Naturalness and Hierarchy. Wilsonian and dimensional renormalization

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Renormalization and renormalization group

Steven Weinberg, **Why the Renormalization Group Is a Good Thing**

"I think that this in the end is what the renormalization group is all about. It's a way of satisfying the Third Law of Progress in Theoretical Physics, which is that **you may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones you'll be sorry**."

Renormalization and renormalization group

RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

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There is a strong intuitive understanding of renormalization, due to Wilson, in terms of the scaling of effective lagrangians. We show that this can be made the basis for a proof of perturbative renormalization. We first study renormalizability in the language of renormalization group flows for a toy renormalization group equation. We then derive an exact renormalization group equation for a four-dimensional $\lambda\phi^4$ theory with a momentum cutoff. We organize the cutoff dependence of the effective lagrangian into relevant and irrelevant parts, and derive a linear equation for the irrelevant part. A lengthy but straightforward argument establishes that the piece identified as irrelevant actually is so in perturbation theory. This implies renormalizability. The method extends immediately to any system in which a momentum-space cutoff can be used, but the principle is more general and should apply for any physical cutoff. Neither Weinberg's theorem nor arguments based on the topology of graphs are needed.

Theory contains an ultimate scale $\Lambda_{phys} \Rightarrow \mathcal{L}_{\Lambda_{phys}}$

Above Λ_{phys} : UV completion needed

Below Λ_{phys} : ok $\mathcal{L}_{\Lambda_{phys}}$ Effective Field Theory

. . . But it seems we have some Problems . . .

Naturalness and Hierarchy problem

- The Higgs mass m_H^2 receives an **unnatural** enormous contribution from the quantum fluctuation, $\Delta m_H^2 \sim \Lambda_{phys}^2$

  Physically : left-over of its UV completion

- Large hierarchy between $m_H^2(\mu_F)$ and $m_H^2(\Lambda_{phys})$

(from now on $\Lambda_{phys} \to \Lambda$)

Useful pedagogical example: ϕ^4 theory in d-dimensions

d = **integer dimension (no dim reg)**

- Wilsonian Effective Action: $S_k[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V_k(\phi)\right]$

Wilson-Polchinski Equation

$$k\frac{\partial}{\partial k} V_k(\phi) = -\frac{k^d}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \ln\left(\frac{k^2 + V_k''(\phi)}{k^2}\right)$$

- UV boundary: $V_\Lambda(\phi) \equiv V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\mu^{4-d}\lambda_0}{4!}\phi^4$

Approximating $V_k(\phi)$ in the rhs as: $V_k(\phi) \to V_0(\phi)$:

$$V_{1l}(\phi) = V_0(\phi) + \underbrace{\frac{1}{2}\int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0\,\phi^2}{k^2}\right)}_{\delta V(\phi)}$$

One-loop effective potential

Radiative correction $\delta V(\phi)$

Radiative correction

$$\delta V(\phi) = \frac{1}{2} \int^{(\Lambda)} \frac{d^d k}{(2\pi)^d} \ln\left(1 + \frac{M^2(\phi)}{k^2}\right) \equiv \delta V_1(\phi) + \delta V_2(\phi)$$

where $M^2(\phi) \equiv m_0^2 + \frac{1}{2}\mu^{4-d}\lambda_0\,\phi^2$

$$\delta V_1(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}} \int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\,(1-t)^{\frac{d}{2}-1}\,t^{-\frac{d}{2}}$$

$$\delta V_2(\phi) \equiv \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)} \left(\frac{\Lambda}{\mu}\right)^{d} \ln\left(1 + \frac{M^2(\phi)}{\Lambda^2}\right)$$

Calculating $\delta V(\phi)$

For any **integer** d:

$$\delta V_1(\phi) = \frac{\mu^d}{d(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{d}{2}}\int_{\frac{M^2}{M^2+\Lambda^2}}^{1} dt\, t^{-\frac{d}{2}}(1-t)^{\frac{d}{2}-1} = \lim_{z\to d}\left[A_1(z) - A_2(z)\right]$$

where z is **complex**, and

$$A_1(z) \equiv F(z)\cdot \overline{B}\left(1-\frac{z}{2},\frac{z}{2}\right) \qquad A_2(z) \equiv F(z)\cdot \overline{B}_i\left(1-\frac{z}{2},\frac{z}{2};\frac{M^2(\phi)}{M^2(\phi)+\Lambda^2}\right)$$

$$F(z) \equiv \frac{\mu^z}{z(4\pi)^{\frac{z}{2}}\Gamma\left(\frac{z}{2}\right)}\left(\frac{M^2(\phi)}{\mu^2}\right)^{\frac{z}{2}}$$

$\overline{B}$ and $\overline{B}_i$ are (the analytic extensions of) the Beta functions, that **separately** have poles for $z = 2, 4, 6, \ldots$

$\delta V_1(\phi)$ **finite** $\Rightarrow$ **the poles of** A_1 **and** A_2 **have to cancel each other**

Cancellation of poles: example $d = 4$

In $d = 4$ dimensions ($z \equiv 4 - \epsilon$), expanding in powers of ϵ and M^2/Λ^2

$$A_1(4-\epsilon) = \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)}$$

$$A_2(4-\epsilon) = -\frac{\mu^{-\epsilon}}{64\pi^2}\left[M^2(\phi)\right]^2\left(\frac{\Lambda^2}{M^2(\phi)} - \log\frac{\Lambda^2}{M^2(\phi)}\right)$$
$$+ \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \cancel{\mathcal{O}(\epsilon)} + \cancel{\mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)}$$

Remember: $\delta V_1(\phi) = \lim_{\epsilon\to 0}\left[A_1(4-\epsilon) - A_2(4-\epsilon)\right]$. Adding $\delta V_2(\phi)$

$$\delta V(\phi) = \delta V_1 + \delta V_2 = \frac{\Lambda^2 M^2(\phi)}{32\pi^2} - \frac{\left[M^2(\phi)\right]^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2(\phi)} + \frac{1}{2}\right) + \cancel{\mathcal{O}\left(\frac{\phi^6}{\Lambda^2}\right)}$$

$$\Rightarrow \quad V_{1l}(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 + \frac{\Lambda^2 M^2}{32\pi^2} - \frac{\left(M^2\right)^2}{64\pi^2}\left(\ln\frac{\Lambda^2}{M^2} + \frac{1}{2}\right)$$

With $\quad \Omega_0 = \Omega + \delta\Omega_\Lambda \quad , \quad m_0^2 = m^2 + \delta m_\Lambda^2 \quad , \quad \lambda_0 = \lambda + \delta\lambda_\Lambda$

Effective Potential

and $\delta\Omega_\Lambda = -\frac{\Lambda^2 m^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$; $\delta m^2_\Lambda = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$

$$\delta\lambda_\Lambda = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right]$$

Renormalized One-Loop Effective Potential (take $\Omega = 0$)

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

No reference whatsoever to ϵ (of course!)

Recap of Dim Reg. Derivation of $V_{1l}(\phi)$

- $\delta V(\phi)$ in **dimensional regularization**. $d \to$ complex value, $d \equiv 4 - \epsilon$:

$$\delta V(\phi) \to \delta V_\epsilon(\phi) \equiv -\frac{\mu^{4-\epsilon}}{2(4\pi)^{2-\frac{\epsilon}{2}}} \left(\frac{M^2(\phi)}{\mu^2}\right)^{2-\frac{\epsilon}{2}} \overline{\Gamma}\left(\frac{\epsilon}{2} - 2\right)$$

$$= \frac{\mu^{-\epsilon}\left[M^2(\phi)\right]^2}{64\pi^2}\left(-\frac{2}{\epsilon} + \gamma + \ln\frac{M^2(\phi)}{4\pi\mu^2} - \frac{3}{2}\right) + \mathcal{O}(\epsilon)$$

$\overline{\Gamma}(-d/2)$ defined for any complex $d \neq 2, 4, 6, \ldots$

- Counterterms in $\overline{MS}$ scheme $\left(\overline{\epsilon} \equiv \epsilon\left(1 + \frac{\epsilon}{2}\ln\frac{e^\gamma}{4\pi}\right)\right)$:

$$\delta\Omega_\epsilon = \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon} \quad , \quad \delta m_\epsilon^2 = \frac{\lambda m^2}{16\pi^2\overline{\epsilon}} \quad , \quad \delta\lambda_\epsilon = \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

- Renormalized Effective Potential (take again $\Omega = 0$) **as before**

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

You probably already guess what DR is and does ... but let us first hear from the Literature ...

"Dim Reg" and "elimination of modes" (Wilson)

Views on Dim Reg and Wilson

1) Typical textbook statement . . . "Dimensional Regularization has no direct physical interpretation" (J. Zinn-Justin - **Quantum field theory of critical phenomena**)

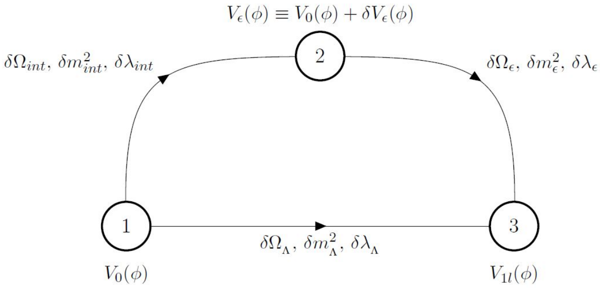
2) Recent ideas (gaining lot of followers)

"Maybe power divergences vanish because the ultimate unknown physical cut-off behaves like dimensional regularization" (M. Farina, D. Pappadopulo and A. Strumia, JHEP 08 (2013) 022)

"Wilsonian computation techniques attribute **physical meaning** to momentum shells of loop integrals" . . . "The naturalness problem can be more generically formulated as a problem of the **Effective Theory Ideology**" (A. Salvio and A. Strumia, JHEP 06 (2014) 080)

Accordingly **DR** should be **endowed with special physical properties** that make it the **correct way** to calculate the quantum fluctuations ... while **Wilson** ... **incorrect** ...

Dim Reg. Physical Meaning? Special Physical Properties?



$$V_0(\phi) = \Omega_0 + \frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 \ ; \ \delta V(\phi) = \lim_{z\to 4}[A_1(z) - A_2(z)] + \frac{\Lambda^4}{64\pi^4}\ln\left(1 + \frac{M^2}{\Lambda^2}\right)$$

$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left[\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right]$$

Dim Reg. Physical Meaning? Special Physical Properties?

Below you see in detail how DR secretly realizes the fine-tuning

$$\delta\Omega_{int} = -\frac{\Lambda^2 m^2}{32\pi^2} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{m^4}{32\pi^2\overline{\epsilon}}\mu^{-\epsilon}$$

$$\delta m^2_{int} = -\frac{\lambda\Lambda^2}{32\pi^2} + \frac{\lambda m^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{\lambda m^2}{16\pi^2\overline{\epsilon}}$$

$$\delta\lambda_{int} = \frac{3\lambda^2}{32\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu^2}\right) - 1\right] - \frac{3\lambda^2}{16\pi^2\overline{\epsilon}}$$

DR has a physical meaning. It implements the Wilsonian iterative elimination of modes for the inclusion of the quantum fluctuations in the Effective Theory, and (**secretly**) realizes the fine-tuning

DR shortcut: "Bubble (3)" is reached starting from "Bubble (2)", while the fine-tuning step "Bubble (1)" $\rightarrow$ "Bubble (2)" is skipped (secretly realized)

Naturalness, Wilson, Dimensional Regularization

What should we then say on those attempts to solve the Naturalness/Hierarchy problem within DR?

Flourishing literature

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Naturalness, Wilson, Dimensional Regularization

What should we then say on those attempts to solve the Naturalness/Hierarchy problem with DR?

- **Classically Scale Invariant BSM**. The theory does not possess mass or length scales ⇒ only dimension four operators
- Dimensional Regularization used ⇒ Scale Invariance only softly broken ⇒ apparently **no fine-tuning needed** . . . seems good . . .
- . . . But . . . we have just shown . . . DR **secretly realizes the fine-tuning**

⇒ **No way to solve the Naturalness/Hierarchy problem with DR**

Let us consider now attempts to solve the NH problem in a RG framework

"Wilson-Polchinski" versus "Perturbative-Renormalized" RG Equations

Scalar Theory : $\mathcal{L}_\Lambda = \frac{1}{2}(\partial_\mu \phi_\Lambda)^2 + \frac{1}{2} m_\Lambda^2 \phi_\Lambda^2 + \frac{\lambda_\Lambda}{4!}\phi_\Lambda^4$

Wilson-Polchinski RG Equations

$$\mu \frac{d\Omega}{d\mu} = -\frac{m^2\mu^2}{16\pi^2} + \frac{m^4}{32\pi^2} \quad ; \quad \mu \frac{dm^2}{d\mu} = -\frac{\lambda\mu^2}{16\pi^2} + \frac{\lambda m^2}{16\pi^2} \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}$$

$\mu \in [0, \Lambda]$ is the running scale. Λ is the UV boundary (physical cut-off)

Define: $\Omega_{\rm cr}(\mu) \equiv \frac{\widetilde{m}^2(\mu)}{16\pi^2}\, \mu\, \delta\mu$ and $\widetilde{\Omega}(\mu - \delta\mu) \equiv \Omega(\mu - \delta\mu) - \Omega_{\rm cr}(\mu)$

$m^2_{\rm cr}(\mu) \equiv \frac{\lambda(\mu)}{16\pi^2}\, \mu\, \delta\mu$ and $\widetilde{m}^2(\mu - \delta\mu) \equiv m^2(\mu - \delta\mu) - m^2_{\rm cr}(\mu)$

Perturbative-Renormalized RG Equations $(\delta\mu \to 0)$

$$\mu \frac{d\widetilde{\Omega}}{d\mu} = \frac{\widetilde{m}^4}{32\pi^2} = \beta_\Omega \quad ; \quad \mu \frac{d\widetilde{m}^2}{d\mu} = \frac{\lambda \widetilde{m}^2}{16\pi^2} = \widetilde{m}^2 \gamma_m \quad ; \quad \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} = \beta_\lambda$$

The Perturbative-Renormalized RG Equations **contain the fine-tuning**

Physically: **Tuning towards the Critical Surface**

RG and the Naturalness/Hierarchy problem

Attempts to solve the Naturalness/Hierarchy problem within RG

Flourishing literature

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- S. Mooij and M. Shaposhnikov, arXiv:2110.15925.

Perturbative-Renormalized RG equations in the Standard Model

Well-known Standard Model perturbative RG equations*

$$\mu \frac{d}{d\mu}\lambda_i = \beta_{\lambda_i} \qquad \mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

λ_i $(i = 1, \ldots, 5)$ are the SM couplings

*similar for SM extensions

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 1 : Quantum Gravity "**miracle**" G.F. Giudice, PoS EPS-HEP2013, 163 (2013)

$$m_H^2(\Lambda) \ll \Lambda^2$$

With the SM perturbative $\gamma_m \Rightarrow$

Apparently no Hierarchy Problem : $m_H^2(\Lambda) \sim m_H^2(\mu_F)$

... But ... remember ... m_H^2 in the above Eq. is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the RG Equation above

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 2 : "**Self-organized criticality**"

J. M. Pawlowski, M. Reichert, C. Wetterich and M. Yamada, Phys. Rev. D **99**, 086010 (2019)

Assumes Quantum Gravity might give a non-perturbative $\gamma_m \sim 2 \;\Rightarrow$

Hierarchy can be tolerated : $m_H^2(\Lambda) \gg m_H^2(\mu_F)$

. . . But . . . remember . . . m_H^2 is the **tuned mass** $\Rightarrow$

Fine-tuning encoded in the above RG Equation

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 3 : $m_H^2(\mu)$ from $\lambda(\mu)$ and $v(\mu)$ . . .

P. H. Chankowski, A. Lewandowski, K. A. Meissner and H. Nicolai, Mod. Phys. Lett. A **30**, 1550006 (2015)

M. Holthausen, K. S. Lim and M. Lindner, JHEP **02**, 037 (2012)

Apparently no large corrections : $m_H^2(\mu_F) \sim 125\,\mathrm{GeV}$

However, same problem as before . . . Tuning encoded in the RG equation for the vev $v(\mu)$ (equivalent to the above RG equation for $m_H^2(\mu)$)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Perturbative-Renormalized RG equations in the Standard Model

Attempt 4 : "Finite formulation" of QFT using RG equations *à la* Callan-Symanzik for the Green's functions . . .

S. Mooij and M. Shaposhnikov, arXiv:2110.05175

S. Mooij and M. Shaposhnikov, arXiv:2110.15925

Apparently no quadratic corrections for the mass m^2 of scalar particles

However . . . Tuning encoded in taking derivatives with respect to m^2 of the Green's functions, until they become finite

Callan has shown that this is just a way of implement the subtraction of Λ^2 and $\log \Lambda$ terms C. G. Callan, Jr., Conf. Proc. C **7507281**, 41-77 (1975)

$\Rightarrow$ **Can't solve the Hierarchy Problem**

Conclusions and Outlook

- No way to solve Naturalness/Hierarchy Problem with DR
- If you use the perturbative RG equations of the SM or of modified versions of it, you can't solve the Naturalness/Hierarchy Problem
- If you take derivatives of your loops integrals to get finite result, you can't pretend to solve the NH problem
- The inclusion of quantum fluctuations in the Effective Theory through the Wilson successive elimination of modes is **PHYSICALLY MANDATORY** (there is no crisis of Effective Field Theory Ideology) ... The consequences of this unavoidable physical starting point have to be further investigated

Thank you for your attention!

Backup Slides

$$\mu \frac{d}{d\mu} m_H^2 = m_H^2 \gamma_m$$

Attempt 5 : hierarchy between M_P and μ_F generated by an instanton configuration contributing to the vev of the Higgs field ...

M. Shaposhnikov and A. Shkerin, Phys. Lett. B **783**, 253 (2018)

M. Shaposhnikov and A. Shkerin, JHEP **10**, 024 (2018)

Apparently Hierarchy explained

however ... quantum corrections calculated with DR, and flow of the parameters studied with the perturbative RG flows ... same problems as before

$\Rightarrow$ **Can't solve the Hierarchy Problem**

- Flow of the theory parameters:

$$\Lambda \frac{d}{d\Lambda}\Omega_0 = -\frac{m_0^2\Lambda^2}{16\pi^2} + \frac{m_0^4}{32\pi^2} \qquad \Lambda \frac{d}{d\Lambda} m_0^2 = -\frac{\lambda_0\Lambda^2}{16\pi^2} + \frac{\lambda_0 m_0^2}{16\pi^2} \qquad \Lambda \frac{d}{d\Lambda}\lambda_0 = \frac{3\lambda_0^2}{16\pi^2}$$

- From the Wegner-Houghton equation for $d = 4$, inserting the expansion $U_k(\phi) = \Omega_k + \frac{1}{2}m_k^2\phi^2 + \frac{1}{4!}\lambda_k\phi^4 + \frac{1}{6!}\lambda_k^{(6)}\phi^6 + \ldots$ we have the flow equations:

$$k\frac{\partial \Omega_k}{\partial k} = -\frac{k^4}{16\pi^2}\log\left(\frac{k^2+m_k^2}{k^2}\right)$$

$$k\frac{\partial m_k^2}{\partial k} = -\frac{k^4}{16\pi^2}\frac{\lambda_k}{k^2+m_k^2}$$

$$k\frac{\partial \lambda_k}{\partial k} = \frac{k^4}{16\pi^2}\frac{3\lambda_k^2}{(k^2+m_k^2)^2}$$

- Under the condition $k^2 \gg m_k^2$, i.e. in the UV regime, they reduce to the bare parameters flow equations.

Critical term

- Finite difference RG equation for the mass:

$$m_0^2(\Lambda - \delta\Lambda) = m_0^2(\Lambda) + \frac{\delta\Lambda}{\Lambda}\frac{\lambda_0(\Lambda)}{16\pi^2}\Lambda^2 - \frac{\delta\Lambda}{\Lambda}\frac{\lambda_0(\Lambda)\, m_0^2(\Lambda)}{16\pi^2} + \mathcal{O}\left(\frac{\delta\Lambda^2}{\Lambda^2}\right)$$

- Subtracted mass parameter at the scale $\Lambda - \delta\Lambda$

$$\widetilde{m}^2(\Lambda - \delta\Lambda) \equiv m_0^2(\Lambda - \delta\Lambda) - m_{\rm cr}^2(\Lambda)$$

  where the *critical mass* $m_{\rm cr}^2$, and the boundary at Λ are given by

$$m_{\rm cr}^2(\Lambda) \equiv \frac{\lambda_0(\Lambda)}{16\pi^2}\Lambda\,\delta\Lambda \qquad \widetilde{m}^2(\Lambda) = m_0^2(\Lambda)$$

- In the limit $\delta\Lambda \to 0$ we recover the perturbative RG equations:

$$\beta_\Omega = \mu\frac{d\Omega}{d\mu} = \frac{m^4}{32\pi^2} \qquad \gamma_m = \frac{1}{m^2}\left(\mu\frac{dm^2}{d\mu}\right) = \frac{\lambda}{16\pi^2} \qquad \beta_\lambda = \mu\frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}$$

- The renormalized RG equations **contain the fine-tuning**: physically, this corresponds to a *tuning towards the critical surface*.

Attempts to a gauge invariant Wilsonian RG

- V. Branchina, K. Meissner and G. Veneziano, The Price of an exact, gauge invariant RG flow equation, Phys. Lett. B **574**, 319-324 (2003)
- S.P. de Alwis, Exact RG Flow Equations and Quantum Gravity, JHEP **03**, 118 (2018)