

Muon g-2 anomaly and new physics in e^+e^- and $\mu^+\mu^-$ final states scattering



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based on:

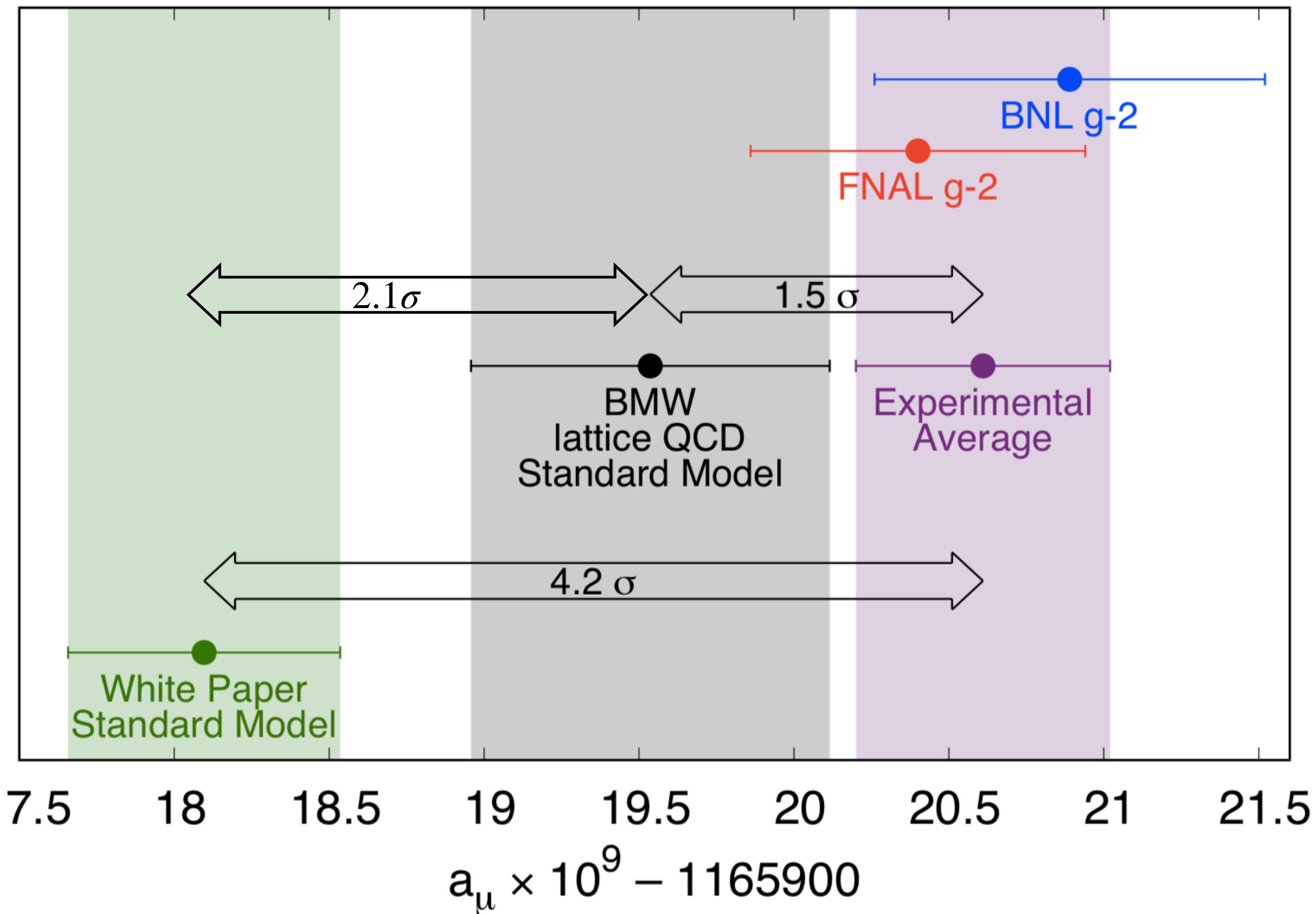
L. Darmé, G²dC and E. Nardi, arXiv:2112.09139, accepted by JHEP

Outline

- Introduction
- The SM estimate [see talk by Knecht]
- Indirect new physics effects: modifying the hadronic cross section
 1. Luminosity determination
 2. The $\sigma(\mu\mu\gamma)$ method
- Solving the a_μ tensions: model and constraints
- Conclusions

Introduction

[see talk by Knecht]



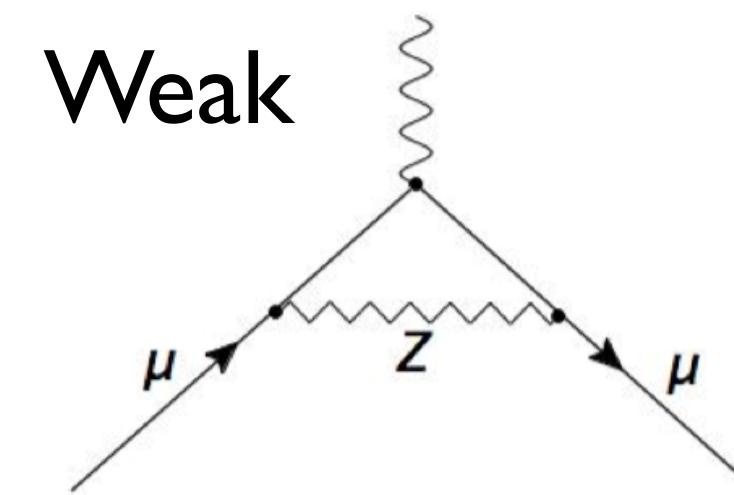
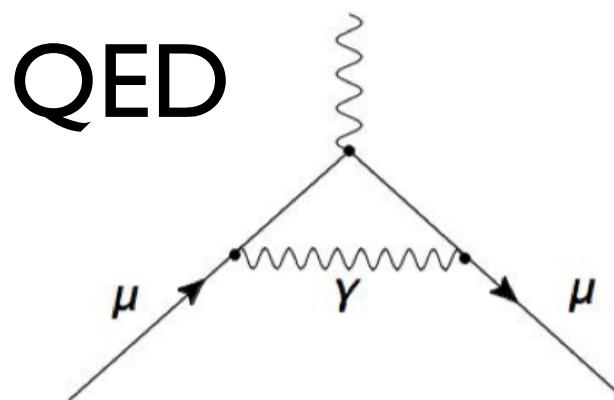
- Experiment vs SM estimate
 $\Delta a_\mu = 251(59) \cdot 10^{-11}$
- SM vs lattice estimate

The SM estimate

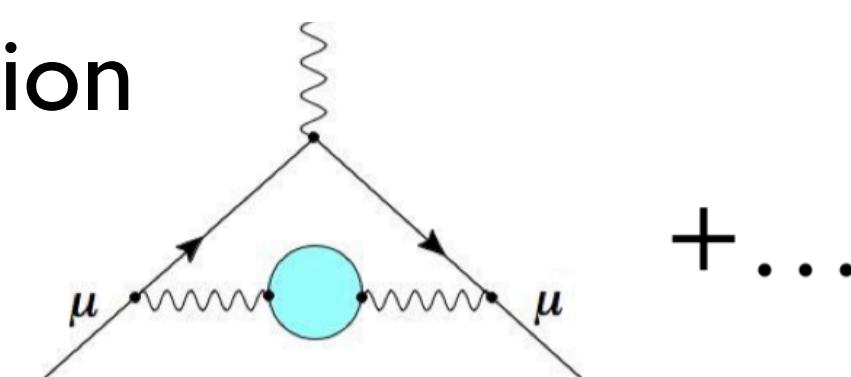
[Aoyama et al, 2006.04822, Phys. Rept. 887 (2020) 1-166]

[see talk by Knecht]

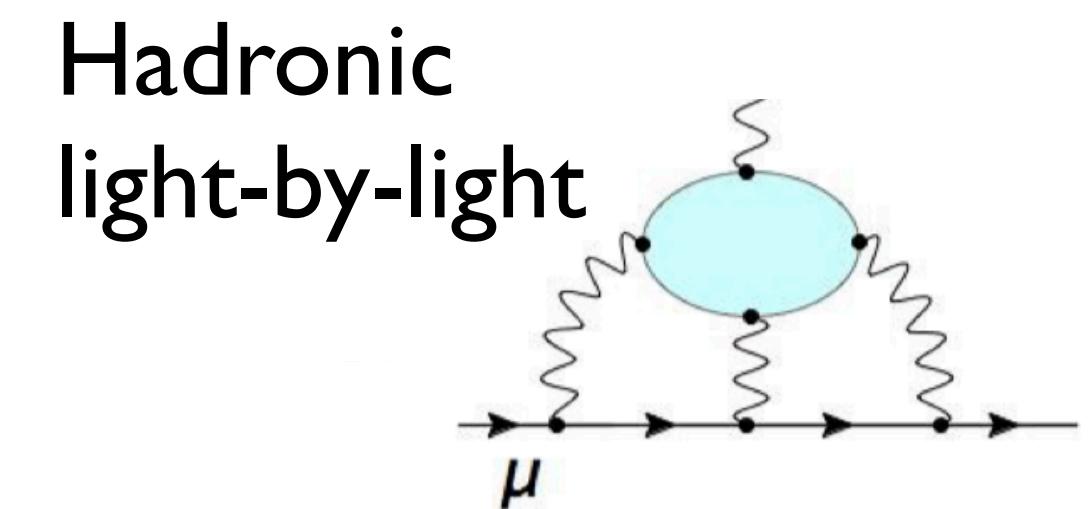
$$116584718.9(1) \cdot 10^{-11}$$



Hadronic vacuum
polarization



+ ...



$$153.6(1.0) \cdot 10^{-11}$$

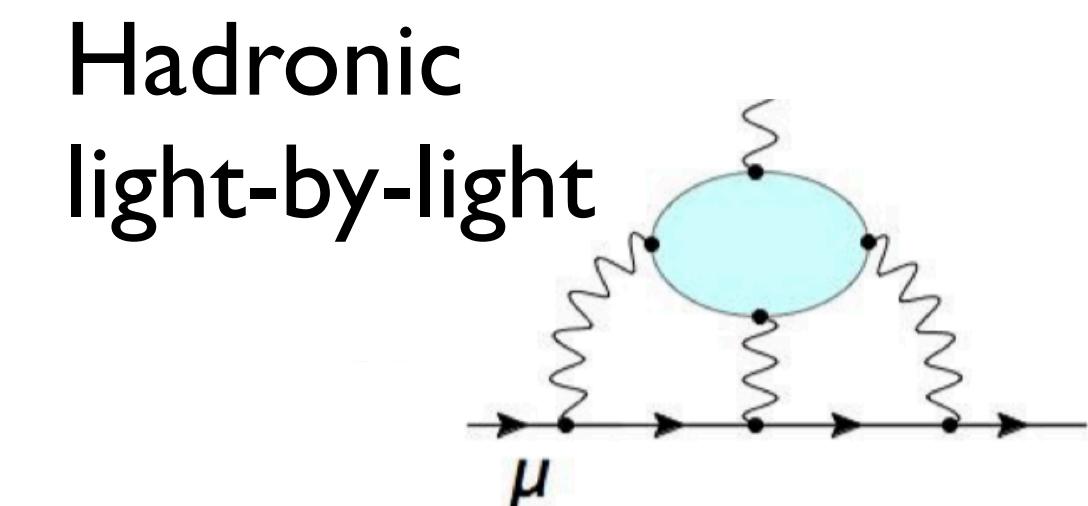
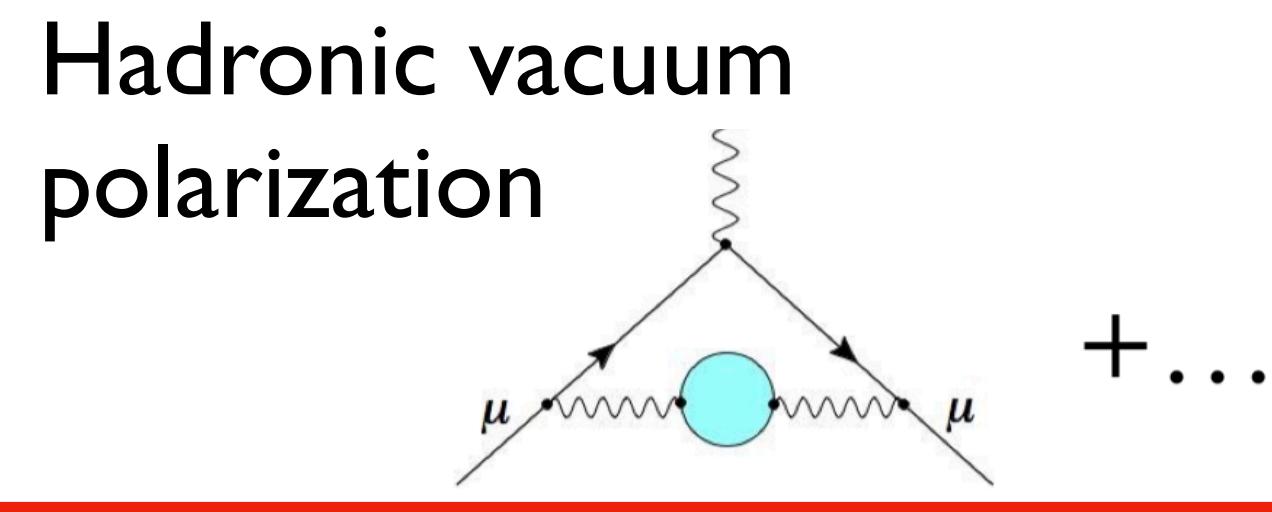
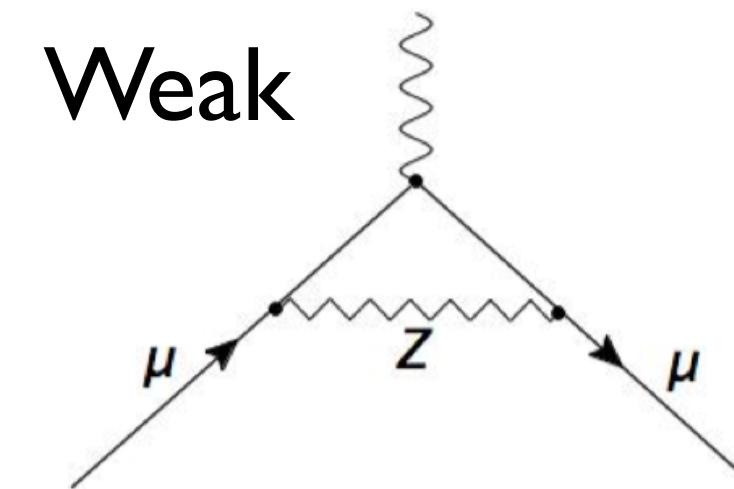
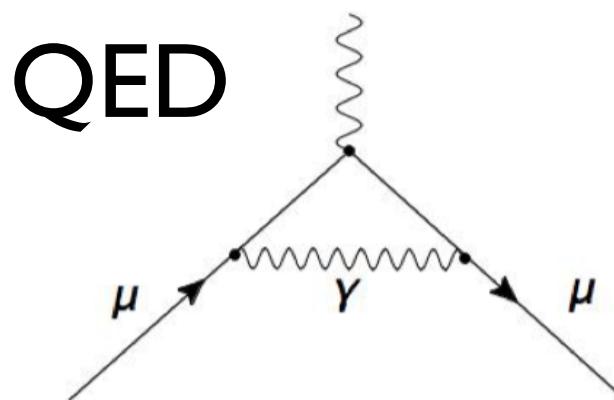
$$92(18) \cdot 10^{-11}$$

The SM estimate

[Aoyama et al, 2006.04822, Phys. Rept. 887 (2020) 1-166]

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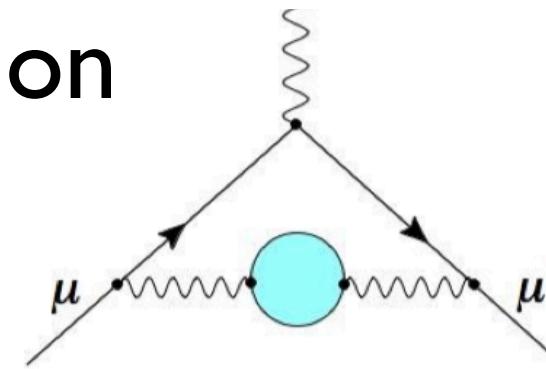
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The SM estimate

[Aoyama et al, 2006.04822, Phys. Rept. 887 (2020) 1-166]

Hadronic vacuum
polarization



[see talk by Knecht]

$6845(40) \cdot 10^{-11}$

Kernel function $\propto s^{-1}$:
lower energies more
important

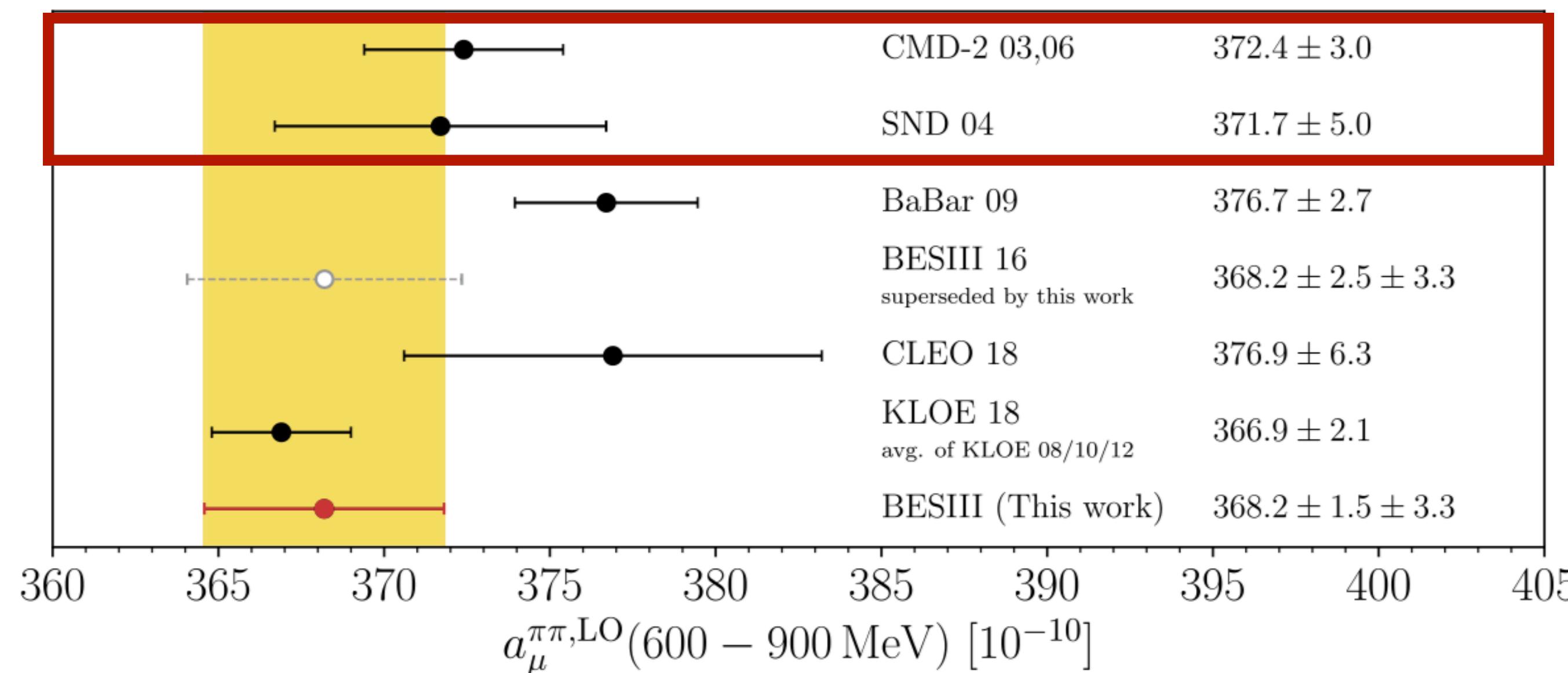
$$a_{\mu}^{LO,HVP} = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds [K(s)] [\sigma_{\text{had}}(s)]$$

$e^+e^- \rightarrow \text{hadrons}$
bare cross section:
experimental input

The SM estimate

The σ_{had} must be measured at all centre of mass energy \sqrt{s} :

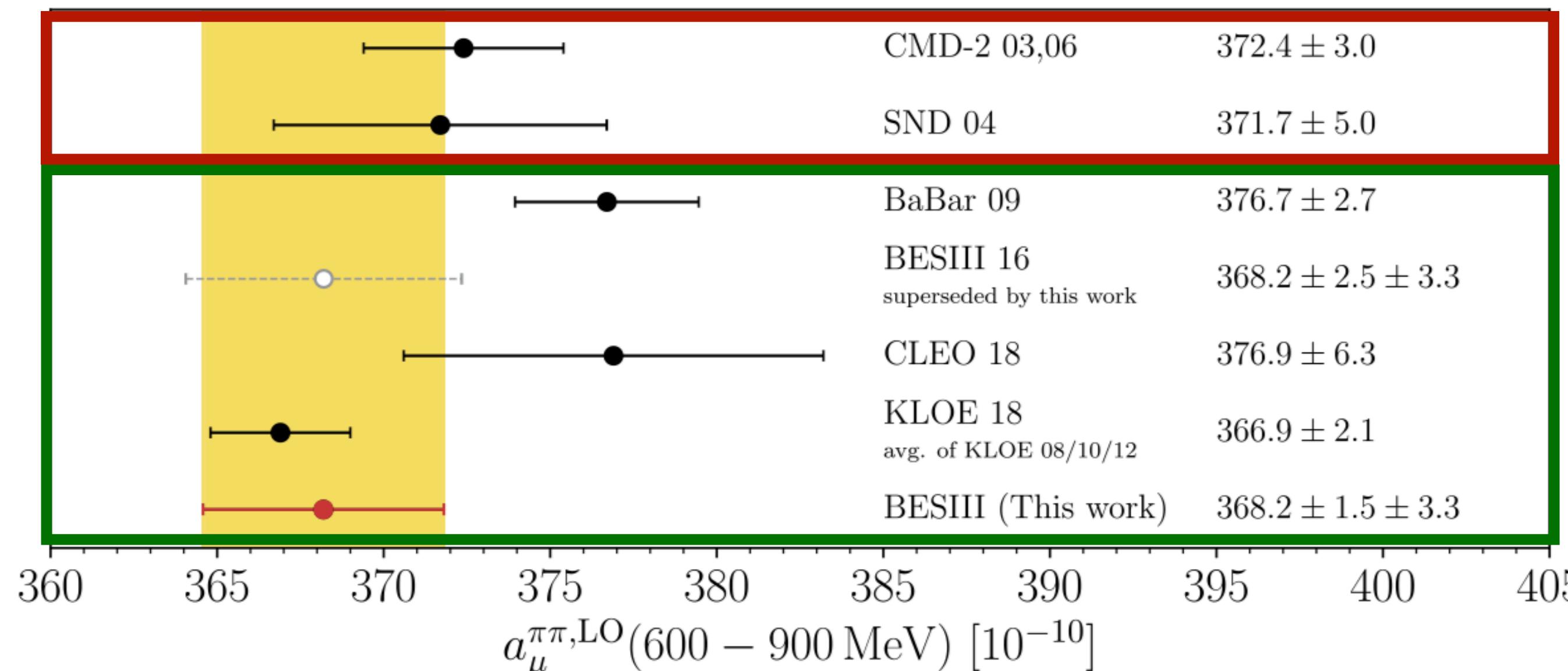
I. Scan analysis by directly varying \sqrt{s} - CMD-2, SND;



The SM estimate

The σ_{had} must be measured at all centre of mass energy \sqrt{s} :

1. Scan analysis by directly varying \sqrt{s} - CMD-2, SND;
2. Use Initial State Radiation to measure the \sqrt{s} of each collision
- KLOE, BaBar, BESIII, CLEO.

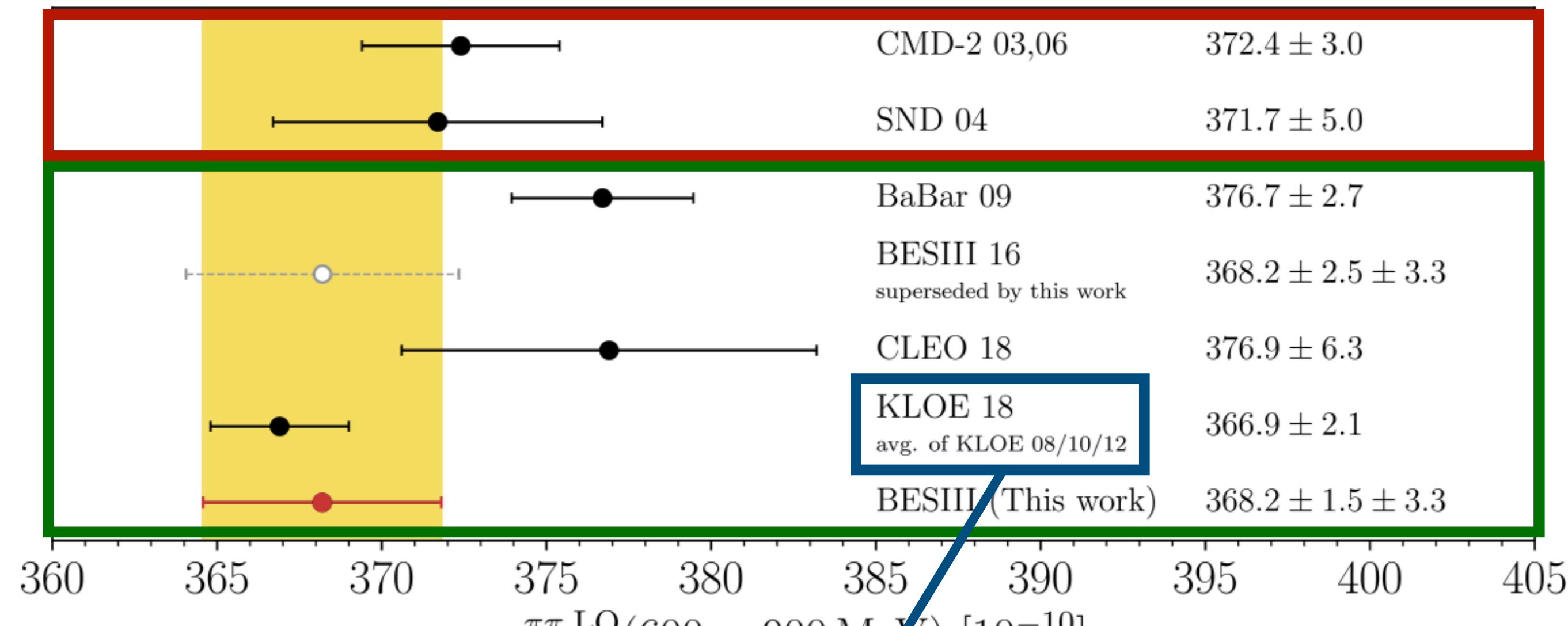


Discrepancies

As a consequence we have the following **discrepancies**:

- **Experiment vs SM data-driven** estimate
- **SM data-driven vs lattice** estimate
- **3σ tension** between **BaBar and KLOE data** used in the SM data-driven estimate

The SM estimate



Three different analysis: KLOE08, KLOE10, KLOE12.

Radiative cross section including ISR photon

$$s \frac{d\sigma(\pi^+\pi^-\gamma)}{ds'} = \sigma_{\pi\pi}^0(s') H(s', s)$$

$s' = M_{\pi\pi}^2 \rightarrow$ di-pion invariant mass

Radiator function
accounting for ISR

The SM estimate

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2. **KLOE10**: measurements in the range $0.1 < s'/\text{GeV}^2 < 0.85$ at $\sqrt{s} = 1 \text{ GeV}$ ($4.5 \cdot \Gamma_\phi$ below the ϕ meson pole). It requires the knowledge of the **luminosity**.

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2. **KLOE10**: measurements in the range $0.1 < s'/\text{GeV}^2 < 0.85$ at $\sqrt{s} = 1 \text{ GeV}$ ($4.5 \cdot \Gamma_\phi$ below the ϕ meson pole). It requires the knowledge of the luminosity.
3. **KLOE12**: relies on the ratio of the number of $\pi^+\pi^-\gamma$ and $\mu^+\mu^-\gamma$ events in the range $0.35 < s'/\text{GeV}^2 < 0.95$. The dependence of the luminosity cancels in the ratio.

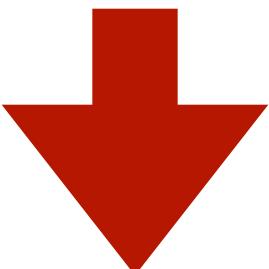
Indirect new physics effects

Can new physics effects impact the hadronic cross section determination?

It is challenging to affect the hadronic cross-section via extra contributions since the hadronic cross sections are very large!

[Passera et al. '08, '09, '10,
Keshavarzi et al '20, Di Luzio '21, ...]

However, the absolute cross section is required!



Key idea: **new physics can enter the channels used to calibrate the luminosity!**

Indirect new physics effects

The Luminosity determination

$$\sigma_{\text{had}} \propto \frac{N_{\text{had}}}{\mathcal{L}_{e^+e^-}}$$

A smaller luminosity implies a larger hadronic cross section

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Total number of $e^+e^- \rightarrow e^+e^-$ events

$$\mathcal{L}_{e^+e^-}^{\text{SM}} = \frac{N_{\text{Bhabha}}}{\sigma_{\text{eff}}^{\text{SM}}}$$

SM prediction

Indirect new physics effects

The Luminosity determination

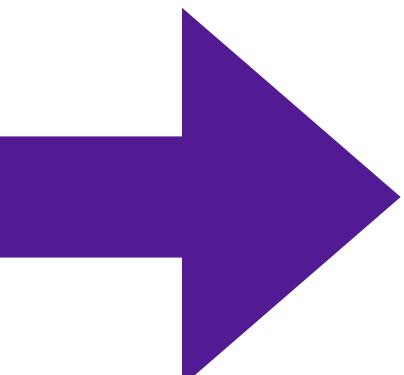
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SM prediction



$$\mathcal{L}_{e^+e^-} = \mathcal{L}_{e^+e^-}^{\text{SM}} \frac{\sigma_{\text{eff}}^{\text{SM}}}{\sigma_{\text{eff}}}$$

Full Bhabha cross section including NP

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}^{\text{SM}}(1 + \delta_R)$$

$$a_\mu^{HVP,LO} \rightarrow a_\mu^{HVP,LO}(1 + \delta_R)$$

Indirect new physics effects

The $\sigma(\mu\mu\gamma)$ method

Measured value

$$\sigma_{\pi^+\pi^-}^0 = \frac{N_{\pi^+\pi^-\gamma_{ISR}}}{N_{\mu^+\mu^-\gamma_{ISR}}} \sigma_{\mu^+\mu^-}^0$$

QED $e^+e^- \rightarrow \mu^+\mu^-$
cross section

Measured value

Indirect new physics effects

The $\sigma(\mu\mu\gamma)$ method

Measured value

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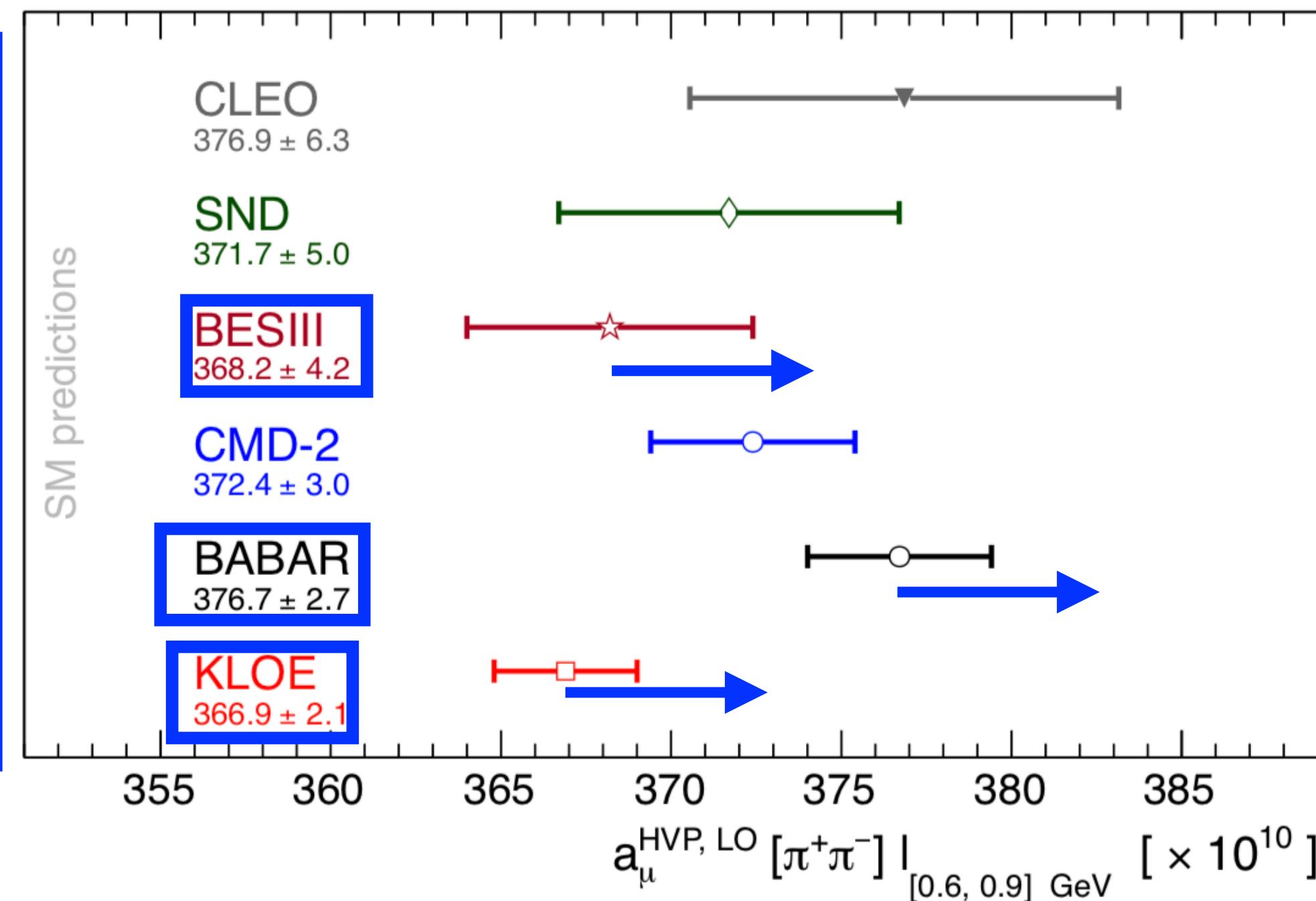
What if we have $\mu^+\mu^-X$ **new physics** events mimicking the $\mu^+\mu^-\gamma$?

$$\sigma_{\pi^+\pi^-}^{0\gamma^*} = \frac{N_{\pi^+\pi^-\gamma_{ISR}}}{N_{\mu^+\mu^-\gamma_{ISR}} - N_{\mu^+\mu^-\gamma_{ISR}}^{NP}} \sigma_{\mu^+\mu^-}^0 \sim \sigma_{\pi^+\pi^-}^0 (1 + \delta_\mu(s'))$$

Indirect new physics effects

New physics mimicking
 $\mu^+ \mu^- \gamma$ final states
modify KLOE12, BaBar
and BESIII analyses.

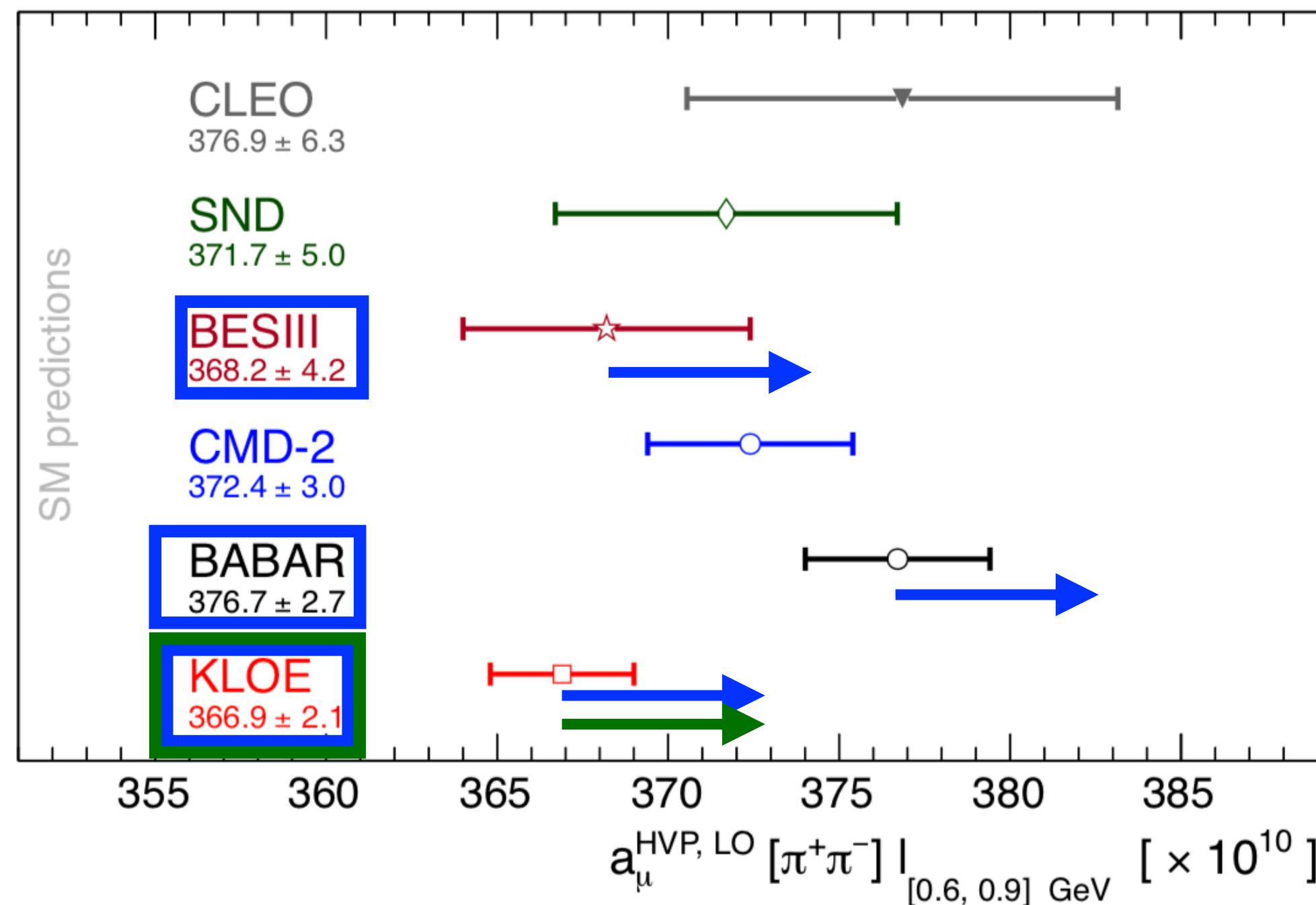
$$\delta_\mu(s') = \frac{\sigma_{\mu\mu X}^{NP} \epsilon^{NP}}{\sigma_{\mu\mu\gamma} \epsilon^{SM}}$$



Indirect new physics effects

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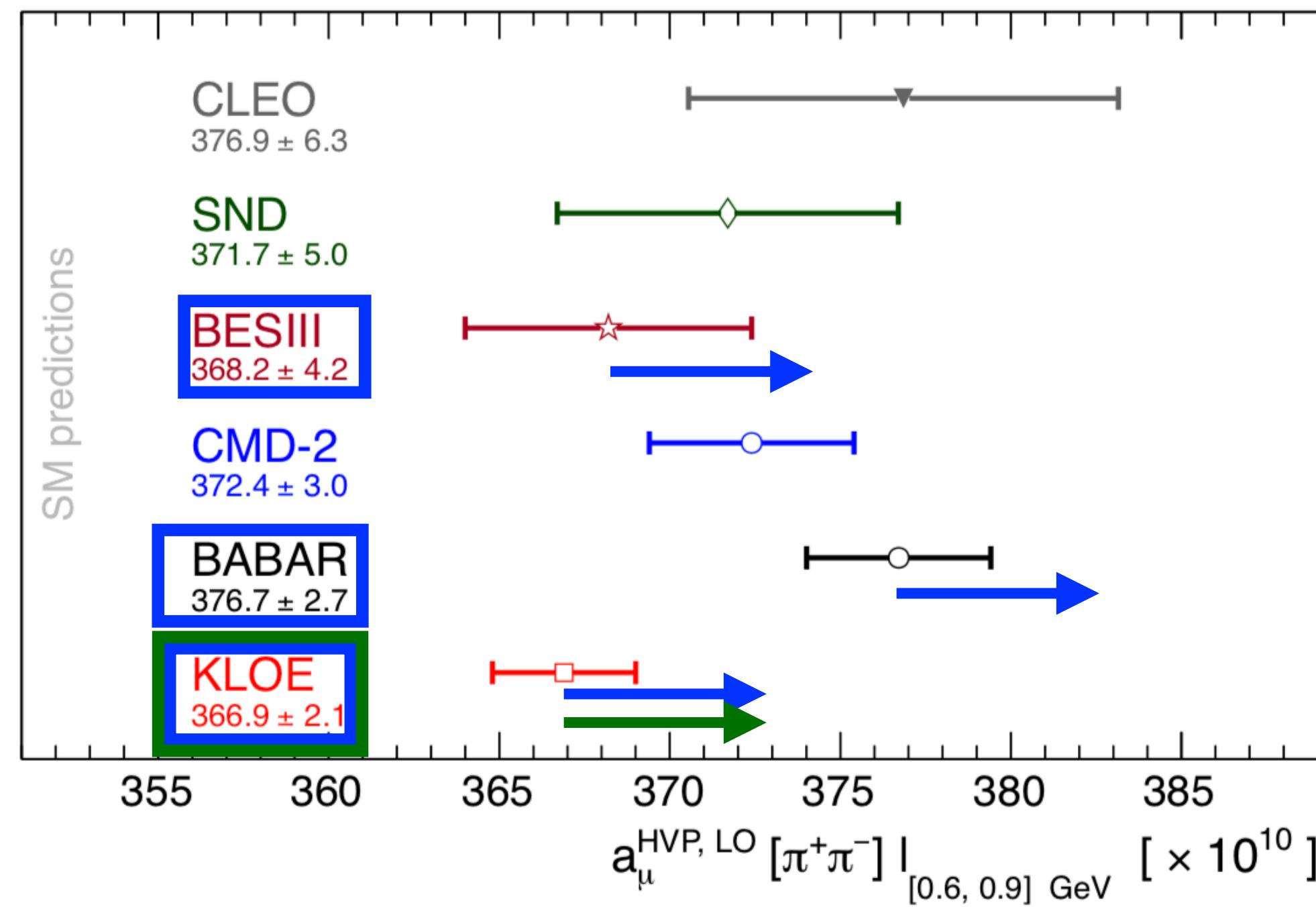
The measurements from KLOE08 and KLOE10 can be modified by new physics entering $e^+e^- \rightarrow e^+e^-$ scattering

$$\delta_R = \frac{\sigma_{e^+e^-}^{NP} \epsilon_{e^+e^-}^{NP}}{\sigma_{e^+e^-}^{SM} \epsilon_{e^+e^-}^{SM}}$$

Indirect new physics effects

New physics mimicking $\mu^+\mu^-\gamma$ final states modify KLOE12, BaBar and BESIII analyses.

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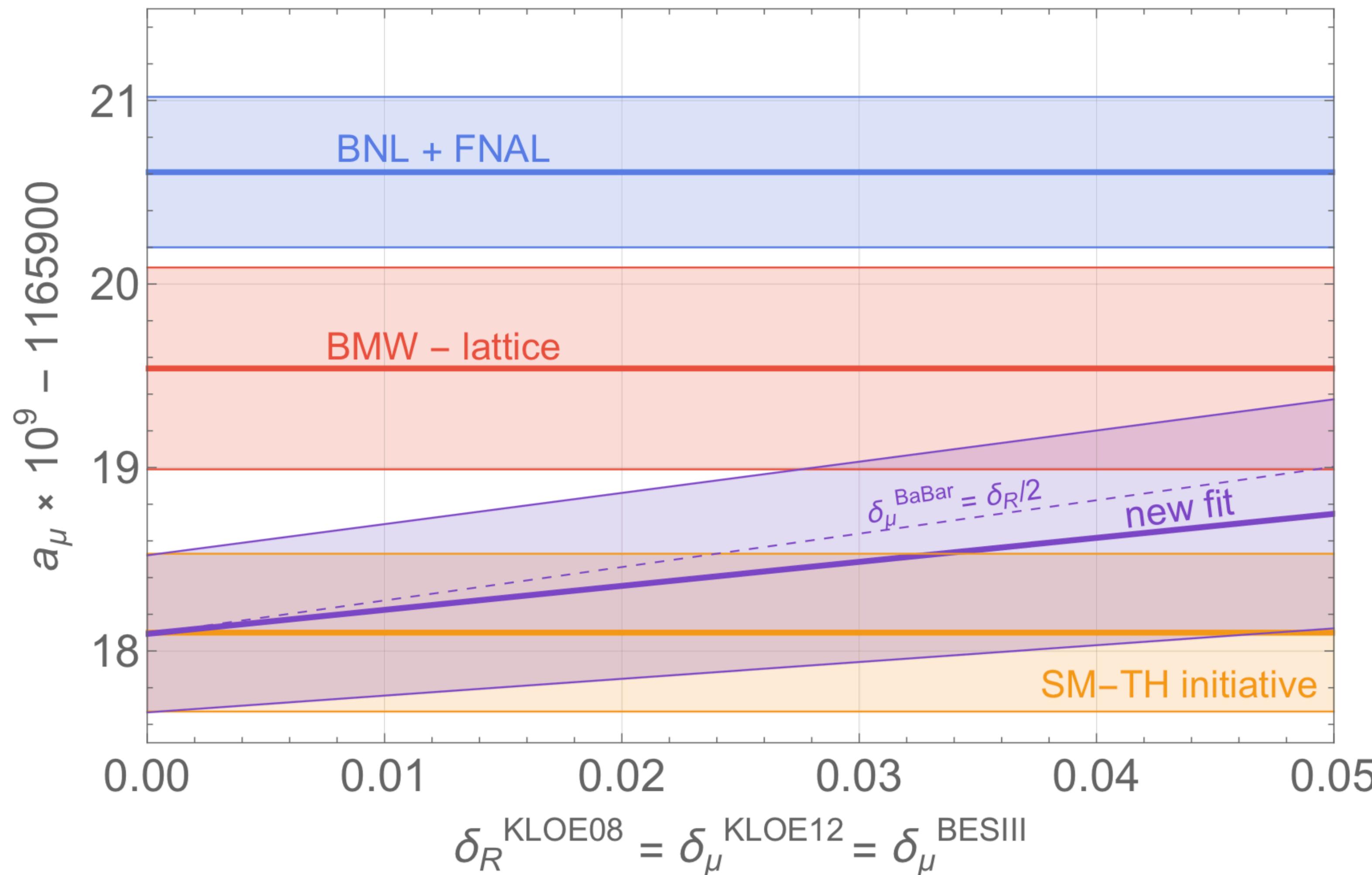


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Flavour universal new physics that modifies the Bhabha scattering is expected to modify the $\gamma\mu\mu X$ process, up to differences related to the muon mass and the experiment. We therefore expect $\delta_R \sim \delta_\mu$.

Indirect new physics effects



Solving the a_u tension

We need a model that fakes Bhabha scattering and $\mu\mu\gamma$ final states!

Dark photon
Field strength kinetic mixing

$$\mathcal{L} \supset -\frac{1}{4} F'^{\mu\nu} F'_{\mu\nu} - \frac{\epsilon}{2c_W} B_{\mu\nu} F'^{\mu\nu} +$$

+ dark Higgs (S) potential +

$$+ \bar{\chi} (iD_\mu \gamma^\mu - m_\chi) \chi +$$
$$+ y_{SL} S \bar{\chi}^c P_L \chi + y_{SR} S \bar{\chi}^c P_R \boxed{\chi} + \dots$$

dirac fermion
dark matter
 $\chi = (\chi_L, \bar{\chi}_R)$

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dirac fermion
dark matter
 $\chi = (\chi_L, \bar{\chi}_R)$

Spectrum after the U(1)
symmetry is broken:

$$g_{\alpha D} q_S v_S$$

$$V$$

$$\sqrt{2\lambda_S} v_S$$

$$S$$

$$m_\chi \pm \sqrt{2} v_S (y_{SR} + y_{SL})$$

$$\begin{matrix} \chi_2 \\ \chi_1 \end{matrix} \neq$$

Solving the a_u tension

In order to generate a significant shift in KLOE's luminosity and to provide additional di-muon events:

1. the dark photon mass must be very close to the KLOE centre of mass energy $\sqrt{s} \simeq 1.02$ GeV;
2. the dark photon must contribute substantially to Bhabha scattering;
3. The dark photon must escape bump searches: the main decay channel must be multibody and include some missing energy;

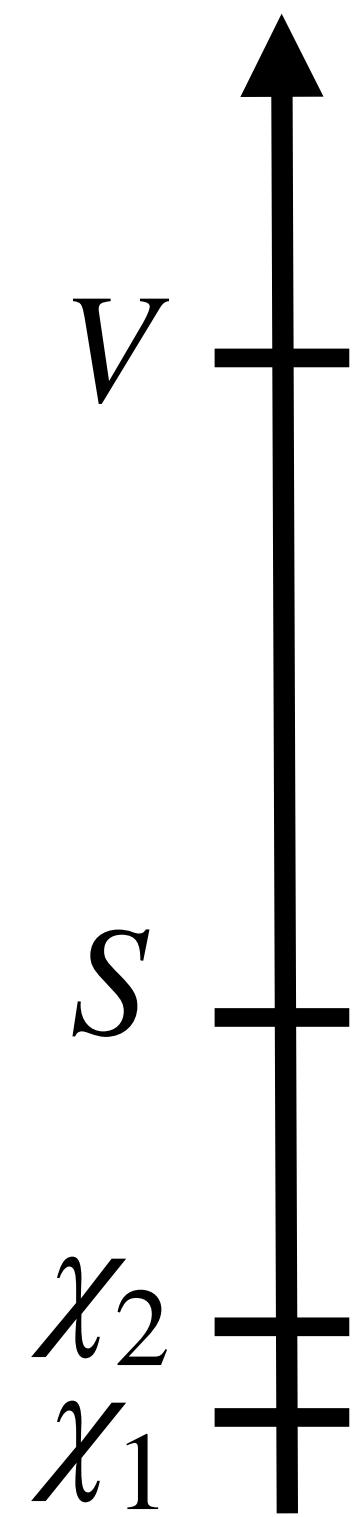
$$m_V \sim 1 \text{ GeV} \gtrsim m_{\chi_2} \gg m_{\chi_1}$$

Spectrum after the U(1) symmetry is broken:

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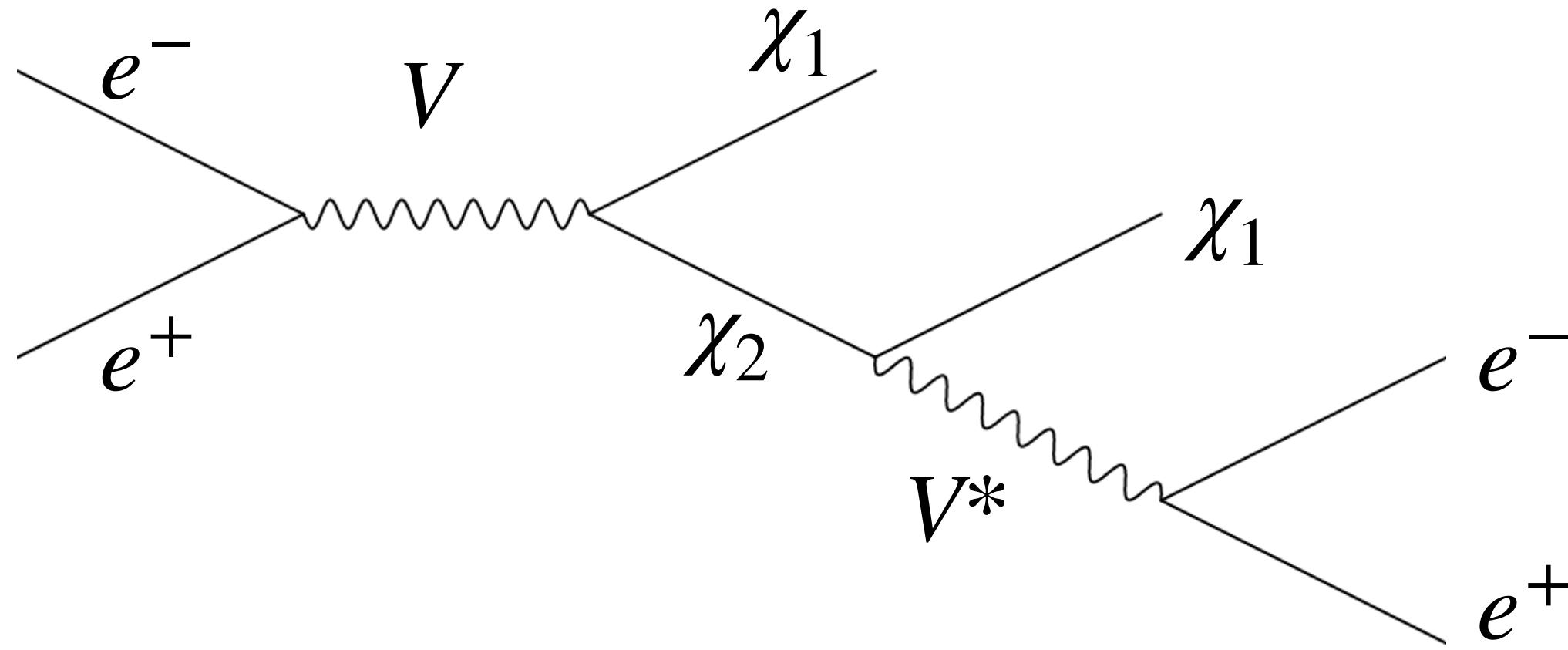
$$\sqrt{2\lambda_S} v_S$$

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Solving the a_u tension

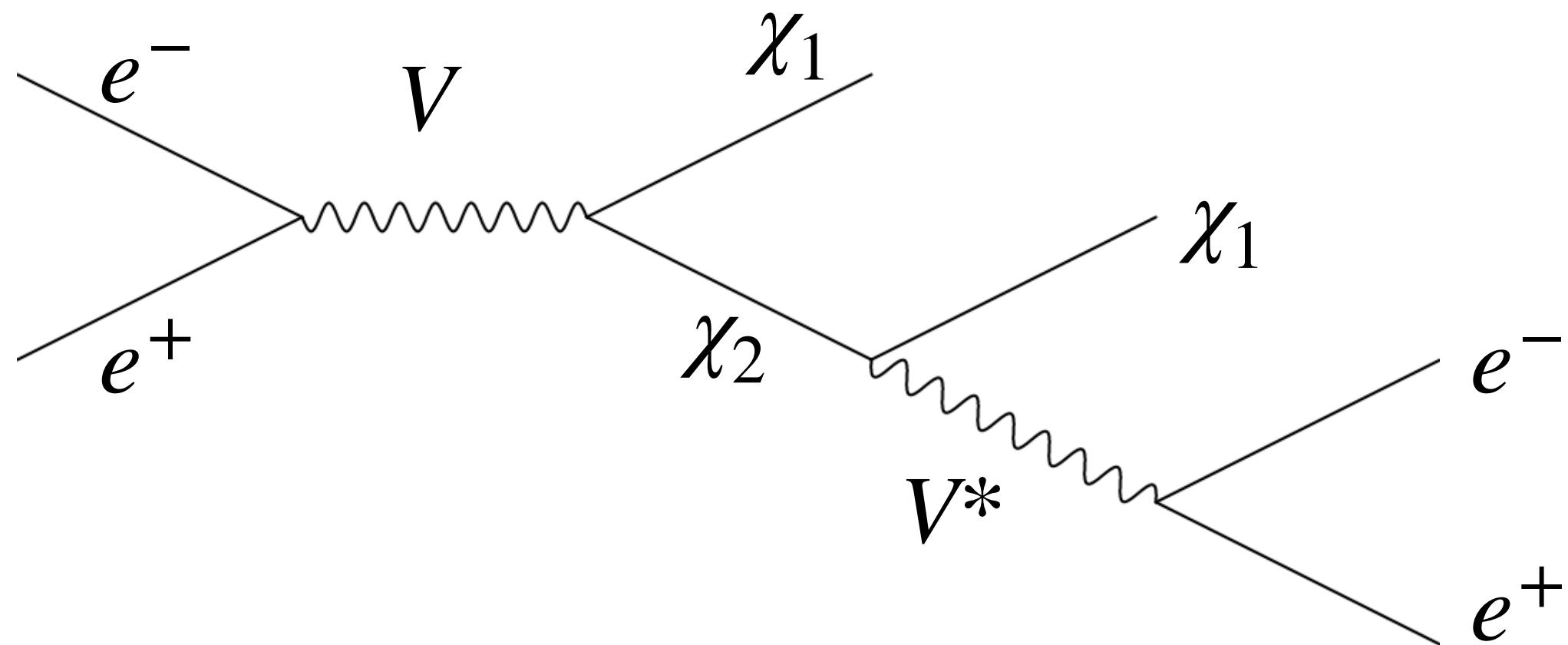
Shifting KLOE08



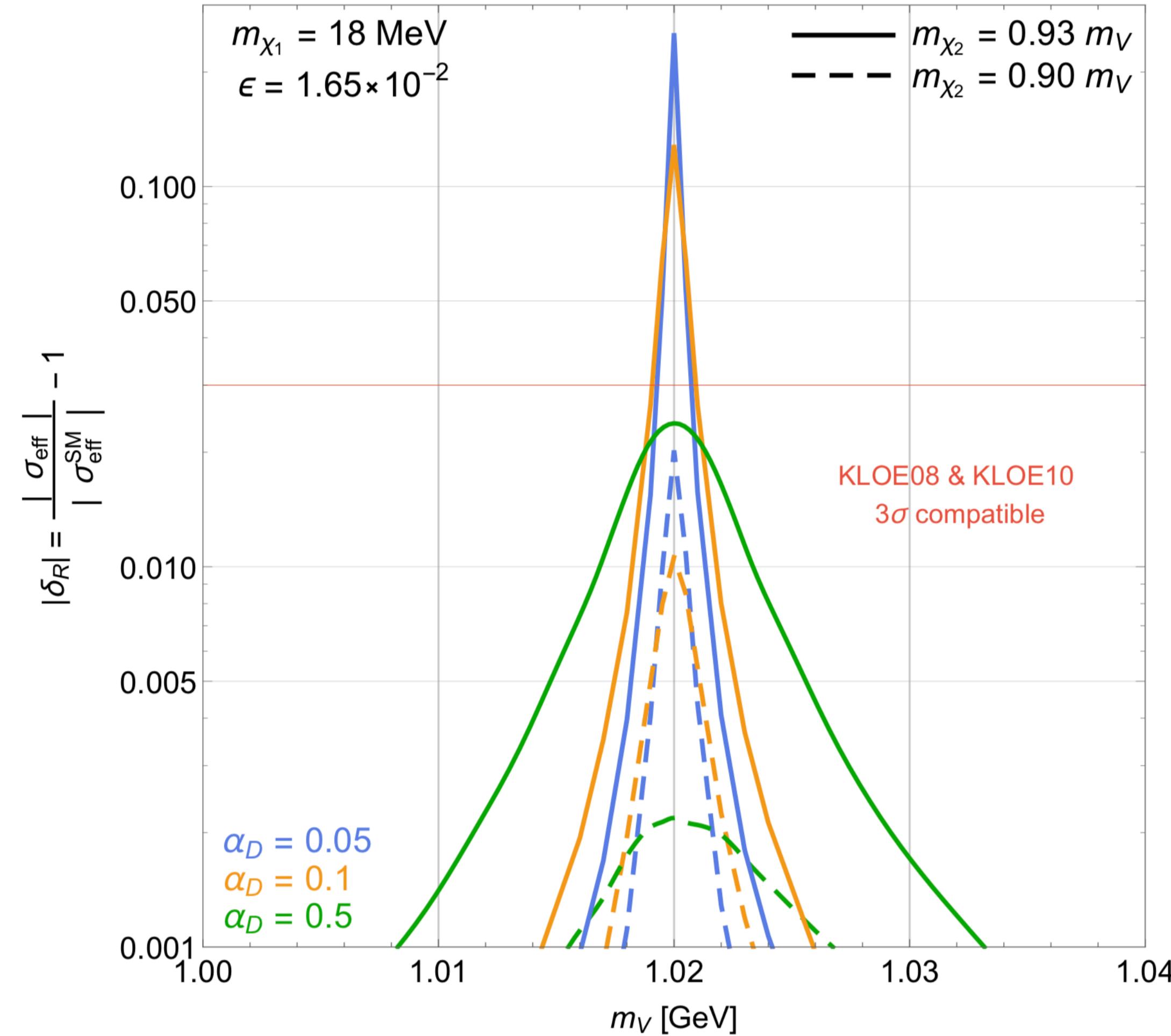
Recast KLOE08 search using
Madgraph.

Solving the a_μ tension

Shifting KLOE08

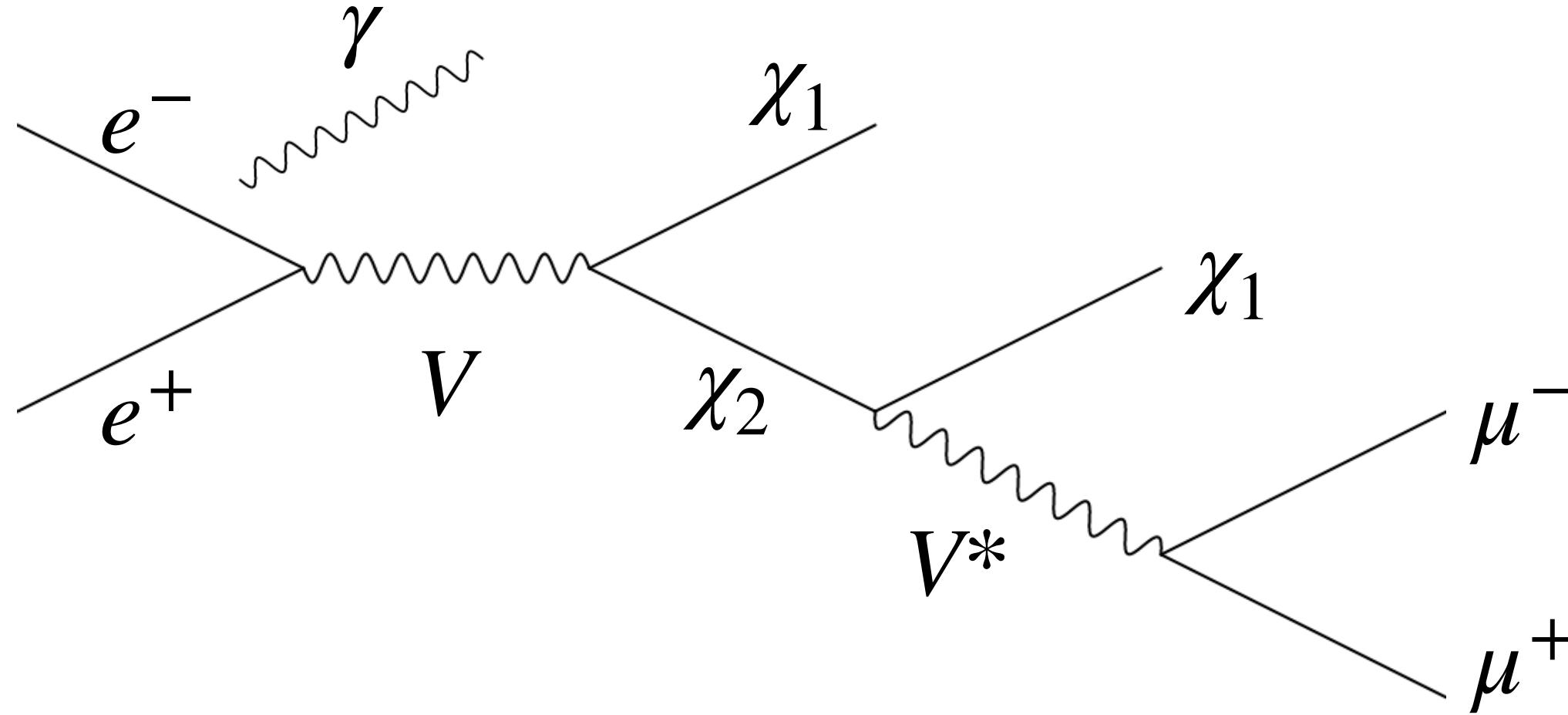


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Solving the a_u tension

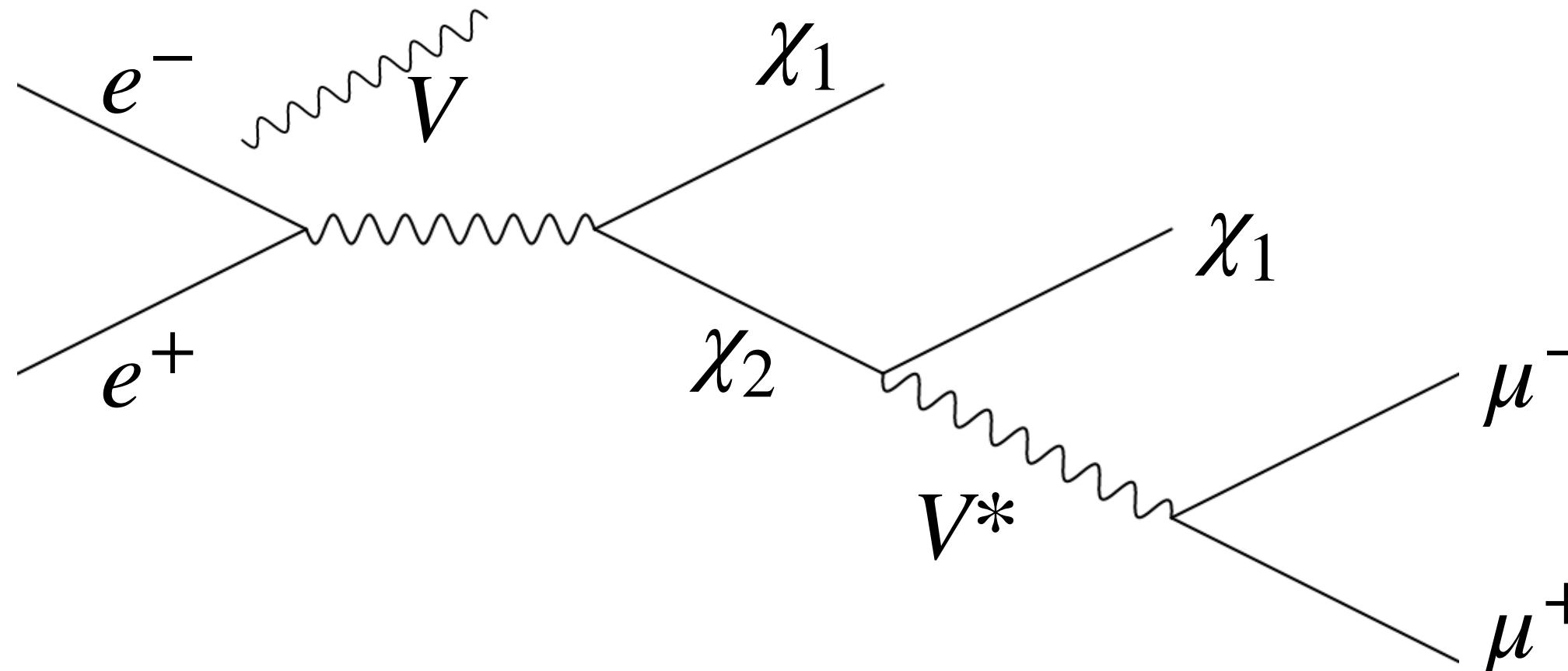
Shifting KLOE I2, BESIII and BaBar



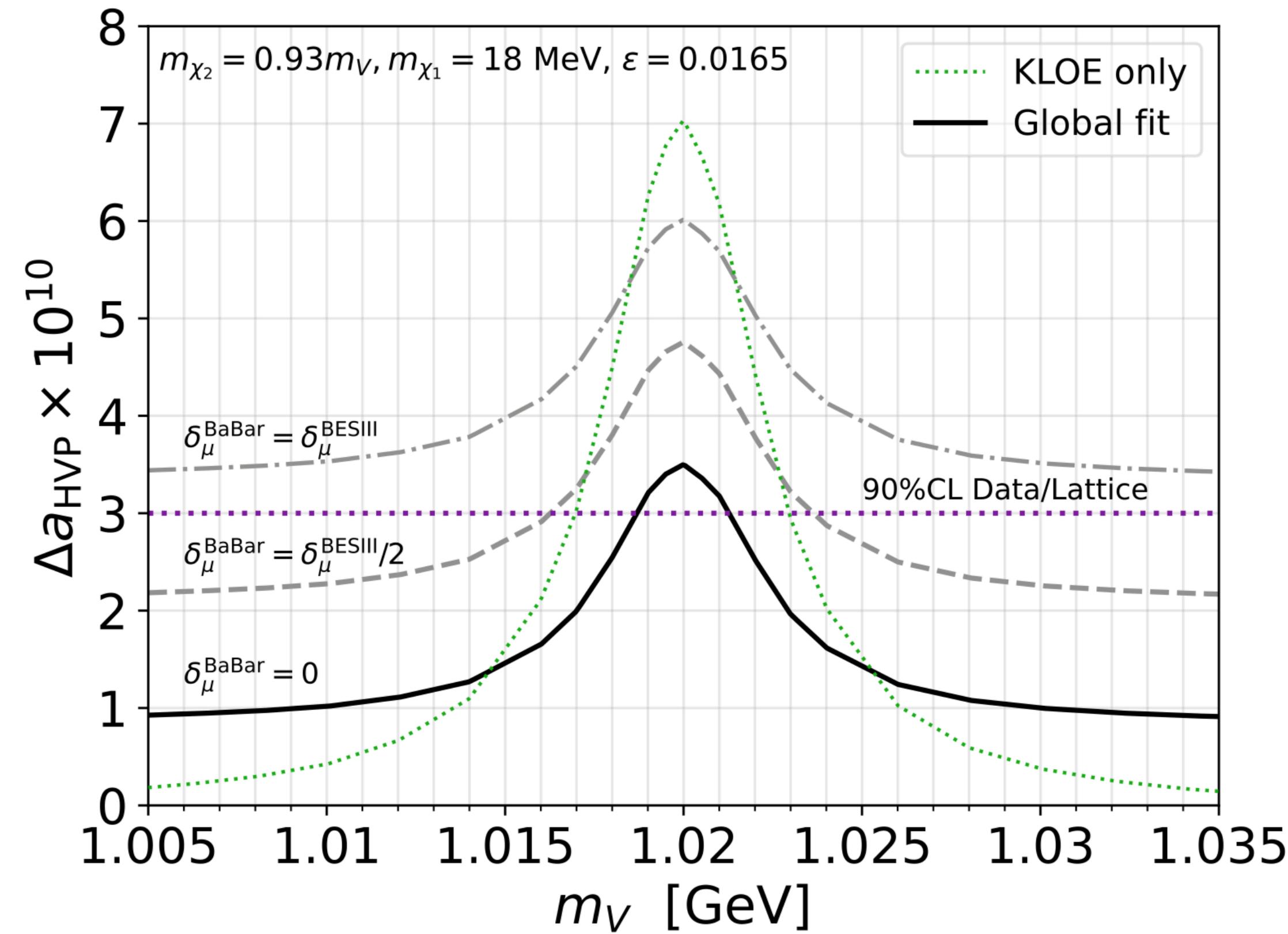
Recast all searches using
Madgraph.

Solving the a_μ tension

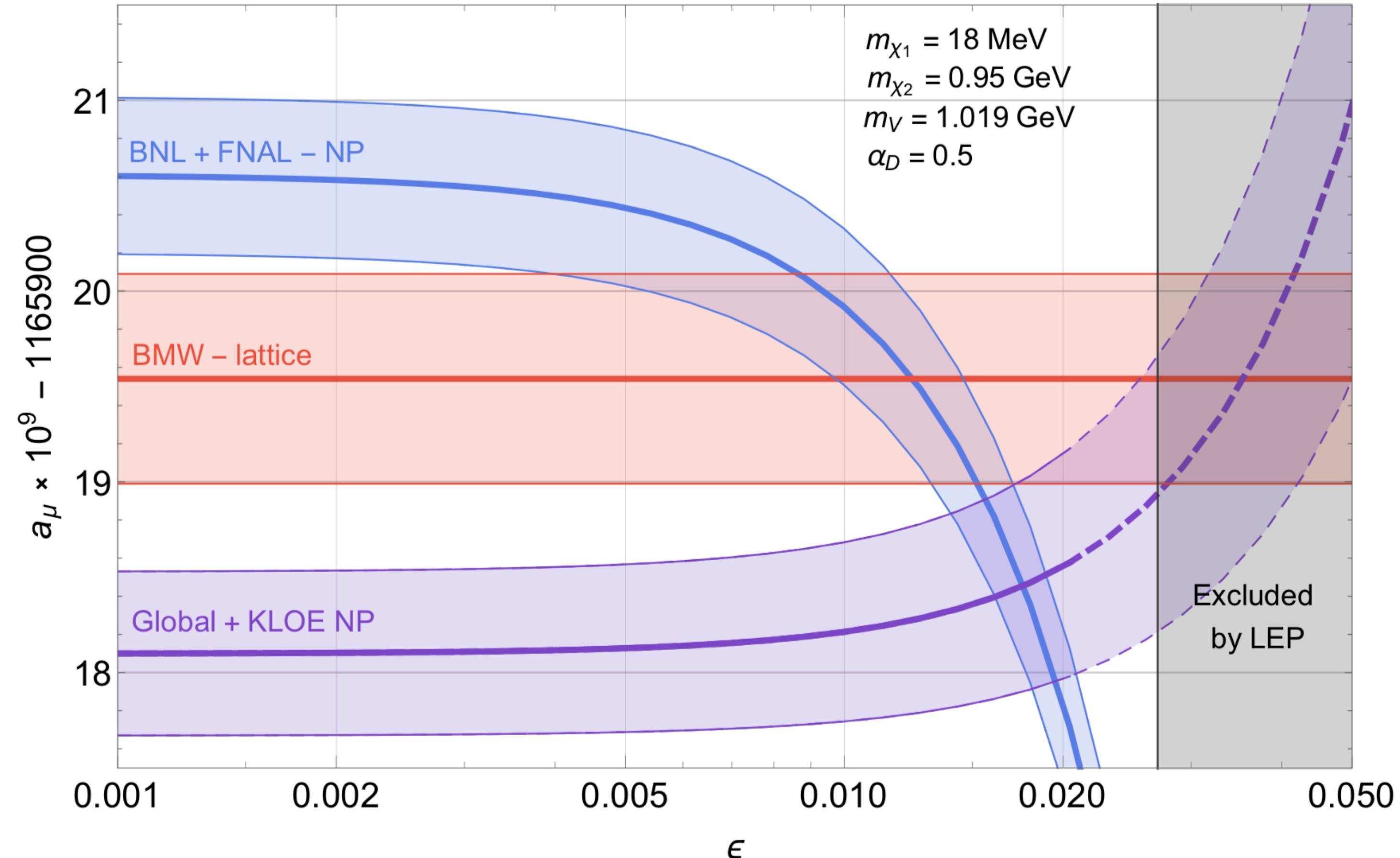
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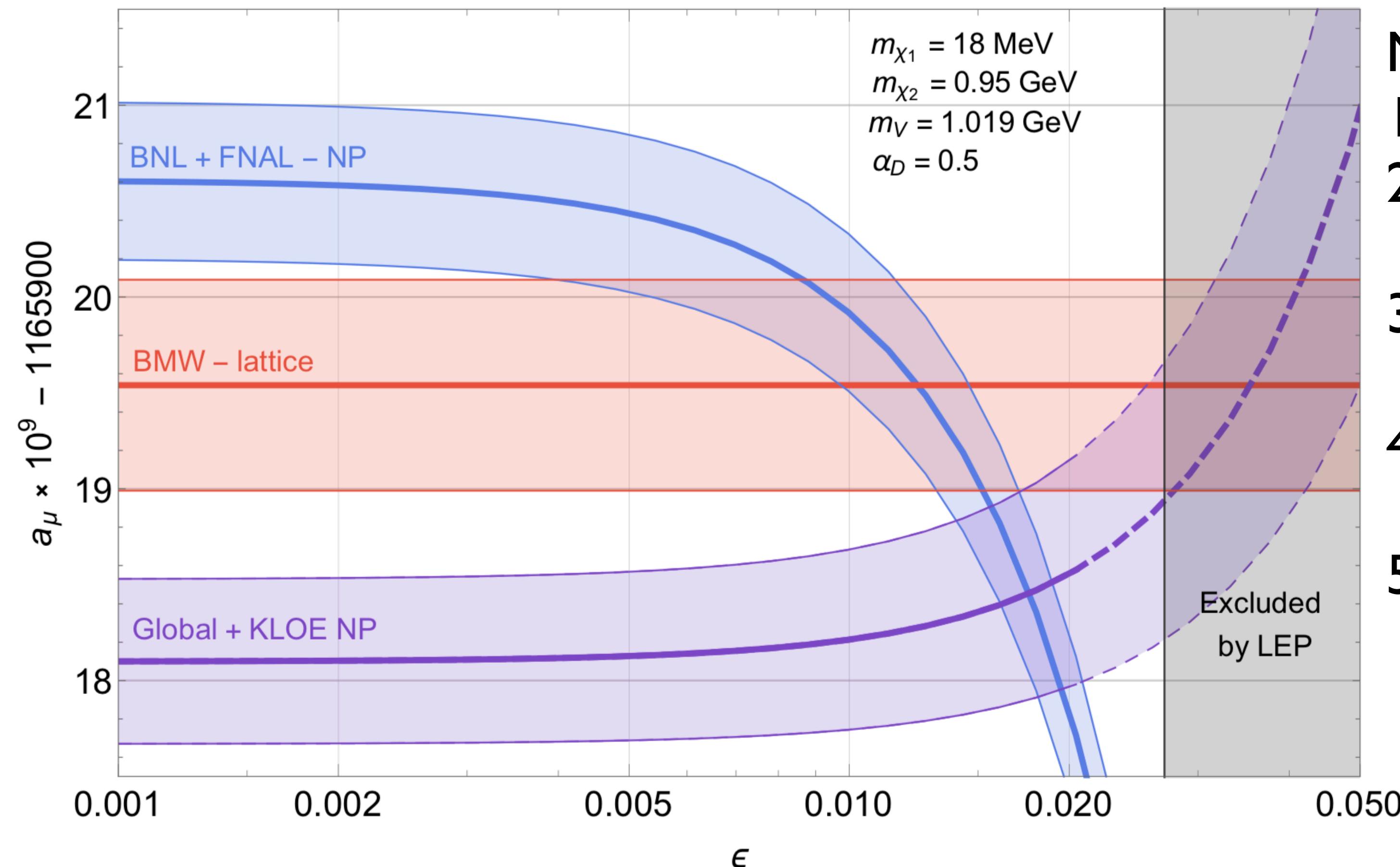
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Solving the a_μ tension



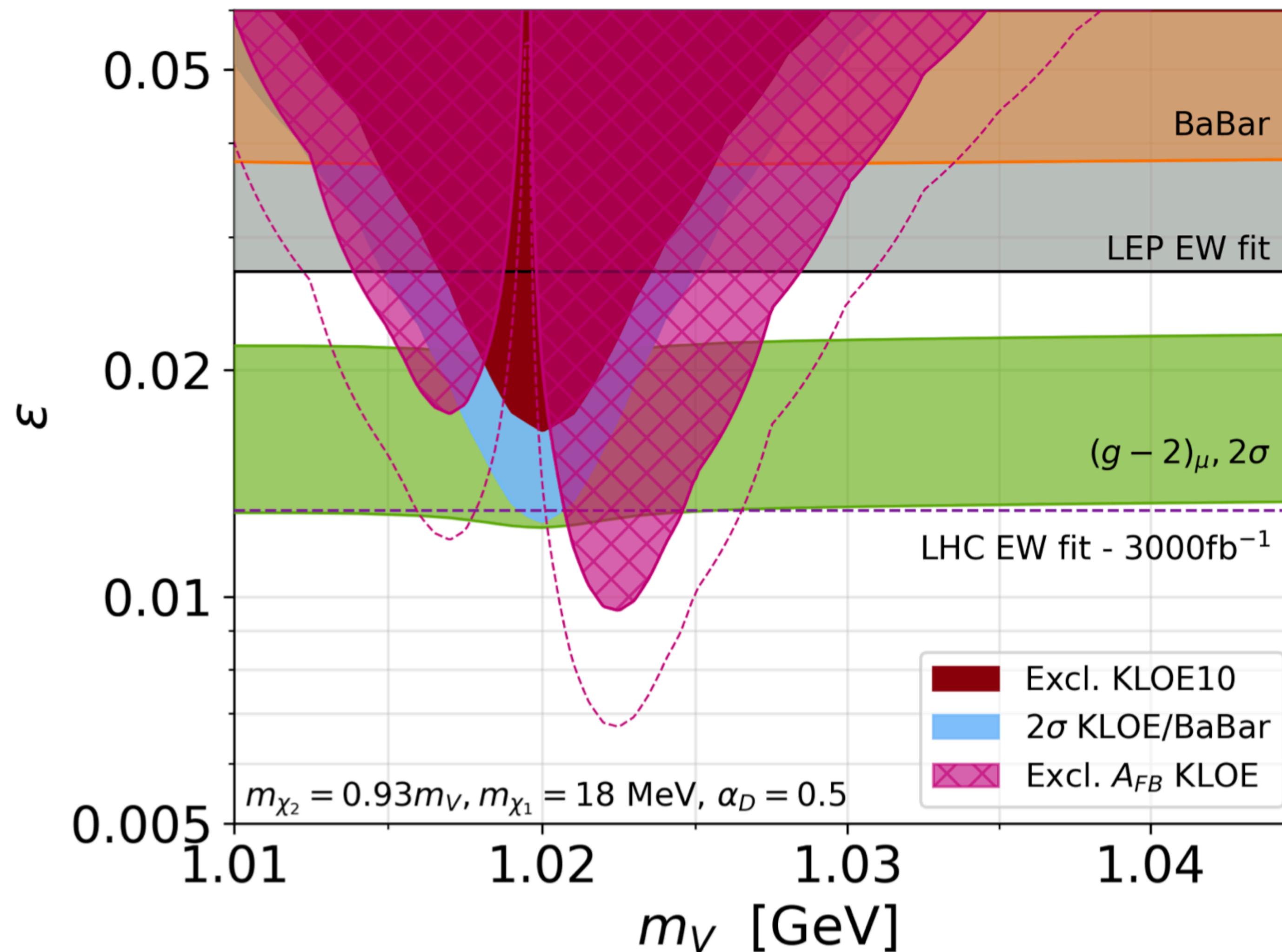
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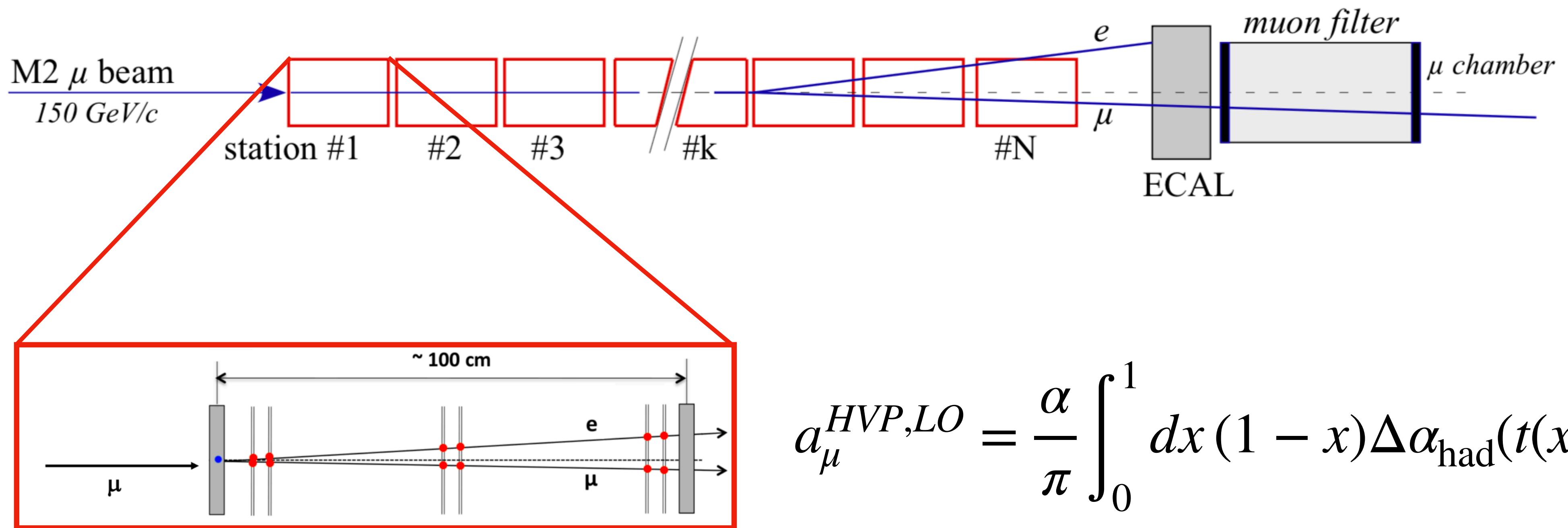
Conclusions

- The **4.2 sigma discrepancy** between the SM prediction for the g-2 and the experimental results is accompanied by other anomalies: **data-driven vs lattice & KLOE vs BaBar** in the data-driven estimate.
- The presence of **new physics indirectly modifies the experimental results used by the data-driven approach, increasing σ_{had}** ;
- **Dark photon models** may shift the σ_{had} measurement of KLOE to be compatible with BaBar, **solving the KLOE vs BaBar discrepancy**;
- The g-2 anomalies can be solved by an **interplay between direct (~75%) and indirect (~25%) contributions**;
- **New measurements by MUonE** will add important **information in order to explain these anomalies**.

Backup



MUonE



$$a_\mu^{HVP,LO} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta \alpha_{\text{had}}(t(x))$$

MUonE

[G²dC, Nardi '22]

