Gap-equations of radiative symmetry breaking in classically scale invariant models

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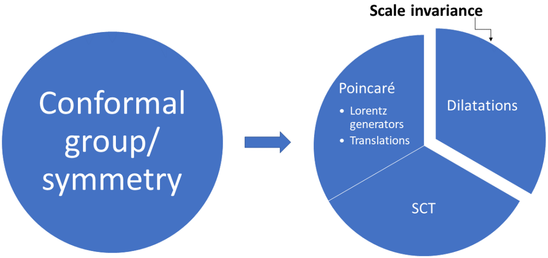
Conformal symmetry and scale invariance



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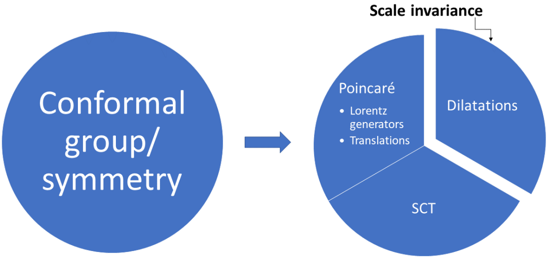

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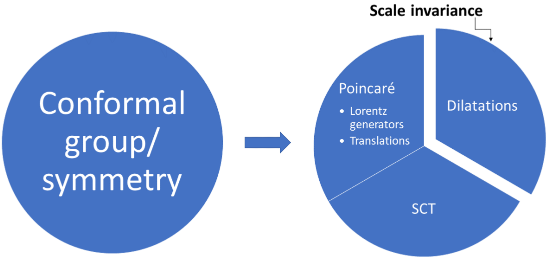

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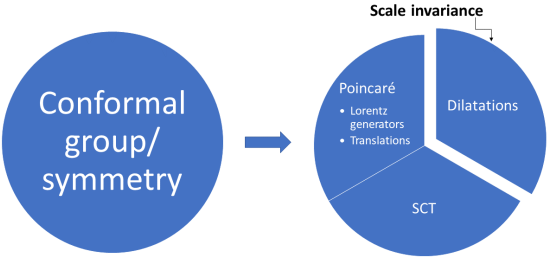

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- Extensions via (gauged) **scalars** [Hel+17]

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$\Rightarrow$ $V^{(0)} \propto \lambda_i$ and $V^{(1)} \propto \lambda_i^2,\ g_j^4$

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$\Rightarrow$ Set the dimensionless coupling at, e.g. M_{Pl} and use RG-flow to evolve until both criticality equations are fulfilled

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$\Rightarrow$ $\lambda_p(\varphi_0) \gtrless 0$ is important for classification

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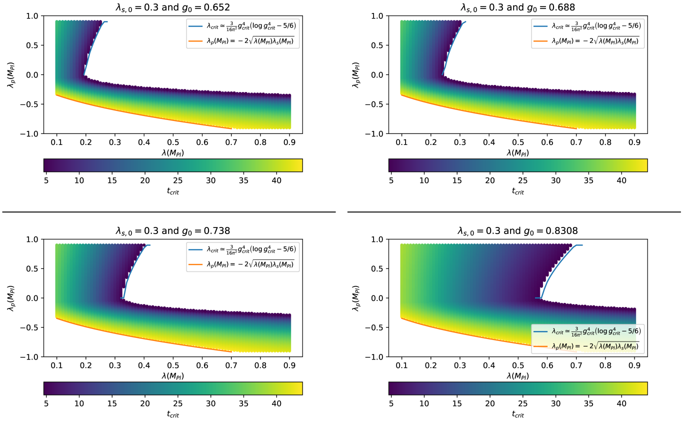

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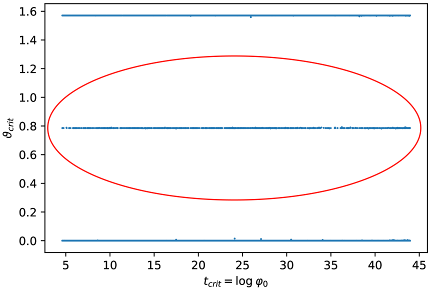

Figure: Two scalars ϕ_1, ϕ_2, with $U_1(1) \times U_2(1)$ and $q_1 = q_2$, $\lambda_{1,0} = \lambda_{2,0}$ and $g_{1,0} = g_{2,0} = 1$ and $\lambda_{p,0} > 0$.

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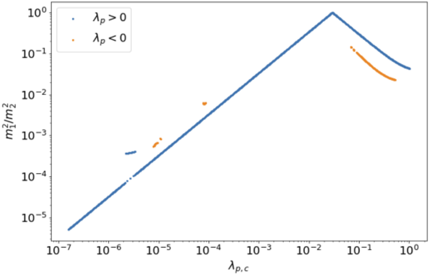

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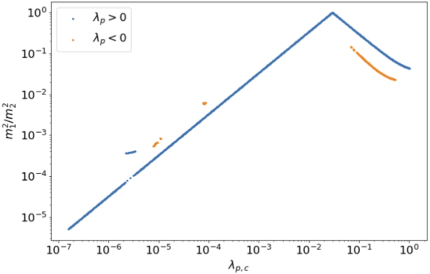

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- Conformal sym. is anomalous, i.e. regulator breaks scale inv. explicitly

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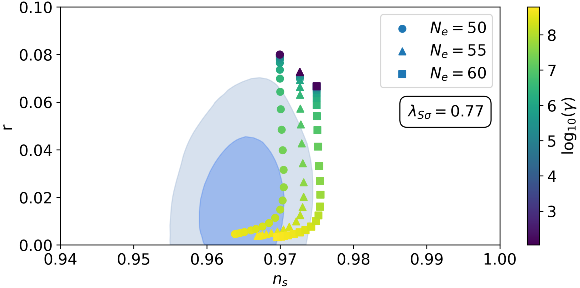
Thank you for your attention!

Backup: Probing via Inflation

- Including gravity ⇒ Inflation potential with 2 external scalars



from 2012.09706, by **Kubo, Kuntz, Lindner, Rezacek, Saake, Trautner**.

[Akr+20] Y. Akrami et al. "Planck 2018 results. X. Constraints on inflation". In: *Astron. Astrophys.* 641 (2020), A10. DOI: 10.1051/0004-6361/201833887. arXiv: 1807.06211 [astro-ph.CO].

[Ams+08] C. Amsler et al. "Review of Particle Physics". In: *Physics Letters B* 667.1 (2008). Review of Particle Physics, pp. 1–6. ISSN: 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2008.07.018. URL: https://www.sciencedirect.com/science/article/pii/S0370269308008435.

[CW73] Sidney R. Coleman and Erick J. Weinberg. "Radiative Corrections as the Origin of Spontaneous Symmetry Breaking". In: *Phys. Rev. D* 7 (1973), pp. 1888–1910. DOI: 10.1103/PhysRevD.7.1888.

[GW76] Eldad Gildener and Steven Weinberg. "Symmetry breaking and scalar bosons". In: *Phys. Rev. D* 13 (12 1976), pp. 3333–3341. DOI: 10.1103/PhysRevD.13.3333. URL: https://link.aps.org/doi/10.1103/PhysRevD.13.3333.

[Hel+17] Alexander J. Helmboldt et al. "Minimal conformal extensions of the Higgs sector". In: *JHEP* 07 (2017), p. 113. DOI: 10.1007/JHEP07(2017)113. arXiv: 1603.03603 [hep-ph].

[Higa] URL: https:////wiki.physik.uzh.ch/cms/latex:tikz.

[Higb] *Higgs Mass*. 2020. URL: https://pdg.lbl.gov/2020/listings/rpp2020-list-higgs-boson.pdf.

[Qui99] Mariano Quiros. *Finite temperature field theory and phase transitions*. 1999. DOI: 10.48550/ARXIV.HEP-PH/9901312. URL: https://arxiv.org/abs/hep-ph/9901312.