N_{Li} \frac{g^2}{16\pi^2 f} a(x) W^i \tilde{W}^i \\
+\;& N_Y \frac{g'^2}{16\pi^2 f} a(x) B\tilde{B} + N_m \frac{g g'}{16\pi^2 f} a(x) W^3 \tilde{B}
\end{aligned}
$$

Derivative coupling will generate mixing term

$$
\begin{aligned}
N_c &= \sum_{f=u,d} T(R_c^f)\chi_A^f(\alpha_c^2 + \frac{1}{3}\beta_c^2) \quad N_Y = \sum_{f=u,d,e} N_c^f \chi_A^f(\alpha_y^2 + \frac{1}{3}\beta_y^2) \\
N_{L1,L2} &= \sum_{f=u,d,e,\nu} N_c^f \chi_A^f(\alpha_l^2 + \frac{1}{3}\beta_l^2) \quad N_{L3} = \sum_{f=u,d,e} N_c^f \chi_A^f(\alpha_l^2 + \frac{1}{3}\beta_l^2) \\
N_m &= \sum_{f=u,d,e} 2T_3^f N_c^f \left(-\chi_A^f \frac{2\beta_l\beta_y}{3} + \chi_V^f(\alpha_y\beta_l + \alpha_l\beta_y)\right) \\
&+ 2T_3^f N_c^f \left[-\chi_A^\nu(\alpha_l\alpha_y + \beta_l\beta_y) + \chi_V^\nu(\alpha_y\beta_l + \alpha_l\beta_y)\right]
\end{aligned}
$$

Comparision with Jeremie paper

- They used type-II THDM

$$\mathcal{L}_Y = -\bar{u}_R Y_u\, q_L \phi_1 - \bar{d}_R Y_d\, q_L \phi_2^\dagger - \bar{e}_R Y_e\, l_L \phi_2^\dagger + h.c.$$

where

$$\phi_1^T = \frac{1}{\sqrt{2}} exp(\frac{iax}{v})(H_1^+ \quad v_1 + ReH_1), \qquad \phi_2^T = \frac{1}{\sqrt{2}} exp(\frac{ia}{vx})(H_2^+ \quad v_2 + ReH_2)$$

$x \rightarrow v_2/v_1$

- Compute χ_V and χ_A to absorb axion from Yukawa term

$$\left| \begin{array}{c} \chi_V^u = 0 \\ \chi_V^d = \frac{1}{x} - x \\ \chi_V^e = 0 \\ \chi_V^\nu = -\frac{1}{2x} \end{array} \right| \begin{array}{c} \chi_A^u = x \\ \chi_A^d = \frac{1}{x} \\ \chi_A^e = \frac{1}{x} \\ \chi_A^\nu = \frac{1}{2x} \end{array} \left| \begin{array}{c} \chi_{q_L} = -\frac{x}{2} \\ \chi_{q_L} = -\frac{x}{2} \\ \chi_{l_L} = -\frac{1}{2x} \\ \chi_{l_L} = -\frac{1}{2x} \end{array} \right| \begin{array}{c} \chi_{u_R} = \frac{x}{2} \\ \chi_{d_R} = \frac{1}{x} - \frac{x}{2} \\ \chi_{e_R} = \frac{1}{2x} \\ \end{array} \right|$$

- Value of (α, β) for SU(3)= (1,0); SU(2)= (1/2, 1/2); $U(1)_Y = \frac{1}{4}(Y_L + Y_R, Y_L - Y_R)$

Comparision with Jeremie paper

$$N_{L_3} = \frac{1}{4}(x + \frac{4}{3x}) \qquad N_{L_1,L_2} = \frac{1}{4}(x + \frac{3}{2x})$$

$$N_c = \frac{1}{2}\left(x + \frac{1}{x}\right) \qquad N_Y = \left(\frac{7x}{12} + \frac{2}{3x}\right)$$

$$N_m = \frac{1}{2}\left(x + \frac{2}{3x}\right)$$

$$\begin{aligned}
\mathcal{L}^{SM}_{agg} \propto\ & N_c \frac{\iota g_s^2}{16\pi^2 f} a(x) G^a \tilde{G}^a + 2N_{L_2} \frac{\iota g^2}{16\pi^2 f} a(x) W^+ \tilde{W}^- \\
+\ & N_{em} \frac{\iota e^2}{16\pi^2 f} a(x) F\tilde{F} + 2(N_0 - s_w^2 N_{em}) \frac{\iota e^2}{16\pi^2 f s_w c_w} a(x) F\tilde{Z} \\
+\ & (N_{L_3} - 2s_w^2 N_0 + s_w^4 N_{em}) \frac{\iota e^2}{16\pi^2 f s_w^2 c_w^2} a(x) Z\tilde{Z}
\end{aligned}$$

Where $N_{em} = N_{L3} + N_Y + N_m$ and $N_0 = N_{L3} + \frac{N_m}{2}$

This is consistent with Jeremie result


Thank you for the attention!

Back-up slide

Toy model for axion

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L \iota \not{D}_L \psi_L + \bar{\psi}_R \iota \not{D}_R \psi_R + |D_{\phi_1}\phi_1|^2 + |D_{\phi_2}\phi_1|^2 - (y\phi_1\bar{\psi}_L\psi_R + h.c.)$$
$$- \lambda_1(f_1^2 - |\phi_1|^2)^2 - \lambda_2(f_2^2 - |\phi_2|^2)^2$$

Gauge tranformation → $\psi_L \to e^{igq_L\theta(x)}\psi_L \qquad \psi_R \to e^{igq_R\theta(x)}\psi_R$

$\phi_i \to e^{igq_{\phi_i}\theta(x)}\phi_i, \quad q_{\phi_1} = q_L - q_R = 2\beta$

Axial transformation → $\psi_L \to e^{ia_L\theta}\psi_L, \quad \psi_R \to e^{-ia_L\theta}\psi_R,$

$\phi_i \to e^{ia_{\phi_i}\theta}\phi_i \quad q_L = -q_R \quad q_{\phi_1} = 2a_L$

The axial and gauge current, using $\phi_i = f_i e^{ia_i(x)/f_i}/\sqrt{2}$

$$J^\mu_A = a_L\overline{\psi}\gamma^\mu\gamma_5\psi + a_{\phi_1}f_1\partial_\mu a_1(x) + a_{\phi_2}f_2\partial_\mu a_2(x) - gA^\mu(q_{\phi_1}a_{\phi_1}f_1^2 + q_{\phi_2}a_{\phi_2}f_2^2)$$
$$J^\mu_G = -g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi + q_{\phi_1}gf_1\partial^\mu a_1(x) + q_{\phi_2}gf_2\partial^\mu a_2(x) - g^2A^\mu(q_{\phi_1}^2 f_1^2 + q_{\phi_2}^2 f_2^2)$$

Divergence of both current at classical level is zero. →

$$\partial_\mu J^\mu_G = 0$$
$$\partial_\mu J^\mu_A = 0$$

Toy model for axion

In $(a,\ \varphi)$ basis

$$a(x) = ca_1(x) - sa_2(x)$$
$$\varphi(x) = sa_1(x) + ca_2(x)$$

Gauge tranformation → $a(x) \to a(x)$ $\qquad \varphi(x) \to \varphi(x) + f\theta_g(x)$

Axial transformation → $a(x) \to a(x) + f\theta$ $\qquad \varphi(x) \to \varphi(x)$

which lead the relation

$$q_{\phi_1} = \frac{sf}{f_1}, \quad q_{\phi_2} = \frac{cf}{f_2}, \quad a_{\phi_1} = \frac{cf}{f_1}, \quad a_{\phi_2} = \frac{-sf}{f_2}, \quad \text{where } f = \sqrt{(q_{\phi_1} f_1)^2 + (q_{\phi_2} f_2)^2}$$

The current in $(a,\ \varphi)$ basis

$$\begin{aligned} J^\mu_A &= a_L \bar{\psi}\gamma^\mu\gamma_5\psi + f\partial_\mu a(x) \\ J^\mu_G &= -g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi + gf\partial^\mu\varphi(x) - g^2 A^\mu f^2 \end{aligned}$$

Direct calculation of Anomaly using one-loop three point function

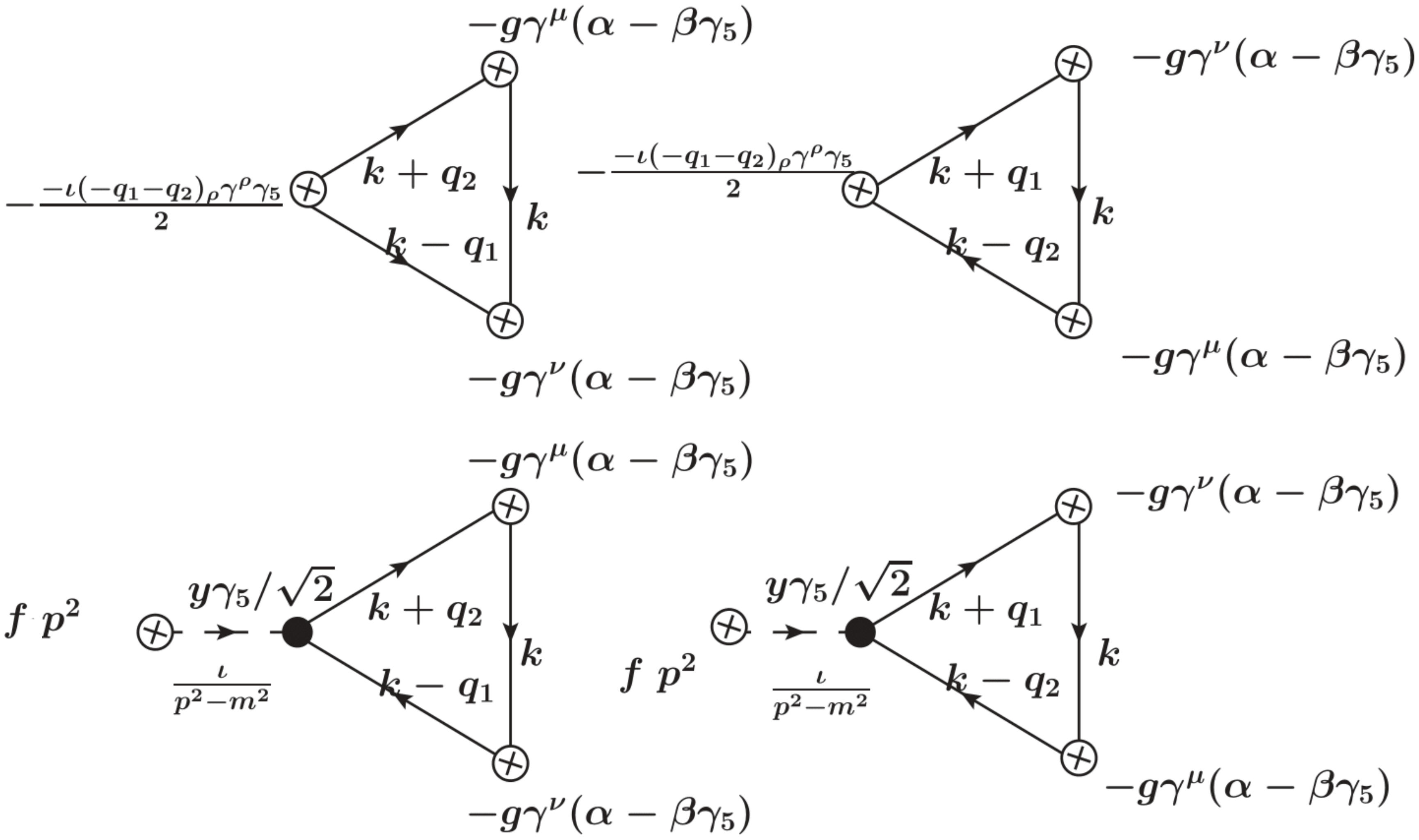
$$
\begin{aligned}
J_A^\mu &= a_L\overline{\psi}\gamma^\mu\gamma_5\psi + f\partial_\mu a(x) \\
J_G^\mu &= -g\bar{\psi}\gamma^\mu(\alpha-\beta\gamma_5)\psi + gf\partial^\mu\varphi(x) - g^2A^\mu f^2
\end{aligned}
$$

$$
\begin{aligned}
\partial_\rho\langle J_G^\rho J_G^\mu J_G^\nu\rangle &= \partial_\rho\langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle + \partial_\rho\langle J_G^{\rho\varphi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle \\
\partial_\rho\langle J_A^\rho J_G^\mu J_G^\nu\rangle &= \partial_\rho\langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle + \partial_\rho\langle J_A^{\rho a} J_G^{\mu\psi} J_G^{\nu\psi}\rangle \\
\partial_\mu\langle J_G^\rho J_G^\mu J_G^\nu\rangle &= \partial_\mu\langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle + \partial_\mu\langle J_A^{\rho\varphi} J_G^{\mu\varphi} J_G^{\nu\psi}\rangle \\
\partial_\nu\langle J_A^\rho J_G^\mu J_G^\nu\rangle &= \partial_\nu\langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi}\rangle + \partial_\nu\langle J_A^{\rho\varphi} J_G^{\mu\psi} J_G^{\nu\varphi}\rangle
\end{aligned}
$$

→ This will give us gauge and axial anomaly.

$$
\begin{aligned}
&\partial_\rho\langle J_G^{\rho a} J_G^{\mu a} J_G^{\nu a}\rangle = 0 \qquad &&\partial_\rho\langle J_G^{\rho\psi} J_G^{\mu a} J_G^{\nu\psi}\rangle = 0, \\
&\partial_\rho\langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu a}\rangle = 0 \qquad &&\partial_\rho\langle J_G^{\rho a} J_G^{\mu a} J_G^{\nu\psi}\rangle = 0, \\
&\partial_\rho\langle J_G^{\rho a} J_G^{\mu\psi} J_G^{\nu a}\rangle = 0 \qquad &&\partial_\rho\langle J_G^{\rho\psi} J_G^{\mu a} J_G^{\nu a}\rangle = 0,
\end{aligned}
$$

Direct calculation of Anomaly using one-loop three point function



Direct calculation of Anomaly using one-loop three point function

$$\text{Tr}\left[\not{p}\gamma_5(\not{k} - \not{q}_1 + m)\gamma^\nu(\alpha - \beta\gamma_5)(\not{k} + m)\gamma^\mu(\alpha - \beta\gamma_5)(\not{k} + \not{q}_2 + m)\right]$$

Using $\not{p}\gamma_5 = (\not{k} + \not{q}_2 - m)\gamma_5 + \gamma_5(\not{k} - \not{q}_1 - m) + 2m\gamma_5$

$$\begin{aligned}
&((k+q_2)^2 - m^2)\,\text{Tr}\left[\gamma_5(\not{k} - \not{q}_1 + m)\gamma^\nu(\alpha - \beta\gamma_5)(\not{k} + m)\gamma^\mu(\alpha - \beta\gamma_5)\right] \\
+\;&((k-q_1)^2 - m^2)\,\text{Tr}\left[\gamma_5\gamma^\nu(\alpha - \beta\gamma_5)(\not{k} + m)\gamma^\mu(\alpha - \beta\gamma_5)(\not{k} + \not{q}_2 + m)\right] \\
+\;&2m\,\text{Tr}\left[\gamma_5(\not{k} - \not{q}_1 + m)\gamma^\nu(\alpha - \beta\gamma_5)(\not{k} + m)\gamma^\mu(\alpha - \beta\gamma_5)(\not{k} + \not{q}_2 + m)\right]
\end{aligned}$$

↓

This trace is exactly same as trace of axion

Quantum correction in divergence of axial current

$$\partial_\rho \langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

$$+ \quad a_{\phi_1}\frac{g^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) + \text{mass term}\right]$$

$$= -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta - if\mathcal{M}_{\mu\nu}(a \to Z\ Z)$$

$$\partial_\rho \langle J_A^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -a_{\phi_1}\left(1 + \frac{m_a^2}{p^2 - m_a^2}\right)\frac{g^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) + \text{mass term}\right]$$

$$= if\left(1 + \frac{m_a^2}{p^2 - m_a^2}\right)\mathcal{M}_{\mu\nu}(a \to Z\ Z)$$

$$\partial_\rho \langle J_A^{\rho} J_G^{\mu} J_G^{\nu} \rangle = -a_{\phi_1}(c_1 - c_2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

where (c_1, c_2) are momentum ambiguity parameter.

Quantum correction in divergence of gauge current

$$\partial_\mu \langle J_A^{\rho\psi}(p) J_G^{\mu\phi}(q_2) J_G^{\nu\psi}(q_1)\rangle = a_{\phi_1}(c_1-2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\nu\rho\alpha\beta}(q_1)_\alpha(q_2)_\beta + a_{\phi_1}\frac{g^2\beta^2}{2\pi^2}\epsilon_{\nu\rho\alpha\beta}q_1^\alpha q_2^\beta\left[\frac{1}{3}+\frac{3m_a^2+7m_z^2}{120m^2}\right]$$

$$\partial_\mu \langle J_A^{\rho\psi}(p) J_G^{\mu\phi}(q_2) J_G^{\nu\psi}(q_1)\rangle = -a_{\phi_1}\frac{g^2\beta^2}{2\pi^2}\epsilon_{\nu\rho\alpha\beta}q_1^\alpha q_2^\beta\left[\frac{1}{3}+\frac{3m_a^2+7m_z^2}{120m^2}\right]$$

$$\partial_\mu \langle J_A^{\rho}(p) J_G^{\mu}(q_2) J_G^{\nu}(q_1)\rangle = a_{\phi_1}(c_1-2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\nu\rho\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

$$\partial_\nu \langle J_A^{\rho\psi}(p) J_G^{\mu\psi}(q_2) J_G^{\nu\psi}(q_1)\rangle = a_{\phi_1}(c_2+2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\rho\alpha\beta}(q_1)_\alpha(q_2)_\beta - a_{\phi_1}\frac{g^2\beta^2}{2\pi^2}\epsilon_{\mu\rho\alpha\beta}q_1^\alpha q_2^\beta\left[\frac{1}{3}+\frac{3m_a^2+7m_z^2}{120m^2}\right]$$

$$\partial_\nu \langle J_A^{\rho\psi}(p) J_G^{\mu\psi}(q_2) J_G^{\nu\phi}(q_1)\rangle = a_{\phi_1}\frac{g^2\beta^2}{2\pi^2}\epsilon_{\mu\rho\alpha\beta}q_1^\alpha q_2^\beta\left[\frac{1}{3}+\frac{3m_a^2+7m_z^2}{120m^2}\right]$$

$$\partial_\nu \langle J_A^{\rho}(p) J_G^{\mu}(q_2) J_G^{\nu}(q_1)\rangle = a_{\phi_1}(c_2+2)\frac{g^2(\alpha^2+\beta^2)}{16\pi^2}\epsilon^{\mu\rho\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

Quantum correction in divergence of gauge current

$$\partial_\rho \langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -q_{\phi_1}(c_1 - c_2)\frac{3g^3(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

$$+ \quad q_{\phi_1}\frac{g^3}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) + \text{mass term}\right]$$

$$= -q_{\phi_1}(c_1 - c_2)\frac{3g^3(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta - if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z)$$

$$\partial_\rho \langle J_G^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -q_{\phi_1}\frac{g^3}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) + \text{mass term}\right] = if\mathcal{M}_{\mu\nu}(\varphi \to Z\ Z)$$

$$\partial_\rho \langle J_G^{\rho} J_G^{\mu} J_G^{\nu} \rangle = -q_{\phi_1}(c_1 - c_2)\frac{3g^3(\alpha^2 + \frac{1}{3}\beta^2)}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}(q_1)_\alpha(q_2)_\beta$$

Effective Lagrangian for Axion decay in SM gauge theory

$$\mathcal{L}_{agg}^{SM} \propto N_c \frac{g_s^2}{16\pi^2 f} a(x) G^a \tilde{G}^a + N_{Li} \frac{g^2}{16\pi^2 f} a(x) W^i \tilde{W}^i$$
$$+ N_Y \frac{g'^2}{16\pi^2 f} a(x) B\tilde{B} + N_m \frac{gg'}{16\pi^2 f} a(x) W^3 \tilde{B}$$

Derivative coupling will generate mixing term

$$N_c = \sum_{f=u,d} T(R_c^f)\chi_A^f(\alpha_c^2 + \frac{1}{3}\beta_c^2) \qquad N_Y = \sum_{f=u,d,e} N_c^f \chi_A^f(\alpha_y^2 + \frac{1}{3}\beta_y^2)$$

$$N_{L1,L2} = \sum_{f=u,d,e,\nu} N_c^f \chi_A^f(\alpha_l^2 + \frac{1}{3}\beta_l^2) \qquad N_{L3} = \sum_{f=u,d,e} N_c^f \chi_A^f(\alpha_l^2 + \frac{1}{3}\beta_l^2)$$

$$N_m = \sum_{f=u,d,e} 2T_3^f N_c^f \left(-\chi_A^f \frac{2\beta_l\beta_y}{3} + \chi_V^f(\alpha_y\beta_l + \alpha_l\beta_y)\right)$$
$$+ 2T_3^f N_c^f \left[-\chi_A^\nu(\alpha_l\alpha_y + \beta_l\beta_y) + \chi_V^\nu(\alpha_y\beta_l + \alpha_l\beta_y)\right]$$

Toy model for axion

In $(a,\, \varphi)$ basis

$$a(x) = c a_1(x) - s a_2(x)$$
$$\varphi(x) = s a_1(x) + c a_2(x)$$

Gauge tranformation → $a(x) \to a(x)$ $\qquad$ $\varphi(x) \to \varphi(x) + f\theta_g(x)$

Axial transformation → $a(x) \to a(x) + f\theta$ $\qquad$ $\varphi(x) \to \varphi(x)$

which lead the relation

$$q_{\phi_1} = \frac{sf}{f_1}, \quad q_{\phi_2} = \frac{cf}{f_2}, \quad a_{\phi_1} = \frac{cf}{f_1}, \quad a_{\phi_2} = \frac{-sf}{f_2}, \quad \text{where } f = \sqrt{(q_{\phi_1} f_1)^2 + (q_{\phi_2} f_2)^2}$$

The current in $(a,\, \varphi)$ basis

$$\begin{aligned} J_A^\mu &= a_L \overline{\psi}\gamma^\mu\gamma_5\psi + f\partial_\mu a(x) \\ J_G^\mu &= -g\bar{\psi}\gamma^\mu(\alpha - \beta\gamma_5)\psi + gf\partial^\mu\varphi(x) - g^2 A^\mu f^2 \end{aligned}$$

The divergence of current in massless case and heavy mass limit

Using $c_1 = -c_2 = 2$ and $m \to \infty$

Divergence of axial current

$$\partial_\rho \langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -a_{\phi_1} \frac{g^2}{4\pi^2} \left(\frac{2\beta^2}{3} \right) \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta$$

$$\begin{aligned} \partial_\rho \langle J_A^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle &= -a_{\phi_1} \frac{g^2}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) \right] \\ &= i f \mathcal{M}_{\mu\nu}(a \to Z\ Z) \end{aligned}$$

Divergence of gauge current

$$\partial_\rho \langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -q_{\phi_1} \frac{g^3 2(\alpha^2 + \frac{1}{3}\beta^2)}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha (q_2)_\beta$$

$$\begin{aligned} \partial_\rho \langle J_G^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle &= -q_{\phi_1} \frac{g^3}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \left[(\alpha^2 + \frac{\beta^2}{3}) \right] \\ &= i f \mathcal{M}_{\mu\nu}(\varphi \to Z\ Z) \end{aligned}$$

$$q_{\phi_1} = 2\beta$$

Using $c_1 = -c_2 = 2$ and exact $m = 0$

$\mathcal{A}_{axial}$

$$\partial_\rho \langle J_A^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -a_{\phi_1} \frac{g^2(\alpha^2 + \beta^2)}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha (q_2)_\beta$$

$$\partial_\rho \langle J_A^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = 0$$

$\mathcal{A}_g$

$$\partial_\rho \langle J_G^{\rho\psi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = -q_{\phi_1} \frac{3g^3(\alpha^2 + \frac{1}{3}\beta^2)}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} (q_1)_\alpha (q_2)_\beta$$

$$\partial_\rho \langle J_G^{\rho\phi} J_G^{\mu\psi} J_G^{\nu\psi} \rangle = 0$$

❖ Axion decay is zero for completely massless fermions.