

Chirally enhanced muon $g-2$ and related observables

with N. McGinnis, K. Hermanek and S. Yoon

[arXiv:2011.11812](https://arxiv.org/abs/2011.11812) [hep-ph]

[arXiv:2103.05645](https://arxiv.org/abs/2103.05645) [hep-ph]

[arXiv:2108.10950](https://arxiv.org/abs/2108.10950) [hep-ph]

[arXiv:2205.14243](https://arxiv.org/abs/2205.14243) [hep-ph]

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Planck 2022, Paris, June 1, 2022

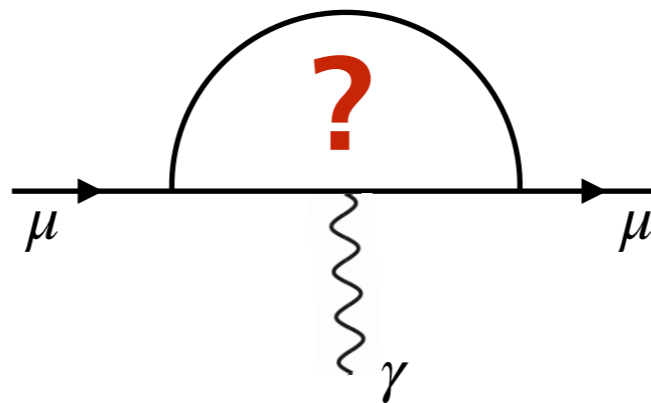
Muon g-2

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (2.51 \pm 0.59) \times 10^{-9}$$

Muon g-2, Fermilab, arXiv:2104.03281 [hep-ex]

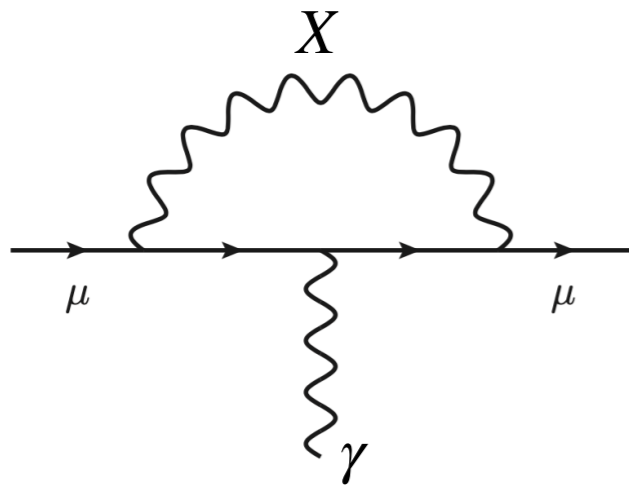
Muon g-2, BNL, arXiv:hep-ex/0602035

SM prediction, T. Aoyama, et al., Phys. Rept. 887, 1-166 (2020)



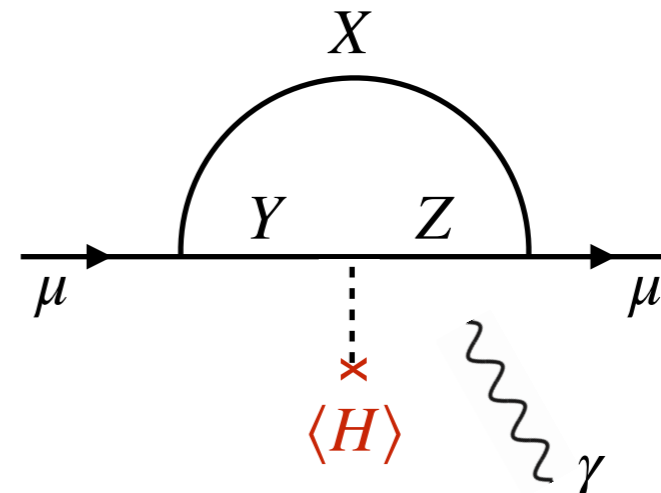
New physics contributions to muon g-2

Typical NP contribution



$$\Delta a_\mu \simeq \frac{\lambda_{NP}^2}{16\pi^2} \frac{m_\mu^2}{m_{NP}^2}$$

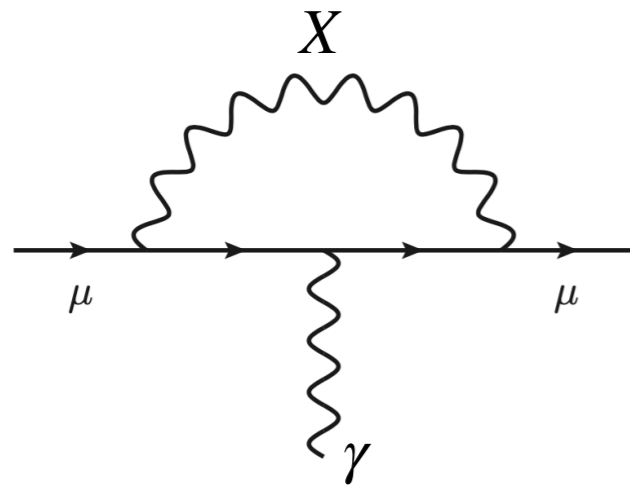
Mass enhanced NP contribution



$$\Delta a_\mu \simeq \frac{\lambda_{NP}^3}{16\pi^2} \frac{m_\mu v}{m_{NP}^2}$$

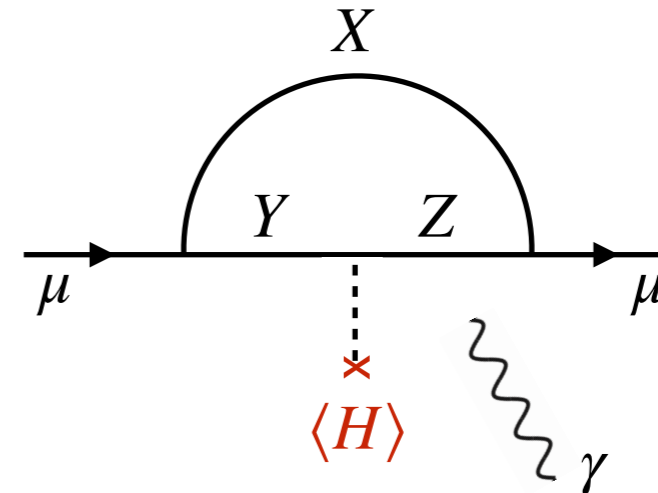
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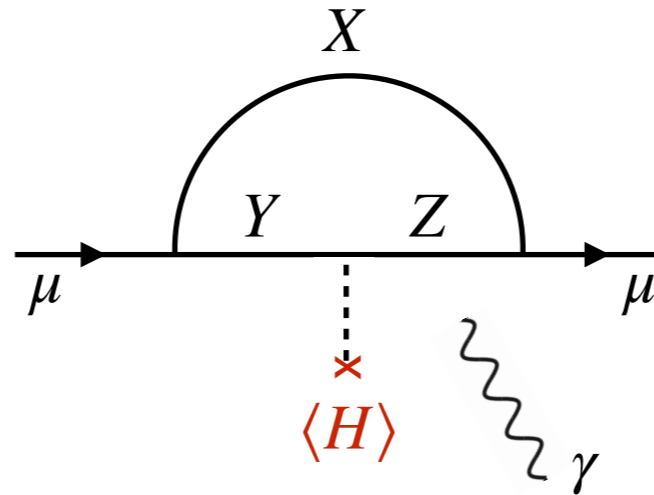
$$\Delta a_\mu \simeq \frac{\lambda_{NP}^3}{16\pi^2} \frac{m_\mu v}{m_{NP}^2}$$

Enhancement:

$$\frac{\lambda_{NP} v}{m_\mu}$$

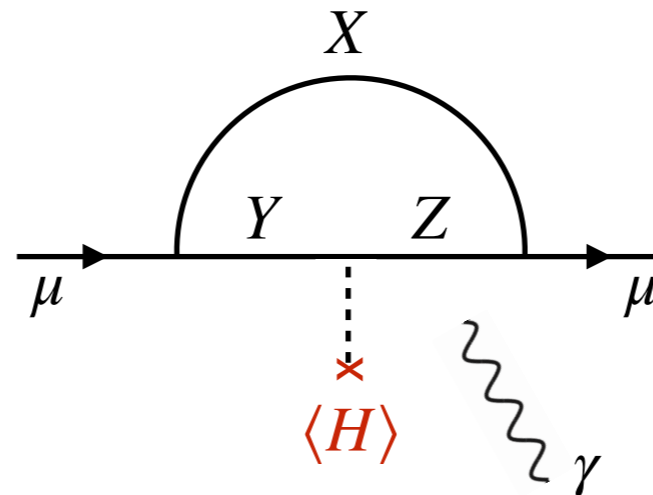
can explain Δa_μ with NP at $\lesssim 10$ TeV (50 TeV) with $\lambda_{NP} \simeq 1$ ($\sqrt{4\pi}$)

Mass enhanced NP contributions to Δa_μ



X, Y, Z can have any quantum numbers (allowing for the loop):

Mass enhanced NP contributions to Δa_μ



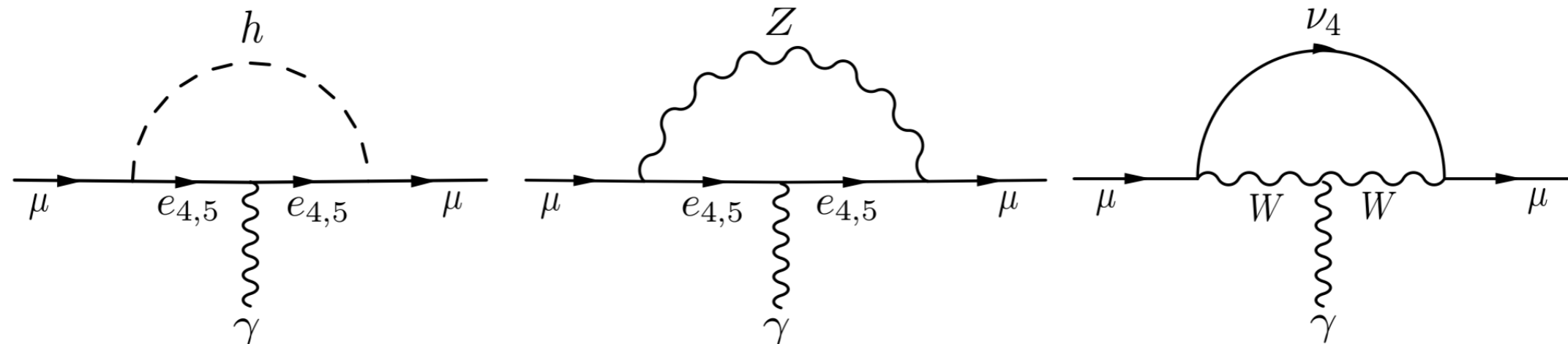
X, Y, Z can have any quantum numbers (allowing for the loop):

- $X = h, Z, W$ and $Y, Z = \text{vectorlike leptons}$

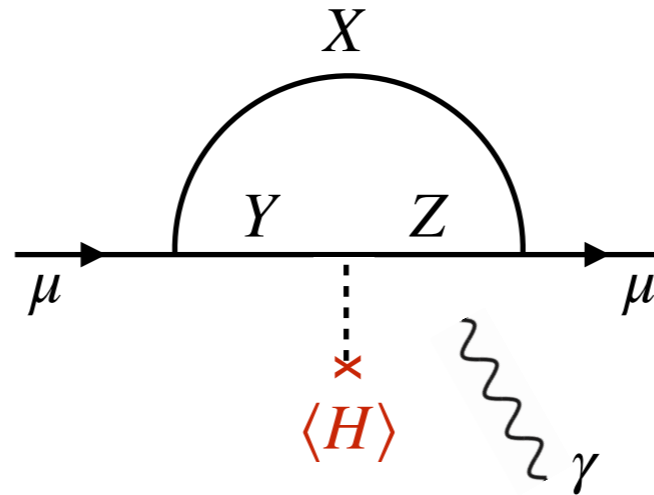
minimal, just SM with new leptons, constrained the most

K. Kannike, M. Raidal, D. M. Straub and A. Strumia, arXiv:1111.2551 [hep-ph]

R. D. and A. Raval, arXiv:1305.3522 [hep-ph]



Mass enhanced NP contributions to Δa_μ



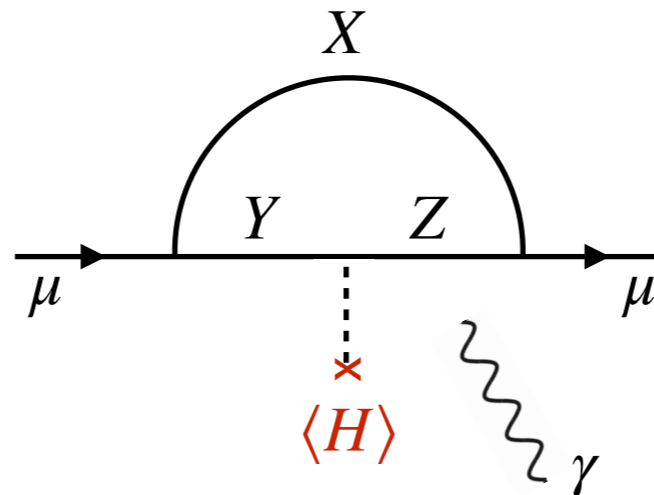
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- $X, Y, Z = 2$ fermions and 1 scalar or 2 scalars and 1 fermion
scalars not participating in EWSB, the most popular, many options, the least constrained
(include models with superpartners)

Mass enhanced NP contributions to Δa_μ



X, Y, Z can have any quantum numbers (allowing for the loop):

- $X = h, Z, W$ and $Y, Z = \text{vectorlike leptons}$
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K. Kannike, M. Raidal, D. M. Straub and A. Strumia, arXiv:1111.2551 [hep-ph]
R. D. and A. Raval, arXiv:1305.3522 [hep-ph]
- $X = h, Z, W, H, A, H^\pm$ and $Y, Z = \text{vectorlike (or SM) leptons}$
e.g. 2HDM with new leptons, interpolates between the other two options
R. D. N. McGinnis and K. Hermanek, arXiv:2011.11812 [hep-ph], arXiv:2103.05645 [hep-ph]
- $X, Y, Z = 2 \text{ fermions and } 1 \text{ scalar}$ or $2 \text{ scalars and } 1 \text{ fermion}$
scalars not participating in EWSB, the most popular, many options, the least constrained
(include models with superpartners)

Type-II 2HDM with L + E (+ N)

General lagrangian describing mixing of the 2nd generation with new leptons:

$$\begin{aligned} \mathcal{L} \supset & \underline{- y_\mu \bar{l}_L \mu_R H_d - \lambda_E \bar{l}_L E_R H_d - \lambda_L \bar{L}_L \mu_R H_d - \lambda \bar{L}_L E_R H_d - \bar{\lambda} H_d^\dagger \bar{E}_L L_R} \\ & - \kappa_N \bar{l}_L N_R H_u - \kappa \bar{L}_L N_R H_u - \bar{\kappa} H_u^\dagger \bar{N}_L L_R \\ & \underline{- M_L \bar{L}_L L_R - M_E \bar{E}_L E_R - M_N \bar{N}_L N_R + h.c.,} \end{aligned}$$

	l_L	e_R	H_u	H_d	$L_{L,R}$	$N_{L,R}$	$E_{L,R}$
$SU(2)_L$	2	1	2	2	2	1	1
$U(1)_Y$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1
Z_2	$+$	$-$	$+$	$-$	$+$	$+$	$-$

Charged lepton mass matrix (after EWSB):

$$(\bar{\mu}_L, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_\mu v_d & 0 & \lambda_E v_d \\ \lambda_L v_d & M_L & \lambda v_d \\ 0 & \bar{\lambda} v_d & M_E \end{pmatrix} \begin{pmatrix} \mu_R \\ L_R^- \\ E_R \end{pmatrix}$$

diagonalizing this matrix leads to:
 two new mass eigenstates, e_4, e_5 ,
 modification of muon couplings,
 and couplings between the muon and e_4, e_5

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At energies much below M_L, M_E :

$$\mathcal{L} \supset -y_\mu \bar{l}_L \mu_R H_d - \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} \bar{l}_L \mu_R H_d H_d^\dagger H_d + h.c.,$$

dim. 6 operator is a new source of muon mass and Yukawa coupling:

$$m_\mu = y_\mu v_d + m_\mu^{LE}$$

$$\lambda_{\mu\mu}^h = (m_\mu + 2m_\mu^{LE})/v$$

$$m_\mu^{LE} \equiv \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} v_d^3$$

and is directly linked to contributions to Δa_μ

Δa_μ in type-II 2HDM with L + E (+ N)

R.D., N. McGinnis, and K. Hermanek, arXiv:2011.11812 [hep-ph]

arXiv:2103.05645 [hep-ph]

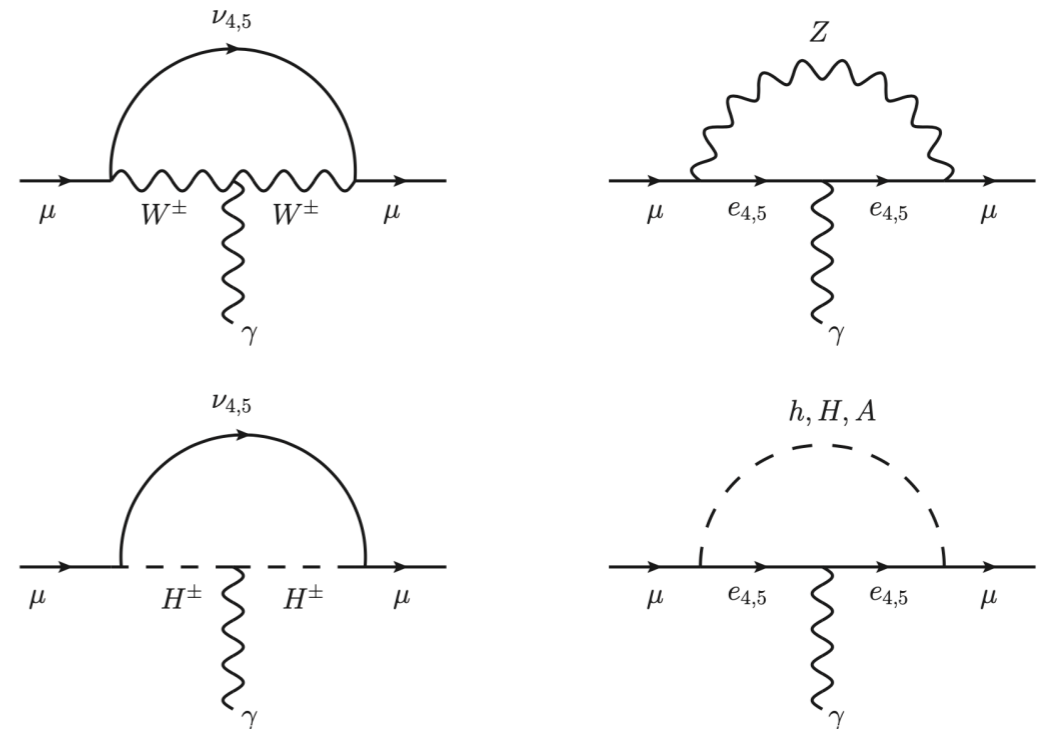
$$\Delta a_\mu^i \simeq \frac{k^i}{16\pi^2} \frac{m_\mu m_\mu^{LE}}{v^2}$$

$m_\mu^{LE} \equiv \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} v_d^3$

$$\left. \begin{aligned} k^W &= 1 \\ k^Z &= -1/2 \\ k^h &= -3/2 \end{aligned} \right\} k^W + k^Z + k^h = -1$$

$$\left. \begin{aligned} k^H &= -(11/12) \tan^2 \beta \\ k^A &= -(5/12) \tan^2 \beta \\ k^{H^\pm} &= (1/3) \tan^2 \beta \end{aligned} \right\} k^H + k^A + k^{H^\pm} = -\tan^2 \beta$$

assuming $M_{L,E} \simeq m_{H,A,H^\pm}$

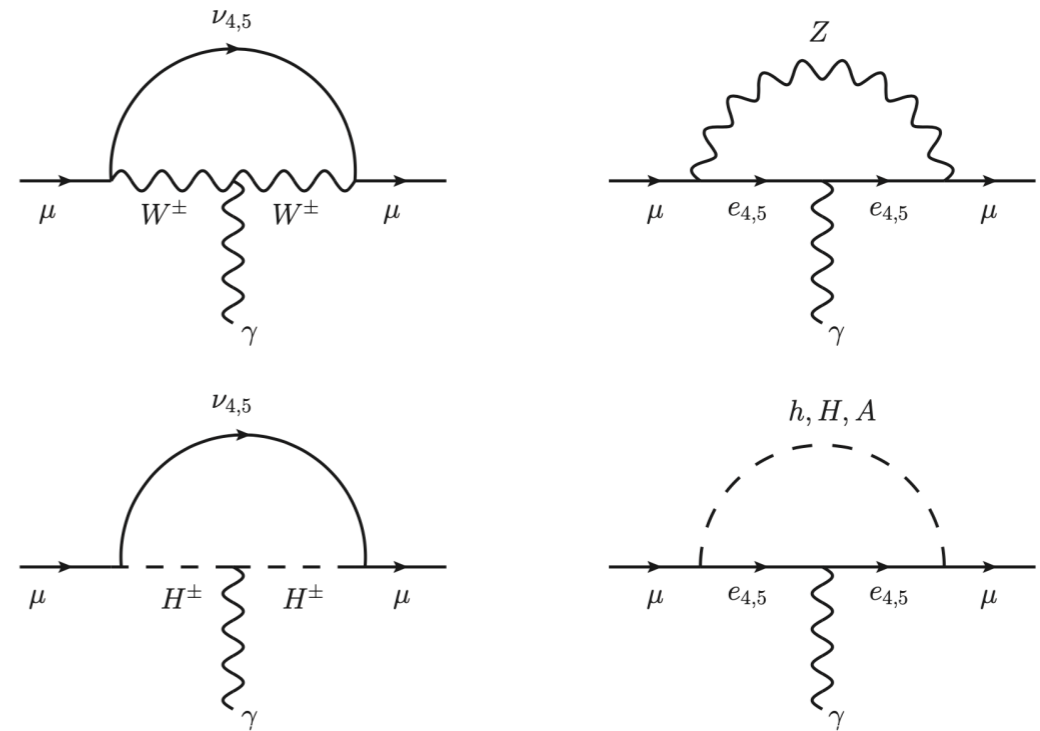


Δa_μ in type-II 2HDM with L + E (+ N)

R.D., N. McGinnis, and K. Hermanek, arXiv:2011.11812 [hep-ph]
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$$\left. \begin{aligned} k^W &= 1 \\ k^Z &= -1/2 \\ k^h &= -3/2 \end{aligned} \right\}$$

$$k^W + k^Z + k^h = -1$$

sufficient to explain Δa_μ with couplings ~ 0.5

$$\left. \begin{aligned} k^H &= -(11/12) \tan^2 \beta \\ k^A &= -(5/12) \tan^2 \beta \\ k^{H^\pm} &= (1/3) \tan^2 \beta \end{aligned} \right\}$$

$$k^H + k^A + k^{H^\pm} = -\tan^2 \beta$$

contributions of H, A, H^\pm to Δa_μ enhanced by $\tan^2 \beta$
would be able to explain even $100 \times \Delta a_\mu$

assuming $M_{L,E} \simeq m_{H,A,H^\pm}$

in the next few slides, I will focus on the SM+L+E

Δa_μ in SM with L + E and $h \rightarrow \mu^+ \mu^-$

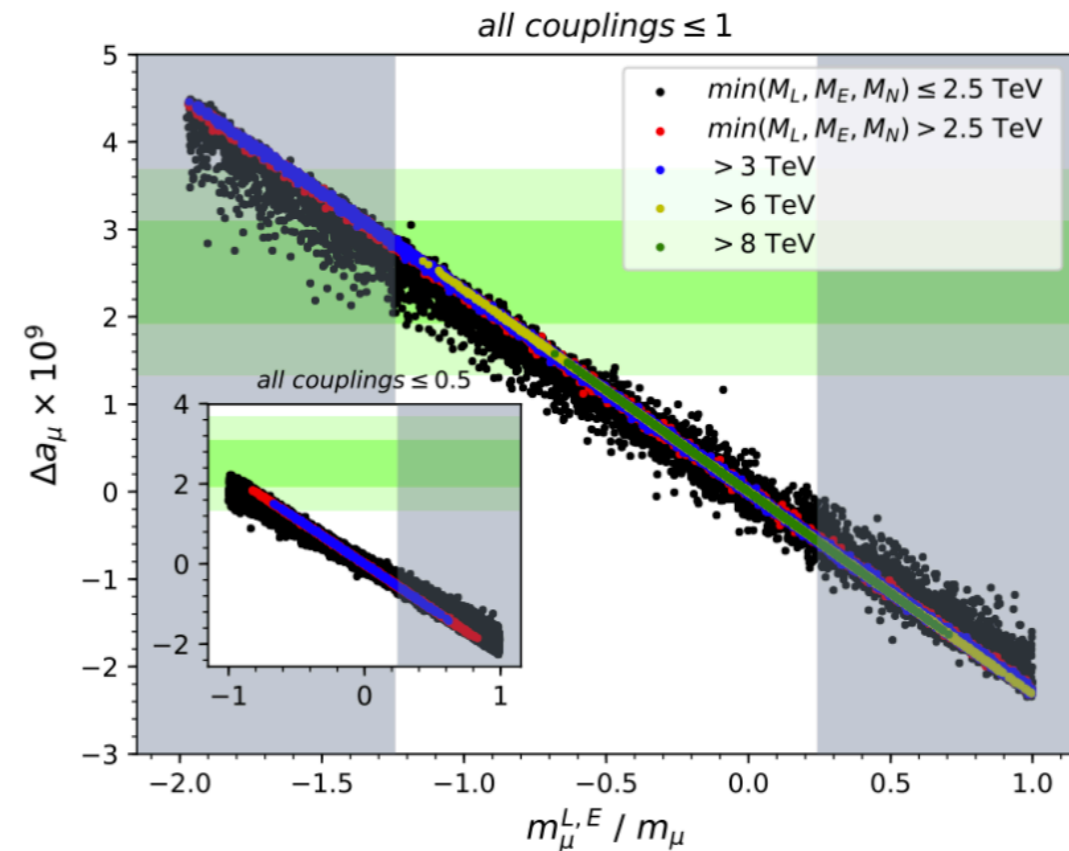
R.D., N. McGinnis, and K. Hermanek, arXiv:2103.05645 [hep-ph]

Δa_μ and muon mass (and thus $h \rightarrow \mu^+ \mu^-$)

highly correlated,

no free parameter for $M_{L,E} \gg M_{EW}$

$$\Delta a_\mu = -\frac{1}{16\pi^2} \frac{m_\mu m_\mu^{LE}}{v^2}$$



Δa_μ in SM with L + E and $h \rightarrow \mu^+ \mu^-$

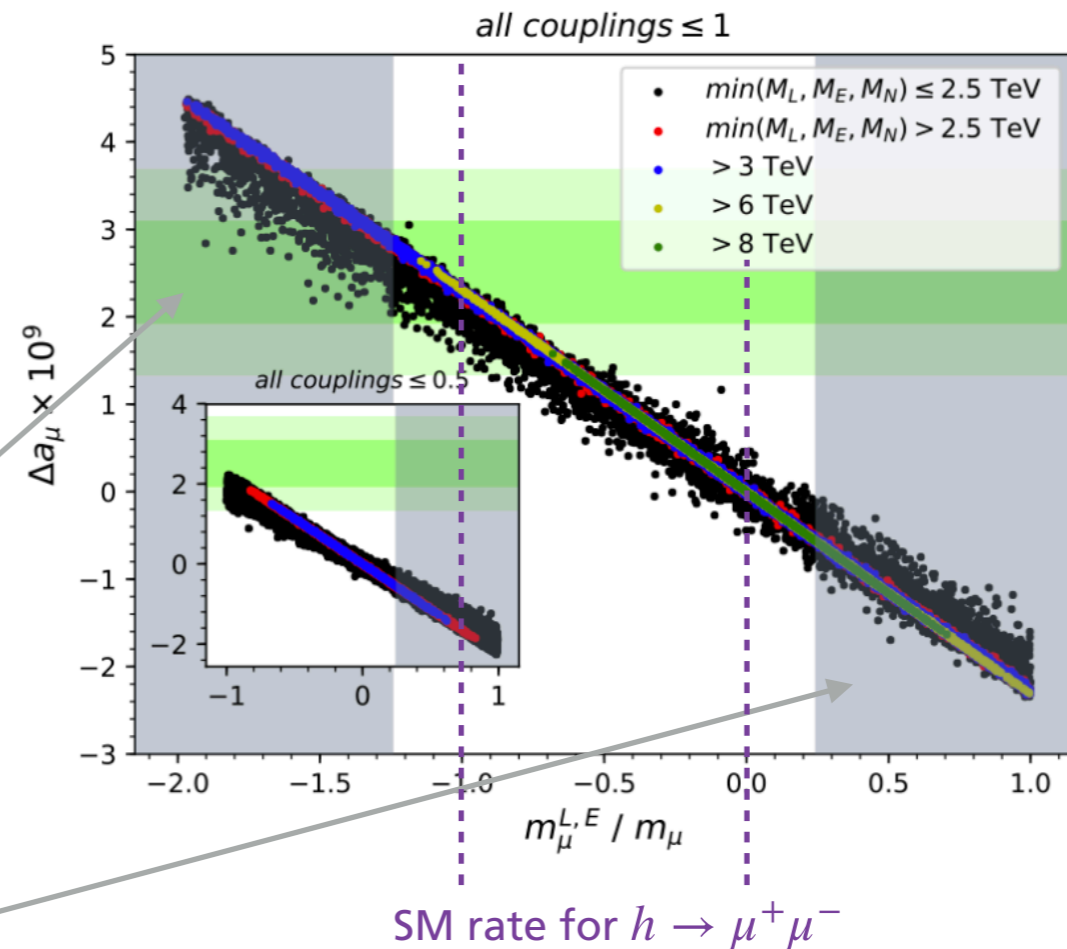
R.D., N. McGinnis, and K. Hermanek, arXiv:2103.05645 [hep-ph]

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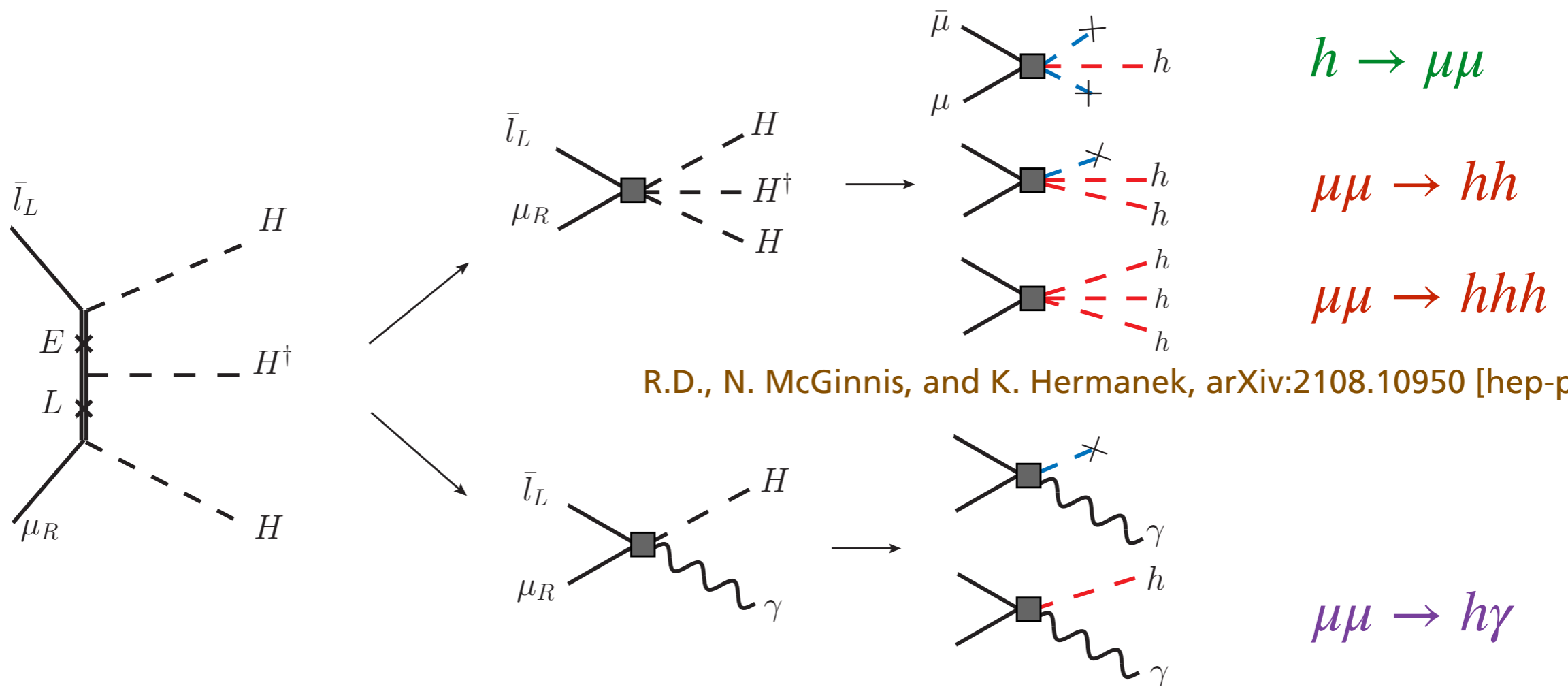
excluded by $R_{h \rightarrow \mu^+ \mu^-} \equiv \frac{BR(h \rightarrow \mu^+ \mu^-)}{BR(h \rightarrow \mu^+ \mu^-)_{SM}} = \left(1 + 2 \frac{m_\mu^{LE}}{m_\mu}\right)^2$

1σ range of Δa_μ predicts $R_{h \rightarrow \mu^+ \mu^-} = 1.32^{+1.40}_{-0.90}$

even if SM rate for $h \rightarrow \mu^+ \mu^-$ is observed it cannot rule out this explanation of Δa_μ

Related observables

New leptons ($\sim 10(s)$ TeV) might be well beyond the reach of (foreseeable) future colliders, but there are related signals:



R.D., N. McGinnis, and K. Hermanek, arXiv:2108.10950 [hep-ph]

requires $\gtrsim 30$ TeV muon collider

D. Buttazzo and P. Paradisi, arXiv:2012.02769 [hep-ph]

W. Yin and M. Yamaguchi, arXiv:2012.03928 [hep-ph]

Di-Higgs and tri-Higgs signals in SM+L+E

Effective lagrangian:

$$\mathcal{L} \supset -y_\mu \bar{l}_L \mu_R H - \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} \bar{l}_L \mu_R H H^\dagger H + h.c.,$$

$$H = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} h \end{pmatrix}$$

is completely fixed by muon mass and g-2:

$$m_\mu = y_\mu v + m_\mu^{LE}$$

$$\Delta a_\mu = -\frac{1}{16\pi^2} \frac{m_\mu m_\mu^{LE}}{v^2}$$

$$m_\mu^{LE} \equiv \frac{\lambda_L \bar{\lambda} \lambda_E}{M_L M_E} v^3$$

Interactions of the muon with SM Higgs boson:

$$\mathcal{L} \supset -\frac{1}{\sqrt{2}} \lambda_{\mu\mu}^h \bar{\mu} \mu h - \frac{1}{2} \lambda_{\mu\mu}^{hh} \bar{\mu} \mu h^2 - \frac{1}{3!} \lambda_{\mu\mu}^{hhh} \bar{\mu} \mu h^3$$

$$\lambda_{\mu\mu}^h = (m_\mu + 2m_\mu^{LE})/v$$

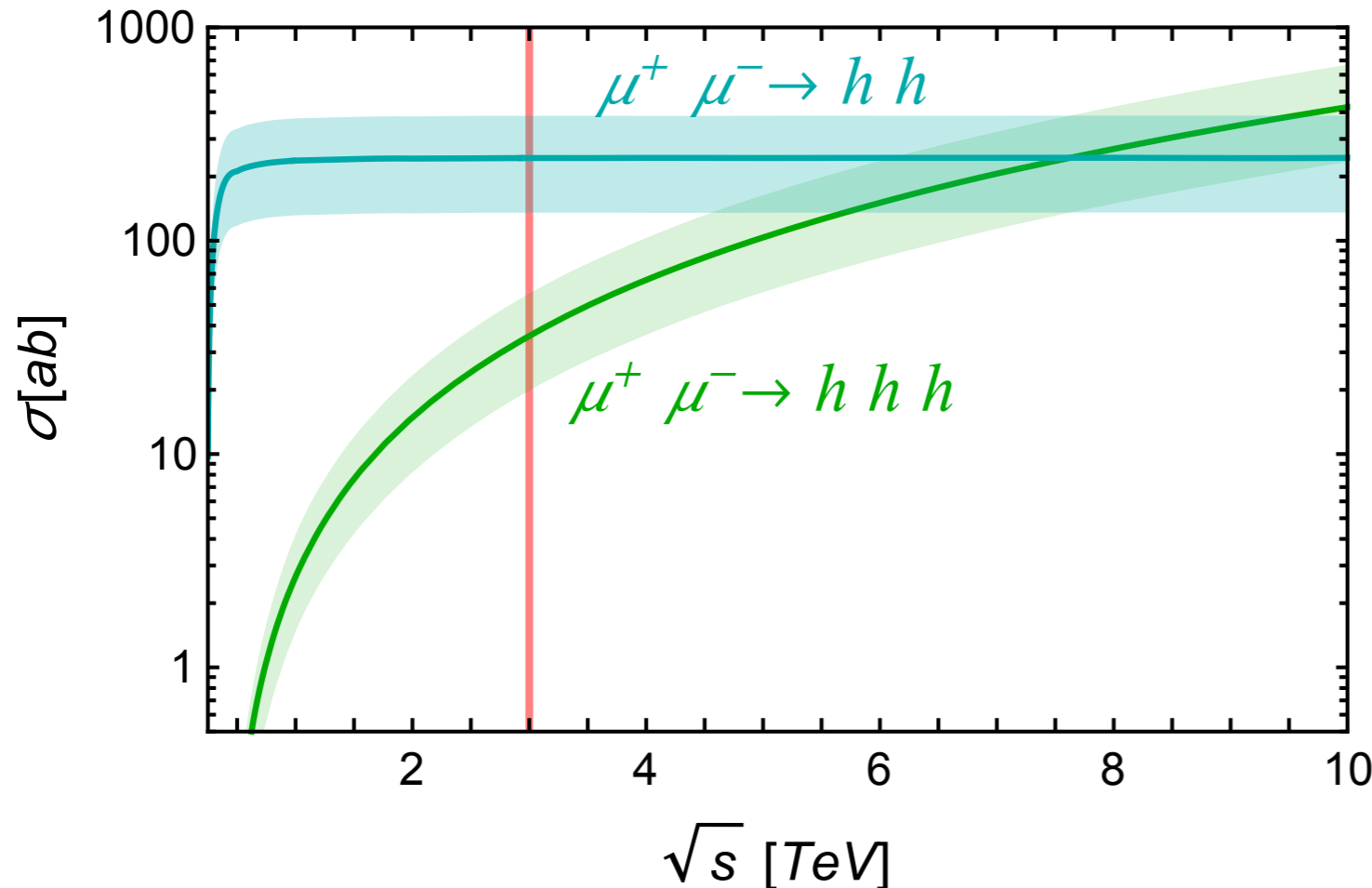
$$\lambda_{\mu\mu}^{hh} = 3 m_\mu^{LE}/v^2,$$

$$\lambda_{\mu\mu}^{hhh} = \frac{3}{\sqrt{2}} m_\mu^{LE}/v^3,$$

are predicted without a free parameter!

Di-Higgs and tri-Higgs signals in SM+L+E

R.D., N. McGinnis, and K. Hermanek, arXiv:2108.10950 [hep-ph]



1σ range of Δa_μ

$$\sigma_{\mu^+\mu^-\rightarrow hh} \simeq \frac{|\lambda_{\mu\mu}^{hh}|^2}{64\pi} = \frac{9}{64\pi} \left(\frac{m_\mu^{LE}}{v^2} \right)^2,$$

$$\sigma_{\mu^+\mu^-\rightarrow hhh} \simeq \frac{|\lambda_{\mu\mu}^{hhh}|^2}{6144\pi^3} s = \frac{3}{4096\pi^3} \left(\frac{m_\mu^{LE}}{v^3} \right)^2 s$$

1 TeV muon collider with 0.2 ab^{-1} could see ~ 50 di-Higgs events

3 TeV muon collider with 1 ab^{-1} could see ~ 30 tri-Higgs events

Connection with μ EDM

Dipole moments and the mass operator

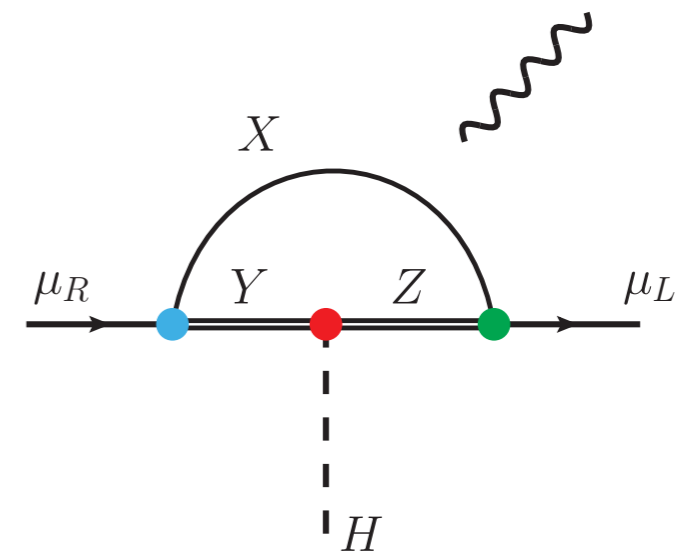
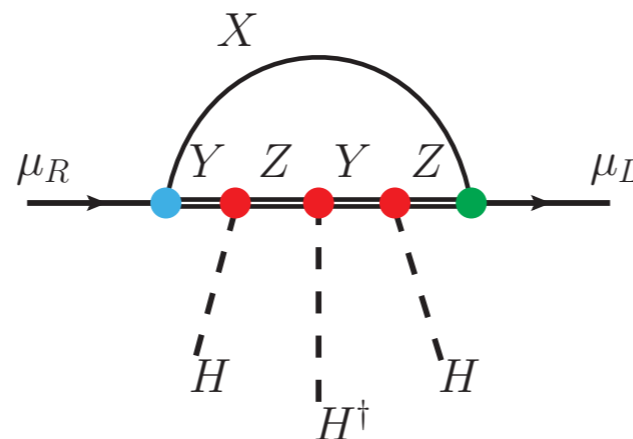
Couplings required for **chiral enhancement** in Δa_μ also generate dim. 6 mass operator:

$$\mathcal{L} \supset - y_\mu \bar{l}_L \mu_R H - C_{\mu H} \bar{l}_L \mu_R H (H^\dagger H) - C_{\mu\gamma} \bar{l}_L \sigma^{\mu\nu} \mu_R H F_{\mu\nu} + h.c.$$

loop models:

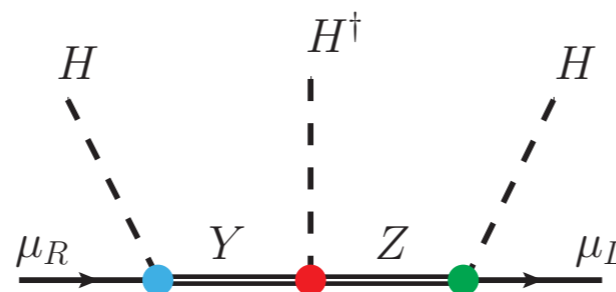
A. Thalappilil and S. Thomas,
arXiv:1411.7362 [hep-ph]

A. Crivellin and M. Hoferichter,
arXiv:2104.03202 [hep-ph]



tree models:

SM or 2HDM with VLs



operators related by a real parameter: $C_{\mu H} = \frac{k}{e} C_{\mu\gamma}$

The muon ellipse

R. D., N. McGinnis, K. Hermanek and S. Yoon, arXiv:2205.14243 [hep-ph]

After electroweak symmetry breaking:

$$\mathcal{L} \supset -m_\mu \bar{\mu} \mu - \frac{1}{\sqrt{2}} (\lambda_{\mu\mu}^h \bar{\mu} P_R \mu h + h.c.) + \frac{\Delta a_\mu e}{4m_\mu} \bar{\mu} \sigma^{\rho\sigma} \mu F_{\rho\sigma} - \frac{i}{2} d_\mu \bar{\mu} \sigma^{\rho\sigma} \gamma^5 \mu F_{\rho\sigma}$$

$$m_\mu = (y_\mu v + C_{\mu H} v^3) e^{-i\phi_{m_\mu}}$$

$$\Delta a_\mu = -\frac{4m_\mu v}{e} \text{Re}[C_{\mu\gamma} e^{-i\phi_{m_\mu}}]$$

$$\lambda_{\mu\mu}^h = (y_\mu + 3C_{\mu H} v^2) e^{-i\phi_{m_\mu}}$$

$$d_\mu = 2v \text{Im}[C_{\mu\gamma} e^{-i\phi_{m_\mu}}]$$

$$R_{h \rightarrow \mu\mu} \equiv \frac{BR(h \rightarrow \mu\mu)}{BR(h \rightarrow \mu\mu)_{SM}} = \left(\frac{v}{m_\mu}\right)^2 |\lambda_{\mu\mu}^h|^2$$

Because the operators are related, we have:

$$C_{\mu H} = \frac{k}{e} C_{\mu\gamma}$$

$$R_{h \rightarrow \mu\mu} = \left(\frac{\Delta a_\mu}{2\omega} - 1\right)^2 + \left(\frac{m_\mu d_\mu}{e\omega}\right)^2$$

$$\omega = m_\mu^2 / \underline{kv^2}$$

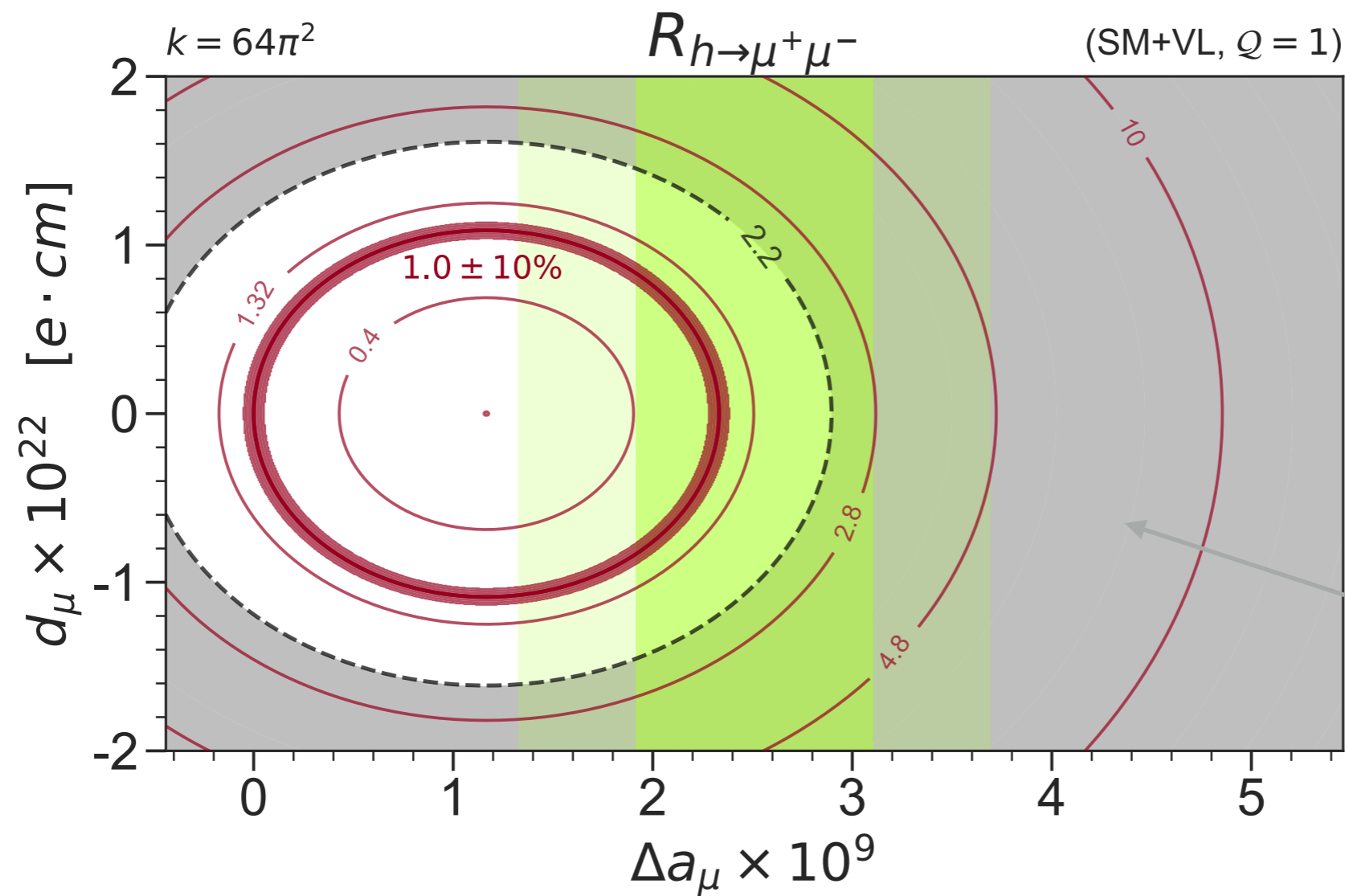
Δa_μ , d_μ and $h \rightarrow \mu\mu$ are highly correlated

Standard model with L + E

VLs with the same quantum numbers as SM leptons:

Relation completely fixed by quantum numbers

$SU(2) \times U(1)_Y$	Q
$\mathbf{2}_{-1/2} \oplus \mathbf{1}_{-1}$	1



$$k = \frac{64\pi^2}{Q}$$

excluded by
 $h \rightarrow \mu\mu$

$|d_\mu| \lesssim 10^{-22} \text{ e} \cdot \text{cm}$ predicted

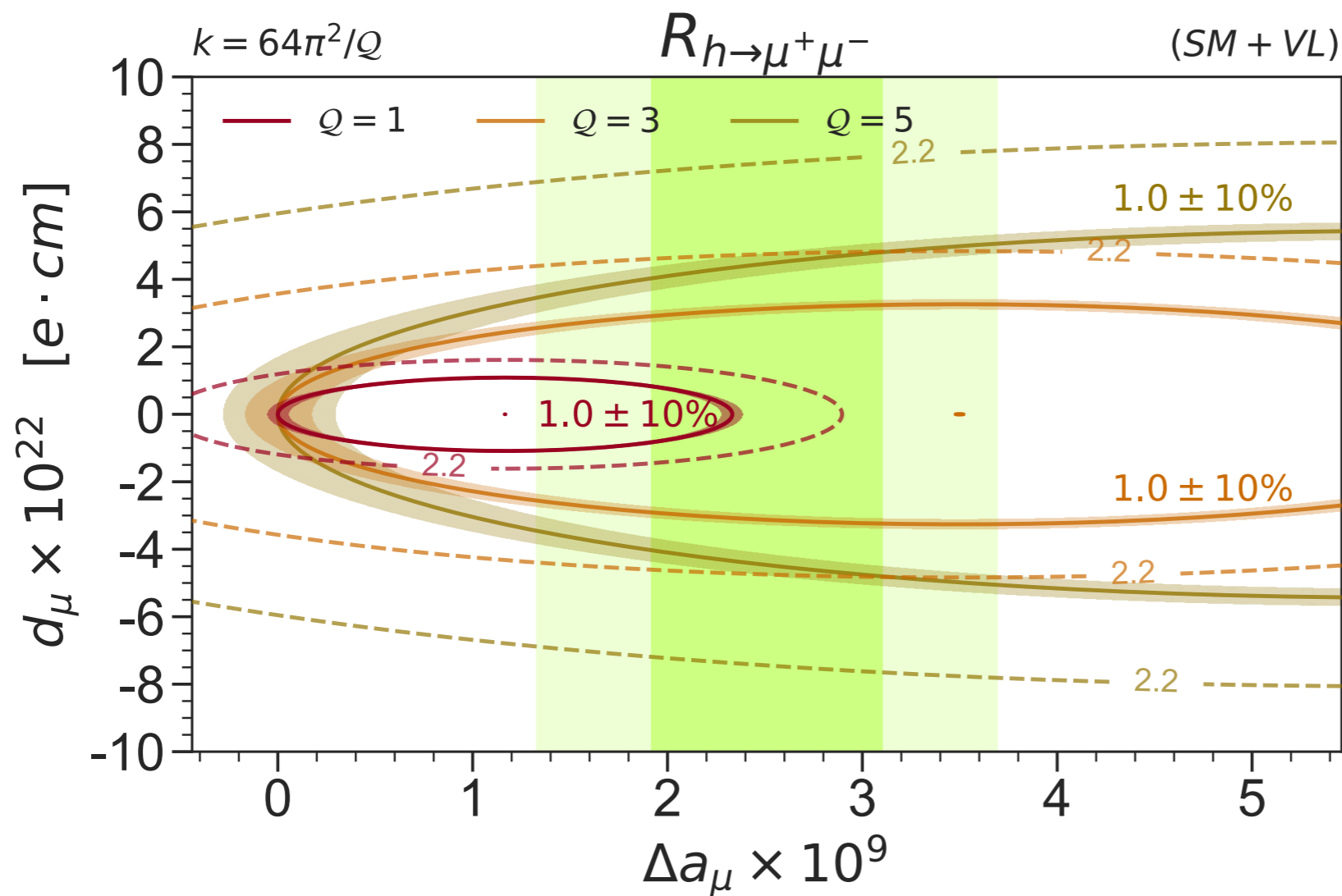
Standard model + VLs

Five possible quantum number assignments for VLs:

(with mass enhanced contributions)

$$k = \frac{64\pi^2}{Q}$$

$SU(2) \times U(1)_Y$	Q
$\mathbf{2}_{-1/2} \oplus \mathbf{1}_{-1}$	1
$\mathbf{2}_{-1/2} \oplus \mathbf{3}_{-1}$	5
$\mathbf{2}_{-3/2} \oplus \mathbf{1}_{-1}$	3
$\mathbf{2}_{-3/2} \oplus \mathbf{3}_{-1}$	3
$\mathbf{2}_{-1/2} \oplus \mathbf{3}_0$	1



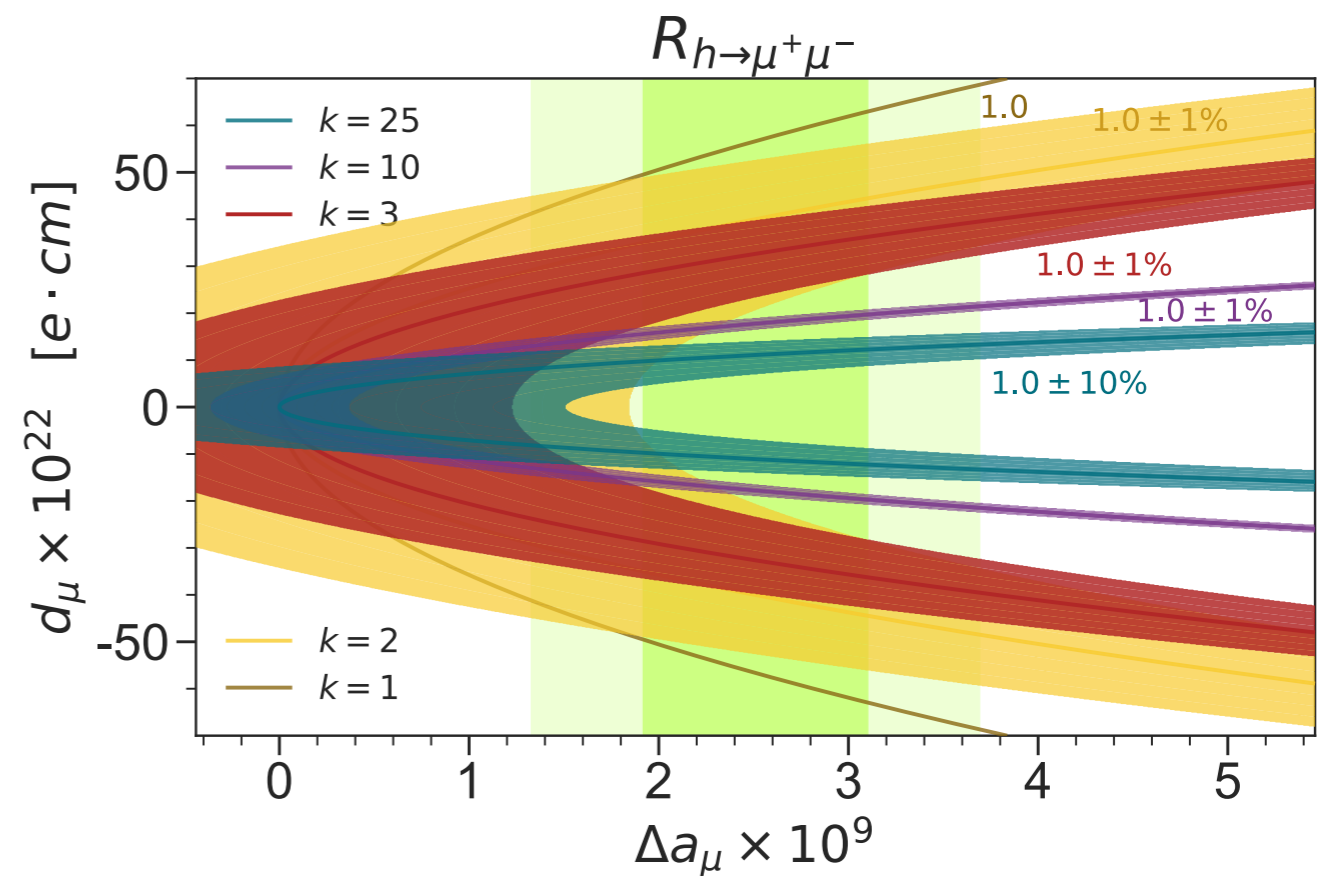
sharp predictions for d_μ once $h \rightarrow \mu\mu$ is precisely measured

Other models

- Models with more scalars participating in EWSB

e.g. 2HDM, type-II:

$$k = \frac{64\pi^2}{Q(1 + \tan^2 \beta)}$$



Other models

- Models with more scalars participating in EWSB

e.g. 2HDM, type-II:

$$k = \frac{64\pi^2}{Q(1 + \tan^2 \beta)}$$

- Loop models:

$$k = \frac{4}{Q} |\lambda_{YZ}|^2$$

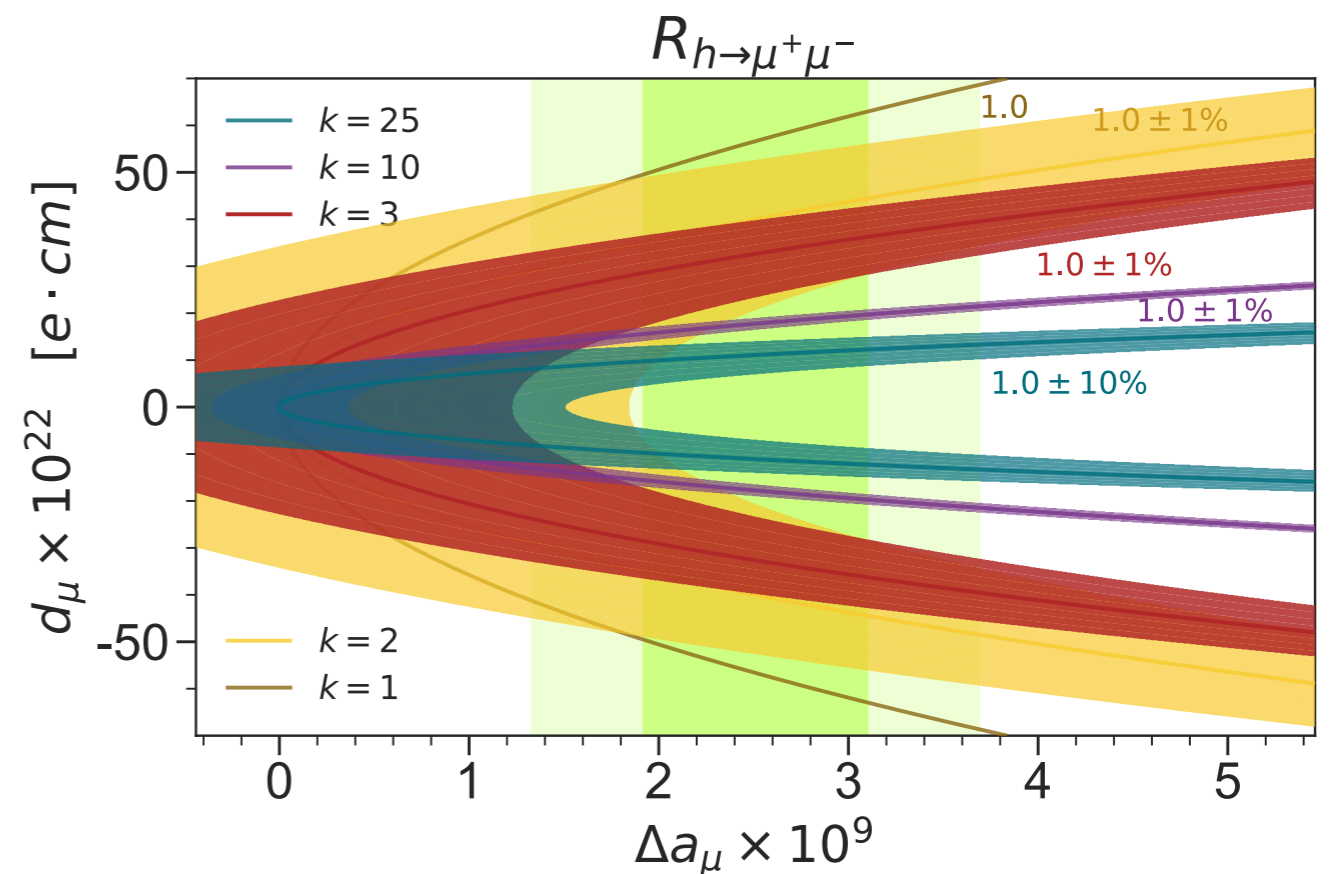
group theoretical factor

e.g.

top quark+leptoquark: $k \simeq 3$

MSSM, bino+smuon (with large mixing): $k \simeq 10 - 50$

In general, larger k means larger coupling and masses of new particles



Impact of future $h \rightarrow \mu\mu$ and μ EDM

All models with chiral enhancement can be parameterized by k :

current limits:

$$|d_\mu| \leq 1.8 \times 10^{-19} \text{ e} \cdot \text{cm}$$

Muon g-2 (BNL)

arXiv:0811.1207 [hep-ex]

indirect limits:

$$|d_\mu| \leq 2 \times 10^{-20} \text{ e} \cdot \text{cm}$$

Y. Ema, T. Gao and M. Pospelov,

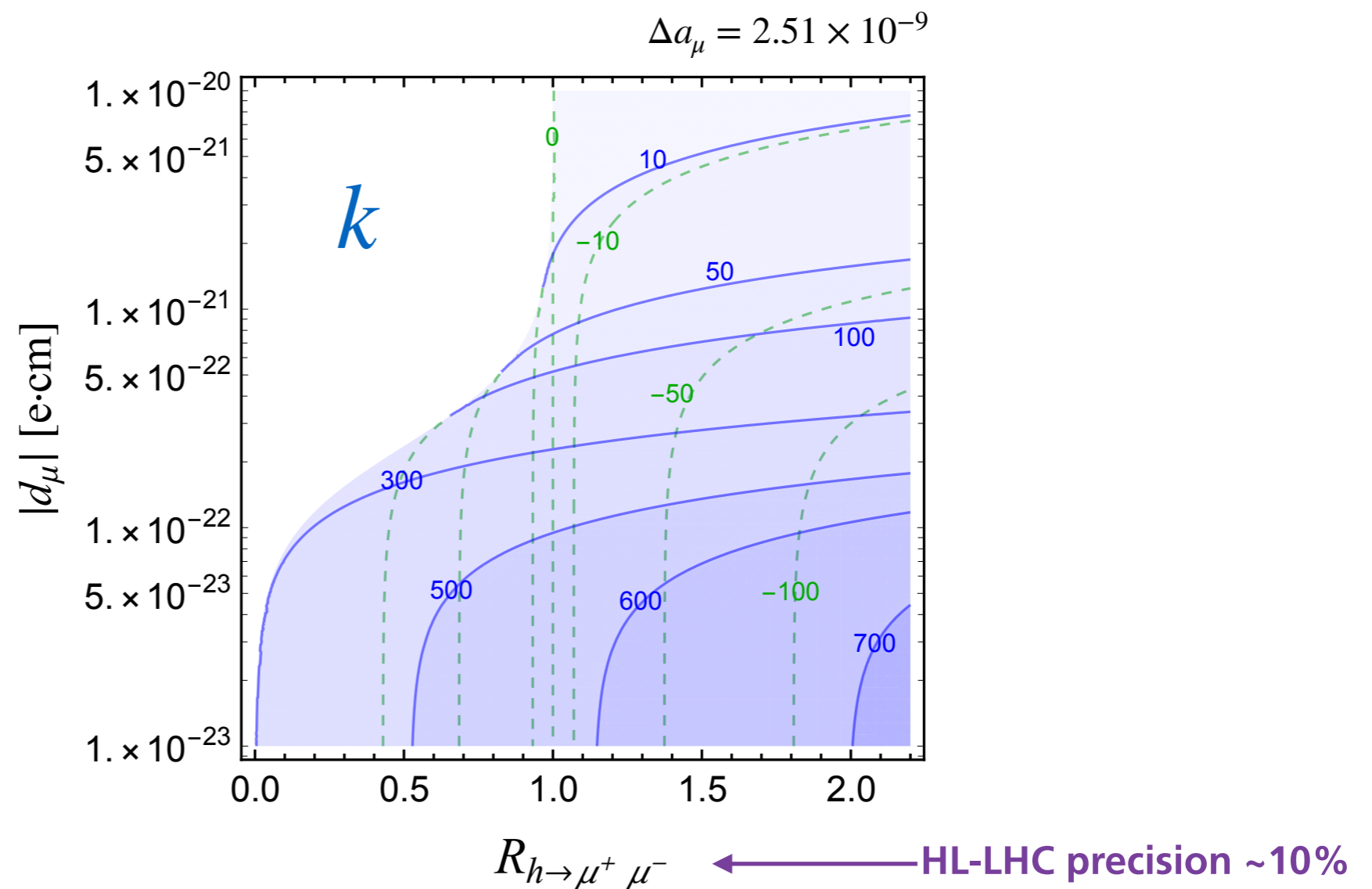
arXiv:2108.05398 [hep-ph]

future sensitivity:

$$|d_\mu| \leq 6 \times 10^{-23} \text{ e} \cdot \text{cm}$$

at Paul Scherrer Institute,

arXiv:2102.08838 [hep-ex]



many models (ranges of parameters) will be tested in near future

Summary

There are many possible explanations of Δa_μ .

Models with chiral enhancement are difficult to test directly since the anomaly can be explained with multi TeV (10s TeV) particles.

Indirect tests include measurements of:

- muon couplings to Z and W (not directly tight to Δa_μ)
relevant especially for models with tree-level mixing, can be tested at Giga-Z or FCC-ee
- muon Yukawa coupling, $h \rightarrow \mu\mu$
a deviation could be seen at the LHC, but SM-like observation would not rule out even tree models
- $\mu\mu \rightarrow hh$ and $\mu\mu \rightarrow hhh$
fixed by Δa_μ in tree models, large rates even at low energy colliders
- μ EDM and correlation of Δa_μ , d_μ and $h \rightarrow \mu\mu$
many models (ranges of parameters) will be tested in near future