Composite fermions in the Veneziano limit via holography

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1. Composite bosons : 2011.03003 – JHEP 03 (2021) 182

2. Composite fermions : 2112.14740 – JHEP 05 (2022) 066

PLANCK 2022, Paris

30 May 2022

Motivations & outline

- *Strong dynamics may naturally explain hierarchy of scales: e.g. composite Higgs with $m_h \sim 0.1$ TeV $<$ $m_* \sim 10$ TeV $\ll$ Λ_{UV}*

- *Composite resonances may be targets for colliders, candidates for dark matter, clues to flavour problem, ...*

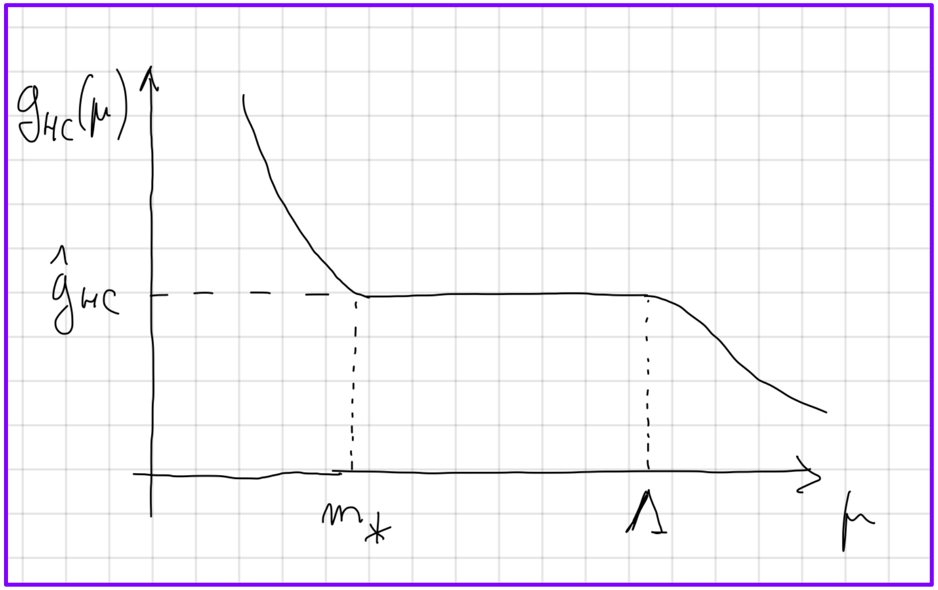
- *The spectrum is model-dependent: consider gauge theory with many colours, and many fermion flavours*

- *Holography: model non-perturbative gauge dynamics in 4D by a weakly-coupled theory of gravity in 5D*

- *A few instances where composite fermions become light*

Relevant scales

Hypercolour (HC) gauge theory:
asymptotically-free,
approximately scale-invariant,
confinement with mass gap m_*



Spontaneous flavour-symmetry breaking:

$$G_F \xrightarrow{f} H_F$$

$$f \simeq \frac{m_*}{g_*} \sim N_C^{1/2} \frac{m_*}{4\pi} \gtrsim 1 \text{ TeV}$$

Partial Compositeness (PC):
mixing SM fermions
with composite operators

$$\mathcal{L}_{PC} = \lambda_i f_i \mathcal{O}_i + \ldots$$

D.B.Kaplan '91,
Contino-Pomarol '04, ++

- Irrelevant operators to
  explain Yukawa hierarchies
  & suppress flavour violation

$$\lambda_i(m_*) \underset{[O_i]>5/2}{\simeq} \lambda_i(\Lambda) \left(\frac{m_*}{\Lambda}\right)^{[O_i]-5/2} \equiv g^* \epsilon_i$$

- Relevant operators to
  explain large couplings
  (top Yukawa)

$$3/2 \leq [O_i] \lesssim 5/2$$

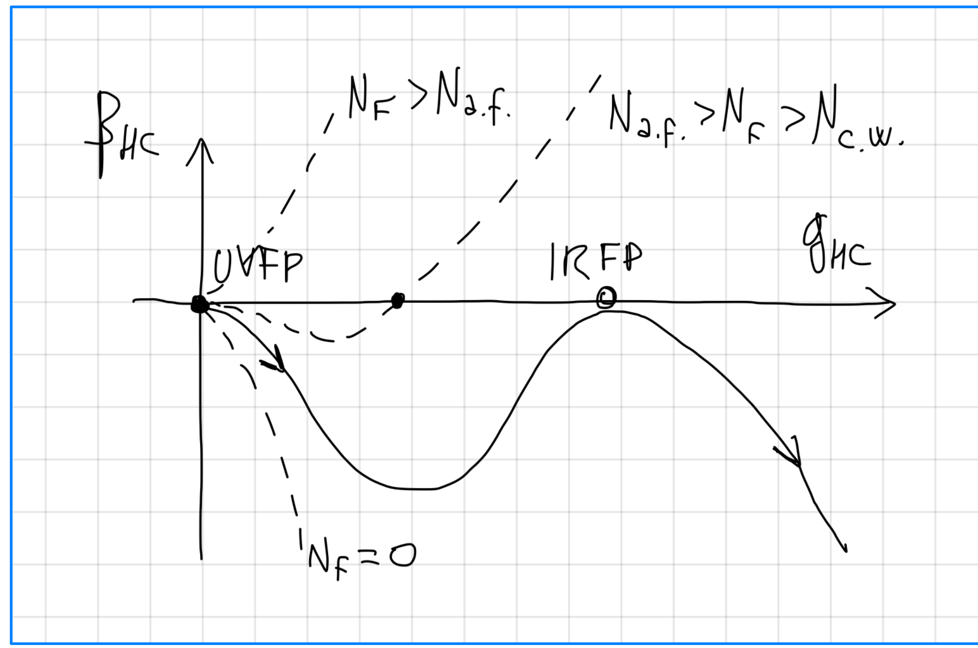
The call for many flavours

TeV physics must respect SM gauge & global symmetries

$$G_F \xrightarrow{f} H_F \supset SU(2)_L \times SU(2)_R \times SU(3)_c \times U(1)_B \times U(1)_L$$

Higgs doublet, with custodial, coupled to quarks, respecting B & L

To approach the lower edge of the conformal window one typically needs $N_F \sim N_C$



Strongly coupled ($g_C^2 N_C \sim 4\pi$)

IR fixed-point requires $N_F / N_C \sim few$

Veneziano limit : $N_C \to \infty \quad \& \quad x_F \equiv \dfrac{N_F}{N_C} \quad \text{constant}$

A minimal model

N_k Weyl fermions in representations R_k $(k = 1, \ldots, n)$ have symmetry

$$G_F = SU(N_1) \times \cdots \times SU(N_n) \times U(1)^{n-1}$$

Optimal choice :	$\psi_i^a \; (a = 1, \ldots, 2N_F)$ fundamental repr. of G_{HC}	χ_{ij} two-index repr. of G_{HC}

Highly preferable that only ψ's carry SM charges through the flavour index a
(preserve asymptotic freedom at large N_c ; postpone Landau poles in SM gauge couplings ; preserve approximate gauge coupling unification)

Case where the fundamental representation is pseudoreal :

$$G_{HC} = Sp(2N_C) \qquad G_F = SU(2N_F) \times U(1) \xrightarrow{f} H_F = Sp(2N_F)$$

Single χ_{ij} necessary and sufficient to implement Partial Compositeness (PC) :

spin-1/2 mesons with SM charges

$$\mathcal{O}_F^{ab} = (\psi_i^a \psi_j^b \chi_{ij})$$

for quark PC $N_F \geq 5$

Model in similar spirit: **Gertov, Nelson, Perko, Walker 2019**
Classification of other relevant cosets: **Ferretti, Karateev 2013, ++**

Gauge-gravity duality

Approximate CFT (with N_C and $\lambda = g^2_C N_C$ large) has holographic description as approximate 5-dim AdS (in the classical & weakly-coupled limit)

$$ds^2 = e^{2A(r)}\mathrm{d}x^2_{1,3} + \mathrm{d}r^2$$
$$AdS: \quad A(r) = r \ , \quad \mu = \mu_0 e^r$$

Maldacena '97

Composite operator O_Φ / global G_F	⟷	5-dim field Φ / gauged G_F
CFT correlator $\langle O_1 O_2 \rangle$	⟷	Bulk correlator $\langle \Phi_1(r)\Phi_2(r)\rangle$ at $r \to \infty$

The mass spectrum for each composite operator is determined by the poles of the associated two-point correlator

$$\Pi_V(q^2)\delta^{AB}(q_\mu q_\nu - \eta_{\mu\nu}q^2) = i\int d^4x\, e^{iq\cdot x}\langle \mathrm{vac}|T\{\mathcal{J}^A_\mu(x)\mathcal{J}^B_\nu(0)\}|\mathrm{vac}\rangle$$

$$\Pi_V(q^2) \underset{\text{large } N_C}{\simeq} \sum_n \frac{f^2_{Vn}}{q^2 - m^2_{Vn}}$$

resonance decay constants (f^2_{Vn})

resonance masses (m^2_{Vn})

Gravity-scalar interplay

GAUGE/GRAVITY SECTOR :

$$T_{\mu\nu} = F_{\mu\rho}F^{\rho}_{\nu} + \ldots \quad \Leftrightarrow \quad g_{\mu\nu}(r)$$

$$\langle \mathrm{tr}(FF)\mathrm{tr}(FF) \rangle \sim N_C^2$$

FLAVOUR SECTOR :

$$(\psi^a \psi^b) \quad \Leftrightarrow \quad \Phi^{ab}(r)$$

$$\langle (\psi^a \psi^a)(\psi^b \psi^b) \rangle \sim N_C N_F$$

In the Veneziano limit, the flavour sector has the same weight as the gravity sector.

Thus, as the scalar Φ(r) develops a non-trivial profile, it strongly back-reacts on the metric.

$$N_C \to \infty$$

$$x_F \equiv \frac{N_F}{N_C}$$

constant

CFT loses scale-invariance ⟷ Departure from AdS: warp factor A(r) ≠ r

Non-zero mass gap m_* ⟷ Geometry ends at some finite value r_0

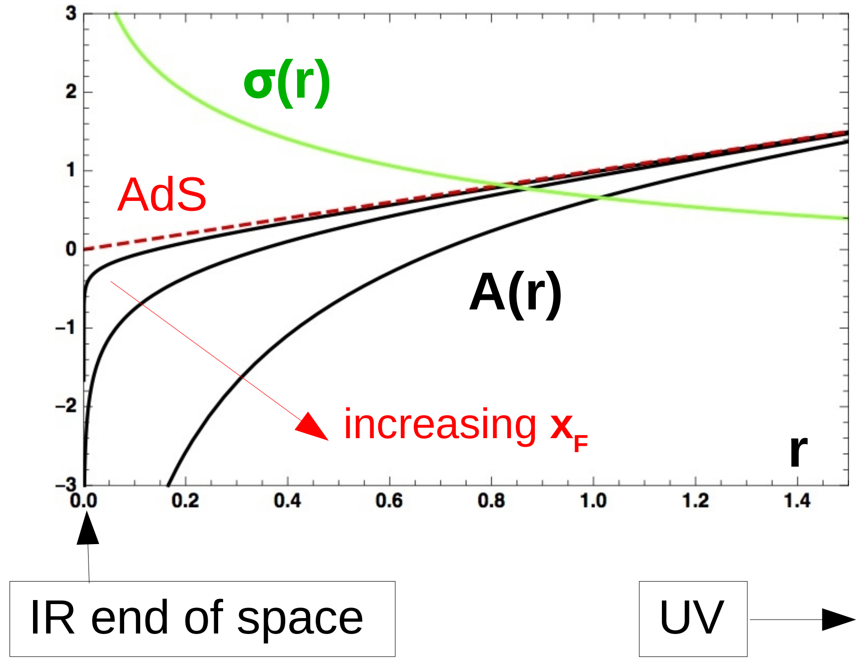
↕ Dynamical correlation !

Flavour SSB: non-zero NGB decay constant f ⟷ Φ(r) breaks gauge symmetry $SU(2N_F)$ spontaneously

Gravity-scalar background

$$g^{MN} = diag(-e^{2A}, e^{2A}, e^{2A}, e^{2A}, 1)^{MN}$$

$$\Phi_{ab} = \sigma \left[\Sigma_{ab}/2 + i\pi_{\hat{A}} (T^{\hat{A}} \Sigma)_{ab} \right] + \dots$$



Equations of motion:

$$\begin{aligned} \partial_r^2 \sigma + 4 \partial_r A \partial_r \sigma - \partial_\sigma \tilde{V}(\sigma) &= 0 \\ 6(\partial_r A)^2 - x_F (\partial_r \sigma)^2 + 2 x_F \tilde{V}(\sigma) &= 0 \end{aligned}$$

V(σ) inspired by **Girardello, Petrini, Porrati, Zaffaroni '99**

Asymptotically AdS : $A(r) \underset{r \to \infty}{\simeq} r \qquad \sigma(r) \underset{r \to \infty}{\simeq} \sigma_\infty e^{-\Delta r}$

UV behaviour of the scalar controls the 'deformation' of the CFT $\quad O_\sigma \Leftrightarrow \sigma$

$$0 < \Delta < 2 \;:\; \text{explicit SB by } \Delta \mathcal{L}_{CFT} \sim \mathcal{O}_\sigma \sigma_\infty \quad [O_\sigma] = 4 - \Delta$$

$$2 < \Delta < 4 \;:\; \text{spontaneous SB by } \langle O_\sigma \rangle \sim \sigma_\infty \quad [O_\sigma] = \Delta$$

Fermion sector

Contino
Pomarol
2004

Source for fermion operators: $O_R^{ab} = \psi^a \bar{\chi} \psi^b \quad \leftrightarrow \quad \Psi_L^{ab}|_{r\to\infty}$

$$\mathcal{S}_\Psi = -N_C^2 \int \mathrm{d}^5 x \sqrt{-g}\, \mathrm{Tr}\left[\frac{1}{2}(\overline{\Psi}\Gamma^M D_M \Psi - \overline{D_M \Psi}\Gamma^M \Psi) + M_\Psi \overline{\Psi}\Psi\right]$$

Spectrum given by the poles of $\quad i\langle O_R(q)\overline{O_R}(-q)\rangle = \lim_{r\to\infty} \frac{\delta^2 S_\Psi^{on-shell}}{\delta\bar{\psi}_L(-q)\delta\psi_L(q)}$

Equation of motion $\quad \left[\partial_r^2 + (\partial_r A + 2M_\Psi)\partial_r - q^2 e^{-2A}\right]\psi_L = 0$

Two consistent IR boundary conditions

$$\psi_L|_{r_{IR}} = 0 \qquad (-)$$
$$\partial_r \psi_L|_{r_{IR}} = 0 \quad (+)$$

UV behaviour dual
to scaling dimension

$$\Psi_L(q,r) \underset{r\to\infty}{\simeq} e^{-(2-M_\Psi)r}\psi_L(q)$$

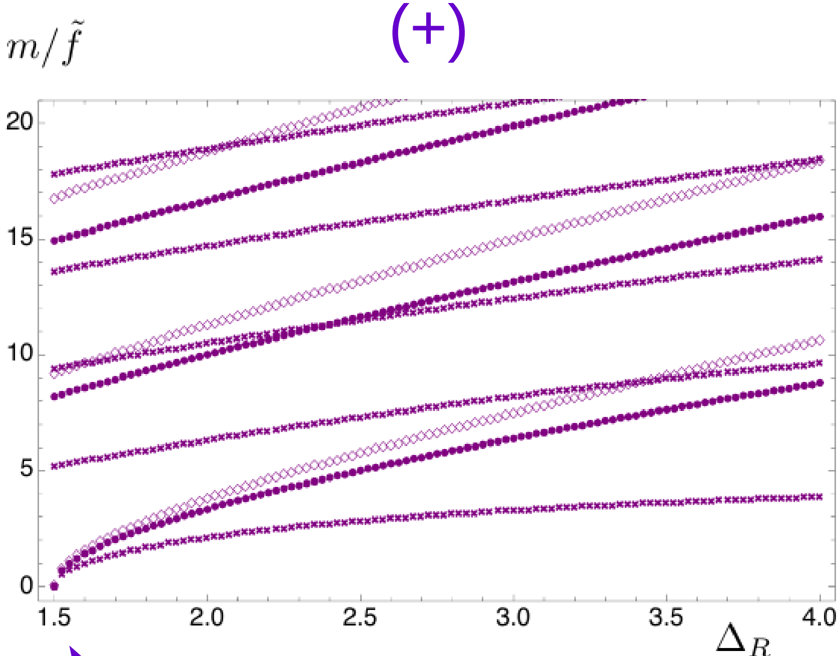
$$\Delta_R \equiv [O_R] = 4 - [\psi_L] = 2 + M_\Psi \underset{unitarity}{\geq} 3/2$$

Fermion spectrum as function of Δ_R

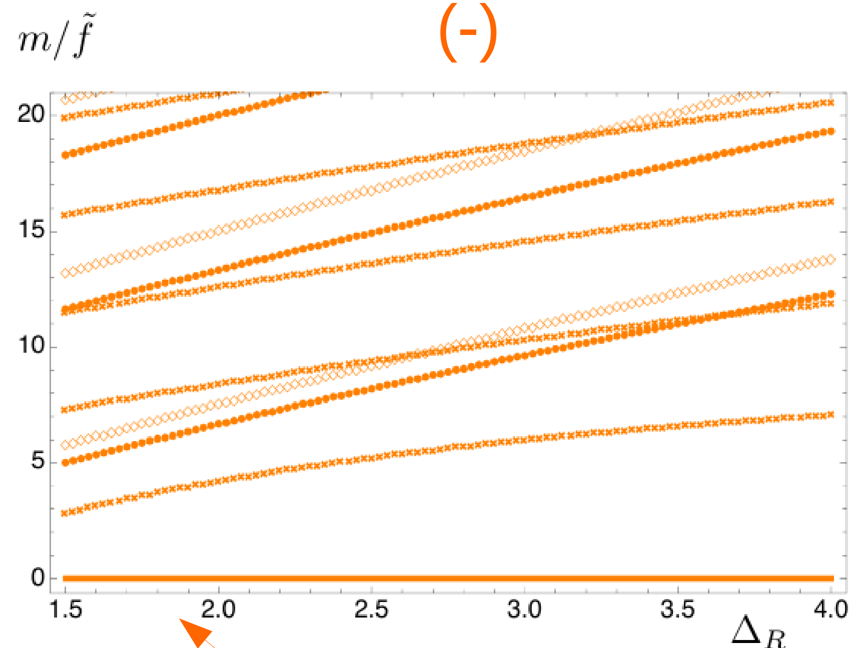
$$\tilde{f} \equiv \frac{f}{\sqrt{N_C}} \gtrsim \frac{1\ \text{TeV}}{\sqrt{N_C}}$$

$\Delta = 2.5$ x_F = 0.5: ◊◊◊ x_F = 2: ●●● x_F = 4: +++

(+)



(-)



As $\Delta_R \to 3/2$, one chiral fermion decouples from the composite sector

There is always a chiral fermion, for any value of Δ_R and x_F

Both options highly non-trivial from the field theory perspective

Fermions in soft-wall models: **Batell-Gherghetta-Sword '08 '09 / Delgado-Diego '09 / Mert Aybat-Santiago '09 / Archer-Huber-Jager '11 / Ahmed-Carmona-Castellano Ruiz-Chung-Neubert '19 / ...**

Adding flavours

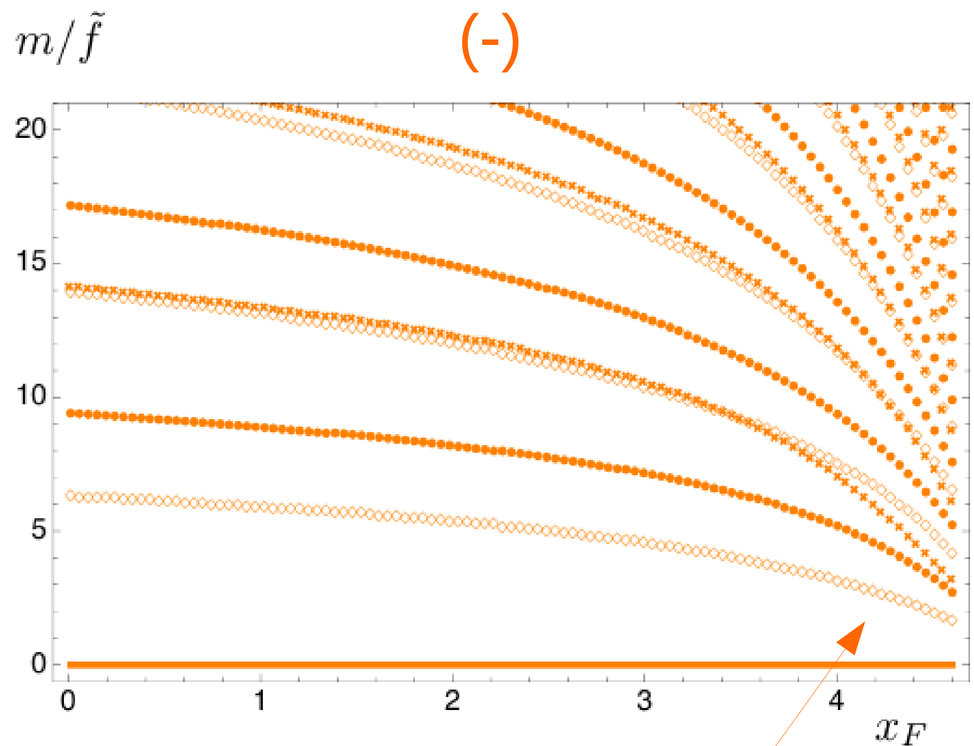
characteristic IR scale : gap between resonances

$$\Lambda_{IR} \equiv \left(\int_0^\infty d\tilde{r}\; e^{-A(\tilde{r})} \right)^{-1}$$

$$x_F \to 2\Delta \quad \Rightarrow \quad \Lambda_{IR} \to 0$$

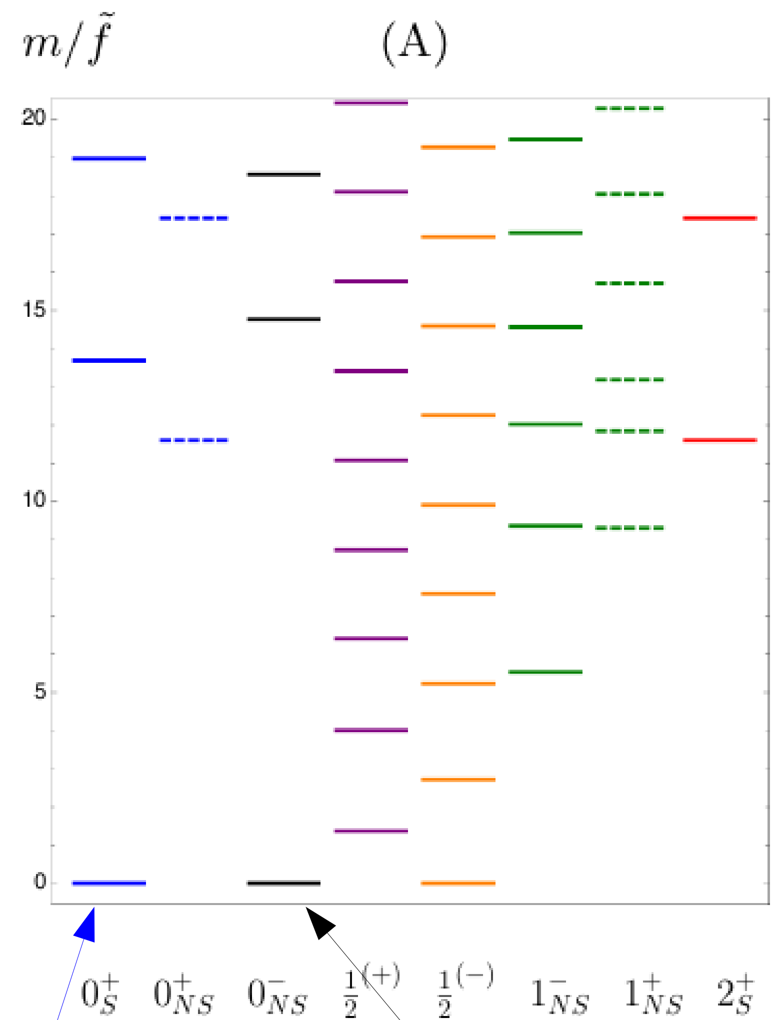
$$\Delta = 2.5$$

$\Delta_R = 1.6$: ◊◊◊ $\Delta_R = 2.5$: ●●● $\Delta_R = 4$: +++



$$m \ll 4\pi \tilde{f}$$

Resonance splitting Λ_{IR} parametrically small with respect to *f*



dilaton

Nambu-Goldstone bosons

PC: holographic RG flow

Partial Compositeness amounts to mix elementary fermion f_L with composite operator O_R

$$S_{PC}[\Lambda] = \int \mathrm{d}^4 q \, [\overline{f_L}(-i\gamma^\mu q_\mu) f_L + (\overline{O_R} \, \lambda(\Lambda) f_L + h.c.)]$$

renormalisation scale Λ

Holographic Wilsonian Renormalisation Group

UV cutoff Λ corresponds to finite radial coordinate *r:*

Duality is assumed to hold along the flow.

$$\Lambda(r)^{-1} \equiv \int_r^\infty d\tilde{r} \; e^{-A(\tilde{r})}$$

Heemskerk, Polchinski 2010; Elander, Isono, Mandal 2011

RG flow equation can be derived as r varies, for an arbitrary background geometry

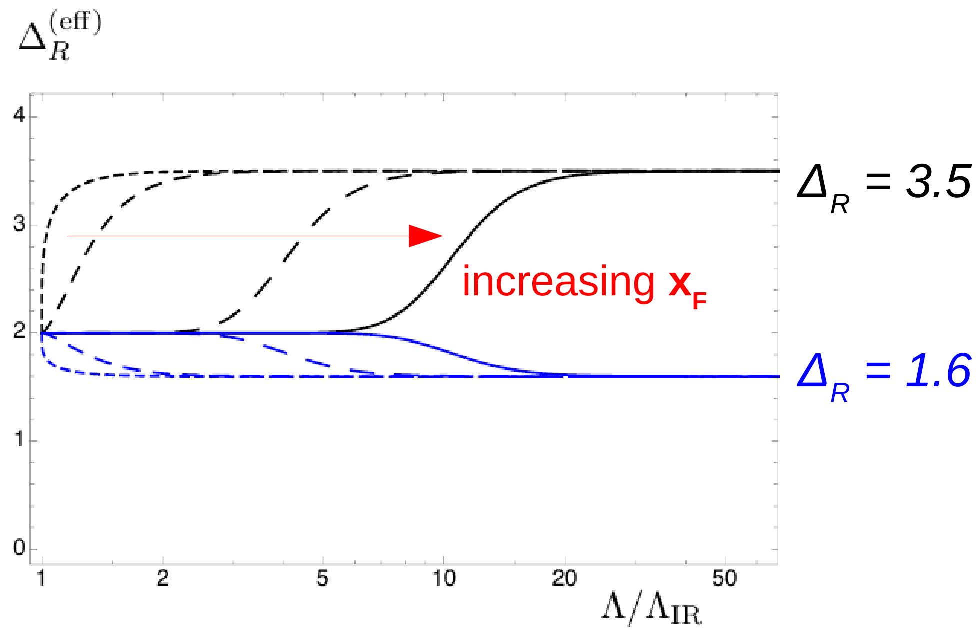
Low-energy limit :

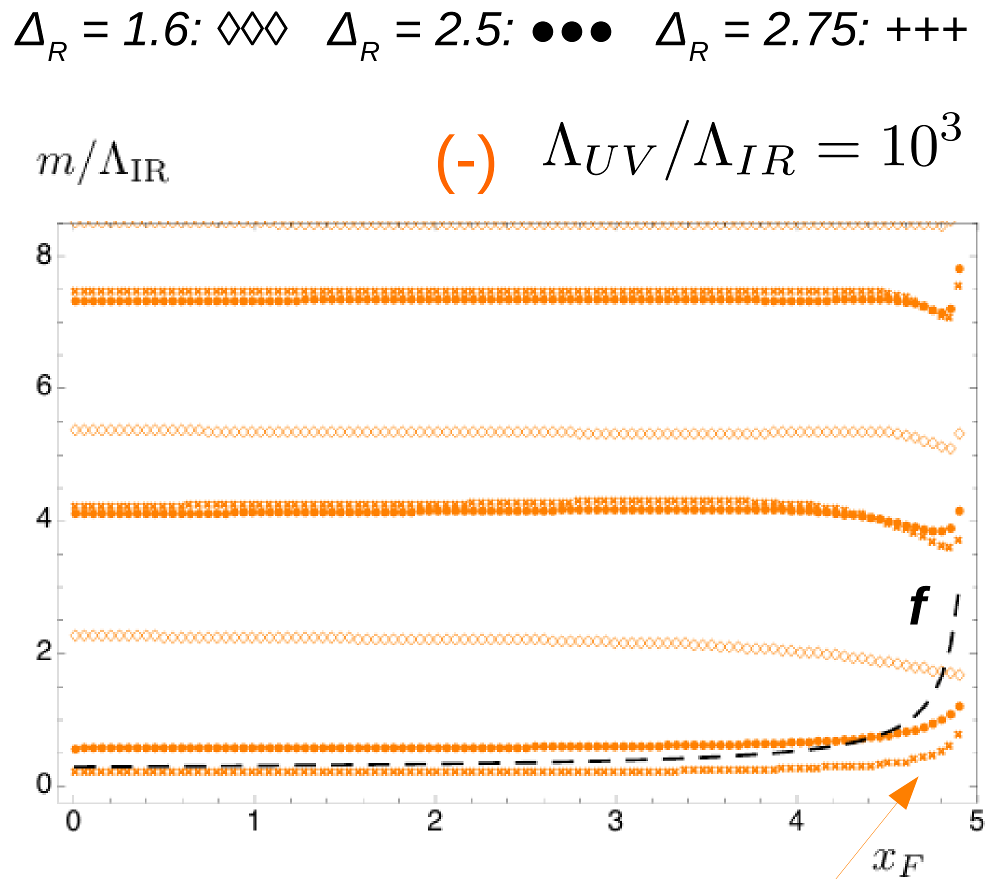
$$\Lambda \partial_\Lambda \tilde{\lambda}_0^2 = 2\left(\Delta_R^{(\mathrm{eff})} - \frac{5}{2}\right) \tilde{\lambda}_0^2 + \tilde{\lambda}_0^4$$

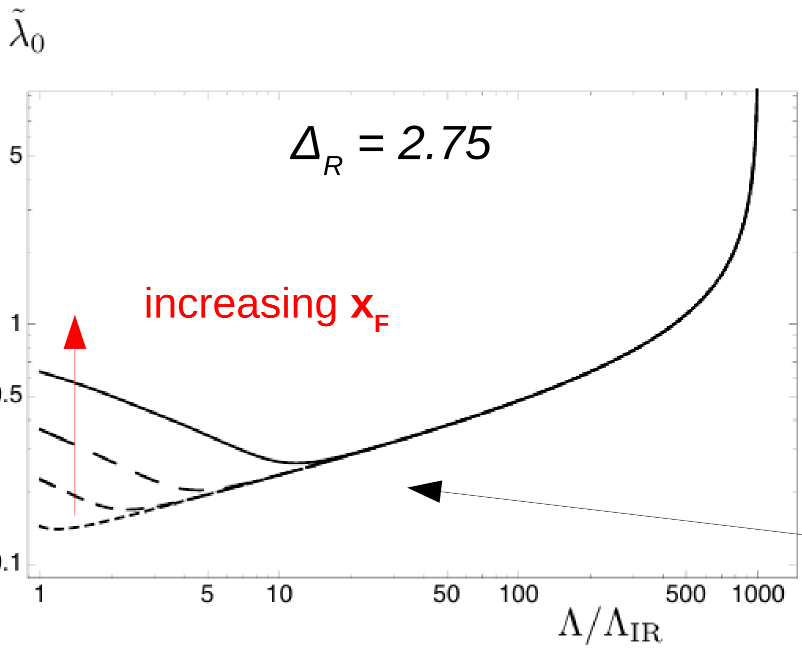
$$\Delta_R^{(\mathrm{eff})}(\Lambda) = \Lambda^{-1} e^A M_\Psi + 2 \underset{AdS}{=} \Delta_R$$

Non-trivial IR fixed point, if the deformation is relevant: $(\tilde{\lambda}_0^2)_{IRFP} = 5 - 2\Delta_R$

Adding flavours in PC







chiral fermion mass lifted proportionally to PC coupling

Irrelevant deformation in the UV can give large PC coupling in the IR

Further breaking scale invariance

Scale invariance may be broken by sources independent from the flavour sector .

This additional breaking can be described by a flavour-singlet bulk scalar :

$$\phi \leftrightarrow O_\phi = (\chi\chi),\ (FF),\ \ldots \quad \Rightarrow \quad \langle O_\phi \overline{O_\phi} \rangle \sim N_C^2$$

$$\phi(r) \underset{r\to\infty}{\simeq} \phi_\infty e^{-\Delta_\phi r}$$

$$0 < \Delta_\phi < 2 \ : \quad \text{explicit SB by } \Delta\mathcal{L}_{CFT} \sim \mathcal{O}_\phi \phi_\infty \quad [O_\phi] = 4 - \Delta_\phi$$

Bulk Yukawa coupling : in our model Ψ^{ab} does not couple to Φ^{ab}, but it does couple to ϕ :

$$S_{\rm Y} = -N_C^2 \int d^5x \sqrt{-g}\ \left[y_5 \phi \overline{\Psi} \Psi \right] \quad \Rightarrow \quad M_\Psi(r) = M_\Psi + y_5 \phi(r)$$

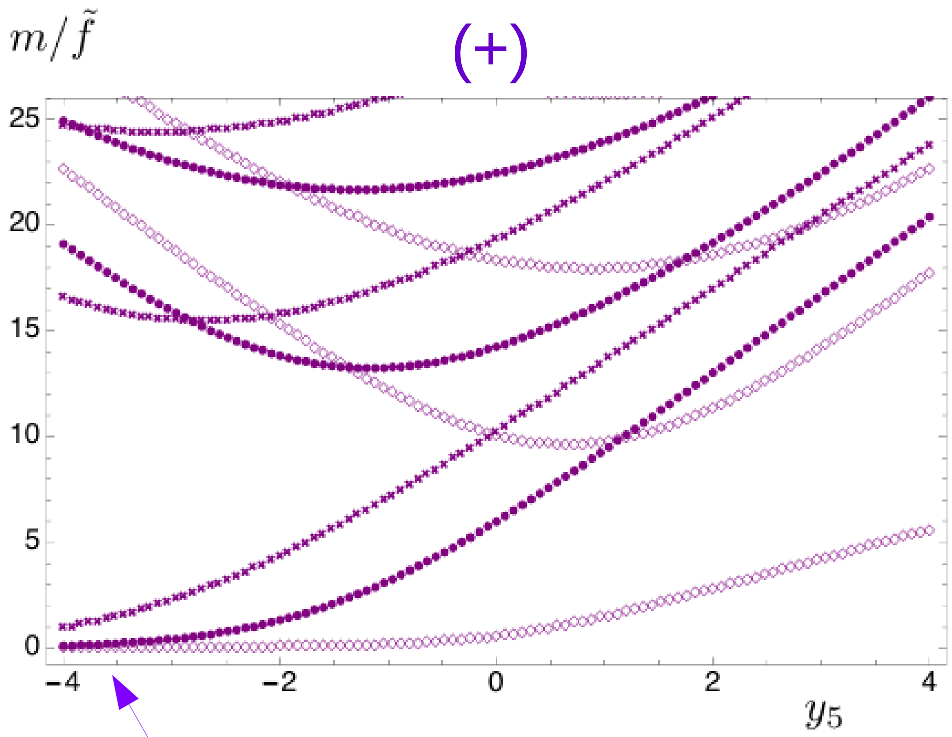
Correspondingly, scaling-dimension of the fermion operator Δ_R becomes scale-dependent

As $\phi(r)$ grows in the IR, the lightest resonances can be strongly affected

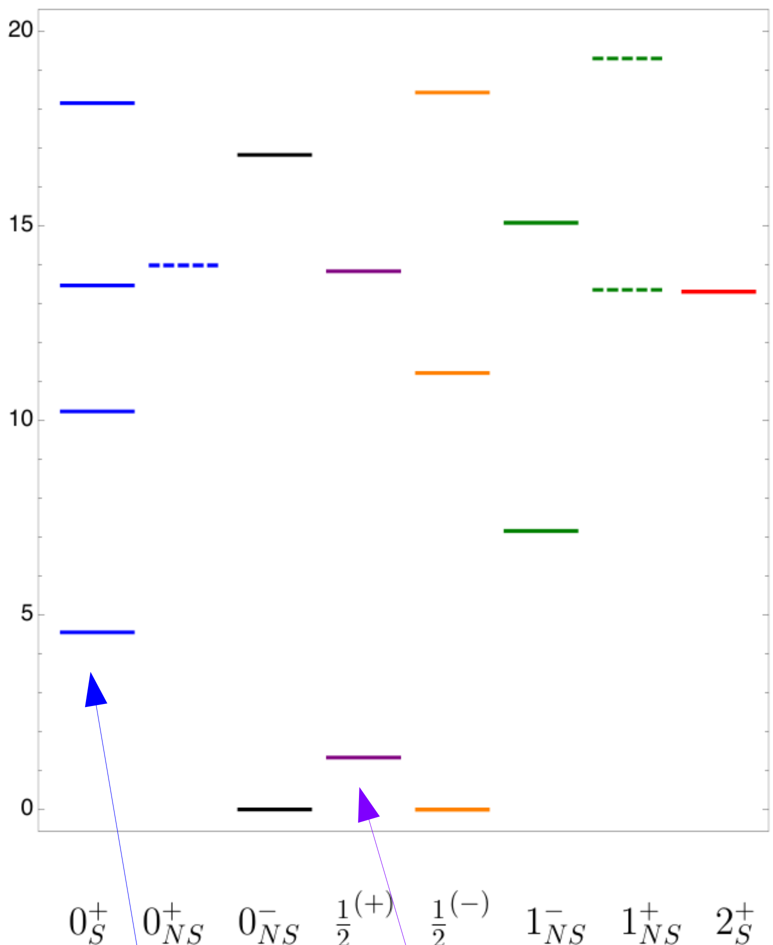
Adding bulk Yukawa

$\Delta = 3 \quad x_F = 1 \quad \Delta_\phi = 1 \quad \phi_\infty = 1.5$

Δ_R = 1.51: ◊◊◊ Δ_R = 2.5: ●●● Δ_R = 4: +++



$m/\tilde{f}$ $\Delta_R = 2.5 \quad y_5 = -2$



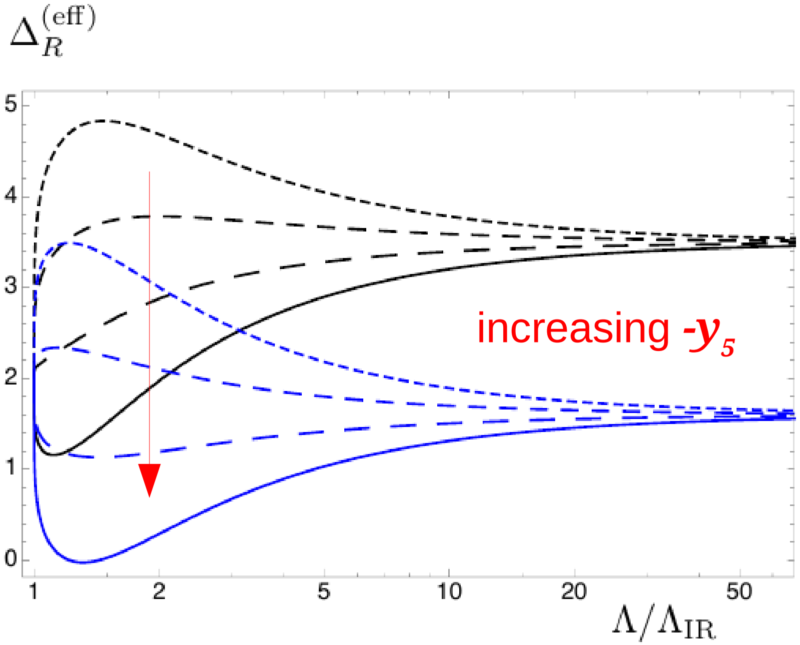
Negative y_5 suppresses ψ_L in the IR, roughly "turning (+) into (-)": the zero-mode becomes light for any value of Δ_R

Light fermion possible even for Δ_R >> 3/2 !

dilaton mass lifted

fermions much lighter than (non-Goldstone) bosons

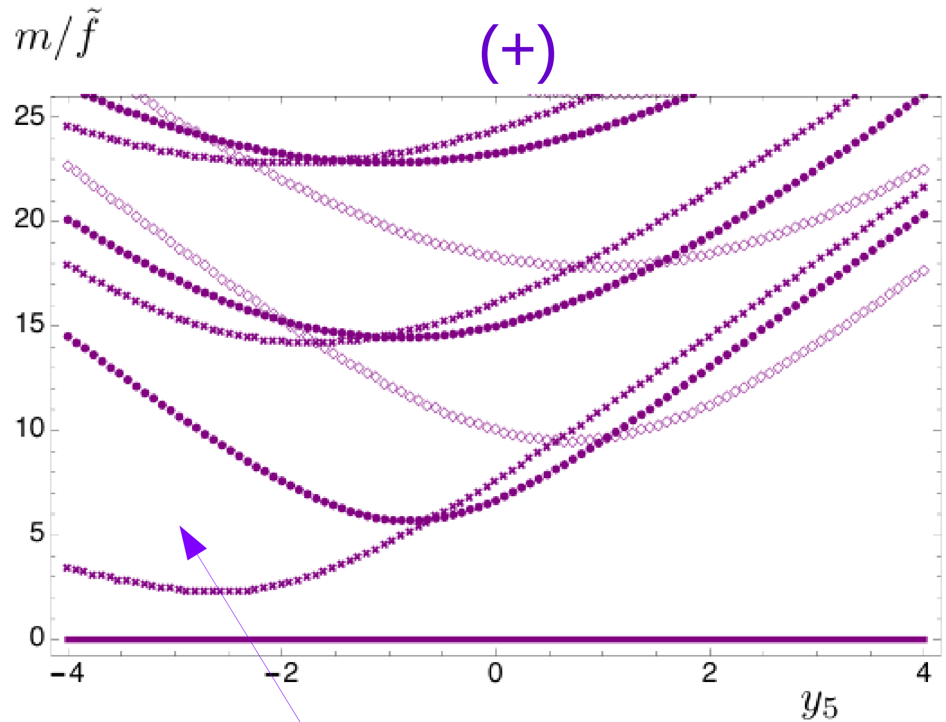
Adding bulk Yukawa in PC



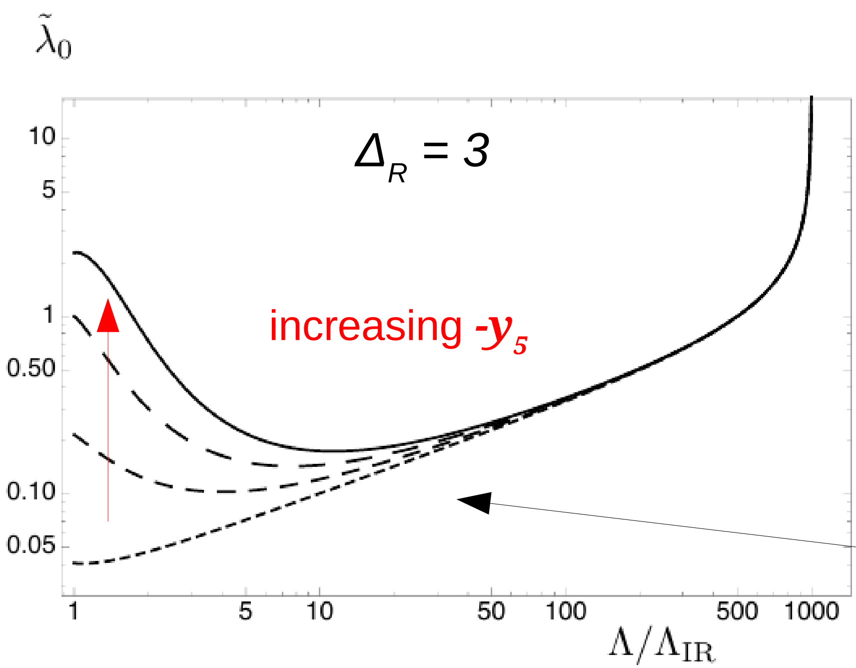
$\Delta_R = 3.5$

$\Delta_R = 1.6$

$\Delta_R = 1.51$: ◊◊◊ $\Delta_R = 2.5$: ●●● $\Delta_R = 3$: +++



light fermion at large $-y_5$ lifted proportionally to PC coupling



Irrelevant deformation in the UV can give large PC coupling in the IR

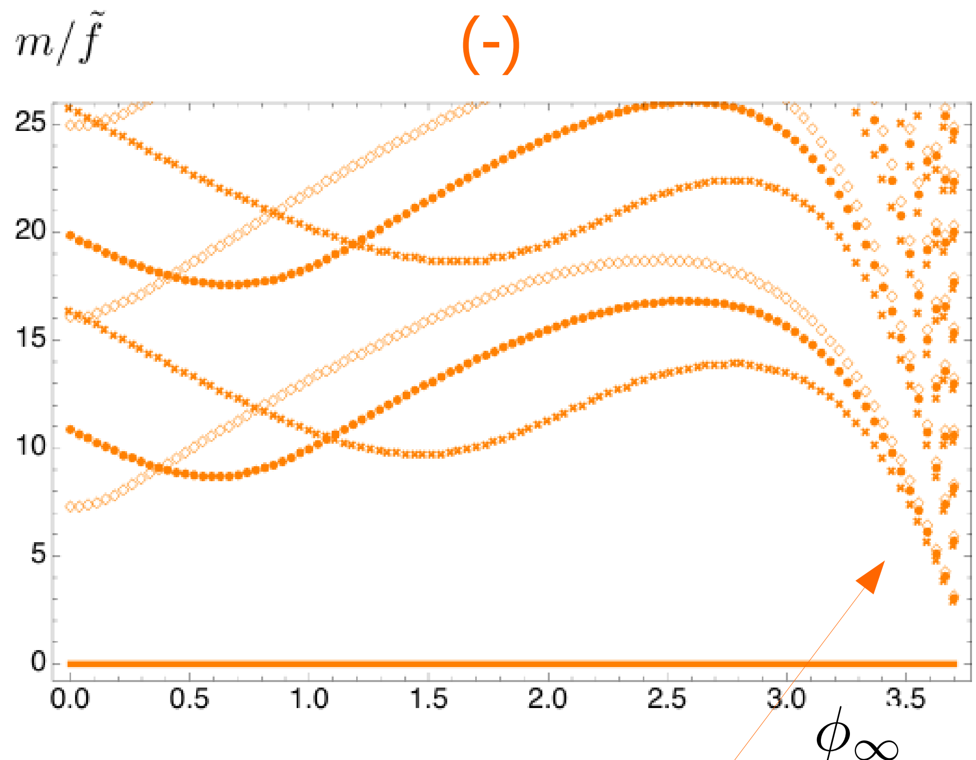
Adding singlet source

$\Delta_\phi \to 0 \qquad \phi_\infty \to \sqrt{3/\Delta_\phi}$

IR spectrum strongly deformed in the limit of
quasi-marginal deformation and large source

$$\Delta = 3 \quad x_F = 1 \quad \Delta_\phi = 0.2$$

Δ$_R$ = 1.6: ◊◊◊ *Δ$_R$ = 2.5:* ●●● *Δ$_R$ = 4:* +++



Resonance splitting Λ_{IR}
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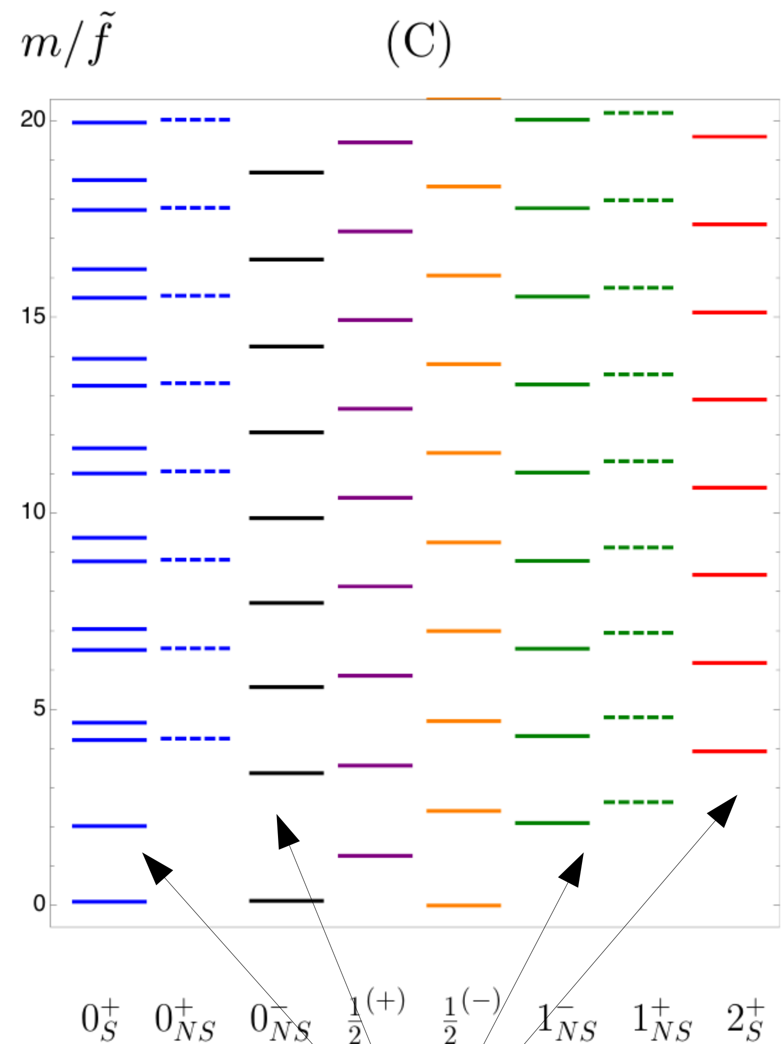
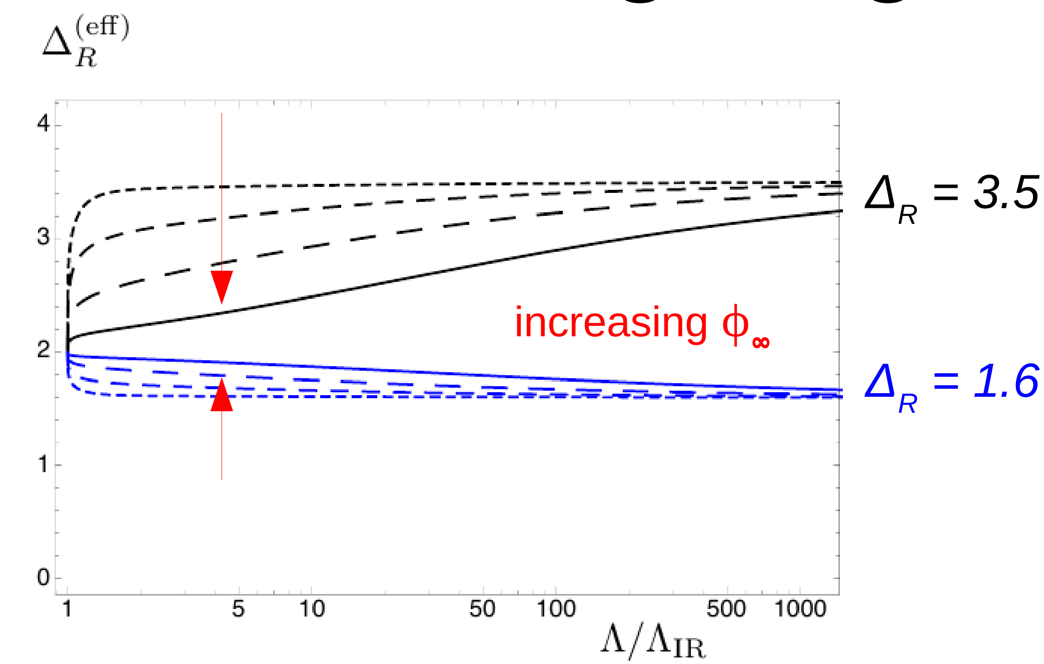
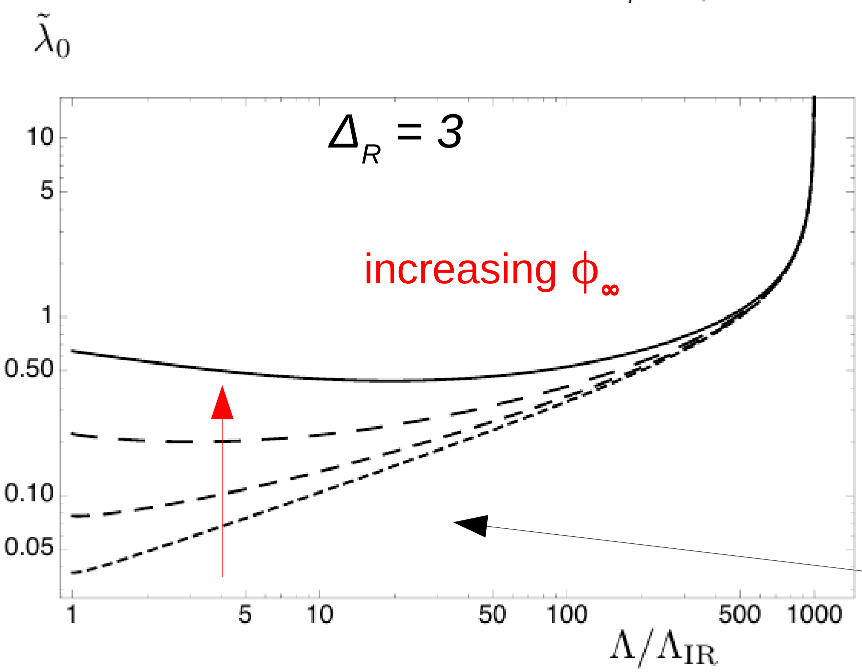


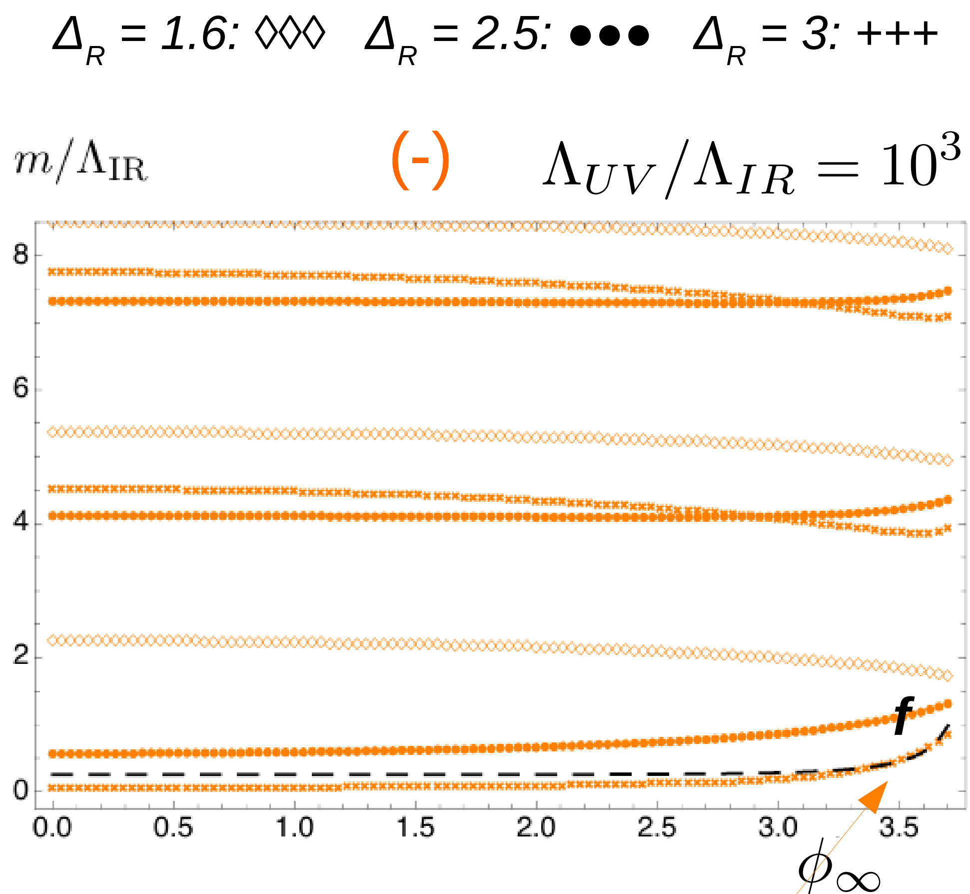
Mass gap decreases also for bosons

$$m \ll 4\pi\tilde{f}$$

Adding singlet source in PC







chiral fermion mass lifted
proportionally to PC coupling

Irrelevant deformation in the UV
can give large PC coupling in the IR

Summary

- Strongly-coupled gauge theories have **rich spectrum of resonances**
- Pheno & theory motivations for **large N_C and large N_F**
- **Holography** to model non-perturbative dynamics
  - Gravity-scalar background $\leftrightarrow$ mass gap & flavour SB
  - Fluctuations around background $\leftrightarrow$ spectrum of resonances
- **Composite fermions may be light** for special values of
  (i) scaling dimensions (ii) number of flavours
  (iii) bulk Yukawas (iv) SB sources
- In some of these limits, **partial-compositeness couplings can be enhanced in the IR,** even for operators irrelevant in the UV