# Higgs-mass constraints on a SUSY solution of the muon $g-2$ anomaly

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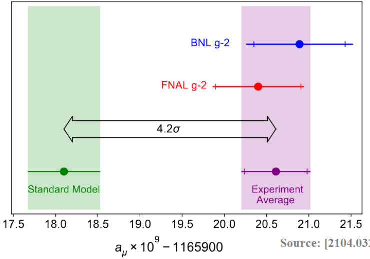
Planck 2022
1 June

# Introduction

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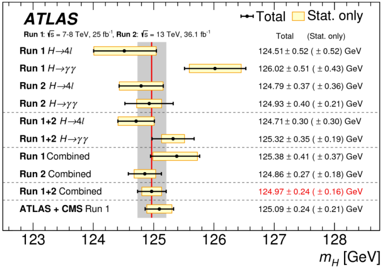



$$a_\mu = (g-2)_\mu / 2$$
$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

New $(g-2)_\mu$ result shows a $4.2\sigma$ deviation from the SM $\Rightarrow$New Physics ?

Plausible scenario : NP shows itself in deviations, but new particles are too heavy

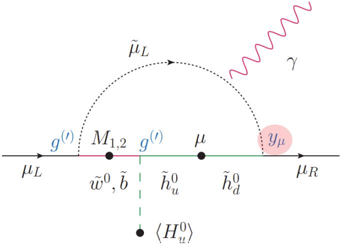
One well-motivated candidate : **supersymmetry**

Infer the hidden structure of a SUSY theory using $(g-2)_\mu$ ?
How does Higgs mass help ?

# SUSY with two Higgs doublets : MSSM

$$\Delta a_\mu^{\mathrm{MSSM}} \propto \frac{g^2\,(g'^2)}{192\pi^2}\,\frac{\overset{\text{muon}}{m_\mu^2}}{\underset{\text{smuon}}{M_{\tilde{\mu}}^2}}\,\frac{\overset{\text{EW gaugino}}{M_{1,2}}\,\overset{\text{Higgsino}}{\mu}}{M_{\tilde{\mu}}^2}\,\frac{\tan\beta}{1+\epsilon_\ell\tan\beta} \times F\left(\frac{M_{1,2}^2}{M_{\tilde{\mu}}^2}, \frac{\mu^2}{M_{\tilde{\mu}}^2}\right)$$

**Enhancement** for large $\tan\beta \equiv v_u/v_d$




Higgsino-muon-smuon coupling $\propto y_\mu^{\mathbf{MSSM}}$

In MSSM, $y_\mu^{\mathrm{MSSM}} \propto m_\mu/(v\cos\beta) \approx m_\mu \tan\beta/v$

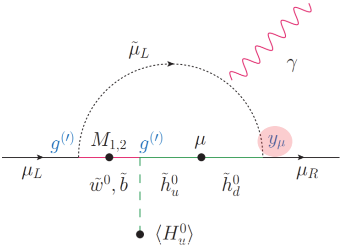
$\Rightarrow \quad \Delta a_\mu$ enhanced by $\tan\beta$ compared to SM

# MSSM and $(g-2)_\mu$ : $\tan\beta$ enhancement, but...

upper bounded

$$\Delta a_\mu^{\rm MSSM} \propto \frac{g^2\,(g'^2)}{192\pi^2}\frac{m_\mu^2}{M_{\tilde\mu}^2}\frac{M_{1,2}\mu}{M_{\tilde\mu}^2}\frac{\tan\beta}{1+\epsilon_\ell\tan\beta}\times F\left(\frac{M_{1,2}^2}{M_{\tilde\mu}^2},\frac{\mu^2}{M_{\tilde\mu}^2}\right)$$

lower bounded




In MSSM, $y_\mu^{\rm MSSM} \propto m_\mu/(v\cos\beta) \approx m_\mu \tan\beta/v$

but also $y_{b,\tau}^{\rm MSSM} \propto m_{b,\tau}/(v\cos\beta)$

$\Rightarrow$ Non-perturbative $y_{b,\tau}^{\rm MSSM}$ for large $\tan\beta$

Upper bound on $\tan\beta$ & $\Delta a_\mu$ constraint

$\Rightarrow$ SUSY particles $< 1$ TeV
**Tension with LHC bounds !**

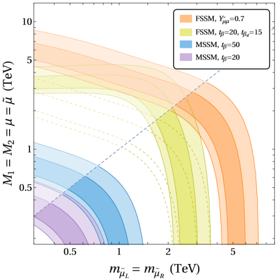
# A possible way out : More Higgs doublets

**Flavorful supersymmetric Standard Model** (FSSM)
Altmannshofer et al., 2104.08293

**Four** Higgs doublets, $H_u, H_d, H'_u, H'_d$ (VEV's : $v_u, v_d, v'_u, v'_d$)

$$\Delta a_\mu^{\rm FSSM} \approx \frac{g^2\,(g'^2)}{192\pi^2}\frac{m_\mu^2}{M_{\tilde\mu}^2}\frac{M_{1,2}\mu_{ud'}}{M_{\tilde\mu}^2}\frac{\tan\beta\tan\beta_d}{1+\epsilon_\ell\tan\beta\tan\beta_d}\times F\left(\frac{M_{1,2}^2}{M_{\tilde\mu}^2},\frac{\mu_{ud'}^2}{M_{\tilde\mu}^2}\right)$$

$\tan\beta_d \equiv v_d/v'_d$, $\tan\beta_u \equiv v_u/v'_u$, $\tan\tilde\beta \equiv \sqrt{\frac{v_u^2+v_u'^2}{v_d^2+v_d'^2}}$




$\tan\beta\tan\beta_d$ enhancement, for $v_d \gg v'_d$

<u>What's better with 4 doublets ?</u>

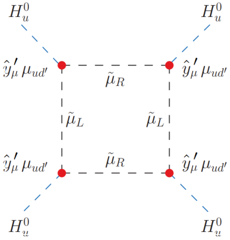
$y_\mu^{\rm FSSM} \propto m_\mu/v'_d = m_\mu/(v\cos\tilde\beta\cos\beta_d)$

Meanwhile, $y_{b,\tau}^{\rm FSSM} \propto m_{b,\tau}/v_d = m_{b,\tau}/(v\cos\tilde\beta\sin\beta_d)$

$\Rightarrow$ $y_\mu \sim \mathcal{O}(1)$ keeping $y_{b,\tau}$ perturbative
**Releases SUSY mass tension !**

# SUSY and Higgs mass

In the FSSM (as in MSSM), **Higgs quartic coupling** ($\lambda$) is not a free parameter, but given by the D-terms
Some of the parameters entering $\Delta a_\mu$ contribute to $\lambda$ at one-loop




$\hat{y}'_\mu$ in **smuon** contribution :

$$\Longrightarrow \quad \Delta\lambda^{\tilde{\mu}} \approx -\frac{\hat{y}'^4_\mu}{96\pi^2}\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right)^4$$

$\hat{y}'_\mu$ in $(g-2)_\mu$ :

$$\Delta a_\mu \approx \frac{\hat{y}'_\mu}{M^2_{\tilde{\mu}}} \times (\cdots)$$

A larger $\hat{y}'_\mu$ gives a larger contribution to $\Delta a_\mu$, but also a larger negative correction to $\lambda \Rightarrow$ needs more positive correction from **stop** :

$$\Delta\lambda^{\tilde{t}} \approx \frac{3y_t^4}{8\pi^2}\left(\ln\frac{M^2_{\tilde{t}}}{Q^2} + \frac{X_t^2}{M^2_{\tilde{t}}} - \frac{X_t^4}{12M^4_{\tilde{t}}}\right)$$

# SUSY and Higgs mass

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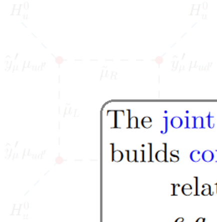


Dominant **smuon** contribution :

$$\Longrightarrow \quad \Delta\lambda^{\tilde{\mu}} \approx -\frac{\hat{y}'^4_\mu}{96\pi^2}\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right)^4$$

The joint constraint on $\Delta a_\mu$ and on $\lambda$, builds correlation between parameters related (or not) to $(g-2)_\mu$, *e.g.* smuon and stop masses

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# Case study with the FSSM W.K. & Pietro Slavich, 2109.15277

**EFT approach to the Higgs quartic coupling calculation**

**SM :** $M_h \Rightarrow \lambda^{SM}(Q_{\text{EW}}) \xrightarrow{\text{3-loop SM RGE}} \lambda^{SM}(M_S)$

**FSSM :** quartic self coupling of the *lightest* Higgs scalar

- Tree-level : $\lambda_0^{\text{FSSM}}(M_S) = \frac{g^2+g'^2}{4}\cos^2(2\tilde{\beta})$
- One-loop :

$$\Delta\lambda^{\text{reg}} + \Delta\lambda^{\tilde{f}} + \Delta\lambda^{H} + \Delta\lambda^{\chi}$$

from SUSY reg. scheme $\overline{\textbf{DR}}$ to SM reg. scheme $\overline{\textbf{MS}}$

sfermion diagrams

heavy Higgs diagrams → tedious diagrammatic calculations...

Higgsino & EW gaugino diagrams

⋆ done in MSSM, but more complicated in FSSM (extended Higgs & Higgsino sector)

# Case study with the FSSM

**EFT approach to the Higgs quartic coupling calculation**

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**FSSM :** quartic self coupling of the *lightest* Higgs scalar

- Tree-level : $\lambda_0^{\mathrm{FSSM}}(M_S) = \frac{g^2+g'^2}{4}\cos^2(2\tilde{\beta})$
- One-loop : Use **general results** in Braathen, Goodsell & Slavich, 1810.09388
  *e.g. sfermion and heavy Higgs contributions*

$$\mathcal{L} \supset -\frac{1}{6}a_{ijk}\Phi_j\Phi_j\Phi_k - \frac{1}{24}\lambda_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l \ (\Phi_i \text{ real})$$

$$\begin{aligned}\frac{\partial^4 V_S^{(1)}}{\partial\Phi_i\partial\Phi_j\partial\Phi_k\partial\Phi_l} =& \frac{1}{16}\tilde{\lambda}_{ijxy}\tilde{\lambda}_{klxy}P_{SS}\left(m_x^2, m_y^2\right) \\ &+ \frac{1}{4}\tilde{\lambda}_{ijxy}a_{kyz}a_{lzx}C_0\left(m_x^2, m_y^2, m_z^2\right) \\ &- \frac{1}{8}a_{ixy}a_{jyz}a_{kzu}a_{lux}D_0\left(m_x^2, m_y^2, m_z^2, m_u^2\right) + (ijkl)\end{aligned}$$

⋆ Complete FSSM threshold correction available on request

# Interplay between $\Delta a_\mu$ & $\lambda$ solutions

**Simplified scenario for numerical analysis :**

- common mass $M_H$ & no mixing between heavy Higgs scalars
- common mass $M_\chi$ for higgsinos and EW gauginos & $M_\chi = \mu_{ud'}$
- common mass $M_{\tilde{\mu}}$ ($M_{\tilde{t}}$) for 1st & 2nd (3rd) generation sfermions

**For a given choice of parameters** $\tan\beta_u, M_\chi, M_{1,2}$, **etc,**
**we are left with : 2 parameters for 2 conditions**

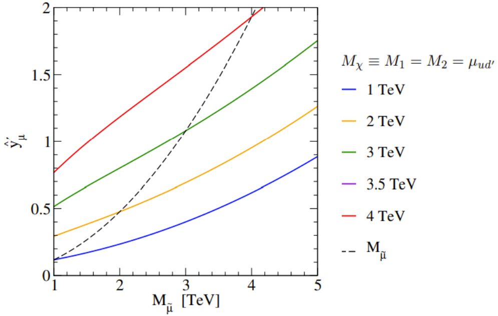
$(g-2)_\mu$ constraint : $\quad \Delta a_\mu(\tan\beta_d) = 251 \times 10^{-11}$

Matching condition : $\quad \lambda^{\mathrm{SM}}(M_{\tilde{t}}) = \lambda^{\mathrm{FSSM}}(\tan\beta_d, M_{\tilde{t}})$

# Results

$\hat{y}'_\mu \sim 1$ **for multi-TeV** $M_{\tilde{\mu}}$

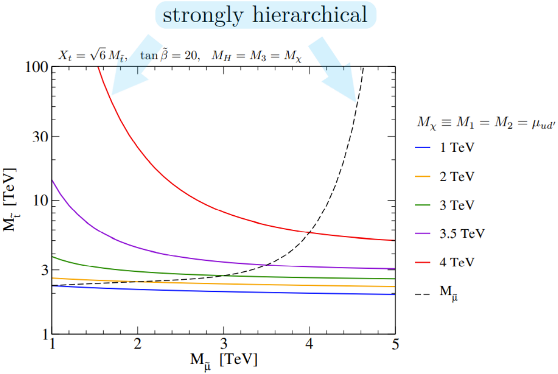
$$\hat{y}'_\mu(M_{\tilde{\mu}}) = \frac{g_\mu(M_{\tilde{\mu}})/\left(\cos\tilde{\beta}\cos\beta_d\right)}{1+\epsilon_l \tan\beta\tan\beta_d}$$




- fixed $M_{\tilde{\mu}}$ : larger $\hat{y}'_\mu$ for larger $M_\chi$
- fixed $M_\chi$ : larger $\hat{y}'_\mu$ for larger $M_{\tilde{\mu}}$

$$\Delta a_\mu \approx \hat{y}'_\mu \frac{m_\mu v_u}{M^2_{\tilde{\mu}}} \frac{g^2\,(g'^2)}{192\pi^2} F\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right)$$

# Stop mass vs. smuon mass




Smuon contribution for large $\hat{y}'_\mu$ :

$$\Delta\lambda^{\tilde{\mu}} \approx -\frac{\hat{y}'^4_\mu}{96\pi^2}\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right)^4$$

when $M_\chi = M_{\tilde{\mu}}$,

$M_{\tilde{\mu}} \nearrow \quad \Rightarrow \quad \hat{y}'_\mu \nearrow$

larger negative $\Delta\lambda^{\tilde{\mu}}$

$\Rightarrow$ need larger positive stop contribution

# Conclusion

**FSSM : a SUSY 4HDM**

- Extra Higgs doublets allow to "decouple" $y_\mu$ from $y_{b,\tau}$
- Joint constraint of $\lambda$ & $\Delta a_\mu$ relates stop and smuon, Higgsino, EW gaugino sectors

**What else can be explored ?**

- Perturbativity of Yukawa couplings at GUT scale
- Flavor constraints in the quark sector
- Heavy SUSY particle production in future colliders ? $\cdots$

**Take-home message**

- The Higgs mass, given as an input, relates parameters relevant to an anomaly, *e.g.* $(g-2)_\mu$, to those that are not

# Thank you !

# Backup

**Superpotential and soft terms :**

$$W_\mu = \mu_{ud}\widehat{H}_u\widehat{H}_d + \mu_{u'd'}\widehat{H}'_u\widehat{H}'_d + \mu_{u'd}\widehat{H}'_u\widehat{H}_d + \mu_{ud'}\widehat{H}_u\widehat{H}'_d$$

$$\begin{aligned}-\mathcal{L}_{\rm soft} \supset & m^2_{uu}H^\dagger_u H_u + m^2_{dd}H^\dagger_d H_d + m^2_{u'u'}H'^\dagger_u H'_u + m^2_{d'd'}H'^\dagger_d H'_d \\ & + \left(m^2_{uu'}H^\dagger_u H'_u + m^2_{dd'}H^\dagger_d H'_d + \text{ h.c. }\right) \\ & + \left(B_{ud}H_u H_d + B_{u'd'}H'_u H'_d + B_{u'd}H'_u H_d + B_{ud'}H_u H'_d + \text{ h.c. }\right)\end{aligned}$$

## Yukawa couplings

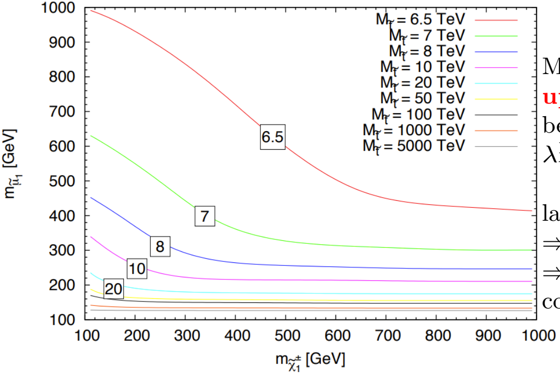
- Ignored flavor-**mixing** couplings (constrained and not relevant)
- Ignored the **first** generation couplings

$$
\begin{aligned}
W_Y = &- y'_c \hat{H}'_u \hat{Q}_2 \hat{U}^c_2 + y'_s \hat{H}'_d \hat{Q}_2 \hat{D}^c_2 + y'_\mu \hat{H}'_d \hat{L}_2 \hat{E}^c_2 \\
&- y'_t \hat{H}'_u \hat{Q}_3 \hat{U}^c_3 + y'_b \hat{H}'_d \hat{Q}_3 \hat{D}^c_3 + y'_\tau \hat{H}'_d \hat{L}_3 \hat{E}^c_3 \\
&- y_t \hat{H}_u \hat{Q}_3 \hat{U}^c_3 + y_b \hat{H}_d \hat{Q}_3 \hat{D}^c_3 + y_\tau \hat{H}_d \hat{L}_3 \hat{E}^c_3
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}} \supset &- y'_c A'_c H'_u Q_2 U^c_2 + y'_s A'_s H'_d Q_2 D^c_2 + y'_\mu A'_\mu H'_d L_2 E^c_2 \\
&- y'_t A'_t H'_u Q_3 U^c_3 + y'_b A'_b H'_d Q_3 D^c_3 + y'_\tau A'_\tau H'_d L_3 E^c_3 \\
&- y_t A_t H_u Q_3 U^c_3 + y_b A_b H_d Q_3 D^c_3 + y_\tau A_\tau H_d L_3 E^c_3
\end{aligned}
$$

# MSSM vs. FSSM

From Badziak et al. 1411.1450




MSSM : The interplay gives an **upper** bound on the stop mass because :

$$\lambda_{\rm tree}^{\rm MSSM} = \frac{g^2+g'^2}{4}\cos^2 2\beta$$

large $\tan\beta$
$\Rightarrow$ small stop mass ($\lambda$ constraint)
$\Rightarrow$ large smuon mass ($\Delta a_\mu$ constraint)

FSSM : **lower** bound on the stop mass, because the tree level $\lambda$ depends on a different $\tan\tilde{\beta}$ ratio