Generating the Weak Scale by Vector-like Quark Condensation

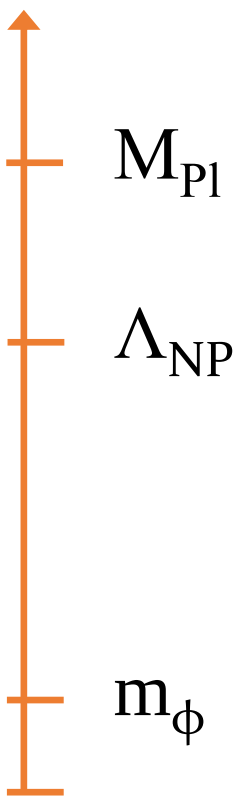
Sophie Klett

In collaboration with Manfred Lindner and Andreas Trautner (Max-Planck-Institut für Kernphysik)

based on arXiv:2205.15323

June 1st 2022, Planck Paris

The Electroweak Hierarchy Problem



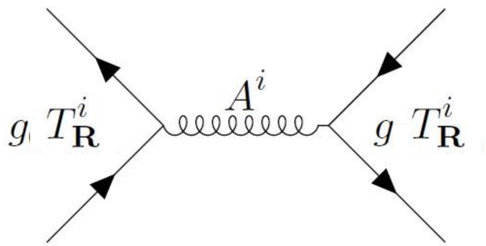
Electroweak (EW) scale sensitive to new physics (NP) quantum corrections:

$$\Delta m_\phi^2 \sim \Lambda_{NP}^2$$

protective symmetry →

- SUSY
- conformal symmetry (CS) → Non-perturbatively created scale in strongly coupled sector
- …

Old Ideas Revisited



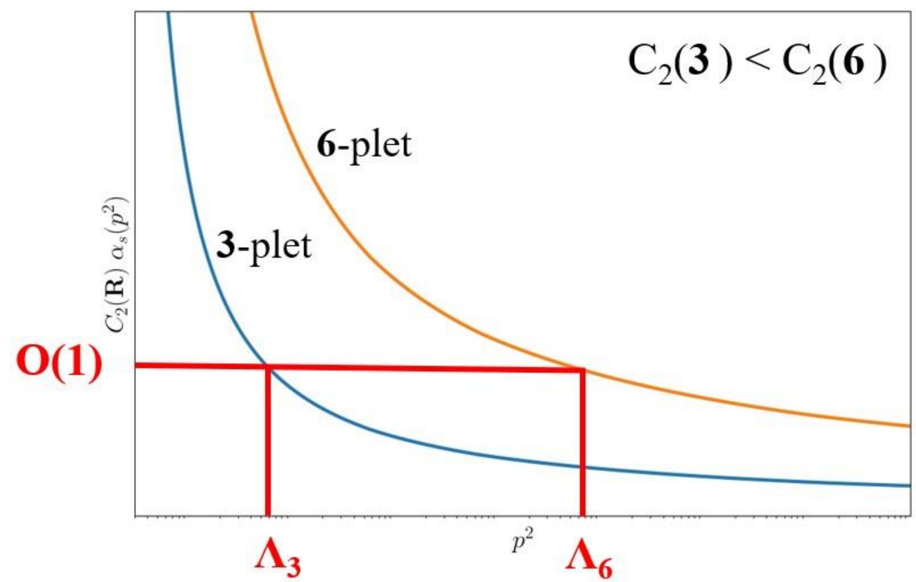
$$\sum_{i=1}^{8} T^i_{\mathbf{R}} T^i_{\mathbf{R}} = C_2(\mathbf{R})$$

→ Color potential proportional to Casimir Invariant of $SU(3)_c$ representation **R**

[Marciano '80], [Zoupanos '83], [Lüst et al '85]

$$C_2(\mathbf{R})\,\alpha_s(\Lambda) \gtrsim \mathcal{O}(1)$$

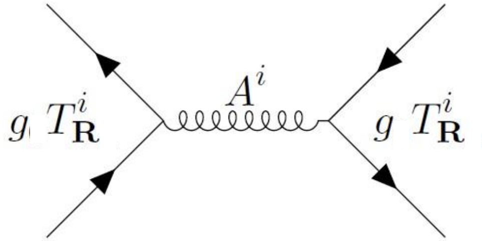
Conjecture: Condensation of high color chiral fermions generates larger scales
→ direct EWSB



Higgsless theory

Old Ideas Revisited

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK



$$\sum_{i=1}^{8} T^i_{\mathbf{R}} T^i_{\mathbf{R}} = C_2(\mathbf{R})$$

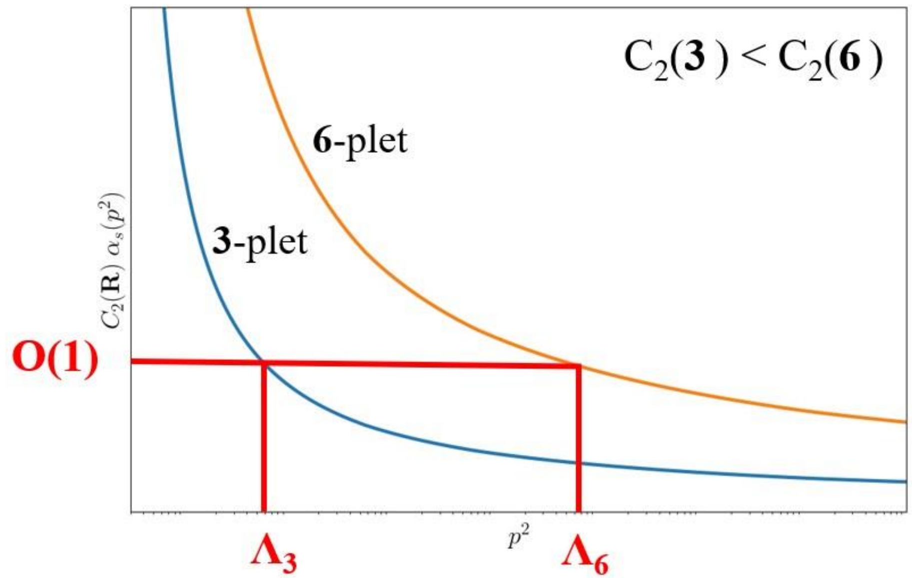
→ Color potential proportional to Casimir Invariant of $SU(3)_c$ representation **R**

[Marciano '80], [Zoupanos '83], [Lüst et al '85]

$$C_2(\mathbf{R})\, \alpha_s(\Lambda) \gtrsim \mathcal{O}(1)$$

Conjecture: Condensation of high color chiral fermions generates larger scales

→ direct EWSB



Higgsless theory

Higgs discovery

Old Ideas … New Ideas

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

NEW: Vector-like (VL) quark $\psi \sim (\mathbf{R}, 1, 0)$ + scalar singlet $S \sim (\mathbf{1}, 1, 0)$

Explicit VL mass > O(1 TeV)

Conformal scalar sector

$\langle \bar{\psi}\psi \rangle$ —Yukawa coupling→ $\langle S \rangle$ —Scalar portal→ $\langle \phi \rangle$

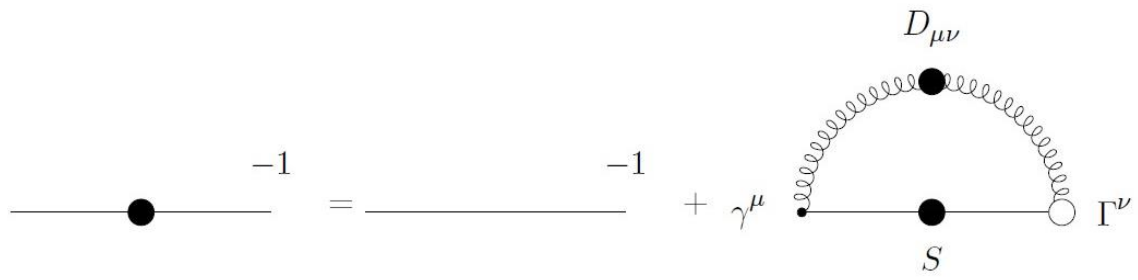
DχSB VEV EWSB

How does the condensate depend on …

…explicit VL quark mass ?

…SU(3) representation?

Dyson-Schwinger Equation



$$S^{-1}(p) = Z_2\,(\not p - Z_m\, m_\mu) - \mathrm{i}\, C_2(\mathbf{R})\, Z_{1\mathrm{F}}\, g^2 \int \frac{d^4k}{(2\pi)^4}\, \gamma_\mu\, S(k)\, \Gamma_\nu(k,p)\, D^{\mu\nu}(p-k)$$

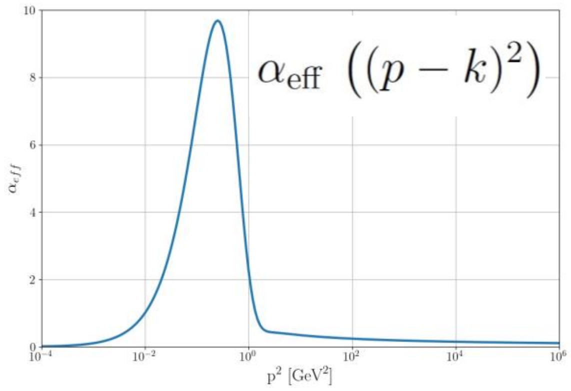
"rainbow approximation"

$$\Gamma_\mu(k,p) = \gamma_\mu$$

bare vertex

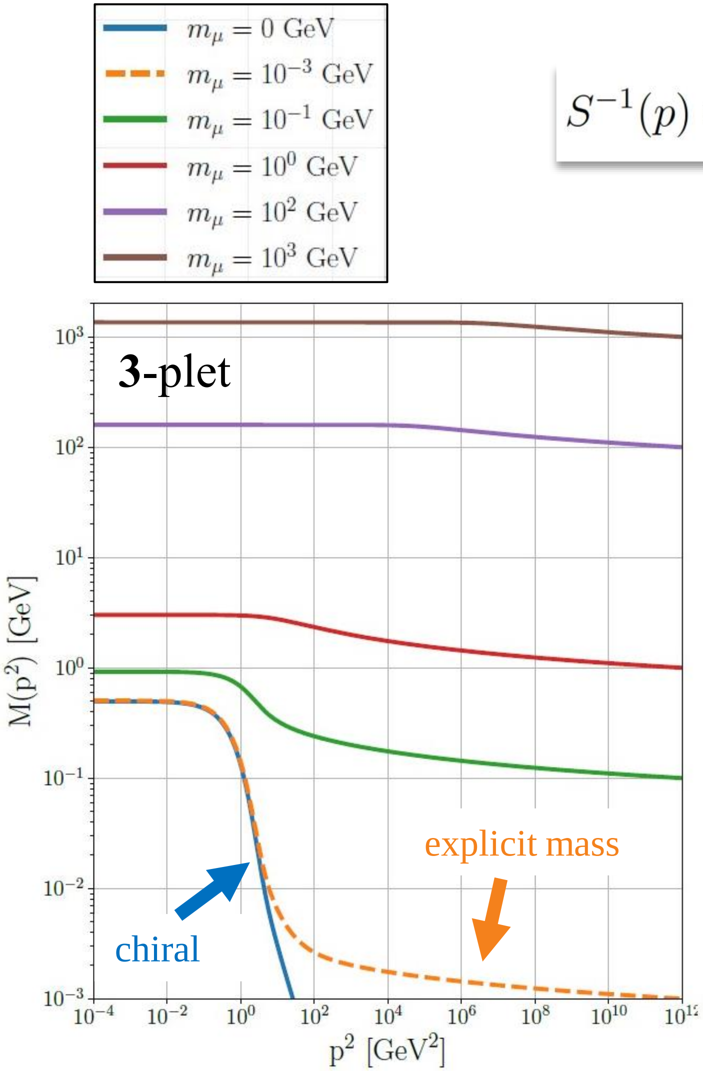
$$D^{\mu\nu}(p-k) \longrightarrow D_0^{\mu\nu}(p-k)$$

free gluon propagator



[Qin et al. , 1108.0603]

Fermion Condensate

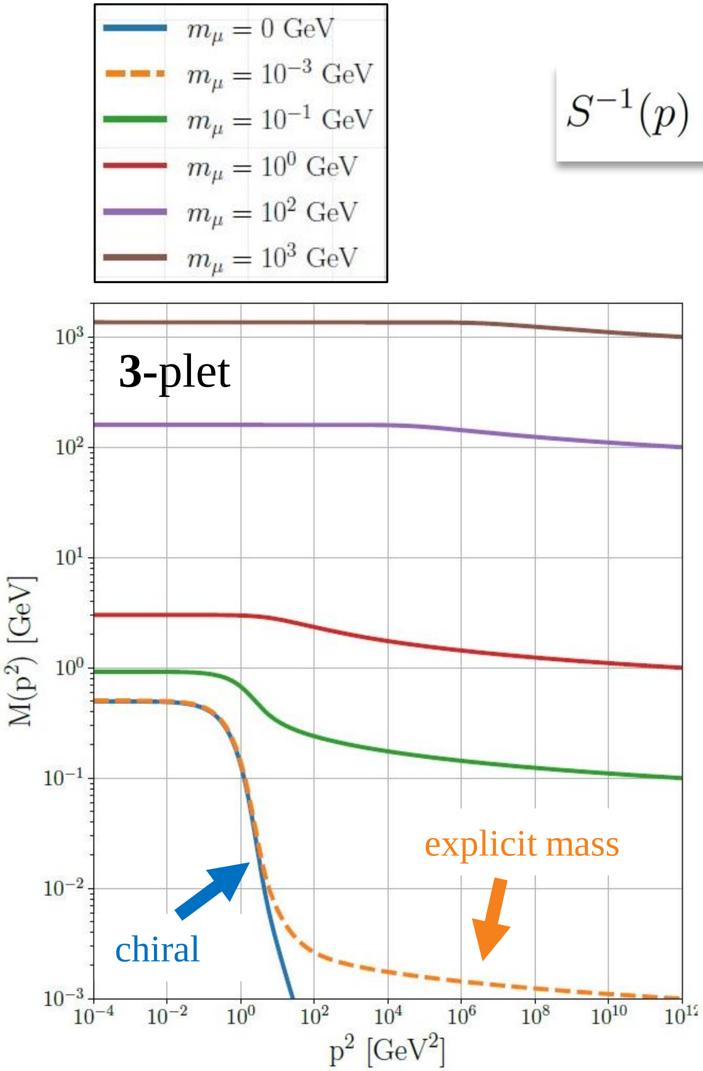


$$S^{-1}(p) \equiv Z^{-1}(p^2)\left[\not{p} - M(p^2)\right]$$

$Z(p^2)$: wavefunction renormalization

$M(p^2)$: dynamical mass function

Fermion Condensate



$$S^{-1}(p) \equiv Z^{-1}(p^2)\left[\not{p} - M(p^2)\right]$$

$Z(p^2)$: wavefunction renormalization

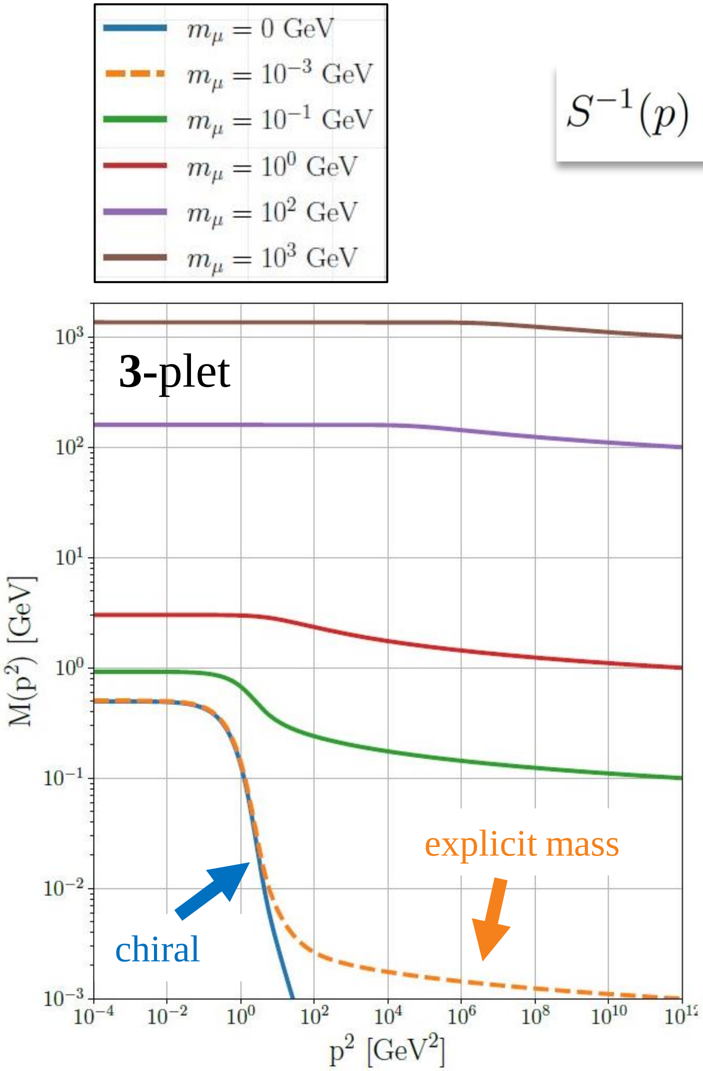
$M(p^2)$: dynamical mass function

Chiral condensate [Maris et al, nucl-th/9708029]

$$-\langle\overline{\psi}\psi\rangle_\mu = \lim_{x\to 0} \mathrm{Tr}\left[S(x)_{m_\mu=0}\right] = \frac{d(\mathbf{R})}{4\pi^2}\int dk^2 \frac{k^2\, Z(k^2)\, M(k^2)}{k^2 + M^2(k^2)}$$

$$\rightarrow\ -\langle\overline{\psi}\psi\rangle_{\mathrm{inv}} = (0.218\,\mathrm{GeV})^3$$

Fermion Condensate



$$S^{-1}(p) \equiv Z^{-1}(p^2)\left[\not{p} - M(p^2)\right]$$

$Z(p^2)$: wavefunction renormalization

$M(p^2)$: dynamical mass function

Chiral condensate

[Maris et al, nucl-th/9708029]

$$-\langle\overline{\psi}\psi\rangle_\mu = \lim_{x\to 0} \mathrm{Tr}\left[S(x)_{m_\mu=0}\right] = \frac{d(\mathbf{R})}{4\pi^2}\int dk^2 \frac{k^2\, Z(k^2)\, M(k^2)}{k^2 + M^2(k^2)}$$

$$\rightarrow \quad -\langle\overline{\psi}\psi\rangle_{\mathrm{inv}} = (0.218\,\mathrm{GeV})^3$$

Massive condensate

$Z(k^2) \approx 1,\ M(k^2) \sim m$

$$-\langle\overline{\psi}\psi\rangle_\mu \sim \int^{\Lambda^2} dk^2 \frac{k^2\, m}{k^2 + m^2} = m\Lambda^2 + m^3 \ln\left(\frac{m^2}{\Lambda^2 + m^2}\right)$$

Not well defined!

Fermion Condensate Beyond Chiral Limit

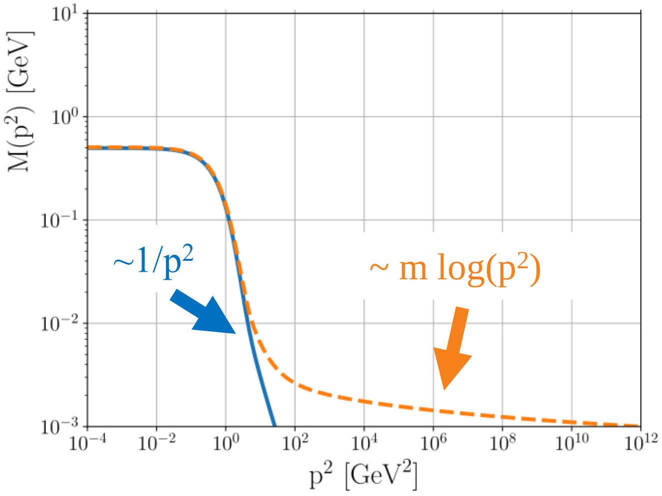
[Lane '74], [Politzer '76], [Wilson '69]

Operator Product Expansion (OPE)

$p \to \infty$

$$M(p^2) \simeq \hat{m}\left[\frac{1}{2}\ln\left(\frac{p^2}{\Lambda_{QCD}^2}\right)\right]^{-\gamma_m} - \frac{2\pi^2\,\gamma_m}{d(\mathbf{R})}\frac{\langle\overline{\psi}\psi\rangle_{\text{inv}}}{p^2}\left[\frac{1}{2}\ln\left(\frac{p^2}{\Lambda_{QCD}^2}\right)\right]^{\gamma_m-1}$$

running hard mass — dynamical part



Extract only dynamical part

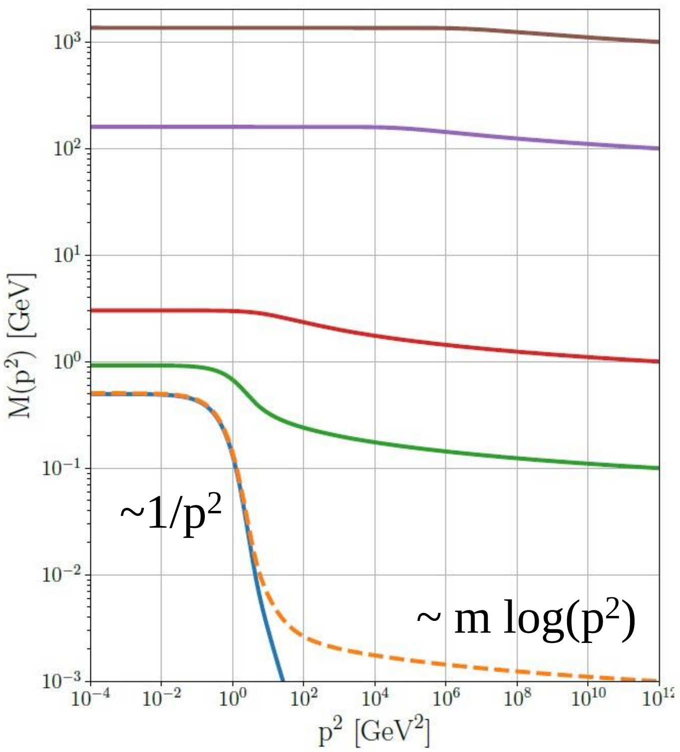
$$\tilde{M}(p^2) := \left(1 - m_\mu \frac{d}{dm_\mu}\right) M(p^2)$$

$$\hat{m} := m_\mu\left[\frac{1}{2}\ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)\right]^{\gamma_m}$$

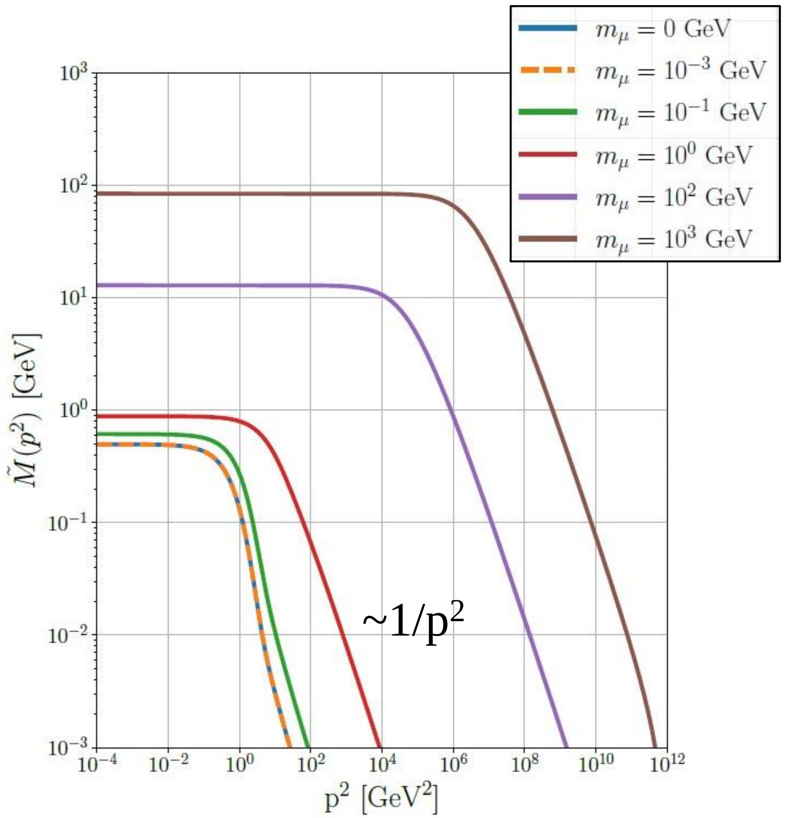
$$\langle\overline{\psi}\psi\rangle_{\text{inv}} := \langle\overline{\psi}\psi\rangle_\mu\left[\frac{1}{2}\ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)\right]^{-\gamma_m}$$

γ_m : anomalous mass dimension

Fermion Condensate Beyond Chiral Limit



Extract dynamical part



OPE

$$\tilde{M}(p^2) \overset{p\to\infty}{\simeq} -\frac{2\pi^2\gamma_m}{d(\mathbf{R})}\frac{C(m_\mu)}{p^2}\left[\frac{1}{2}\ln\left(\frac{p^2}{\Lambda_{\mathrm{QCD}}^2}\right)\right]^{\gamma_m-1}$$

Need this!

$$C(m_\mu) := \left(1 - m_\mu\frac{d}{dm_\mu}\right)\langle\overline{\psi}\psi\rangle^{m_\mu}_{\mathrm{inv}}$$

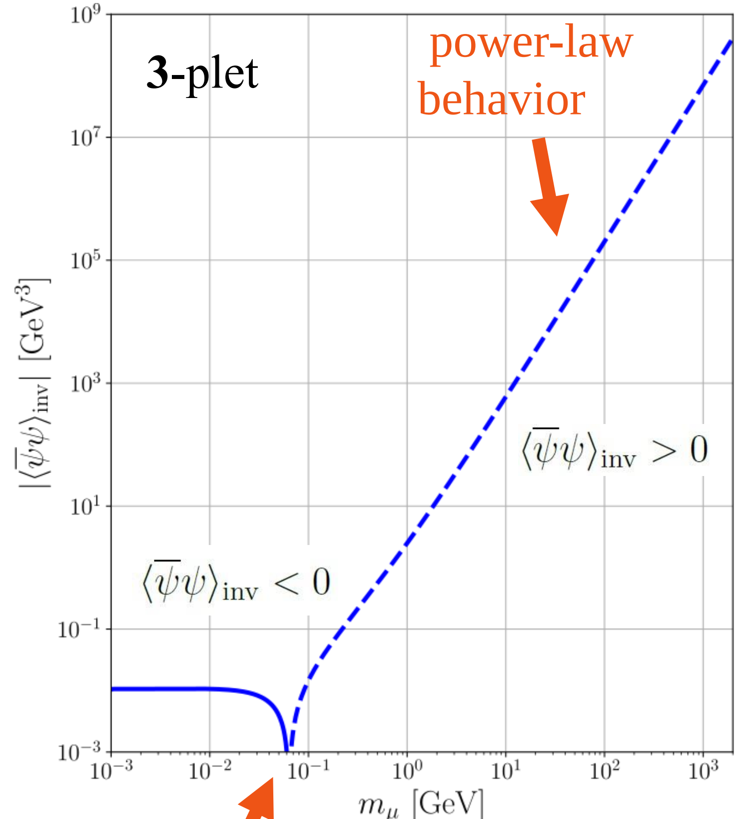
Fermion Condensate Beyond Chiral Limit

Rewrite as DE: $$-\frac{C(m_\mu)}{m_\mu^2} = \frac{d}{dm_\mu}\left[\frac{\langle\overline{\psi}\psi\rangle_{\rm inv}^{m_\mu}}{m_\mu}\right]$$

Initial conditions

$$\epsilon = 0.001\,\text{GeV} \approx \text{up-quark}$$

$$-\langle\overline{\psi}\psi\rangle_{\rm inv}^{m_\mu=\epsilon} = (0.218\,\text{GeV})^3$$



zero crossing $\sim \Lambda_{QCD}$

empirical relation for $m_\mu > 1$ GeV :

$$\langle\overline{\psi}\psi\rangle_{\rm inv} = (c_1\,\text{GeV})^{3-c_2} \times m_\mu^{c_2} \approx (3.7\,\text{GeV})^{1/2}\, m_\mu^{5/2}$$

@ $m_\mu = 1$ TeV :

$$\langle\overline{\psi}\psi\rangle_{\rm inv} \approx (415\,\text{GeV})^3$$

Exotic Quarks in High Color Representations

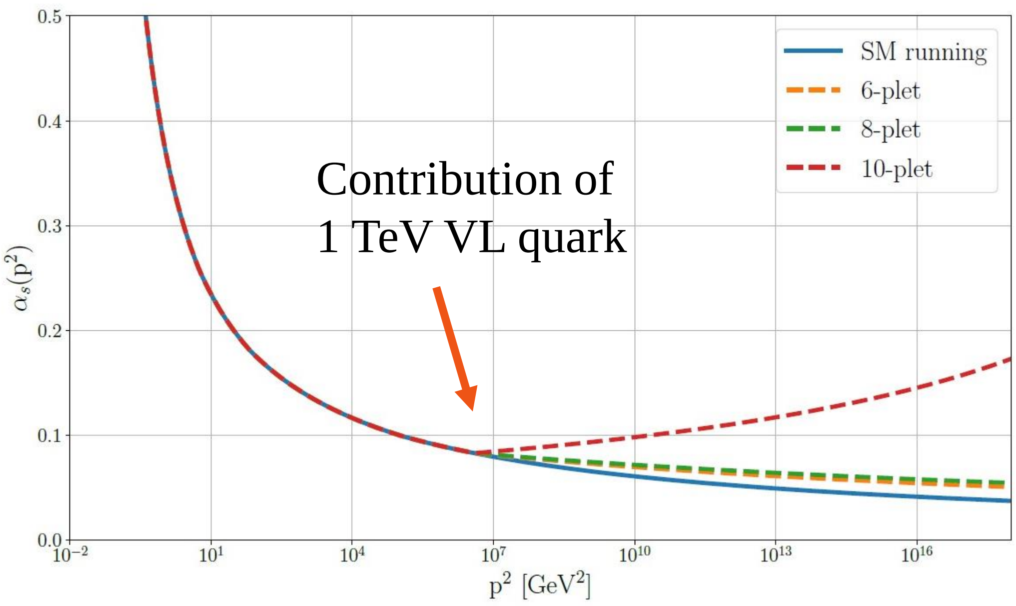
$$\langle\overline{\psi}\psi\rangle_{\text{inv}} = (c_1\ \text{GeV})^{3-c_2} \times m_\mu^{c_2}$$

Rep R	(p,q)	$C_2(\mathbf{R})$	$T(\mathbf{R})$	γ_m	$-\langle\overline{\psi}\psi\rangle_{\text{inv}}$	(c_1, c_2) Method 1	(c_1, c_2) Method 2
3	(1, 0)	4/3	1/2	12/21	$(0.218\ \text{GeV})^3$	(3.71, 2.53)	(44.40, 2.66)
6	(2, 0)	10/3	5/2	30/21	$(0.337\ \text{GeV})^3$	(62.32, 2.30)	(163.73, 2.41)
8	(1, 1)	3	3	27/21	$(0.363\ \text{GeV})^3$	(91.71, 2.34)	(317.52, 2.45)

chiral condensate

~ 5/2

massive condensate



Contribution of 1 TeV VL quark

$$\alpha_s(p^2) = \frac{1}{2\pi\, b\, \ln\left(\frac{p^2}{\Lambda_{\text{QCD}}^2}\right)}$$

$$b = \left(\tfrac{11}{3} C_2(\mathbf{8}) - \tfrac{2}{3} n_{\text{F}} - \tfrac{4}{3} T(\mathbf{R})\, n_{\text{V}}\right) / (8\pi^2)$$

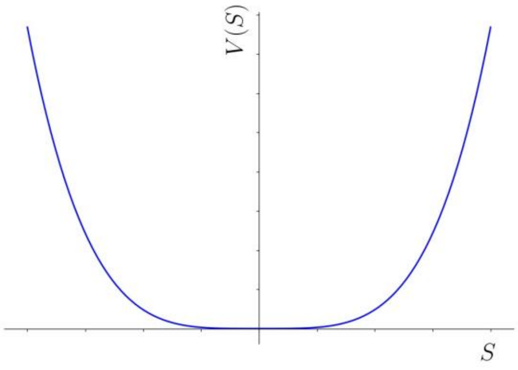
Triggering Electroweak Symmetry Breaking

VL mass technically natural

$$\mathcal{L}_{\mathrm{VLF}} = \overline{\psi}\left(\mathrm{i}\not{D} - m_\psi - y\, S\right)\psi$$

$$V(\phi, S) = \lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{\phi S} S^2(\phi^\dagger\phi)$$

Scale invariant scalar potential

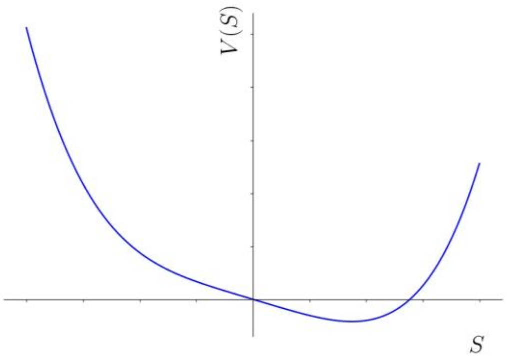


$\psi \sim (\mathbf{3}, 1, 0)$

$S \sim (\mathbf{1}, 1, 0)$

condensation

$$V_{\mathrm{eff}}(\phi, S) = \lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{\phi S} S^2(\phi^\dagger\phi) - y\langle\overline{\psi}\psi\rangle_{\mathrm{inv}} S$$



Induces VEVs for scalar fields → Triggers EWSB

Triggering Electroweak Symmetry Breaking

Benchmark point (**3**-plet) :

$m_\psi = 1.5\,\text{TeV}$ → $\langle\overline{\psi}\psi\rangle_{\text{inv}} \approx (590\,\text{GeV})^3$

$\lambda_\phi = 0.130\,,\quad \lambda_S = 0.695\,,\quad \lambda_{\phi S} = 0.100\,,\quad y = 0.210$

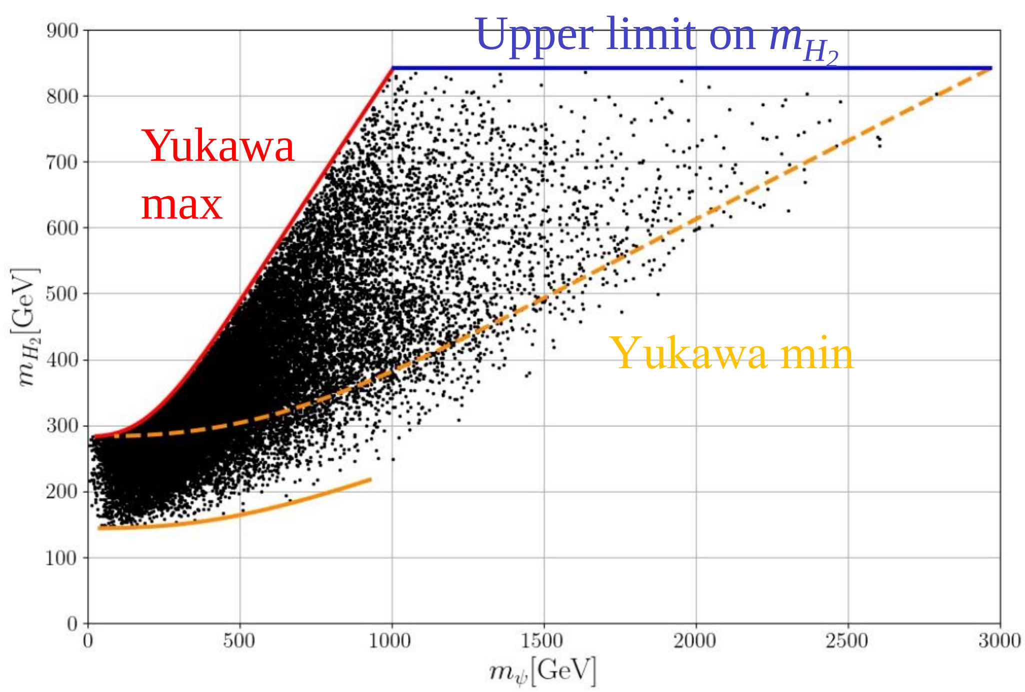
Natural couplings ~ O(0.1-1)

$m_{H_1} = 125.1\,\text{GeV}$

$m_{H_2} = 574.7\,\text{GeV}$

$\tan(2\theta) = 6.3 \times 10^{-2}$

Within current limits [1901.09966, 1803.09678]



→ Solves hierarchy problem

$y, \lambda_S, \lambda_{\phi S} \in [0.1, 1.5]$ and $\lambda_\phi = 0.13$

Conclusion

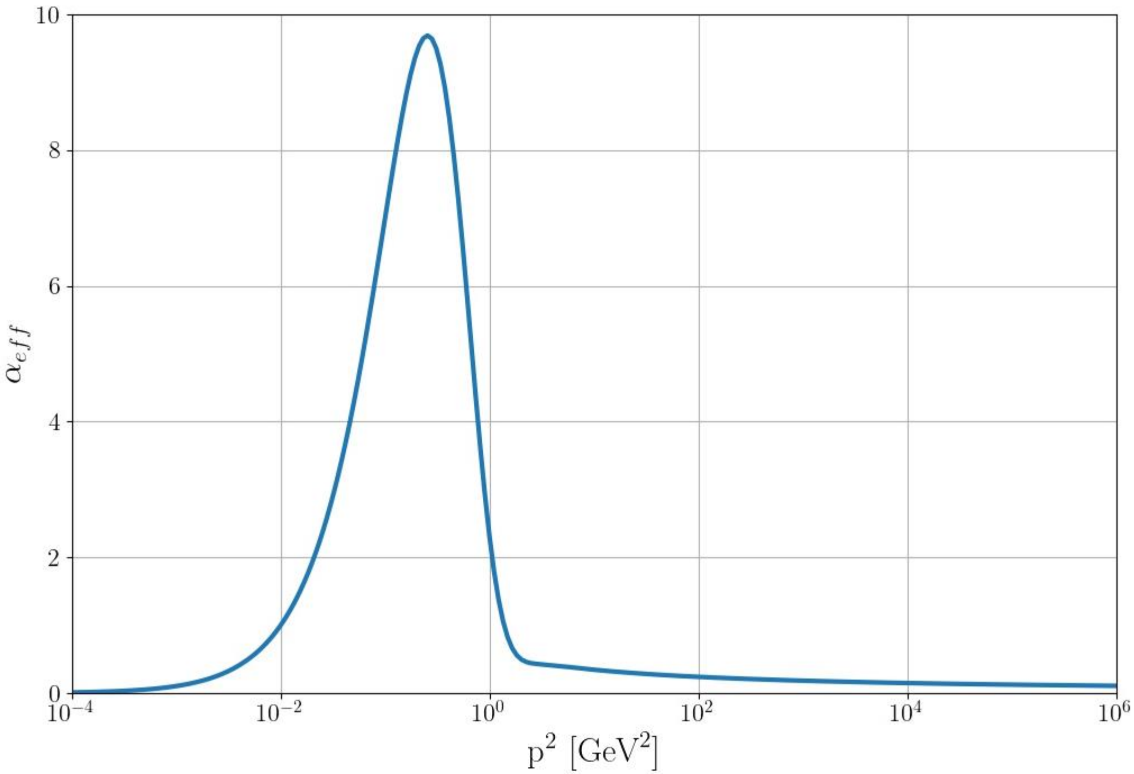
MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

- Numerical Solution of fermion DSE within rainbow approximation
- Extracted the expectation value of fermion two-point function as a function of explicit fermion mass and SU(3) representation
  → absolute value increases with explicit fermion mass and representation
- If VL quark couples to scalar the condensate can induce a tadpole in the scalar potential and induce a scalar VEV even though the scalar potential can be originally scale invariant → trigger EWSB
- Presented simplest realization of mechanism which requires a scalar singlet and a VL quark in low representation of $SU(3)_c$ → solves hierarchy problem
- VL quarks could form stable baryons that can be DM candidates [De Luca, Mitridate, Redi, Smirnov, Strumia, 1801.01135]

Backup Slides

Backup Slides

$$\alpha_{\text{eff}}(k^2) = 2\pi \frac{D}{\omega^4} k^2 \exp\left(-\frac{k^2}{\omega^2}\right) + \frac{2\pi\,\gamma_m}{\ln\left[\tau + (1+\frac{k^2}{\Lambda^2_{\text{QCD}}})^2\right]} \left[1 - \exp\left(\frac{-k^2}{4\,m_\perp^2}\right)\right]$$



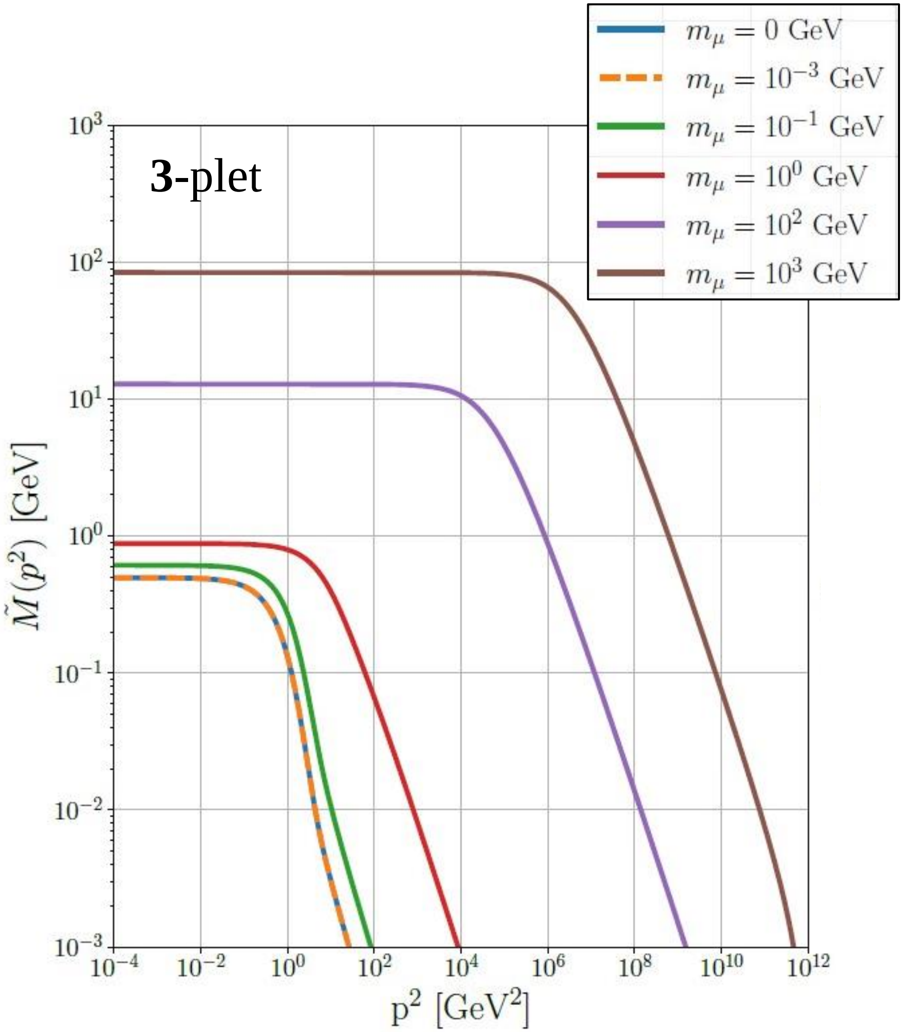
$$m_\perp = 0.5\,\text{GeV}, \;\; \tau := e^2 - 1,$$

$$\Lambda_{\text{QCD}} = 0.234\,\text{GeV}, \;\; \gamma_m = \frac{12}{33-2n_F},$$

$$n_F = 6.$$

$$\omega = 0.5\,\text{GeV} \text{ and } D = 1.024\,\text{GeV}^2$$

Fermion Condensate Beyond Chiral Limit



OPE

$$\tilde{M}(p^2) \overset{p\to\infty}{\simeq} -\frac{2\pi^2\gamma_m}{d(\mathbf{R})}\frac{C(m_\mu)}{p^2}\left[\frac{1}{2}\ln\left(\frac{p^2}{\Lambda^2_{\mathrm{QCD}}}\right)\right]^{\gamma_m-1}$$

Extract: $C(m_\mu) := \left(1 - m_\mu \frac{d}{dm_\mu}\right)\langle\overline{\psi}\psi\rangle^{m_\mu}_{\mathrm{inv}}$

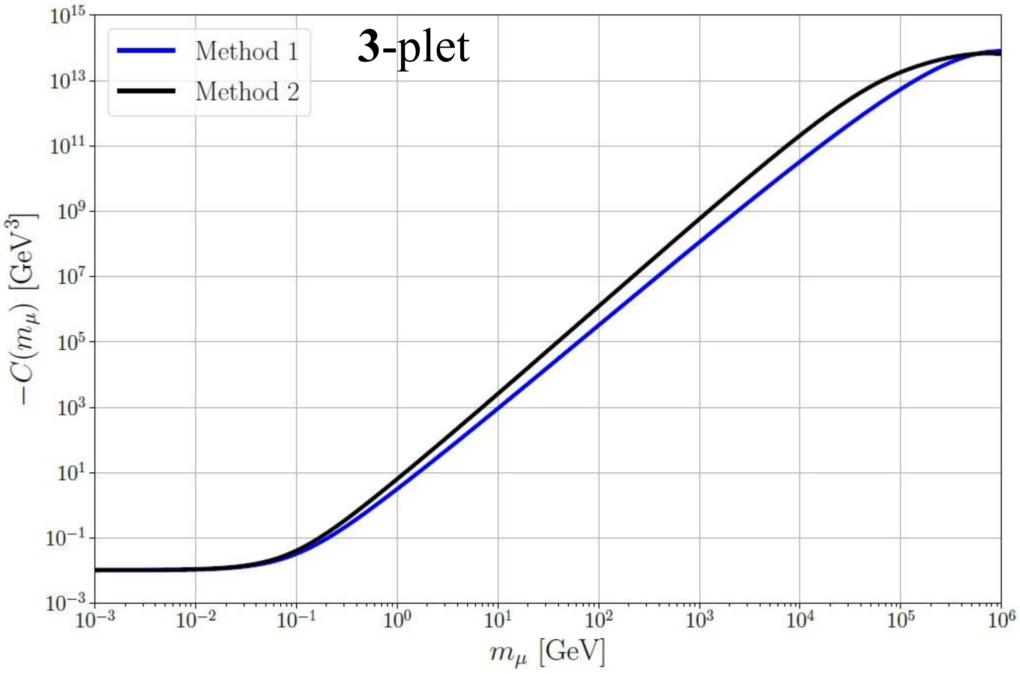
Method 1:

Calculate trace over fermion propagator with $\tilde{M}(p^2)$ instead of $M(p^2)$

Method 2:

Infer $C(m_\mu)$ from large momentum behavior of $\tilde{M}(p^2)$

Fermion Condensate Beyond Chiral Limit



Initial conditions:

$$\epsilon = 0.001\,\text{GeV} \approx \text{up-quark}$$

$$-\langle\overline{\psi}\psi\rangle_{\text{inv}}^{m_\mu=\epsilon} = (0.218\,\text{GeV})^3$$

$$C(m_\mu) := \left(1 - m_\mu \frac{d}{dm_\mu}\right) \langle\overline{\psi}\psi\rangle_{\text{inv}}^{m_\mu}$$

Extract VEV of two-point function

$$-\frac{C(m_\mu)}{m_\mu^2} = \frac{d}{dm_\mu}\left[\frac{\langle\overline{\psi}\psi\rangle_{\text{inv}}^{m_\mu}}{m_\mu}\right]$$

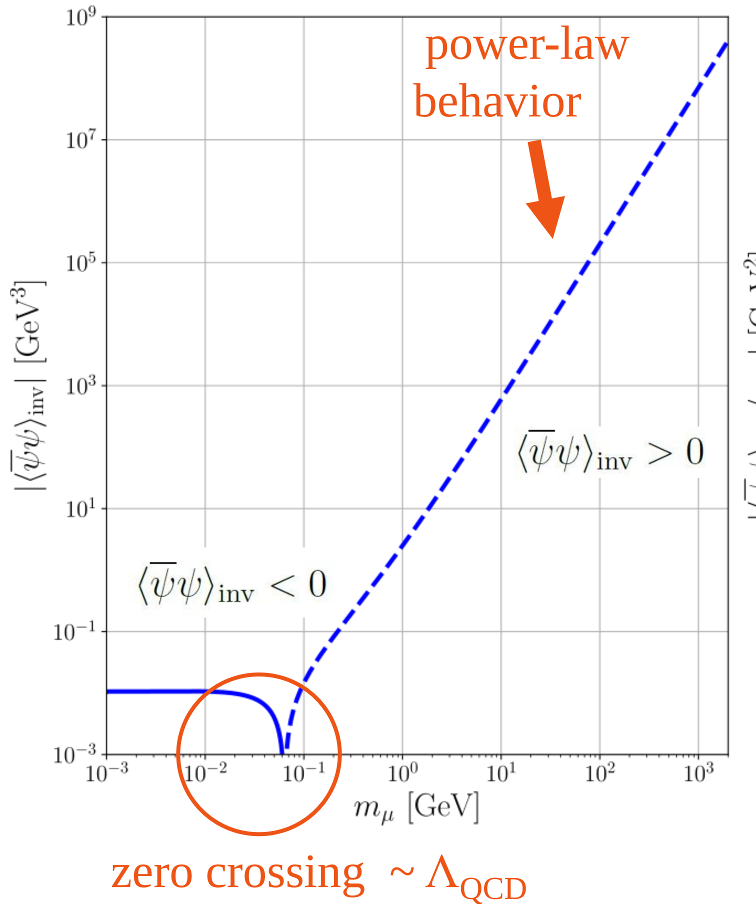
Solve with initial condition

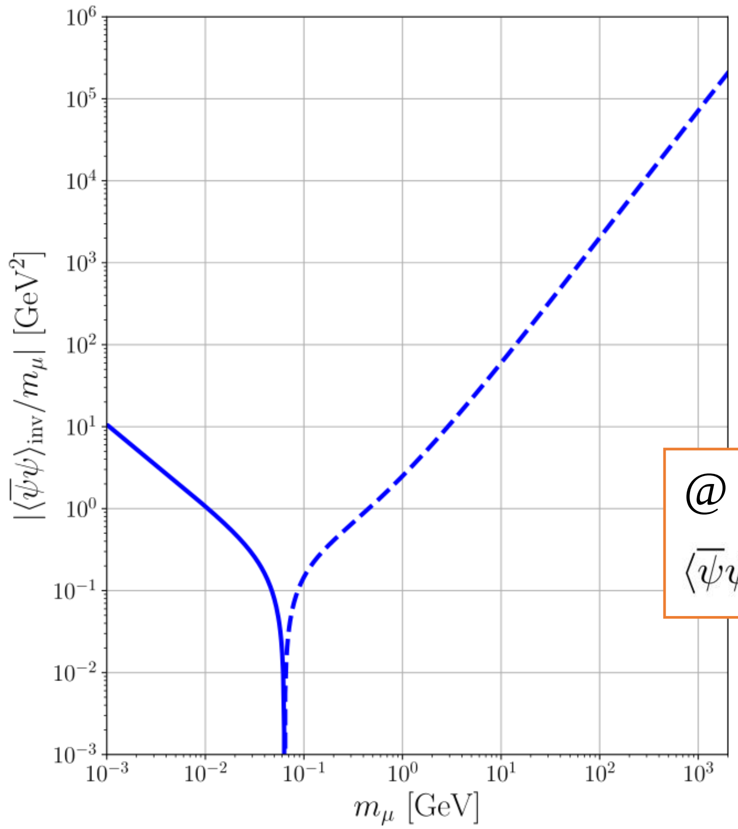
$$-\frac{\langle\overline{\psi}\psi\rangle_{\text{inv}}^{m_\mu}}{m_\mu} = \left[-\frac{\langle\overline{\psi}\psi\rangle_{\text{inv}}^{m_\mu}}{m_\mu}\right]_{m_\mu=\epsilon} + \int_\epsilon^{m_\mu} \frac{C(m_\mu)}{m_\mu^2}\, dm_\mu$$

Fermion Condensate Beyond Chiral Limit

empirical relation for $m_\mu > 1$ GeV

$$\langle\overline{\psi}\psi\rangle_{\text{inv}} = (c_1\ \text{GeV})^{3-c_2} \times m_\mu^{c_2} \approx (3.7\ \text{GeV})^{1/2}\, m_\mu^{5/2}$$





@ $m_\mu = 1$ TeV :

$\langle\overline{\psi}\psi\rangle_{\text{inv}} \approx (415\ \text{GeV})^3$

Triggering Electroweak Symmetry Breaking

Benchmark point : $m_\psi = 1.5\,\text{TeV}$ → $\langle\overline{\psi}\psi\rangle_{\text{inv}} \approx (590\ \text{GeV})^3$

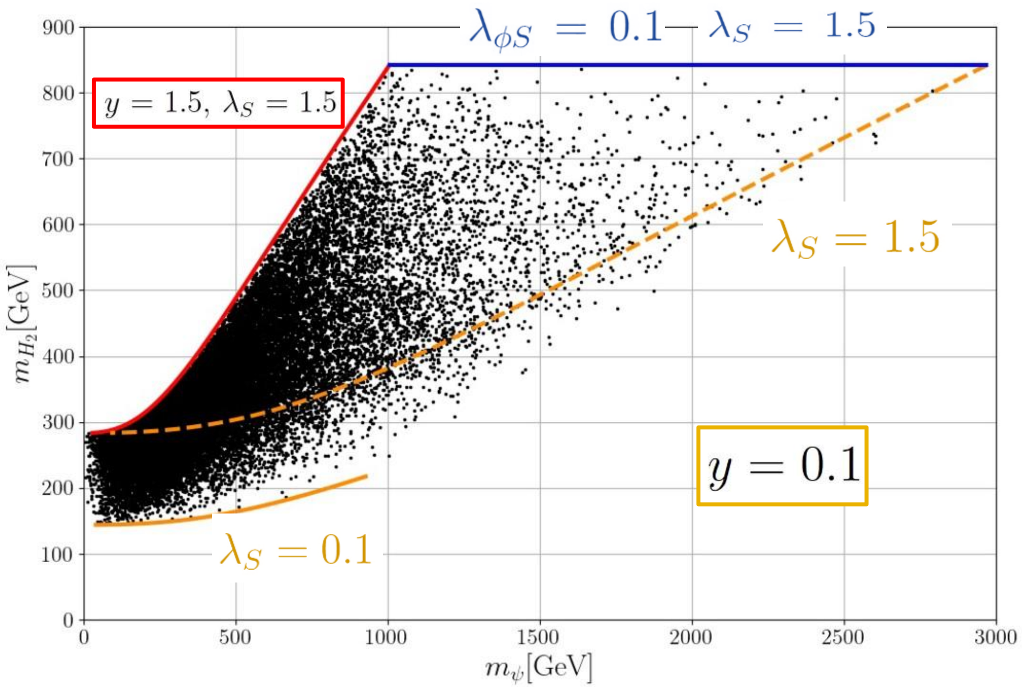
$$\lambda_\phi = 0.130\,, \quad \lambda_S = 0.695\,, \quad \lambda_{\phi S} = 0.100\,, \quad y = 0.210$$

$$m_{H_1} = 125.1\ \text{GeV}$$

$$m_{H_2} = 574.7\,\text{GeV}$$

$$\tan(2\theta) = 6.3 \times 10^{-2}$$

Within current limits
[1901.09966, 1803.09678]



$y, \lambda_S, \lambda_{\phi S} \in [0.1, 1.5]$ and $\lambda_\phi = 0.13$

NP scales: $m_{H_2} \approx \left(\frac{6\,\lambda_\phi\,\lambda_S}{\lambda_{\phi S}}\right)^{1/2} \times v$

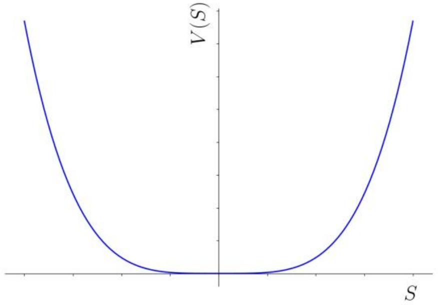
$$m_\psi \approx \frac{m_{H_2}^{\frac{6}{5}}}{(3.7\,\text{GeV})^{\frac{1}{5}}} \times \left(\frac{1}{3\,\lambda_S}\right)^{\frac{3}{5}} \left(\frac{4\,\lambda_\phi\,\lambda_S - \lambda_{\phi S}^2}{4\,y\,\lambda_\phi}\right)^{\frac{2}{5}}$$

Triggering Electroweak Symmetry Breaking

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

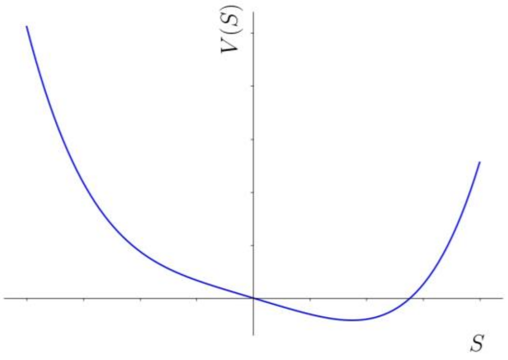
$$\mathcal{L}_{\text{VLF}} = \overline{\psi}\left(\mathrm{i}\not{D} - m_\psi - y\,S\right)\psi$$

$$V(\phi, S) = \lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{\phi S} S^2(\phi^\dagger\phi)$$



condensation

$$V_{\text{eff}}(\phi, S) = \lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{\phi S} S^2(\phi^\dagger\phi) - y\langle\overline{\psi}\psi\rangle_{\text{inv}} S$$



$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$S(x) = w + s(x)$$

$$w^2 = \left(\frac{4\,y\,\lambda_\phi\,\langle\overline{\psi}\psi\rangle_{\text{inv}}}{4\,\lambda_\phi\,\lambda_S - \lambda_{\phi S}^2}\right)^{\frac{2}{3}}, \qquad \frac{v^2}{w^2} = \frac{\lambda_{\phi S}}{2\lambda_\phi}$$

Induced VEVs

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}\begin{pmatrix} h, & s \end{pmatrix}\begin{pmatrix} 2\lambda_\phi v^2 & -\lambda_{\phi S} v w \\ -\lambda_{\phi S} v w & 3\lambda_S w^2 - \frac{1}{2}\lambda_{\phi S} v^2 \end{pmatrix}\begin{pmatrix} h \\ s \end{pmatrix}$$

$$m_{H_1}^2 \approx \left(2\,\lambda_\phi - \frac{\lambda_{\phi S}^2}{3\,\lambda_S}\right)v^2$$

SM Higgs

$$m_{H_2}^2 \approx 3\,\lambda_S\,w^2$$

New Scalar