Automatic generation of EFT operators

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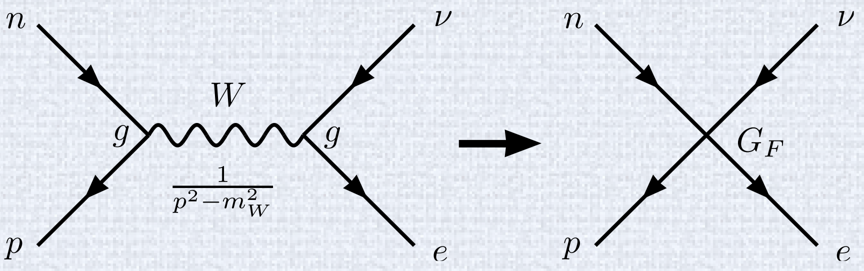
Effective field Theories (EFTs)

The appeal of EFTs

Model independent

The effects of physics at high energies can be understood in terms of effective operators involving only light fields

Fermi theory



The full theory can be matched to an effective theory by integrating out the heavy degrees of freedom and introducing a series of local non-renormalizable operators of dimension d suppressed by powers of (new physics scale)$^{-1}$

$$\left(\frac{1}{\Lambda}\right)^d$$

Basis of operators ↔ **Group theory**

A basis of operators is needed (up to some mass dimension). Fields are usually representations of one or more symmetry groups (Lorentz, gauge, flavor, ...) so finding all independent operators is mostly a group theory problem

Important example: SMEFT

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SO(3,1)$
Q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	$\left(\frac{1}{2},0\right)$
u^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	$\left(\frac{1}{2},0\right)$
d^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	$\left(\frac{1}{2},0\right)$
L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$\left(\frac{1}{2},0\right)$
e^c	$\mathbf{1}$	$\mathbf{1}$	1	$\left(\frac{1}{2},0\right)$
H	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	$(0,0)$
G	$\mathbf{8}$	$\mathbf{1}$	0	$(1,0)$
W	$\mathbf{1}$	$\mathbf{3}$	0	$(1,0)$
B	$\mathbf{1}$	$\mathbf{1}$	0	$(1,0)$

... in a nutshell

¿ **What are the effective interactions between the Standard Model fields allowed by gauge and Lorentz symmetries?** ?

The SM supplemented by these interactions is called the Standard Model effective field theory (SMEFT)

Redundancies: EOMs

Non-renormalizable operators may have derivatives, and combinations of some of these operators might be redundant

Equations of motion (EOMs)

Few field redefinitions

$$\Phi_i \rightarrow \Phi_i' \equiv f(\Phi_j)$$

leave the Lagrangian invariant

However, the S-matrix does not change under very general field redefinitions

Consequence: operators proportional to the classical equation of motion

$$\frac{\delta \mathscr{L}_{ren.}}{\delta \Phi_i} - \partial_\mu \left[\frac{\delta \mathscr{L}_{ren.}}{\delta \left(\partial_\mu \Phi_i \right)} \right]$$

can be dropped

In practice:

Lehman Martin 1510.00372

For every field $\Phi = \phi, \psi, \mathcal{F}_{\mu\nu}$ add a tower of extra fields $\partial X, \partial^2 X$, ... which are independent of X

BUT

Retain only the highest spin part of $\partial^n X$

Redundancies: IBPs

Integration by parts

For some operators, $\int_{\mathcal{M}} \mathcal{O}\, d^4x = 0$

In the language of differential forms, these redundant operators are associated exact to exact differential forms

(by assumption)

$$\omega^{(4),\mathrm{red}} = d\omega^{(3)} \longrightarrow \int_{\mathcal{M}} \omega^{(4),\mathrm{red}} = \int_{\mathrm{Boundary}(\mathcal{M})} \omega^{(3)} = 0$$

but ...

Henning, Lu, Melia, Murayama 1512.03433

we need to be careful: for some 3-forms, $d\omega^{(3),\mathrm{red}} = 0$ and we shouldn't consider them because *dd=0*, so these account for identically null 4-forms

Which are these 3-forms? $\omega^{(3),\mathrm{red}} = d\omega^{(2)}$

We have a recursive process, which ends when we reach 0-forms

Translation into language of operators:

The total number of non-redundant operators up to dimension *d* is:

$$\left(\#\mathcal{O}^{dim\leq d}\right) - \left(\#\mathcal{O}_{\mu}^{dim\leq d-1}\right) + \left(\#\mathcal{O}_{[\mu\nu]}^{dim\leq d-2}\right) - \left(\#\mathcal{O}_{[\mu\nu\rho]}^{dim\leq d-3}\right) + \left(\#\mathcal{O}_{[\mu\nu\rho\sigma]}^{dim\leq d-4}\right)$$

μ, ν, ρ, σ are Lorentz completely anti-symmetrized indices

Counting operators

Credit: Guilhem Vellut (https://www.flickr.com/photos/o_0/33725509613/)

Rapid progress in recent years

Using a **mathematical tool called the Hilbert series**, it became possible to count all SMEFT operators up to very high dimensions

!

Benvenuti, Feng, Hanany, He hep-th/0608050
Feng, Hanany, He hep-th/0701063
Hanany, Jenkins, Manohar, Torri 1010.3161
Lehman, Martin 1503.07537, 1510.00372
Henning, Lu, Melia, Murayama 1512.03433
...

Sample

Dim 5

$$6 H^2 L^2 + 6 H^{*2} L^{*2}$$

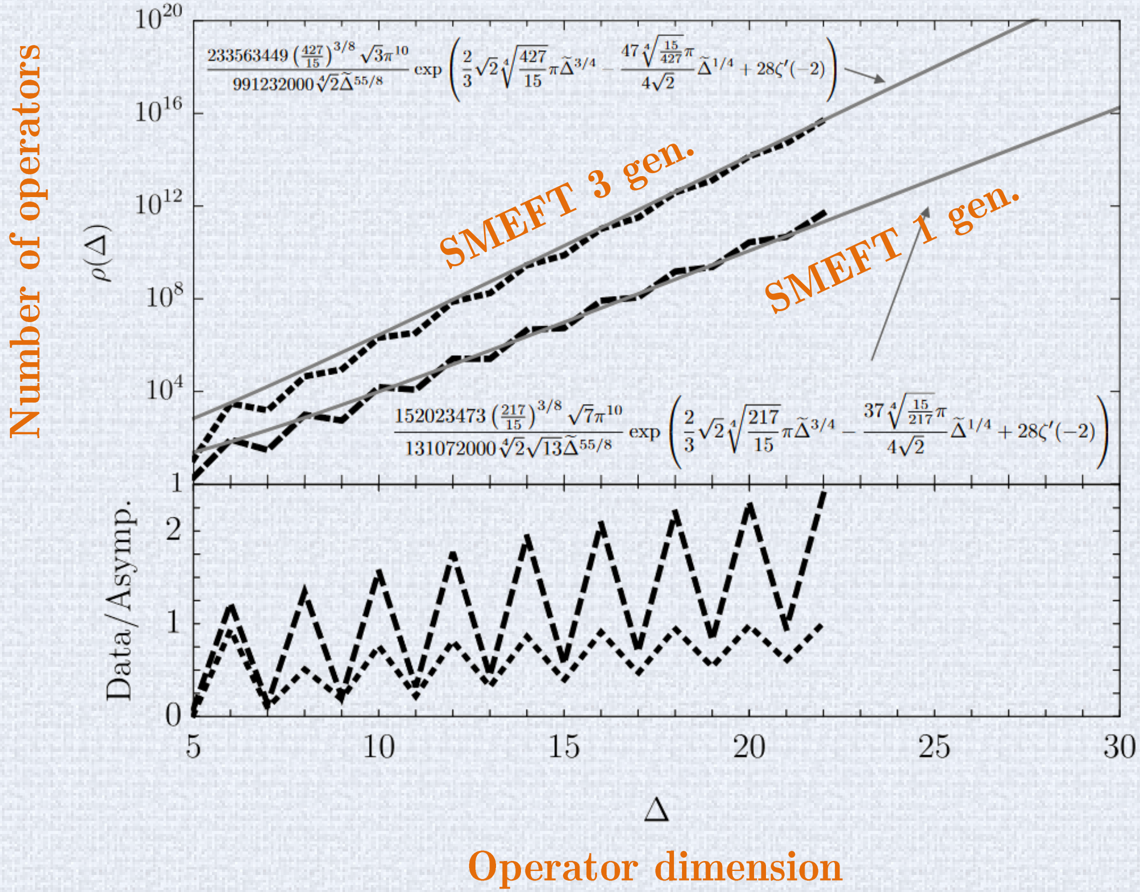
Dim 6

$$G^3 + 57 L Q^3 + 45 d^2 d^{*2} + 81 d e d^* e^* + 36 e^2 e^{*2} + G^{*3} + B^2 H H^* + G^2 H H^* + 9 B e L H^* + 9 B d Q H^* + 9 d G Q H^* +$$
$$H B^{*2} H^* + H G^{*2} H^* + 9 e H L H^{*2} + 9 d H Q H^{*2} + H^3 H^{*3} + 81 d L d^* L^* + 81 e L e^* L^* + 81 d Q e^* L^* + 9 H B^* e^* L^* +$$
$$9 H^2 e^* H^* L^* + 45 L^2 L^{*2} + 81 e L d^* Q^* + 162 d Q d^* Q^* + 9 H B^* d^* Q^* + 81 e Q e^* Q^* + 9 H d^* G^* Q^* + 9 H^2 d^* H^* Q^* +$$
$$162 L Q L^* Q^* + 90 Q^2 Q^{*2} + 57 L^* Q^{*3} + 81 L Q d^* u^* + 54 Q^2 e^* u^* + 9 B^* H^* Q^* u^* + 9 G^* H^* Q^* u^* + 9 H H^{*2} Q^* u^* +$$
$$162 e^* L^* Q^* u^* + 162 d^* Q^{*2} u^* + 81 d^* e^* u^{*2} + H B^* H^* W^* + 9 H e^* L^* W^* + 9 H d^* Q^* W^* + 9 H^* Q^* u^* W^* + H H^* W^{*2} + W^{*3} +$$
$$9 B H Q u + 9 G H Q u + 162 e L Q u + 162 d Q^2 u + 9 H^2 Q H^* u + 81 d L^* Q^* u + 54 e Q^{*2} u + 162 d d^* u^* u + 81 e e^* u^* u +$$
$$81 L L^* u^* u + 162 Q Q^* u^* u + 81 d e u^2 + 45 u^{*2} u^2 + B H H^* W + 9 e L H^* W + 9 d Q H^* W + 9 H Q u W + H H^* W^2 + W^3 +$$
$$9 d H d^* H^* \partial + 9 e H e^* H^* \partial + 18 H L H^* L^* \partial + 18 H Q H^* Q^* \partial + 9 d H^{*2} u^* \partial + 9 H^2 d^* u \partial + 9 H H^* u^* u \partial + 2 H^2 H^{*2} \partial^2$$

Format of each term: (#operators) x (field combinations)

- The **Hilbert series method counts operators**
  It does not build them explicitly
- This method also does not indicate where to apply the **derivatives**

Rapid progress in recent years



Melia, Pal 2010.08560

The traditional way

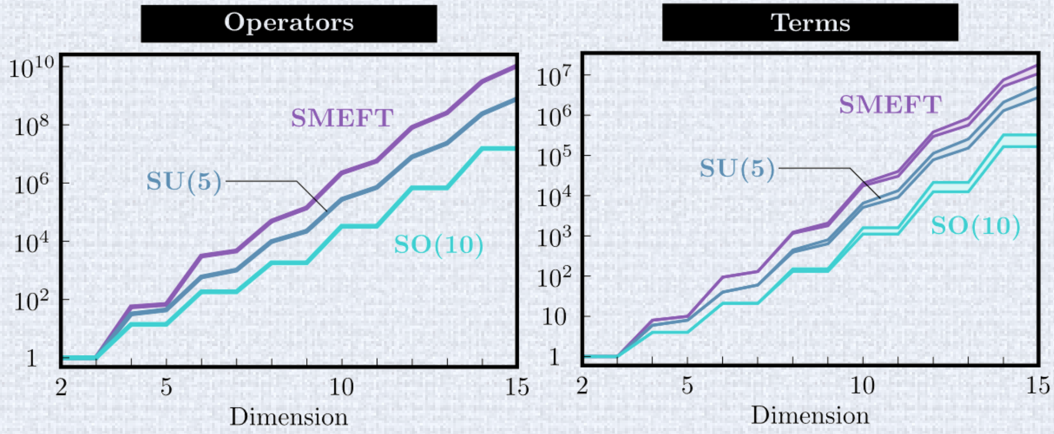
The Hilbert series gained prominence only in recent years

For decades, physicists have been building models and listing operators taking **all combinations of fields, and picking out the ones which are gauge and Lorentz invariant** (the ***traditional method***)

Can it be used to **reproduce the Hilbert series counting**?
Yes. There are programs doing that.

BasisGen Criado 1901.03501

Sym2Int RF 1703.05221, 1907.12584
more on it later



RF 1907.12584

- **Viable to high dimensions**
- Works out of the box with **any group, representations**
- Yields **more information** than just the number of operators, namely **permutation symmetries** of flavor indices
- **Can't tell where to apply derivatives** (same as HS method)

QQQL in SMEFT

> When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 5 new operators arise in the four-fermion sector.

Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884
(a.k.a. the "*Warsaw paper*")

7 years later (2017)
v3 in arXiv of the same work

> When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 4 new operators arise in the four-fermion sector.

The culprit

Easy to tackle this kind of problem systematically
(see extra slides)

Building operators explicitly

Credit: Guilhem Vellut (https://www.flickr.com/photos/o_0/33725509613/)

Known results for SMEFT

SMEFT dim 6

1986-2017

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu \varphi\right)^\star \left(\varphi^\dagger D_\mu \varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \widetilde{G}}$	$\varphi^\dagger \varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \widetilde{W}}$	$\varphi^\dagger \varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \widetilde{B}}$	$\varphi^\dagger \varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \widetilde{W} B}$	$\varphi^\dagger \tau^I \varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

(plus the 4-fermion operators)

Buchmüller, Wyler NPB 268 (1986) 621
Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884

Known results for SMEFT

SMEFT dim 7

2014

$1: \psi^2 H^4 + \text{h.c.}$	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n(H^\dagger H)$

$2: \psi^2 H^2 D^2 + \text{h.c.}$	
$\mathcal{O}_{LHD}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}L^iC(D^\mu L^j)H^m(D_\mu H^n)$
$\mathcal{O}_{LHD}^{(2)}$	$\epsilon_{im}\epsilon_{jn}L^iC(D^\mu L^j)H^m(D_\mu H^n)$

$3: \psi^2 H^3 D + \text{h.c.}$	
$\mathcal{O}_{LHDe}$	$\epsilon_{ij}\epsilon_{mn}\left(L^iC\gamma_\mu e\right)H^jH^mD^\mu H^n$

$4: \psi^2 H^2 X + \text{h.c.}$	
$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}\left(L^iC\sigma_{\mu\nu}L^m\right)H^jH^nB^{\mu\nu}$
$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I\epsilon)_{mn}\left(L^iC\sigma_{\mu\nu}L^m\right)H^jH^nW^{I\mu\nu}$

$5: \psi^4 D + \text{h.c.}$	
$\mathcal{O}_{LL\bar{d}uD}^{(1)}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iCD^\mu L^j)$
$\mathcal{O}_{LL\bar{d}uD}^{(2)}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iC\sigma^{\mu\nu}D_\nu L^j)$
$\mathcal{O}_{\bar{L}QddD}^{(1)}$	$(QC\gamma_\mu d)(\bar{L}D^\mu d)$
$\mathcal{O}_{\bar{L}QddD}^{(2)}$	$(\bar{L}\gamma_\mu Q)(dCD^\mu d)$
$\mathcal{O}_{ddd\bar{e}D}$	$(\bar{e}\gamma_\mu d)(dCD^\mu d)$

$6: \psi^4 H + \text{h.c.}$	
$\mathcal{O}_{LLL\bar{e}H}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^jCL^m)H^n$
$\mathcal{O}_{LLQ\bar{d}H}^{(1)}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{LLQ\bar{d}H}^{(2)}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{LL\bar{Q}uH}$	$\epsilon_{ij}(\bar{Q}_m u)(L^mCL^i)H^j$
$\mathcal{O}_{\bar{L}QQdH}$	$\epsilon_{ij}(\bar{L}_m d)(Q^mCQ^i)\tilde{H}^j$
$\mathcal{O}_{\bar{L}dddH}$	$(dCd)(\bar{L}d)H$
$\mathcal{O}_{\bar{L}uddH}$	$(\bar{L}d)(uCd)\tilde{H}$
$\mathcal{O}_{Leu\bar{d}H}$	$\epsilon_{ij}(L^iC\gamma_\mu e)(\bar{d}\gamma^\mu u)H^j$
$\mathcal{O}_{\bar{e}QddH}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$

Lehman 1410.4193

Known results for SMEFT

SMEFT dim 8

2020

$10: \psi^2 X H^3 + \text{h.c.}$	
$Q^{(1)}_{leWH^3}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H(H^\dagger H)W^I_{\mu\nu}$
$Q^{(2)}_{leWH^3}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)H(H^\dagger\tau^I H)W^I_{\mu\nu}$
Q_{leBH^3}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H(H^\dagger H)B_{\mu\nu}$
Q_{quGH^3}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}(H^\dagger H)G^A_{\mu\nu}$
$Q^{(1)}_{quWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}(H^\dagger H)W^I_{\mu\nu}$
$Q^{(2)}_{quWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}(H^\dagger\tau^I H)W^I_{\mu\nu}$
Q_{quBH^3}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}(H^\dagger H)B_{\mu\nu}$
Q_{qdGH^3}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H(H^\dagger H)G^A_{\mu\nu}$
$Q^{(1)}_{qdWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H(H^\dagger H)W^I_{\mu\nu}$
$Q^{(2)}_{qdWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)H(H^\dagger\tau^I H)W^I_{\mu\nu}$
Q_{qdBH^3}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H(H^\dagger H)B_{\mu\nu}$

$11: \psi^2 H^2 D^3$	
$Q^{(1)}_{l^2H^2D^3}$	$i(\bar{l}_p\gamma^\mu D^\nu l_r)(D_{(\mu}D_{\nu)}H^\dagger H)$
$Q^{(2)}_{l^2H^2D^3}$	$i(\bar{l}_p\gamma^\mu D^\nu l_r)(H^\dagger D_{(\mu}D_{\nu)}H)$
$Q^{(3)}_{l^2H^2D^3}$	$i(\bar{l}_p\gamma^\mu\tau^I D^\nu l_r)(D_{(\mu}D_{\nu)}H^\dagger\tau^I H)$
$Q^{(4)}_{l^2H^2D^3}$	$i(\bar{l}_p\gamma^\mu\tau^I D^\nu l_r)(H^\dagger\tau^I D_{(\mu}D_{\nu)}H)$
$Q^{(1)}_{e^2H^2D^3}$	$i(\bar{e}_p\gamma^\mu D^\nu e_r)(D_{(\mu}D_{\nu)}H^\dagger H)$
$Q^{(2)}_{e^2H^2D^3}$	$i(\bar{e}_p\gamma^\mu D^\nu e_r)(H^\dagger D_{(\mu}D_{\nu)}H)$
$Q^{(1)}_{q^2H^2D^3}$	$i(\bar{q}_p\gamma^\mu D^\nu q_r)(D_{(\mu}D_{\nu)}H^\dagger H)$
$Q^{(2)}_{q^2H^2D^3}$	$i(\bar{q}_p\gamma^\mu D^\nu q_r)(H^\dagger D_{(\mu}D_{\nu)}H)$
$Q^{(3)}_{q^2H^2D^3}$	$i(\bar{q}_p\gamma^\mu\tau^I D^\nu q_r)(D_{(\mu}D_{\nu)}H^\dagger\tau^I H)$
$Q^{(4)}_{q^2H^2D^2}$	$i(\bar{q}_p\gamma^\mu\tau^I D^\nu q_r)(H^\dagger\tau^I D_{(\mu}D_{\nu)}H)$
$Q^{(1)}_{u^2H^2D^3}$	$i(\bar{u}_p\gamma^\mu D^\nu u_r)(D_{(\mu}D_{\nu)}H^\dagger H)$
$Q^{(2)}_{u^2H^2D^3}$	$i(\bar{u}_p\gamma^\mu D^\nu u_r)(H^\dagger D_{(\mu}D_{\nu)}H)$
$Q^{(1)}_{d^2H^2D^3}$	$i(\bar{d}_p\gamma^\mu D^\nu d_r)(D_{(\mu}D_{\nu)}H^\dagger H)$
$Q^{(2)}_{d^2H^2D^3}$	$i(\bar{d}_p\gamma^\mu D^\nu d_r)(H^\dagger D_{(\mu}D_{\nu)}H)$
$Q_{udH^2D^3}$ + h.c.	$i(\bar{u}_p\gamma^\mu D^\nu d_r)(\widetilde{H}^\dagger D_{(\mu}D_{\nu)}H)$

(plus many more)

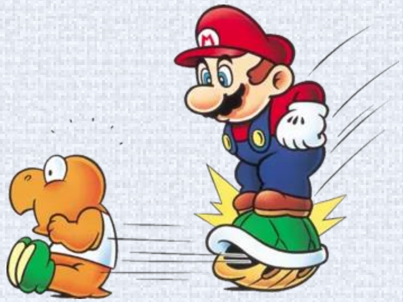
Murphy 2005.00059
Li, Ren, Shu, Xiao, Yu, Zheng, 2005.00008

SMEFT dim 9

2020

Li, Ren, Xiao, Yu, Zheng, 2007.07899

Going off-shell



Matching of Lagrangians: after integrating out heavy fields at some scale, one must ensure that the EFT and the UV theory are physically equivalent at that scale.

It is convenient to **compare the two theories off-shell**. Without EOM relations, there are more operators. It would be useful to have a **Green basis** of operators

SMEFT dim 6 — Gherardi, Marzocca, Venturini, 2003.12525

SMEFT dim 8 (bosons) — Chala, Díaz-Carmona, Guedes 2112.12724

PLANCK 2022
Paris, May 30 - June 3, 2022

Talk by
Álvaro Díaz-Carmona

Note: The methods (Hilbert series, traditional) and computer codes which count operators can help. Knowing their number = knowing when to stop looking for more independent operators

GroupMath

A Mathematica package for the group theory computations

RF 2011.01764

Basis-independent functions

Adjoint | Casimir | ConjugateIrrep | DynkinIndex | DimR | PermutationSymmetryOfInvariants | ReduceRepProduct | RepName | RepsUpToDimN | Weights | TriangularAnomalyValue | ...

Basis-dependent functions

IrrepInProduct | RepMatrices | Invariants

Permutation group functions

DecomposeSnProduct | DrawYoungDiagram | GenerateStandardTableaux | HookContentFormula | LittlewoodRichardsonCoefficients | SnClassCharacter | SnClassOrder | SnIrrepDim | SnIrrepGenerators | ...

Symmetry breaking functions

DecomposeRep | FindAllEmbeddings | MaximalSubgroups | RegularSubgroupProjectionMatrix | SubgroupEmbeddingCoefficients

GROUPMATH

Group theory code for Mathematica

GroupMath is a Mathematica package containing several functions related to Lie Algebras and the permutation group. For now, it is still a work in progress, so it not fully documented.

However, it inherits much of its code from the Susyno package, so some of GroupMath's function have already described in this link. Over the years, group theory functions were added to the Susyno program (whole aim is to calculate renormalization group equations), however it became clear at some point that such code would be interesting on its own, so GroupMath was created.

Note that the latest version of the Sym2Int code requires GroupMath.

References

GroupMath has not been described in any publication yet, however it inherits much of its code from Susyno: Computer Physics Communications 183 (2012) 2298.

Installing the code

GroupMath can be obtained from this page:

Download

(GroupMath 0.11)

Sym2Int

«Symmetries to Interactions»

A Mathematica package to list the operators in a model
Works out of the box for any gauge group and representations

RF 1703.05221, 1907.12584

```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", 3};
fld2 = {"d", {3, 1, -1/3}, "R", "C", 3};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", 3};
fld4 = {"e", {1, 1, -1}, "R", "C", 3};
fld5 = {"L", {1, 2, -1/2}, "L", "C", 3};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};

savedResults = GenerateListOfCouplings[SM, MaxOrder → 6];
```

Sym2Int

«Symmetries to Interactions»

A Mathematica package to list the operators in a model
Works out of the box for any gauge group and representations

RF 1703.05221, 1907.12584

A name to the model (e.g. SM)

```
gaugeGroup[SM] ^= {SU3, SU2, U1};
```

The gauge group (e.g. SU(3) x SU(2) x U(1))

```
fld1 = {"u", {3, 1, 2/3}, "R", "C", 3};
fld2 = {"d", {3, 1, -1/3}, "R", "C", 3};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", 3};
fld4 = {"e", {1, 1, -1}, "R", "C", 3};
fld5 = {"L", {1, 2, -1/2}, "L", "C", 3};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};
```

The fields, i.e. the irreps under the gauge and Lorentz groups, including #flavors

```
savedResults = GenerateListOfCouplings[SM, MaxOrder → 6];
```

Max dimension of interactions (e.g.: 6)

Example: SMEFT up to dim 6

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	H* H	2	True	1	1		
2	L* e H	4	False	9	1		
3	Q* d H	4	False	9	1		
4	u* Q H	4	False	9	1		
5	H* H* H H	4	True	1	1	{H*, H}	{□□, □□}
6	L L H H	5	False	6	1	{L, H}	{□□, □□}
7	F1 F1 F1	6	False	1	1	F1	□□□
8	F2 F2 F2	6	False	1	1	F2	□□□
9	𝒟 𝒟 H* H* H H	6	True	2	2	{H*, H}	2 {□□, □□} + 2 {□×□, □×□} − 2 {□×□, □□}
10	𝒟 H* L* L H	6	True	18	2		
11	𝒟 H* e* e H	6	True	9	1		
12	𝒟 H* Q* Q H	6	True	18	2		
13	𝒟 H* d* d H	6	True	9	1		
14	𝒟 H* u* u H	6	True	9	1		
15	F3* L* e H	6	False	9	1		
16	F3* Q* d H	6	False	9	1		
17	F2* L* e H	6	False	9	1		
18	F2* Q* d H	6	False	9	1		
19	F1* Q* d H	6	False	9	1		

Example: SMEFT up to dim 6

42	𝒟 u⋆ d H H	6	False	9	1	H	□ × □
43	u⋆ Q H F1	6	False	9	1		
44	u⋆ Q H F2	6	False	9	1		
45	u⋆ Q H F3	6	False	9	1		
46	u u d e	6	False	81	1	u	□□ + [□/□]
47	u d Q L	6	False	81	1		
48	u Q Q e	6	False	54	1	Q	□□
49	Q Q Q L	6	False	57	1	Q	[□/□/□] + [□□/□] + □□□
50	H⋆ L⋆ e H H	6	False	9	1	H	□□
51	H⋆ Q⋆ d H H	6	False	9	1	H	□□
52	H⋆ u⋆ Q H H	6	False	9	1	H	□□
53	H⋆ H⋆ H⋆ H H H	6	True	1	1	{H⋆, H}	{□□□, □□□}

Dimension	# real operators	# real terms	# types of real operators
2	1	1	1
3	0	0	0
4	55	7	7
5	12	2	2
6	3045	84	72

Sym2Int upgrade
Ongoing work

Currently, Sym2Int does not compute the operators explicitly

Compute distinct contractions of Lorentz indices (fermions indices, vector indices)

Compute distinct contractions of gauge indices

GroupMath can do this, although it might be worth considering alternative forms (for human readability of the results)

Get a "maximal basis" of operators

Use them as a basis for a vector space. Everything becomes Linear Algebra (involving in some cases very large vectors/matrices)

Account for EOMs

Account for IBPs

Relations among vectors

With them we may get
(1) All Operators
(2) Use IBPs
(3) Use EOMs
(4) Use IBPs and EOMs

Interface with other programs:

FeynRules — Alloul, Christensen, Degrande, Duhr, Fuks, 1310.1921
MatchMakerEFT — Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787

Example: EOMs relations

Replace in each operators $\mathcal{O}$ **the expression** $\partial^n \Phi$ by a new expression where the part removable by EOMs is segregated

$$\partial_\mu \partial_\nu \phi = \begin{pmatrix} \partial_0\partial_0\phi & \partial_0\partial_1\phi & \partial_0\partial_2\phi & \partial_0\partial_3\phi \\ \partial_0\partial_1\phi & \partial_1\partial_1\phi & \partial_1\partial_2\phi & \partial_1\partial_3\phi \\ \partial_0\partial_2\phi & \partial_1\partial_2\phi & \partial_2\partial_2\phi & \partial_2\partial_3\phi \\ \partial_0\partial_3\phi & \partial_1\partial_3\phi & \partial_2\partial_3\phi & \partial_3\partial_3\phi \end{pmatrix}$$

Change variables

$$\begin{pmatrix} \frac{K[1]}{4}+\frac{K[3]}{4}+\frac{K[6]}{4}+\frac{R[1]}{4} & K[9] & K[8] & K[7] \\ K[9] & -\frac{K[1]}{4}+\frac{3K[6]}{4}-\frac{K[3]}{4}-\frac{R[1]}{4} & K[5] & K[4] \\ K[8] & K[5] & -\frac{K[1]}{4}+\frac{3K[3]}{4}-\frac{K[6]}{4}-\frac{R[1]}{4} & K[2] \\ K[7] & K[4] & K[2] & \frac{3K[1]}{4}-\frac{K[3]}{4}-\frac{K[6]}{4}-\frac{R[1]}{4} \end{pmatrix}$$

The **EOM removes the R[...] components** and leaves all the K[...] components:

$$\partial_\mu \partial^\mu \phi = \mathrm{R}[1]$$

So in this case we can just set R[1]=0 and see what **relations appear between the operators** in the "maximal basis"

Challenges

Performance/Speed!

Each operator is a polynomial in many variables. And there are many operators.

Repeated fields

Operators with repeated fields (such as the Weinberg operator $LLHH$) are quite complicated to handle. Note that in these cases one cannot treat gauge and Lorentz indices separately.

Flavor

Couplings become tensors in flavor space and they can have complex symmetries (e.g. $QQQL$) Flavor, gauge and Lorentz indices cannot be treated separately.

Do things in full generality

Making a code work for any operator/gauge group/representation is not easy

Stay tunned

The good news: building such a code seems doable. E.g.: SMEFT with 3 generations looks doable up to at least dimension 10. Still, a work in progress.

Summary

Credit: Guilhem Vellut (https://www.flickr.com/photos/o_0/33725509613/)

Summary

Effective field theories (EFTs) are an important tool in the exploration of potential new physics

From a list of fields, and some symmetries, we want to get a basis of operators

There has been a remarkable progress in recent years on counting and building explicitly operators...

...but there is still no efficient code to do so

Ongoing work to make **GroupMath** + **Sym2Int** not just list, but also <u>build explicitly EFT operators</u>

Thank you


Extra slides
Credit: Guilhem Vellut (https://www.flickr.com/photos/o_0/33725509613/)

$QQQL$ in SMEFT

		QQQ	L
	$SU(3)_C$	$\{1,1,1\}$	$\{1\}$
	$SU(2)_L$	$\{2,1\}$	$\{1\}$
Lorentz group	$SU(2)_l$	$\{2,1\}$	$\{1\}$
	$SU(2)_r$	$\{3\}$	$\{1\}$
	Grassmann	$\{1,1,1\}$	$\{1\}$
	Total symmetry	$\{1,1,1\}^2 \times \{2,1\}^2 \times \{3\} = \{3\} + \{2,1\} + \{1,1,1\}$	$\{1\}^5 = \{1\}$

RF 1907.12584

Relevant symmetry if $S_3 \times S_1$ (the S_1 we could ignore)

Color: 3 triplets contract completely anti-symmetrically ($\{1,1,1\}$)

$SU(2)_L$: 3 doublets (QQQ) can form a pair of doublets (to couple with L)
They transforms as $\{2,1\}$ under S_3 (more on this later)

$SU(2)_l$: Same as with $SU(2)_L$

$SU(2)_R$: The Q's (and the L) are left-handed spinors, so they are singlets of $SU(2)_r$.
Therefore the contractions of this group transform trivially ($\{3\}$) under S_3

Grassmann: Fermion components anti-commute, so we multiply the resulting symmetry by the completely antisymmetric representation of S_3 ($\{1,1,1\}$)

$QQQL$ in SMEFT

	QQQ	L
$SU(3)_C$	$\ydiagram{1,1,1}$	$\ydiagram{1}$
$SU(2)_L$	$\ydiagram{2,1}$	$\ydiagram{1}$
Lorentz group { $SU(2)_l$	$\ydiagram{2,1}$	$\ydiagram{1}$
Lorentz group { $SU(2)_r$	$\ydiagram{3}$	$\ydiagram{1}$
Grassmann	$\ydiagram{1,1,1}$	$\ydiagram{1}$
Total symmetry	$\ydiagram{1,1,1}^2 \times \ydiagram{2,1}^2 \times \ydiagram{3} = \ydiagram{3} + \ydiagram{2,1} + \ydiagram{1,1,1}$	$\ydiagram{1}^5 = \ydiagram{1}$

RF 1907.12584

Relevant symmetry if $S_3 \times S_1$ (the S_1 we could ignore)

Colo

$SU(2$

$SU(2$

Grassm

as with

$U(2)_L$

 # Operators

$$\left[\mathcal{S}\left(\ydiagram{3}, n_Q\right) + \mathcal{S}\left(\ydiagram{2,1}, n_Q\right) + \mathcal{S}\left(\ydiagram{1,1,1}, n_Q\right)\right] \mathcal{S}\left(\ydiagram{1}, n_L\right) = \frac{n_L n_Q \left(2n_Q^2 + 1\right)}{3}$$

 # Terms

1