# A Green's Basis for the Bosonic SMEFT to Dimension 8

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# Outline

# Motivation

## Current status of High Energy Physics

Some remarks regarding the experimental anomalies found over the last years:

- The SM of particles cannot explain all experimental observations.
- No particles or fields have been found from EW scale to TeV.

$$\left.\begin{array}{c} \text{New Physics} \mapsto \Lambda \sim \mathcal{O}(\text{TeV}) \\ \\ \textit{Nothing} \\ \\ \text{Standard Model} \mapsto v \sim 246\,\text{GeV} \end{array}\right\} \text{SMEFT}$$

# Motivation

## SMEFT Lagrangian

The SMEFT extends the SM with all Lorentz and gauge invariant operators with dimension greater than 4, but for now we only consider operators up to dimension 8.

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)}$$

Operators can be grouped into classes according to their fields:

| Class | Dimension $(D=4)$ | Example |
|---|---|---|
| H4 | 4 | $\left(\phi^\dagger\phi\right)^2$ |
| H4D2 | 6 | $\left(\phi^\dagger\phi\right) D^2 \left(\phi^\dagger\phi\right)$ |
| H4D4 | 8 | $\left(D_\mu\phi^\dagger D_\nu\phi\right)\left(D_\nu\phi^\dagger D_\mu\phi\right)$ |
| H8, H6D2, BH4D2, WH4D2, G2H4, WBH2D2, ... | | |

# Physical and redundant operators

## Transformations within a class

All operators are related to others of the same class through algebraic transformations, integration by parts and other relations:

$$0 = \partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} \qquad \text{Bianchi}$$

$$0 = D_\mu(\mathcal{O}_1\mathcal{O}_2) = D_\mu\mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_1 D_\mu\mathcal{O}_2 \qquad \text{IBP}$$

For example:

$$D_\mu \left[ \left(\phi^\dagger\phi\right)^2 D^\mu \left(\phi^\dagger\phi\right) \right] = 2\left(\phi^\dagger\phi\right) D_\mu \left(\phi^\dagger\phi\right) D^\mu \left(\phi^\dagger\phi\right) + \left(\phi^\dagger\phi\right)^2 \left(D^2\phi^\dagger\phi + \phi^\dagger D^2\phi\right) + 2\left(\phi^\dagger\phi\right)^2 \left(D_\mu\phi^\dagger D^\mu\phi\right)$$

# Physical and redundant operators

## Transformation to other classes

Some operators are related to others of different classes through the equations of motion:

$$D^2\phi^i = \mu^2\phi^i - 2\lambda(\phi^\dagger\phi)\phi^i$$

These are called redundant operators. Non-redundant operators are called physical. Applying the equations of motion to a redundant operator leads to physical operators, for example:

$$\left(\phi^\dagger\phi\right)^2\left(\phi^\dagger D^2\phi + D^2\phi^\dagger\phi\right) = 2\mu^2\left(\phi^\dagger\phi\right)^3 - 4\lambda\left(\phi^\dagger\phi\right)^4$$

# Physical and redundant operators

## Physical basis and Green's Basis

Classes of operators have a finite number of independent members. This means that a **basis** can be created.

- They are called physical basis for physical operators and Green's basis for physical and redundant operators.
- The (63) physical and (69) redundant $\mathcal{O}_i^{(6)}$ are listed in [1008.4884] Grzadkowski et al. 2010 and [2003.12525] Gherardi, Marzocca, and Venturini 2020.

| | | | | | |
|---|---|---|---|---|---|
| $H^6$ | 1+0 | $H^4D^2$ | 2+2 | $H^2D^4$ | 0+1 |
| $X^3$ | 4+0 | $X^2H^2$ | 8+0 | $H^2XD^2$ | 0+2 |
| $\psi^2H^3$ | 3+0 | $X^2D^2$ | 0+3 | $\psi^2D^3$ | 0+5 |
| $4\ell$ | 3+0 | $\psi^2XH$ | 8+0 | $\psi^2XD$ | 0+30 |
| $4q$ | 12+0 | $\psi^2DH^2$ | 8+14 | $\psi^2HD^2$ | 0+12 |
| Semilep. | 10+0 | $B$ and $L$ violating | | | 4+0 |

# Physical and redundant operators

## Physical basis and Green's Basis (II)

A Green's basis was derived for dimension 6, but latest results point towards order $v^4/\Lambda^4$ in the SMEFT.

- The (993) physical $\mathcal{O}_i^{(8)}$ are listed in [2005.00059] Murphy 2020.
- There isn't a complete list for the (1649) redundant operators.
- In our work, we built a Green's basis for dimension 8 bosonic operators. The basis consists in 89+86 operators [2112.12724] Chala, Díaz-Carmona, and Guedes 2022.

Note that there is always some freedom of choice, thus different Green's bases could coexist.

# Physical and redundant operators

## Working in momentum space

For simplicity, it is convenient to work in momentum space, i.e: setting $\partial_\mu \to -ip_\mu$. We can find a 1 to 1 mapping to all the previous concepts and relations. For example:

$$\left.\begin{array}{c} 0 = D_\mu(\mathcal{O}_1\mathcal{O}_2) = D_\mu\mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_1 D_\mu \mathcal{O}_2 \\ 0 = p_\mu^{(1+2)} = p_\mu^{(1)} + p_\mu^{(2)} \end{array}\right\} \text{IBP} \Leftrightarrow \text{mom. cons.}$$

$$\left.\begin{array}{c} D^2\phi^i = \mu^2\phi^i - 2\lambda(\phi^\dagger\phi)\phi^i \\ p_\phi^2 = \mu^2 - 2\lambda(\phi^\dagger\phi) \end{array}\right\} \text{EoM} \Leftrightarrow \text{on-shell field}$$

# Physical and redundant operators

## Off-shell vs on-shell

The last relation leads to a crucial fact:

We can work with physical operators through onshell processes and with redundant operators through offshell processes.

Thus, there are two different approaches with their pros and cons:

| On-shell | Off-shell |
|---|---|
| Connected diagrams | 1PI diagrams |
| Fewer independent structures | More independent structures |
| Diagrams with bridges | Only vertices and loops |

# Physical and redundant operators

## Off-shell method

In the off-shell approach we study the 1PI amplitudes generated by a set of operators $\{\mathcal{O}_i\}_{i=1\ldots N}$ for some processes:

$$\mathcal{A}\left(a \to b\right) = c_i \sum_{\alpha \in I} f^i_\alpha\left(\vec{g}\right) \kappa_\alpha \quad \vec{g} = \left(g_1, g_2, g_3, \lambda\right)$$

$\kappa_\alpha$ are the kinematic invariants, *i.e:* $p_2 \cdot \varepsilon_4\,, \quad \epsilon_{\mu\nu\delta\theta} p_1^\mu p_2^\nu p_3^\delta \varepsilon_3^\theta\,, \ldots$

$\{\kappa_\alpha\}$ is formed by contractions of the momenta and polarization vectors with the proper mass dimension for each process.

$f^i_\alpha$ are the amplitude coefficients (complex numbers) coming from the Feynman rules.

# Physical and redundant operators

## Independence of operators

Algebraic relations condition the set of kinematic invariants $\{\kappa_\alpha\}$ and tweak the amplitude coefficients $f^i_\alpha$, but these are all linear transformations that don't affect the amplitude.

$$\mathcal{A} = c_i \sum_{\alpha \in I} f^i_{\ \alpha}(\vec{g})\, \kappa_\alpha = c_i \sum_{\beta \in J} h^i_{\ \beta}(\vec{g})\, \kappa_\beta \qquad \vec{g} = (g_1, g_2, g_3, \lambda)$$

We can determine the dependancy of a set of operators in momentum space by checking the rank of the matrix $M_{i\alpha} \equiv f^i_{\ \alpha}$

$$Rank\,(M) = N \Rightarrow \{\mathcal{O}_i\}_{i=1\ldots N} \text{ indep.}$$

If a process exists such that the amplitude coefficients matrix has maximum rank, then we can say the operators are independent.

# Building a Green's basis

## Generation of new redundant operators.

All **classes** and **number** of independent operators are inferred through `Sym2Int` [1703.05221] Fonseca 2017 and `Basisgen` [1901.03501] Criado 2019

| | | | | | |
|---|---|---|---|---|---|
| $\phi^8$ | 1+0 | $\phi^6 D^2$ | 2+2 | $\phi^4 D^4$ | 3+10 |
| $X^3\phi^2$ | 6+0 | $X^2\phi^4$ | 10+0 | $\phi^2 D^6$ | 0+1 |
| $X^4$ | 18+0 | $W^2\phi^2 D^2$ | 6+13 | $X\phi^2 D^4$ | 0+6 |
| $X^2 X'^2$ | 21+0 | $WB\phi^2 D^2$ | 6+13 | $B\phi^2 D^2$ | 3+9 |
| $X^3 X'$ | 4+0 | $G^2\phi^2 D^2$ | 3+9 | Total | 89+86 |

Redundant operators were **generated** by applying transformations (except EoM) to other known operators.

# Building a Green's basis

## Securing the basis and going on-shell

**Independence** of operators is ensured by working in momentum space and computing the adecuate processes. For example:

$$\text{Class } B^2\phi^2 D^2 \to \{\mathcal{O}_i\}_{i=1\ldots12} \to Rank(M_{\varphi^0\varphi^0\to BB}) = 12$$

Inserting the **EoM** into the redundant operators transforms the Green's basis to a physical basis.

$$\mathcal{O}_R \to b\mathcal{O}_P \Rightarrow \mathcal{L} \supset c_P\mathcal{O}_P + c_R\mathcal{O}_R \to (c_P + bc_R)\mathcal{O}_P$$

In general: $c_i^{phys} \to c_i^{phys} + \sum_j b_j c_j^{red}$ for all physical WC.

# Results and applications

## Results

- In [2112.12724] Chala, Díaz-Carmona, and Guedes 2022, we feature the explicit form of 89 physical and 86 redundant independent operators, classified by their field content.
- The relation of the redundant operators to the physical ones is explicitly shown.

| | | | | |
|---|---|---|---|---|
| $\phi^4 D^4$ | $(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi)$ | $\mathcal{O}^{(1)}_{\phi^4}$ | $(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi)$ | $\mathcal{O}^{(2)}_{\phi^4}$ |
| | $(D^\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D_\nu\phi)$ | $\mathcal{O}^{(3)}_{\phi^4}$ | $D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger D^2\phi + \text{h.c.})$ | $\mathcal{O}^{(4)}_{\phi^4}$ |
| | $D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger \mathrm{i} D^2\phi + \text{h.c.})$ | $\mathcal{O}^{(5)}_{\phi^4}$ | $(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger D_\mu\phi) + \text{h.c.}$ | $\mathcal{O}^{(6)}_{\phi^4}$ |
| | $(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger \mathrm{i} D_\mu\phi) + \text{h.c.}$ | $\mathcal{O}^{(7)}_{\phi^4}$ | $(D^2\phi^\dagger\phi)(D^2\phi^\dagger\phi) + \text{h.c.}$ | $\mathcal{O}^{(8)}_{\phi^4}$ |
| | $(D^2\phi^\dagger\phi)(\mathrm{i} D^2\phi^\dagger\phi) + \text{h.c.}$ | $\mathcal{O}^{(9)}_{\phi^4}$ | $(D^2\phi^\dagger D^2\phi)(\phi^\dagger\phi)$ | $\mathcal{O}^{(10)}_{\phi^4}$ |
| | $(\phi^\dagger D^2\phi)(D^2\phi^\dagger\phi)$ | $\mathcal{O}^{(11)}_{\phi^4}$ | $(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger D^2\phi) + \text{h.c.}$ | $\mathcal{O}^{(12)}_{\phi^4}$ |
| | $(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger \mathrm{i} D^2\phi) + \text{h.c.}$ | $\mathcal{O}^{(13)}_{\phi^4}$ | | |
| $X^3\phi^2$ | $f^{ABC}(\phi^\dagger\phi)G^{A,\nu}_\mu G^{B,\rho}_\nu G^{C,\mu}_\rho$ | $\mathcal{O}^{(1)}_{G^3\phi^2}$ | $f^{ABC}(\phi^\dagger\phi)G^{A,\nu}_\mu G^{B,\rho}_\nu \tilde{G}^{C,\mu}_\rho$ | $\mathcal{O}^{(1)}_{G^3\phi^2}$ |
| | $\epsilon^{IJK}(\phi^\dagger\phi)W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$ | $\mathcal{O}^{(1)}_{W^3\phi^2}$ | $\epsilon^{IJK}(\phi^\dagger\phi)W^{I\nu}_\mu W^{J\rho}_\nu \widetilde{W}^{K\mu}_\rho$ | $\mathcal{O}^{(2)}_{W^3\phi^2}$ |
| | $\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)B^\nu_\mu W^{J\rho}_\nu W^{K\mu}_\rho$ | $\mathcal{O}^{(1)}_{W^2B\phi^2}$ | $\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(\widetilde{B}^{\mu\nu}W^J_{\nu\rho}W^{K\rho}_\mu + B^{\mu\nu}W^J_{\nu\rho}\widetilde{W}^{K\rho}_\mu)$ | $\mathcal{O}^{(2)}_{W^2B\phi^2}$ |
| $X^2\phi^4$ | $(\phi^\dagger\phi)^2 G^A_{\mu\nu}G^{A\mu\nu}$ | $O^{(1)}_{G^2\phi^4}$ | $(\phi^\dagger\phi)^2 \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$ | $O^{(2)}_{G^2\phi^4}$ |
| | $(\phi^\dagger\phi)^2 W^I_{\mu\nu}W^{I\mu\nu}$ | $\mathcal{O}^{(1)}_{W^2\phi^4}$ | $(\phi^\dagger\phi)^2 \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$ | $\mathcal{O}^{(2)}_{W^2\phi^4}$ |
| | $(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)W^I_{\mu\nu}W^{J\mu\nu}$ | $\mathcal{O}^{(3)}_{W^2\phi^4}$ | $(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)\widetilde{W}^I_{\mu\nu}W^{J\mu\nu}$ | $\mathcal{O}^{(4)}_{W^2\phi^4}$ |
| | $(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)W^I_{\mu\nu}B^{\mu\nu}$ | $\mathcal{O}^{(1)}_{WB\phi^4}$ | $(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$ | $\mathcal{O}^{(2)}_{WB\phi^4}$ |
| | $(\phi^\dagger\phi)^2 B_{\mu\nu}B^{\mu\nu}$ | $\mathcal{O}^{(1)}_{B^2\phi^4}$ | $(\phi^\dagger\phi)^2 \widetilde{B}_{\mu\nu}B^{\mu\nu}$ | $\mathcal{O}^{(2)}_{B^2\phi^4}$ |
| $X\phi^2D^4$ | $i(D_\nu\phi^\dagger\sigma^I D^2\phi - D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$ | $\mathcal{O}^{(1)}_{W\phi^2D^4}$ | $(D_\nu\phi^\dagger\sigma^I D^2\phi + D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$ | $\mathcal{O}^{(2)}_{W\phi^2D^4}$ |
| | $i(D_\rho D_\nu\phi^\dagger\sigma^I D^\rho\phi - D^\rho\phi^\dagger\sigma^I D_\rho D_\nu\phi)D_\mu W^{I\mu\nu}$ | $\mathcal{O}^{(3)}_{W\phi^2D^4}$ | | |
| | $i(D_\nu\phi^\dagger D^2\phi - D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$ | $\mathcal{O}^{(1)}_{B\phi^2D^4}$ | $(D_\nu\phi^\dagger D^2\phi + D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$ | $\mathcal{O}^{(2)}_{B\phi^2D^4}$ |
| | $i(D_\rho D_\nu\phi^\dagger D^\rho\phi - D^\rho\phi^\dagger D_\rho D_\nu\phi)D_\mu B^{\mu\nu}$ | $\mathcal{O}^{(3)}_{B\phi^2D^4}$ | | |
| $X\phi^4D^2$ | $\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W^I_{\mu\nu}$ | $\mathcal{O}^{(1)}_{W\phi^4D^2}$ | $\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)\widetilde{W}^I_{\mu\nu}$ | $\mathcal{O}^{(2)}_{W\phi^4D^2}$ |
| | $\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)W^K_{\mu\nu}$ | $\mathcal{O}^{(3)}_{W\phi^4D^2}$ | $\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)\widetilde{W}^K_{\mu\nu}$ | $\mathcal{O}^{(4)}_{W\phi^4D^2}$ |
| | $(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger\sigma^I\phi + \text{h.c.})$ | $\mathcal{O}^{(5)}_{W\phi^4D^2}$ | $(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger \mathrm{i}\sigma^I\phi + \text{h.c.})$ | $\mathcal{O}^{(6)}_{W\phi^4D^2}$ |
| | $\epsilon^{IJK}(D_\mu\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J D_\nu\phi)W^{K\mu\nu}$ | $\mathcal{O}^{(7)}_{W\phi^4D^2}$ | $\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\nu}$ | $\mathcal{O}^{(1)}_{B\phi^4D^2}$ |
| | $\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)\widetilde{B}_{\mu\nu}$ | $\mathcal{O}^{(2)}_{B\phi^4D^2}$ | $(\phi^\dagger\phi)D_\nu B^{\mu\nu}(D_\mu\phi^\dagger \mathrm{i}\phi + \text{h.c.})$ | $\mathcal{O}^{(3)}_{B\phi^4D^2}$ |

# Results and applications

## Reduction of Lagrangian to a physical basis (I)

Functional methods perform matching by integrating out the heavy fields. A basis is not needed in this case but can be very useful.

For example, if we extend the SM with a real vector triplet $\mathcal{W} \sim (1,3)_0$, where

$$\mathcal{L}_{NP} = \frac{1}{2}\Big[D_\mu \mathcal{W}_\nu^\dagger D^\nu \mathcal{W}^\mu - D_\mu \mathcal{W}_\nu^\dagger D^\mu \mathcal{W}^\nu + m_{\mathcal{W}}^2 \mathcal{W}_\mu^\dagger \mathcal{W}^\mu + (g_{\mathcal{W}}^\phi \mathcal{W}^\mu \phi^{I\dagger} \sigma^I i D_\mu \phi + \text{h.c.})\Big]$$

and we want to integrate it out to the SM scale.

# Applications

## Reduction of Lagrangian to a physical basis (II)

Matching with functional methods yields the following output:

$$\begin{aligned}
\mathcal{L}_{\mathsf{EFT}}^{(8)} = \frac{(g_{\mathcal{W}}^{\phi})^2}{m_{\mathcal{W}}^4}\Big[& 2(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + 4(D_\nu\phi^\dagger D^\nu D^\mu\phi)(D_\mu\phi^\dagger\phi) \\
&- 2(D_\mu\phi^\dagger D_\nu\phi)(\phi^\dagger D^\mu D^\nu\phi) - 4(D_\mu\phi^\dagger\phi)(D^\mu D_\nu\phi^\dagger D^\nu\phi) \\
&+ 2(D_\mu\phi^\dagger D_\nu\phi)(D^\mu D^\nu\phi^\dagger\phi) - 4(D_\mu\phi^\dagger D^\mu\phi)(D_\nu\phi^\dagger D^\nu\phi) \\
&+ 2(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + \frac{1}{2}(\phi^\dagger D_\mu D_\nu\phi)(\phi^\dagger D^\mu D^\nu\phi) \\
&- 2(D_\nu D_\rho\phi^\dagger D^\nu D^\rho\phi)(\phi^\dagger\phi) + (D_\mu D_\nu\phi^\dagger\phi)(\phi^\dagger D^\mu D^\nu\phi) \\
&- 4(\phi^\dagger D_\rho\phi)(D_\nu\phi^\dagger D^\rho D^\nu\phi) + 2(\phi^\dagger D_\nu D_\mu\phi)(D^\mu\phi^\dagger D^\nu\phi) \\
&+ \frac{1}{2}(D_\mu D_\nu\phi^\dagger\phi)(D^\mu D^\nu\phi^\dagger\phi) + 4(D_\rho D_\nu\phi^\dagger D^\rho\phi)(D^\nu\phi^\dagger\phi) \\
&- 2(D_\nu D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi) - \frac{1}{2}(\phi^\dagger D_\nu D_\mu\phi)(\phi^\dagger D^\mu D^\nu\phi) \\
&+ 2(D_\rho D_\nu\phi^\dagger D^\nu D^\rho\phi)(\phi^\dagger\phi) - (D^\nu D^\mu\phi^\dagger\phi)(\phi^\dagger D_\mu D_\nu\phi) - \frac{1}{2}(D_\nu D_\mu\phi^\dagger\phi)(D^\mu D^\nu\phi^\dagger\phi)\Big].
\end{aligned}$$

# Applications

## Reduction of Lagrangian to a physical basis (III)

By using `matchmakereft` [2112.10787] Carmona et al. 2021 we can match $\mathcal{L}_{\mathsf{EFT}}^{(8)}$ at tree-level to the SMEFT where we embedded our Green's basis.

$$\mathcal{L}_{\mathsf{EFT}}^{(8)} = \frac{(g_{\mathcal{W}}^{\phi})^2}{m_{\mathcal{W}}^4}\left[2\mathcal{O}_{\phi^4}^{(1)} + 2\mathcal{O}_{\phi^4}^{(2)} - 4\mathcal{O}_{\phi^4}^{(3)} - \frac{1}{4}g_2^2\mathcal{O}_{W^2\phi^4}^{(1)} + \frac{1}{2}g_1 g_2\mathcal{O}_{WB\phi^4}^{(1)} + \frac{3}{4}g_1^2\mathcal{O}_{B^2\phi^4}^{(1)} - 2g_2\mathcal{O}_{W\phi^4D^2}^{(1)} + 6g_1\mathcal{O}_{B\phi^4D^2}^{(1)} + 2g_1\mathcal{O}_{B\phi^4D^2}^{(3)}\right].$$

We get an equivalent Lagrangian with simpler and fewer operators.

# Conclusions

## A Green's Basis for the Bosonic SMEFT to Dimension 8

- The SMEFT extends the SM with **all possible operators**, but they are related through algebraic relations, IBP or Equations of Motion.
- We have presented a Green's basis for bosonic interactions up to dimension 8. Computation and algebraic manipulation was simplified by **working in momentum space**.
- The Green's basis operators are explicitly translated to a physical basis via the Equations of Motion. **Renormalization** can also be achieved.
- This basis is essential for **off-shell matching methods**, and can be extended for fermionic operators.

# Conclusions

## A Green's Basis for the Bosonic SMEFT to Dimension 8

# Thanks for your attendance!

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*[2112.12724] Chala, Díaz-Carmona, and Guedes 2022*.

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# APPENDIX

# Removing redundancies

## Other transformations and identities

By using algebraic transformations and other relations, the redundant operators are expressed in terms of physical operators:

$$[D_\mu, D_\nu]\,\phi = -i\frac{g_1}{2}B_{\mu\nu}\phi - i\frac{g_2}{2}\sigma^I W^I_{\mu\nu}\phi$$
$$[D_\mu, D_\nu]\,W^{I\rho\lambda} = g_2\epsilon^{IJK}W^J_{\mu\nu}W^{K\rho\lambda}$$
$$[D_\mu, D_\nu]\,G^{A\rho\lambda} = g_3 f^{ABC}G^B_{\mu\nu}G^{C\rho\lambda}$$

$$0 = \partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu} \qquad \text{Bianchi}$$
$$0 = D_\mu(\mathcal{O}_1\mathcal{O}_2) = D_\mu\mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_1 D_\mu\mathcal{O}_2 \qquad \text{IBP}$$

# Applications

## Integrating out a scalar singlet to one loop

SM extended with a heavy singlet scalar $\mathcal{S}$ with a $\mathbb{Z}_2$ symmetry such that $\mathcal{S} \to -\mathcal{S}$. The New Physics Lagrangian is:

$$\mathcal{L}_{NP} = \frac{1}{2}(D_\mu \mathcal{S})(D^\mu \mathcal{S}) - \frac{1}{2} m_{\mathcal{S}}^2 \mathcal{S}^2 - \lambda_{\mathcal{S}\phi} \mathcal{S}^2 \phi^\dagger \phi - \lambda_{\mathcal{S}} \mathcal{S}^4$$

and can be matched at 1L with our Green's basis just presented.

$$\frac{c_{\phi^6}^{(1)}}{\Lambda^4} = \frac{1}{1920\, m_{\mathcal{S}}^4 \pi^2} \lambda_{\mathcal{S}\phi}^2 \left(5\lambda_{\mathcal{S}\phi} - 8\lambda\right), \qquad \frac{c_{\phi^4}^{(3)}}{\Lambda^4} = \frac{1}{960\, m_{\mathcal{S}}^4 \pi^2} \lambda_{\mathcal{S}\phi}^2$$

# Applications

## Integrating out a scalar quadruplet to one loop

SM extended with a $SU(2)_L$ quadruplet scalar $\Theta$ with $Y = 1/2$ :

$$\mathcal{L}_{NP} = D_\mu\Theta^\dagger D^\mu\Theta - m_\Theta^2\Theta^\dagger\Theta - \lambda_\Theta(\phi^\dagger\sigma^I\phi)C^\alpha_{I\beta}\tilde{\phi}^\beta\epsilon_{\alpha\gamma}\Theta^\gamma + h.c.$$

and can be matched at 1L as well.

$$\frac{c^{(1)}_{B^4}}{\Lambda^4} = \frac{7g_1^4}{92160\, m_\Theta^4\, \pi^2}\,, \qquad \frac{c^{(1)}_{\phi^6}}{\Lambda^4} = \frac{|\lambda_\Theta|^2}{3\, m_\Theta^2} + \frac{-6440\, g_1^2\, |\lambda_\Theta|^2 + 103040\, |\lambda_\Theta|^2 \lambda}{80640\, m_\Theta^4\, \pi^2}\,,$$

$$\frac{c^{(2)}_{B^4}}{\Lambda^4} = \frac{g_1^4}{92160\, m_\Theta^4\, \pi^2}\,, \qquad \frac{c^{(2)}_{\phi^6}}{\Lambda^4} = -\frac{|\lambda_\Theta|^2}{2\, m_\Theta^2} + \frac{+3640\, g_1^2 |\lambda_\Theta|^2 - 655200\, |\lambda_\Theta|^2\, \lambda}{483840\, m_\Theta^4\, \pi^2}\,,$$

$$\frac{c^{(1)}_{\phi^4}}{\Lambda^4} = \frac{4480\, |\lambda_\Theta|^2 - 3g_1^4}{40320\, m_\Theta^4\, \pi^2}\,, \qquad \frac{c^{(2)}_{\phi^4}}{\Lambda^4} = \frac{3g_1^4 + 1120\, |\lambda_\Theta|^2}{40320\, m_\Theta^4\, \pi^2}\,, \qquad \frac{c^{(3)}_{\phi^4}}{\Lambda^4} = -\frac{|\lambda_\Theta|^2}{18\, m_\Theta^4\, \pi^2}\,,$$

$$\frac{c^{(1)}_{B^2\phi^4}}{\Lambda^4} = \frac{1960\, g_1^2 |\lambda_\Theta|^2 - 3g_1^6}{322560\, m_\Theta^4\, \pi^2}\,, \qquad \frac{c^{(1)}_{B\phi^4 D^2}}{\Lambda^4} = -\frac{g_1^5}{13440\, m_\Theta^4\, \pi^2}\,.$$

# Appendix

## Applications

- Embedded in `matchmakereft` [2112.10787] Carmona et al. 2021, this Green's basis can be applied to simplify redundant terms in a Lagrangian after insertion of the EoM.

$$\mathcal{L}_{NP} = \sum_{n=4}^{8} \sum_{i=1}^{\text{arbitrary}} a_i^{(n)} \mathcal{O}_i^{(n)} \equiv \sum_{n=4}^{8} \sum_{\mathcal{R}_j^{(n)} \in \text{GB}} c_j^{(n)} \mathcal{R}_j^{(n)}$$

- We can also use it to integrate out heavy fields in a UV theory and match the Lagrangian to the SMEFT in the IR.

# Appendix

## Subsequent lines of work

- Computing the anomalous dimension matrix. Bosonic sector has been computed in [2205.03301] Das Bakshi et al. 2022:

$$
\begin{array}{c}
\dot{c}^{(1)}_{W^2H^4} \\ \dot{c}^{(2)}_{W^2H^4} \\ \dot{c}^{(3)}_{W^2H^4} \\ \dot{c}^{(4)}_{W^2H^4}
\end{array}
\overset{\begin{array}{cccc} c^{(1)}_{BH^4D^2} & c^{(2)}_{BH^4D^2} & c^{(1)}_{WH^4D^2} & c^{(2)}_{WH^4D^2} \end{array}}{
\begin{pmatrix}
-\frac{g_1 g_2^2}{8} & 0 & \frac{g_1^2 g_2}{3} + \frac{41 g_2^3}{24} - 5 g_2 \lambda & 0 \\
0 & -\frac{g_1 g_2^2}{8} & 0 & \frac{g_1^2 g_2}{3} + \frac{15 g_2^3}{8} - 5 g_2 \lambda \\
-\frac{g_1 g_2^2}{8} & 0 & -\frac{5 g_1^2 g_2}{24} & 0 \\
0 & -\frac{g_1 g_2^2}{8} & 0 & -\frac{5 g_1^2 g_2}{24}
\end{pmatrix}}
$$

- Including fermionic operators to make a complete dimension 8 Green's basis.