5D perspectives on the QCD axion

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Strong CP problem

$$\mathcal{L}_{QCD} \supset \theta G\tilde{G}$$

CP-odd term

Basis independent: $\bar{\theta} = \theta + \arg(\det \mathcal{M}_q)$

Observable effect:

Neutron electric dipole moment $d_N \simeq (5 \times 10^{-16} e \cdot \mathrm{cm})\bar{\theta}$

$$|d_N| \lesssim 3 \times 10^{-26} e \cdot \mathrm{cm} \quad \Rightarrow \quad \bar{\theta} \lesssim 10^{-10}$$

Why is $\bar{\theta}$ so small?

$\bar{\theta}$ does not appear to be "anthropic"

Our Universe possible for $0 \lesssim \bar{\theta} \lesssim 0.1$ [Lee,Meissner,Olive,Shifman,Vonk: 2006.12321]

Dynamical Solution: PQ mechanism and axion

Axion Lagrangian: $$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \underbrace{\frac{a}{f_a}\frac{\alpha_s}{8\pi}}_{\text{dim 5 term}} G\tilde{G} + \frac{1}{4} a \underbrace{g_{a\gamma\gamma}}_{\text{diphoton coupling}} F\tilde{F} + \frac{1}{f_a} \underbrace{J^\mu \partial_\mu a}_{\text{axial current coupling}}$$

Axion mass: $$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i\int d^4x\, \langle 0|\mathrm{T}\left[\frac{1}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}(x),\ \frac{1}{32\pi^2}G^b_{\rho\sigma}\widetilde{G}^{b\rho\sigma}(0)\right]|0\rangle$$

topological susceptibility

$$\Rightarrow \quad m_a^2 f_a^2 \sim \underbrace{\tfrac{1}{8}}_{\text{light-quark contribution}}\Lambda^4_{\rm QCD}$$

[or precisely, $m_a = 5.70(7)\ \mu\mathrm{eV}\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)$]

[Cortona, Hardy, Pardy Vega, Villadoro, 1511.02867]

Axion quality: Gravitational violation of $\mathrm{U}(1)_{PQ}$ $\quad \frac{c_n}{M_P^{n-4}}\phi^n + h.c.$

Terms must be suppressed to very high order! $(c_n \sim 1, n \geq 10)$

[however, if only gravitational instantons $c_n \sim e^{-S} \longrightarrow \quad S \geq 200$]

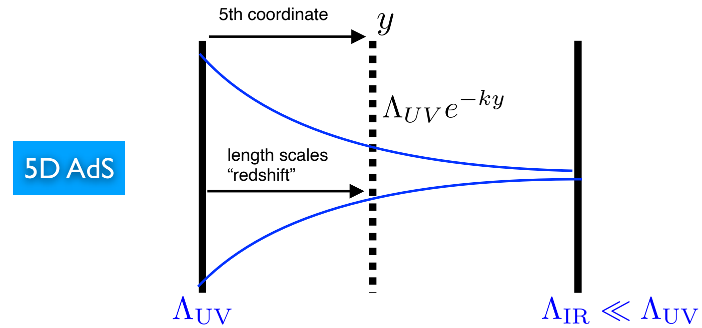
Questions

- Origin of the PQ breaking potential and spontaneous breaking scale f_a ?
- How to solve the axion quality problem?
- Can the QCD axion mass be different?

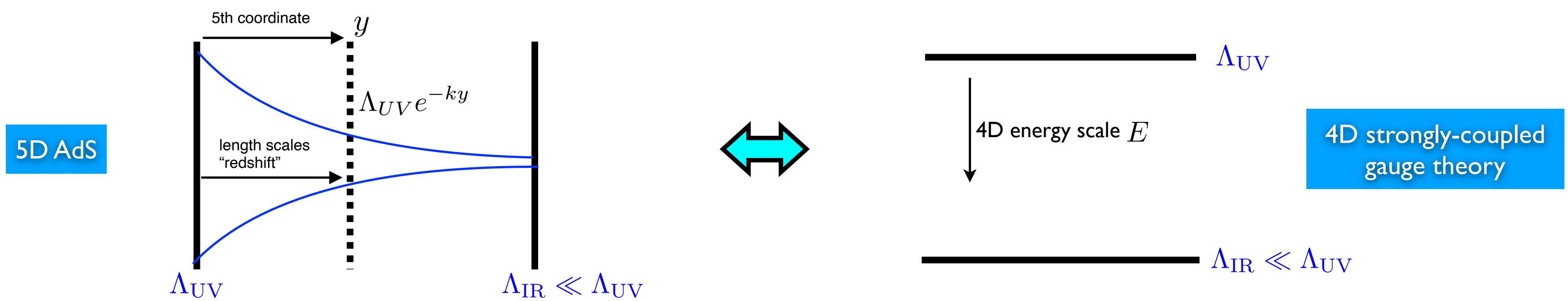
⇨ Use 5th dimension to address these questions!

Why use the 5th dimension?

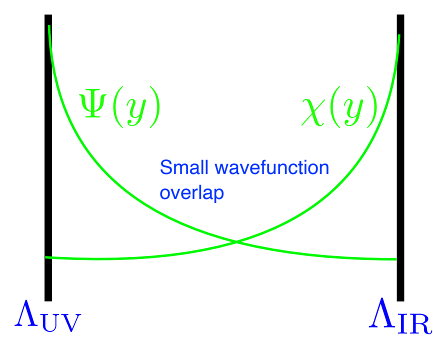
◆ Can generate a hierarchy of scales!



◆ "Warped" dimension is dual to 4D strong dynamics!



◆ Can generate small couplings!



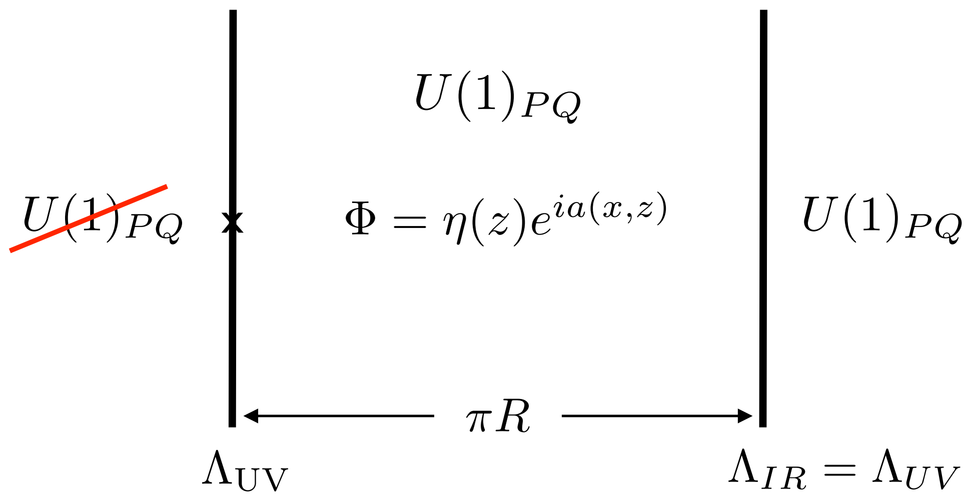
I. Axion Quality Problem [Cox, TG, Nguyen 1911.09385]

5D metric: $ds^2 = A^2(z)(dx^2 + dz^2) \equiv g_{MN}dx^M dx^N$ $\qquad A(z) = \frac{1}{kz}$

AdS curvature scale

"slice of AdS"

[Randall, Sundrum '99]
[Dienes, Dudas, TG '99]
[Choi '03; Flacke, Gripaios, March-Russell, Maybury '06]



Explains PQ scale

$m_\Phi^2 = \Delta(\Delta - 4)k^2$

"Bulk mass"

PQ symmetry breaking ⇒

$$\eta(z) = k^{3/2}(\lambda(kz)^{4-\Delta} + \sigma(kz)^{\Delta})$$

"Bulk VEV"

$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}(kz_{UV})^{\Delta-4},$$

"explicit" breaking

$$\sigma = \sqrt{v_{IR}^2 - \frac{\Delta}{2\lambda_{IR}}}(kz_{IR})^{-\Delta} \equiv \sigma_0\,(kz_{IR})^{-\Delta}$$

"spontaneous" breaking

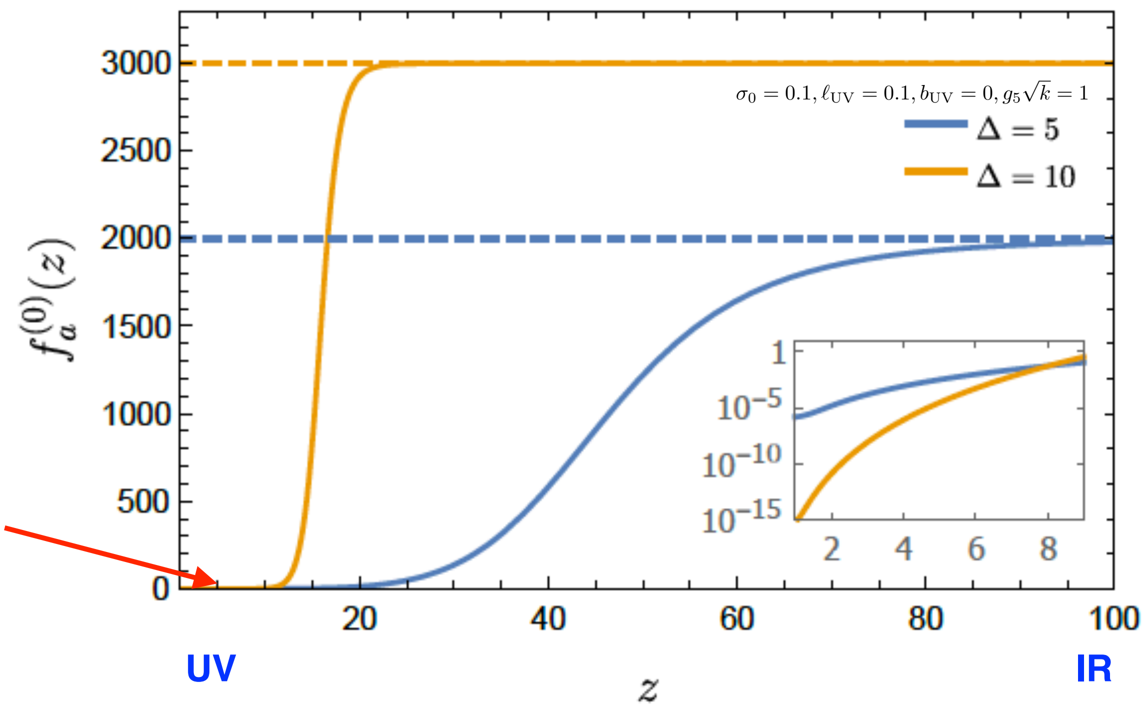
UV PQ-breaking $(\lambda \neq 0)$

$$U_{UV}(\Phi) \supset -2\ell_{UV} k^{5/2} \eta \cos(a) = -2\ell_{UV} k^{5/2} \eta \left(1 - \frac{1}{2}a^2 + \dots\right)$$

$(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq z_{IR} \frac{k^{3/2}}{\eta(z)} \sqrt{\Delta - 1} \left(\frac{z}{z_{IR}}\right)^{\Delta} \left[1 + \frac{2\lambda(\Delta-2)(k z_{UV})^{\Delta}(kz)^{2(2-\Delta)}}{\ell_{UV} + 2\sigma_0(\Delta-2)(z_{UV}/z_{IR})^{\Delta}}\right]$$

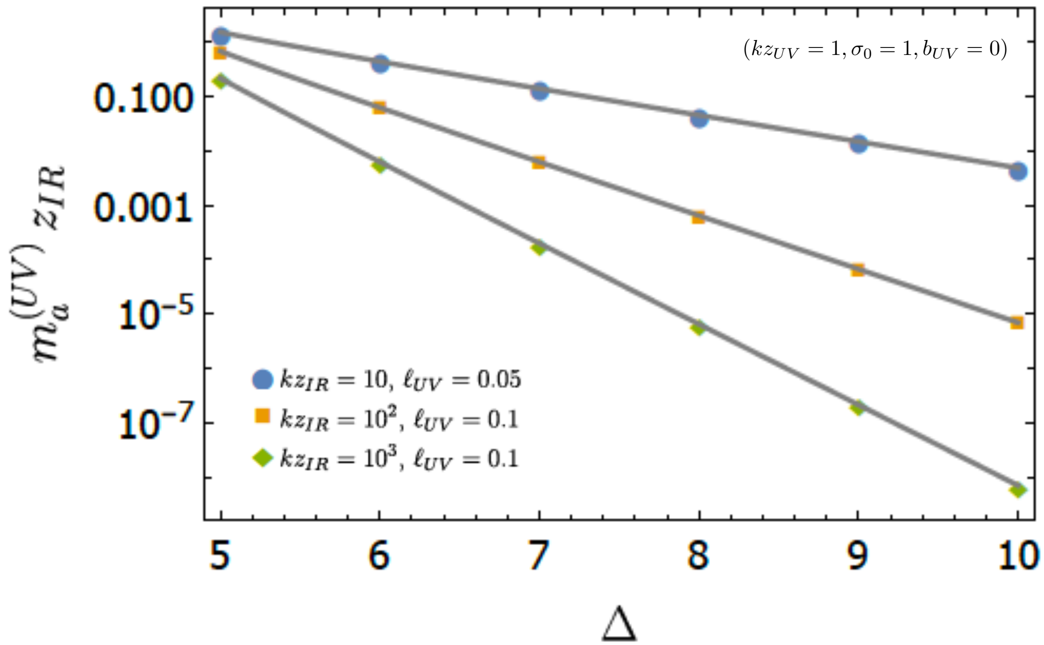
[Cox, TG, Nguyen 1911.09385]



Axion profile suppressed!

UV axion mass $(\lambda \neq 0)$

[Cox,TG, Nguyen 1911.09385]



UV axion mass suppressed for large Δ

Bulk Chern-Simons term: $-\frac{\kappa}{32\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^a_{NP} G^a_{QR}$ ← generates axion-gluon coupling

$$(m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})}\left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^{\Delta}\left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4} F_a^2$$

where
$$F_a \simeq \frac{1}{\kappa}\frac{\sigma_0}{\sqrt{\Delta-1}} z_{IR}^{-1}$$

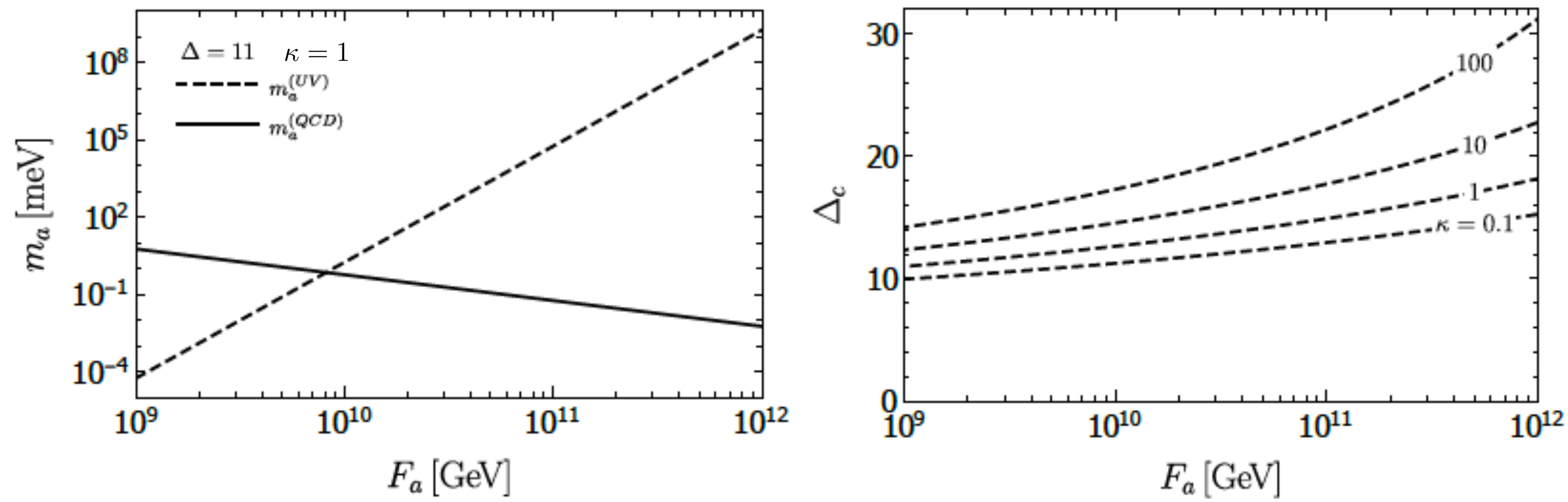
Axion potential:

$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - (m_a^{(UV)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \delta\right)$$

relative phase

Require: $(m_a^{(UV)})^2 \lesssim 10^{-10}\,(m_a^{(QCD)})^2$

[Cox, TG, Nguyen 1911.09385]



10^9 GeV $\lesssim F_a \lesssim 10^{12}$ GeV ⇨ $\Delta_c \gtrsim 10$ ← **Solves axion-quality problem!**

Holographic (AdS/CFT) interpretation:

5D		4D
local $U(1)_{PQ}$ symmetry	$\leftrightarrow$	global $U(1)_{PQ}$ symmetry
$\Phi,\ m_\Phi^2 = \Delta(\Delta-4)k^2$	$\leftrightarrow$	$\mathcal{O}$, dimension Δ
$\Lambda_{\rm UV} e^{-\pi k R} = \Lambda_{\rm IR}$	$\leftrightarrow$	$\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\frac{8\pi^2}{b_{CFT} g^2}}$
σ	$\leftrightarrow$	$\langle \mathcal{O} \rangle$
λ	$\leftrightarrow$	$\lambda \Phi \mathcal{O}$

Holographic interpretation:

5D axion,
local U(1) PQ symmetry ⟺ 4D composite axion, accidental global U(1) PQ symmetry

Example: [Gavela, Ibe, Quilez, Yanagida: 1812.08174]

New strong gauge group

	$SU(5)$	$SU(3)_c$
$\psi_{\bar{5}}$	$\bar{\mathbf{5}}$	$\mathbf{R}$
ψ_{10}	$\mathbf{10}$	$\mathbf{R}$

Global symmetries

	$SU(n)_{\bar{5}}$	$SU(n)_{10}$	$U(1)_{B-L} \equiv U(1)_{PQ}$
$\psi_{\bar{5}}$	$\square$	$\mathbf{1}$	-3
ψ_{10}	$\mathbf{1}$	$\square$	1

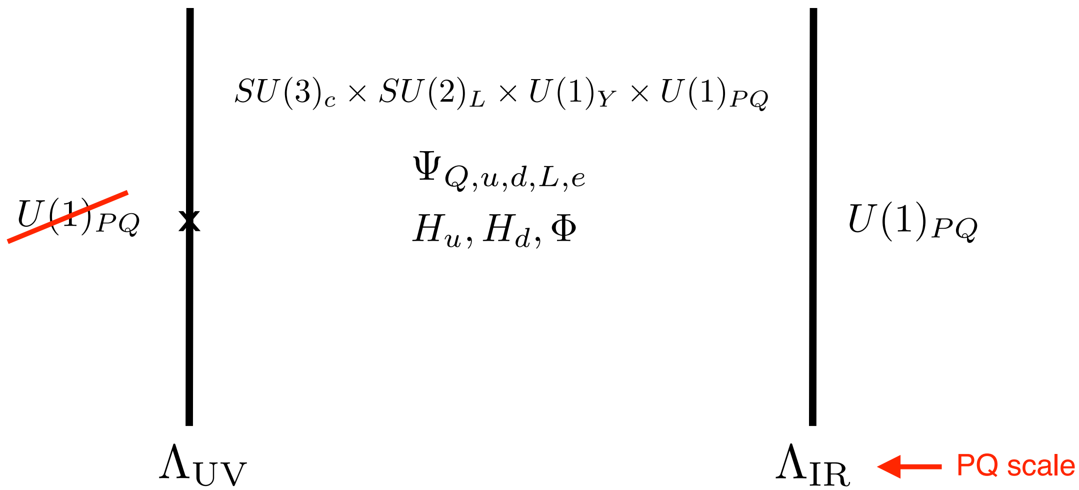
Chiral condensate: $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} \rangle \sim \Lambda_5^6 \quad \Longrightarrow \quad SU(n)_{\bar{5}} \times SU(n)_{10} \longrightarrow G \supset SU(3)_c$

PQ condensate: $\langle \bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10} \rangle \sim \Lambda_5^9$ ← dimension 9! (due to gauge invariance and chirality)

$$\Rightarrow \quad \mathcal{L}_{PQ} = c\,\frac{(4\pi)^2}{2!\,4!}\left(\frac{N}{5}\right)^9 \frac{f_a^9}{M_{\rm Pl}^5}\, e^{-i\frac{10}{N}a/f_a} + \text{h.c.}$$

Flavored Warped Axion [Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]

Consider DFSZ-like axion model with bulk Standard Model fermions:



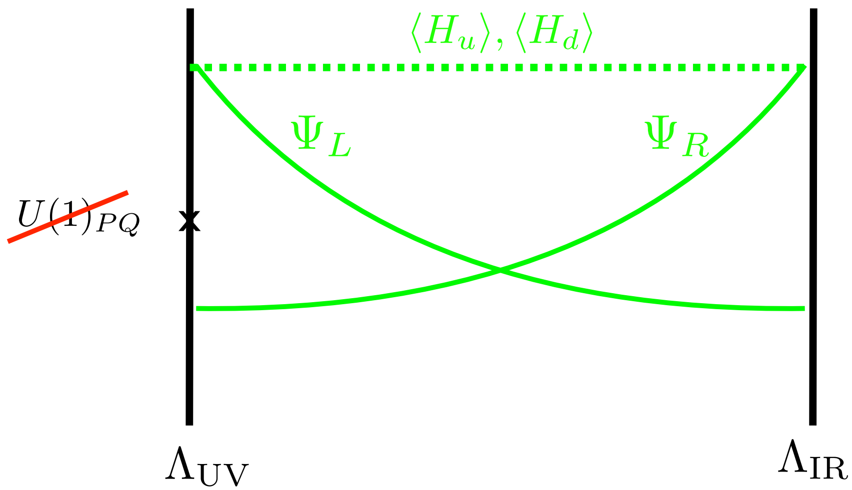
constant bulk Higgs vevs

Bulk VEVs: $H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{a_u(x,z)}{v_u}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{a_d(x,z)}{v_d}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \Phi = \eta(z) e^{ia(x,z)}$

Bulk Yukawa couplings: $-2\int_{z_{UV}}^{z_{IR}} d^5x \sqrt{-g}\, \frac{1}{\sqrt{k}} \left(y^{(5)}_{u,ij} \bar{Q}_i U_j H_u + y^{(5)}_{d,ij} \bar{Q}_i D_j H_d + y^{(5)}_{e,ij} \bar{L}_i E_j H_d + \text{h.c.} \right)$

Bulk fermion mass: $m_{\Psi_i} = c_{\Psi_i} k$

Bonus feature: explains fermion mass hierarchy [TG, Pomarol '00]



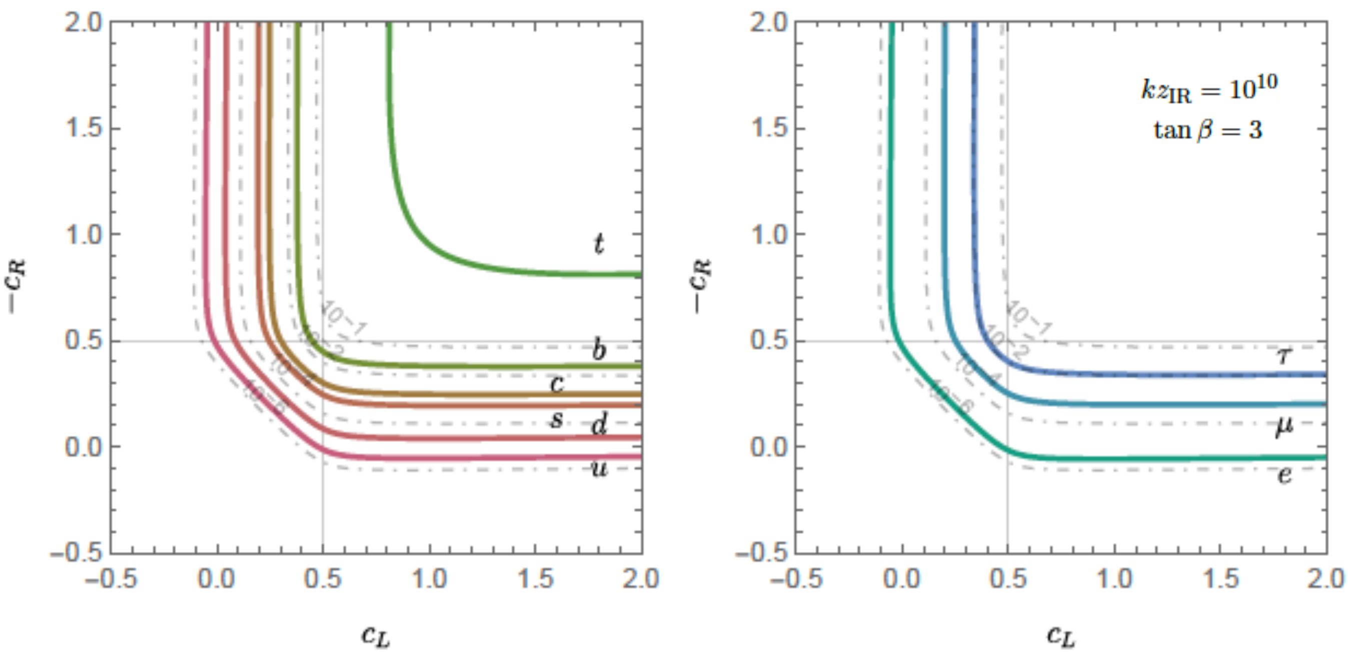
← PQ scale

$$m_u^{ij} = y_{u,ij}^{(5)} \frac{\sqrt{2} v_u}{\sqrt{k}} \int_{z_{UV}}^{z_{IR}} \frac{dz}{(kz)^5} \, f^0_{Q_{iL}}(z) f^0_{U_{jR}}(z)$$

$$f^0_{Q_{iL}}(z) = \mathcal{N}_{Q_i} (kz)^{2-c_{Q_i}}$$
$$f^0_{U_{iR}}(z) = \mathcal{N}_{U_i} (kz)^{2+c_{U_i}}$$

{ bulk fermion mass parameters

(similarly $m^{ij}_{d,e}$)



Axion-fermion couplings:

$$i\int d^4x\,\frac{\partial_\mu a^0}{2F_a}\left(\bar{u}_i\gamma^\mu\left((c_u^V)_{ij}-(c_u^A)_{ij}\gamma^5\right)u_j\right)$$

$$A_L^u m_u^{ij} A_R^{u\dagger} = m_{u_i}$$

(similarly for down-type quarks and leptons)

Overlap between axion and fermion profiles

$$(c_u^{V,A})_{ij} = X_{H_u}\int_{z_{UV}}^{z_{IR}}\frac{dz}{(kz)^4}\,g_a^0(z)\left((A_R^u)_{ik}(f^0_{U_kR})^2(A_R^{u\dagger})_{kj} \mp (A_L^u)_{ik}(f^0_{Q_kL})^2(A_L^{u\dagger})_{kj}\right)$$

Axion profile:

$$f_a^0(z) = \frac{1}{F_a}\left(1+g_a^0(z)\right)$$

z-dependent part of axion profile depends on Δ

SM fermion profiles:

$$f^0_{Q_iL}(z) = \mathcal{N}_{Q_i}(kz)^{2-c_{Q_i}}\,,$$
$$f^0_{U_iR}(z) = \mathcal{N}_{U_i}(kz)^{2+c_{U_i}}\,,$$
$$f^0_{D_iR}(z) = \mathcal{N}_{D_i}(kz)^{2+c_{D_i}}\,.$$

(similarly for leptons)

Parameter count:

9 $c_{Q_i}, c_{u_i}, c_{d_i}$ parameters - (6 quark masses + 2 CKM) = 1 free parameter $(c_{Q_3}+c_{u_3})$

6 c_{L_i}, c_{e_i} parameters - (3 charged lepton masses + 2 PMNS) = 1 free parameter $(c_{L_3}+c_{e_3})$

(assuming, for simplicity, PMNS generated in charged lepton sector)

{Majorana neutrino-axion model [Cox, TG, Nguyen: 2107.14018]}

Numerical results

Flavor-violating couplings:

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

$F^A_{u,d,\ell} \approx \mathcal{O}(F^V_{u,d,\ell})$

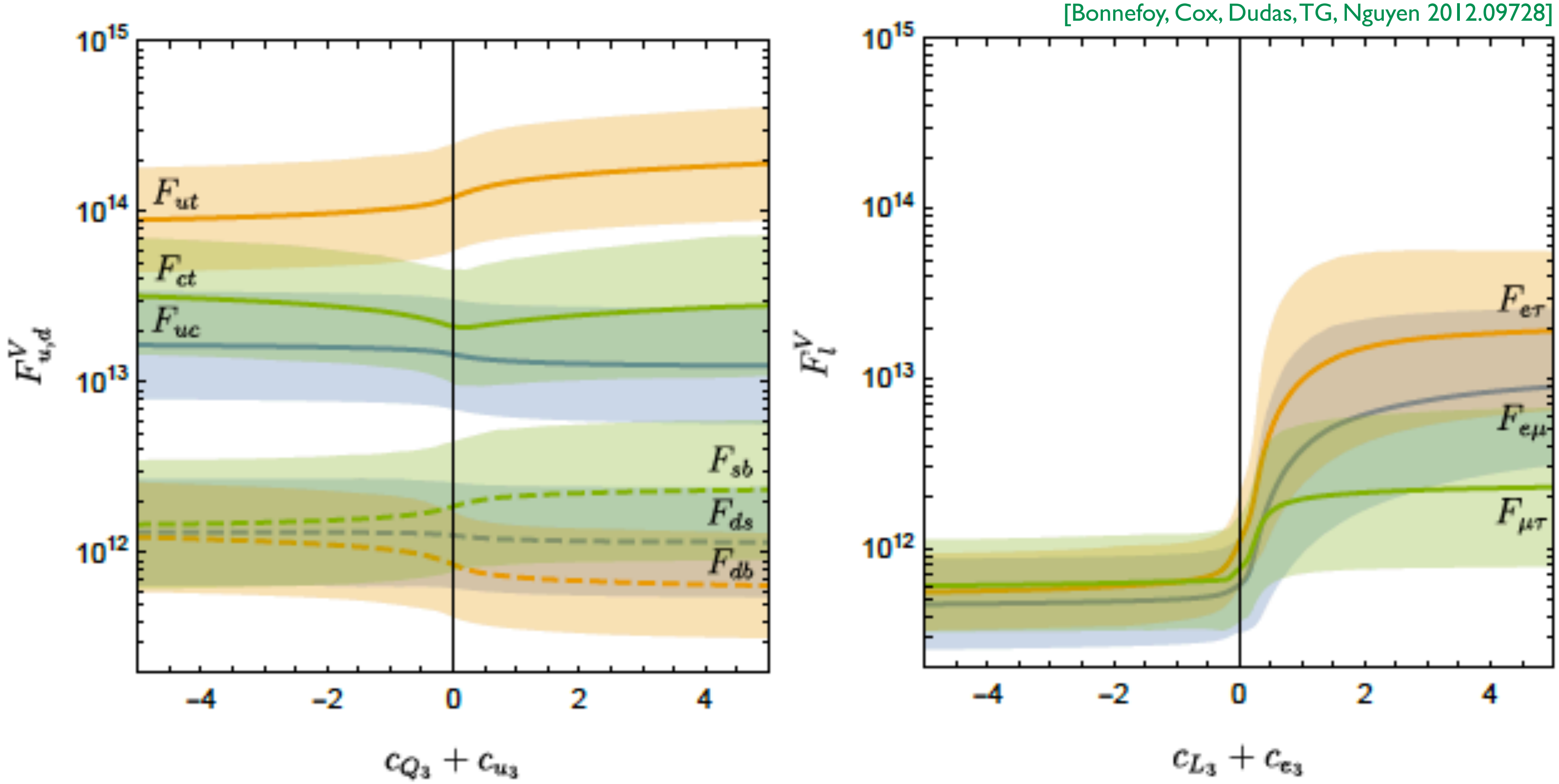
$kz_{\rm IR} = 10^{10}, g_5^2 k = 1$

$\Delta = 10 \quad \sigma_0 = 3$

$F_a \simeq 10^9\,{\rm GeV}.$

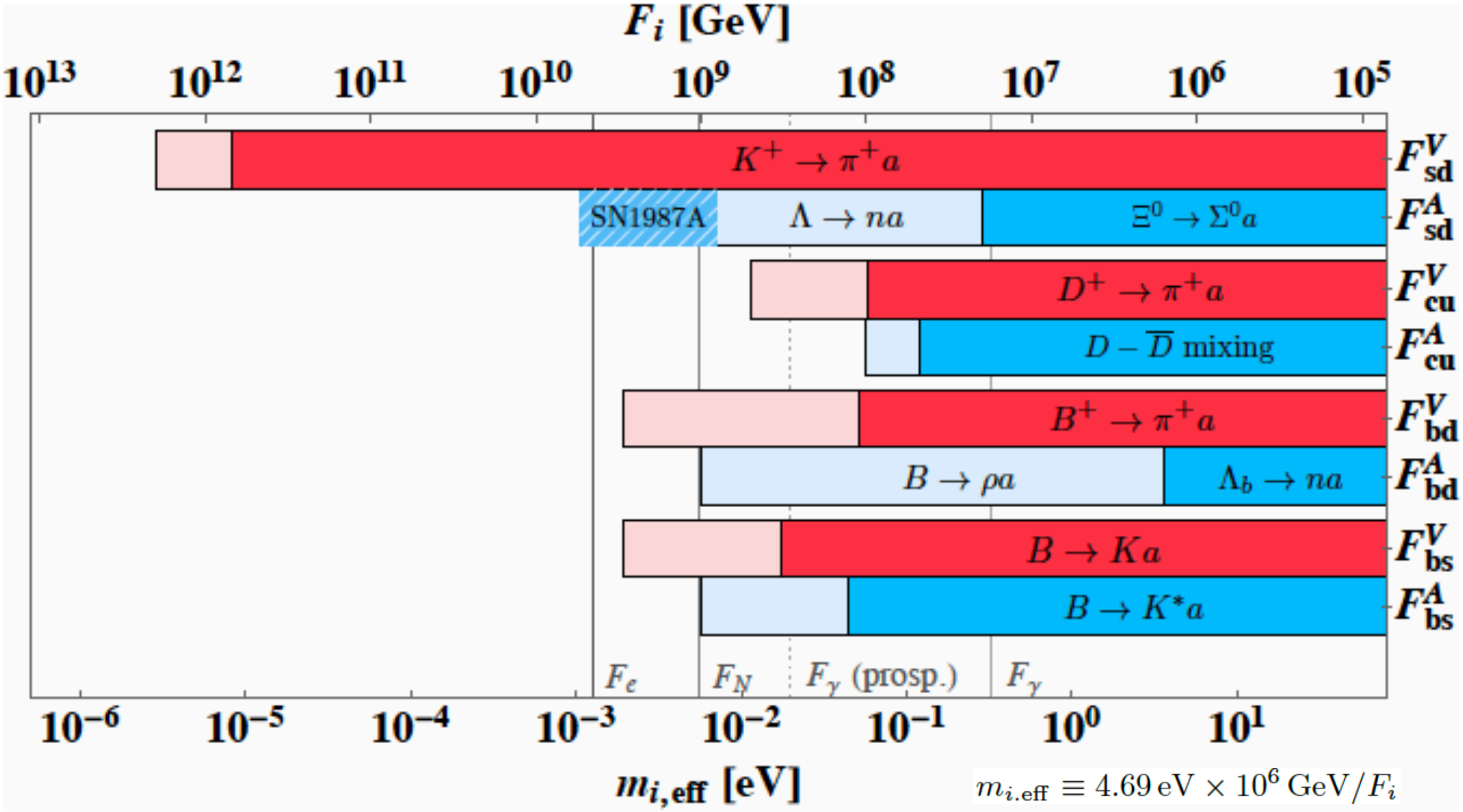
Scan over $y^{(5)}_{u,d,e} \sim 1$

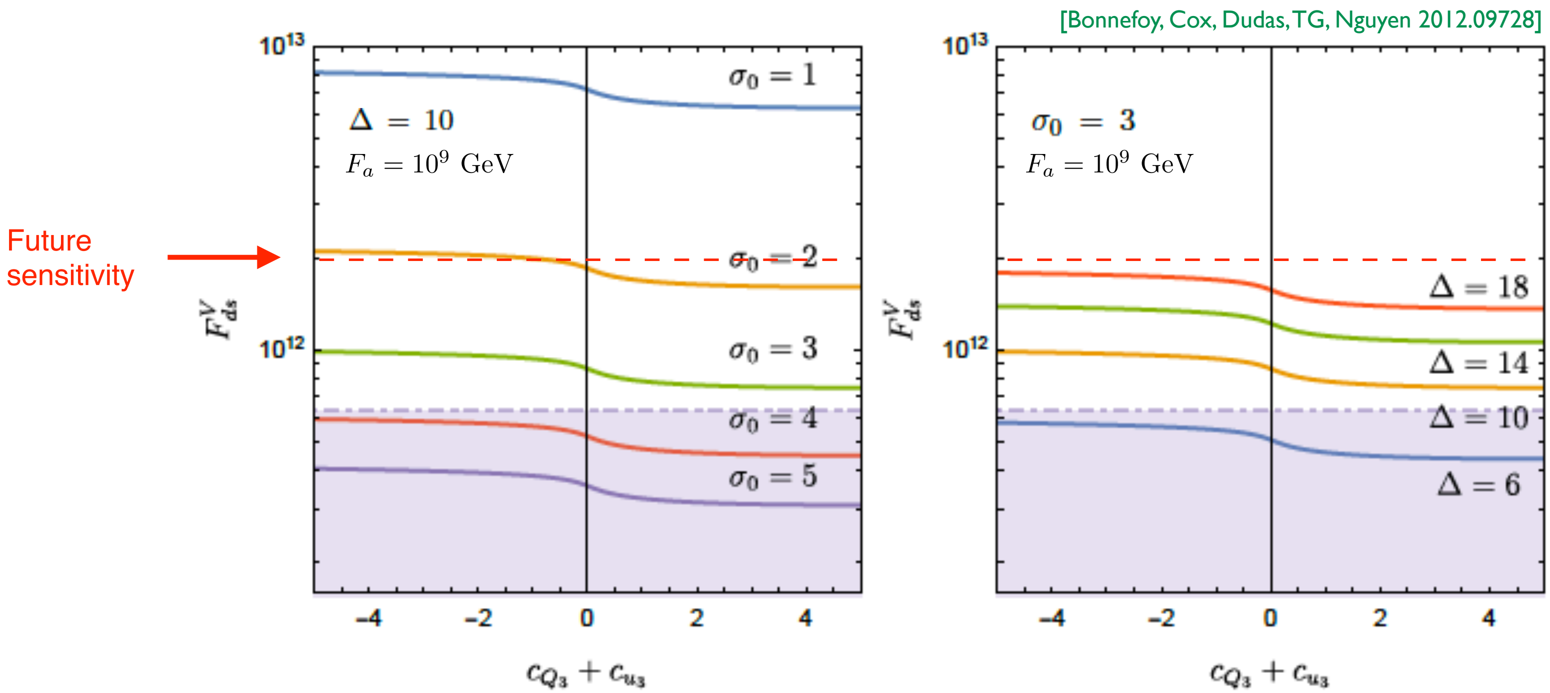
[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]



Limits on axion-quark couplings

[Martin Camalich, Pospelov, Vuong, Ziegler, Zupan: 2002.04623]





Experimental limits : $(F^V_d)_{12} \gtrsim 6.8 \times 10^{11}\,\text{GeV}$ $(K^+ \to \pi^+ a \text{ decays})$

[Martin Camalich, Pospelov, Vuong, Ziegler, Zupan 2002.04623]

⇨ $\sigma_0 \gtrsim 4, \quad \Delta \gtrsim 6$

2. Axion mass from 5D small instantons

[TG, Khoze, Pomarol, Shirman: 2001.05610]

QCD axion mass:

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i \int d^4x \, \langle 0 | \mathrm{T} \left[\frac{1}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}(x), \; \frac{1}{32\pi^2} G^b_{\rho\sigma} \widetilde{G}^{b\rho\sigma}(0) \right] |0\rangle$$

topological susceptibility

Dilute instanton gas approximation

$$\mathcal{T} \propto \int \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}}$$

QCD asymptotically free ⇒ $\mathcal{T} \propto \Lambda^4_{QCD}$ "Large instantons" $\rho \sim 1/\Lambda_{QCD}$

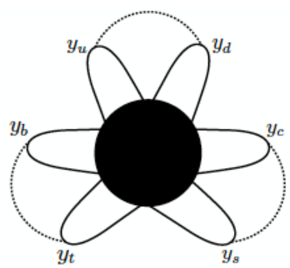
Fermion zero modes: $(\rho m_f)^{N_f} \longrightarrow$ suppression $\frac{\prod_f m_f}{\Lambda_{QCD}^{N_f}}$

⇒ $$m^2_{a,QCD} = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

How to enhance axion mass?

- Change QCD coupling in UV $\alpha_s(1/\rho) \sim 1$ "Small instantons" $\rho \sim 1/\Lambda_{UV}$

- Close fermion loops with Higgs boson



$$\kappa_f = \frac{y_u}{4\pi}\frac{y_d}{4\pi}\frac{y_c}{4\pi}\frac{y_s}{4\pi}\frac{y_t}{4\pi}\frac{y_b}{4\pi} \approx 10^{-23} \qquad \left(\text{otherwise } \frac{m_u m_d m_c m_s m_b m_t}{\Lambda_{UV}^6}\right)$$

$$\Rightarrow \quad \boxed{m_a^2 f_a^2 \sim \frac{1}{8}\Lambda_{\rm QCD}^4 + \Lambda_I^4} \qquad \text{where } \Lambda_I \gg \Lambda_{\rm QCD}$$

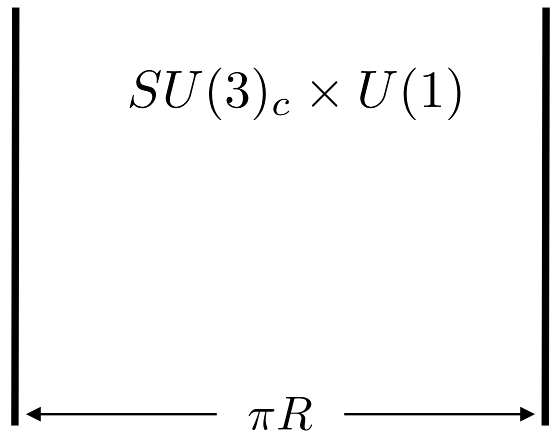
new contribution

Use 5th dimension to make QCD axion heavy!

QCD in 5D

Flat space 5D metric: $ds^2 = dx^2 + dy^2$

$$S_5 = -\int d^4x \int_0^L dy \left(\frac{1}{4g_5^2} \mathrm{Tr}[G_{MN}^2] + \frac{b_{CS}}{32\pi^2} \varepsilon^{MNRST} B_M \mathrm{Tr}[G_{NR} G_{ST}] + \frac{1}{4g_5^2} F_{MN}^2 + \dots \right),$$



$A_\mu^{(0)}(x)$ QCD gluon

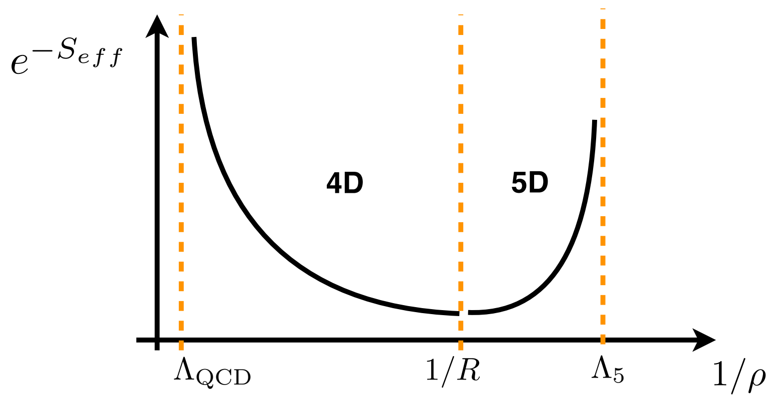
$B_5^{(0)}(x) \equiv a(x)$ QCD axion

5D instanton: $A_\mu^a(x,y) = A_\mu^{(I)a}(x) = \frac{2\eta_{a\mu\nu}(x-x_0)_\nu}{(x-x_0)^2+\rho^2}, \quad A_5^a(x,y) = 0$

$$S_5^{(I)} = \frac{8\pi^3 R}{g_5^2} = \frac{2\pi}{\alpha_s}$$ Finite action

5D small instantons

Fluctuations + Kaluza-Klein contributions



small instantons!

$$\int_{1/\Lambda_5}^{R} \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 e^{-S_{\rm eff}} \equiv \frac{K}{R^4} \quad \propto m_a^2 f_a^2$$

$$\Rightarrow \quad S_{\rm eff} = \frac{2\pi}{\alpha_s(1/R)} - 3\xi(R/\rho)\frac{R}{\rho} + b_0 \ln\frac{R}{\rho}$$

power law term!

$\xi(R/\rho) \sim 1/3$

$R/\rho \gtrsim 20$

$$\Rightarrow \quad K \simeq C[3] \left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 (\Lambda_5 R)^{3-b_0} e^{-\frac{2\pi}{\alpha_s(1/R)} + \Lambda_5 R}$$

power law contribution can overcome suppression

Valid up to $\frac{g_5^2 \Lambda_5}{24\pi^3} \sim 1$ or $\Lambda_5 R \lesssim \frac{6\pi}{\alpha_s}$

Axion mass from 5D small instantons

Assume boundary Standard Model fermions $(b_0 = 7)$ and QCD in bulk

Yukawa coupling suppression from Higgs loops

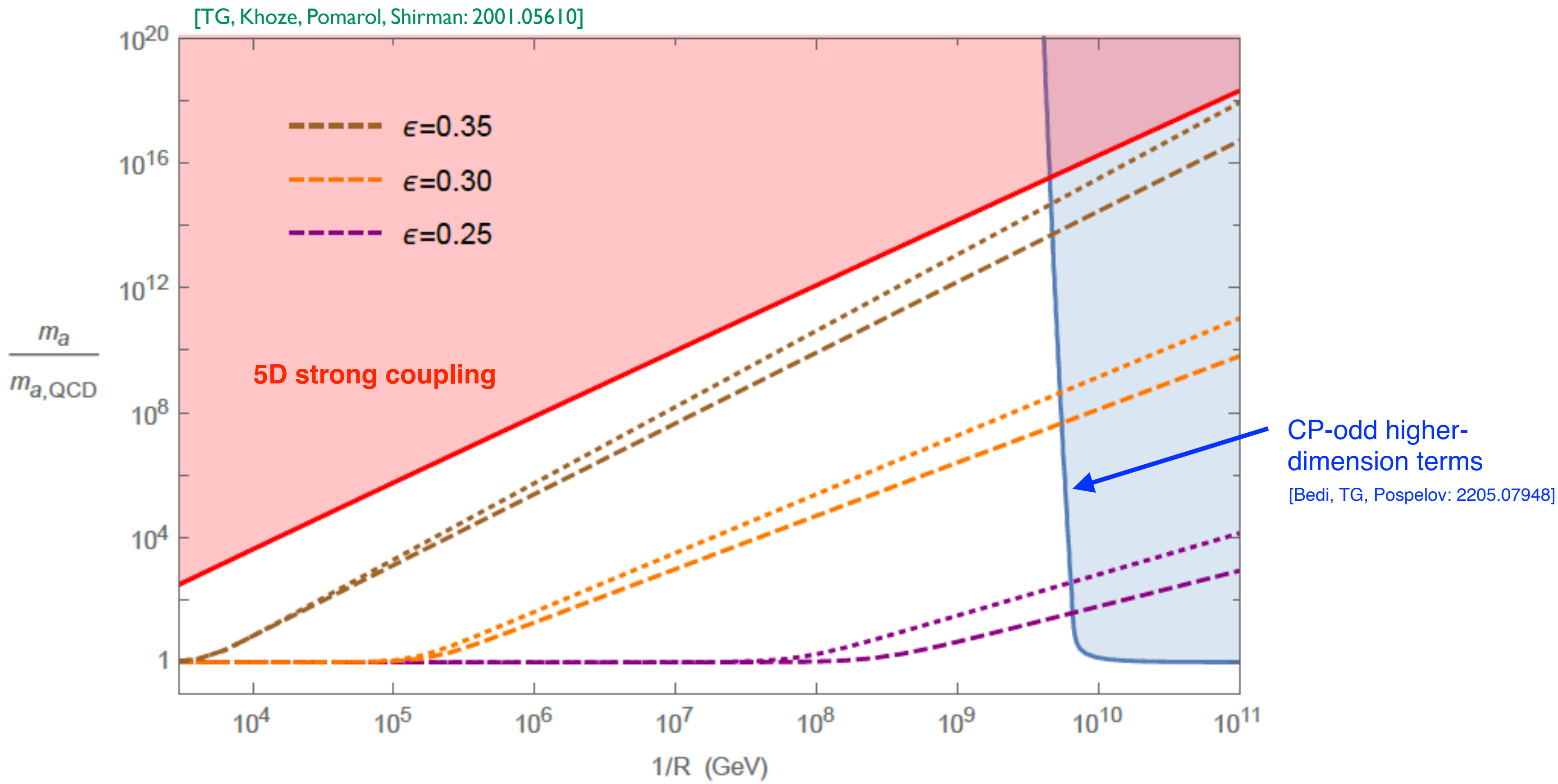
5D enhancement

$$\frac{m_a}{m_{a,QCD}} \simeq \sqrt{2\kappa_f C[3]} \left(\frac{2\pi}{\alpha_s(1/R)}\right)^3 \frac{(m_u+m_d)}{\sqrt{m_u m_d}} \frac{1}{m_\pi f_\pi R^2} \frac{e^{-\frac{1}{2}\left(\frac{2\pi}{\alpha_s(1/R)} - \Lambda_5 R\right)}}{(\Lambda_5 R)^{\frac{1}{2}(b_0-3)}}$$

Write $\quad \Lambda_5 R = \frac{6\pi\varepsilon}{\alpha_s(1/R)} \quad$ where $\varepsilon \lesssim 1$ (perturbativity limit)

Positive exponent: $\quad \frac{2\pi}{\alpha_s(1/R)} - \Lambda_5 R < 0 \quad \Rightarrow \quad \varepsilon \gtrsim 0.14$

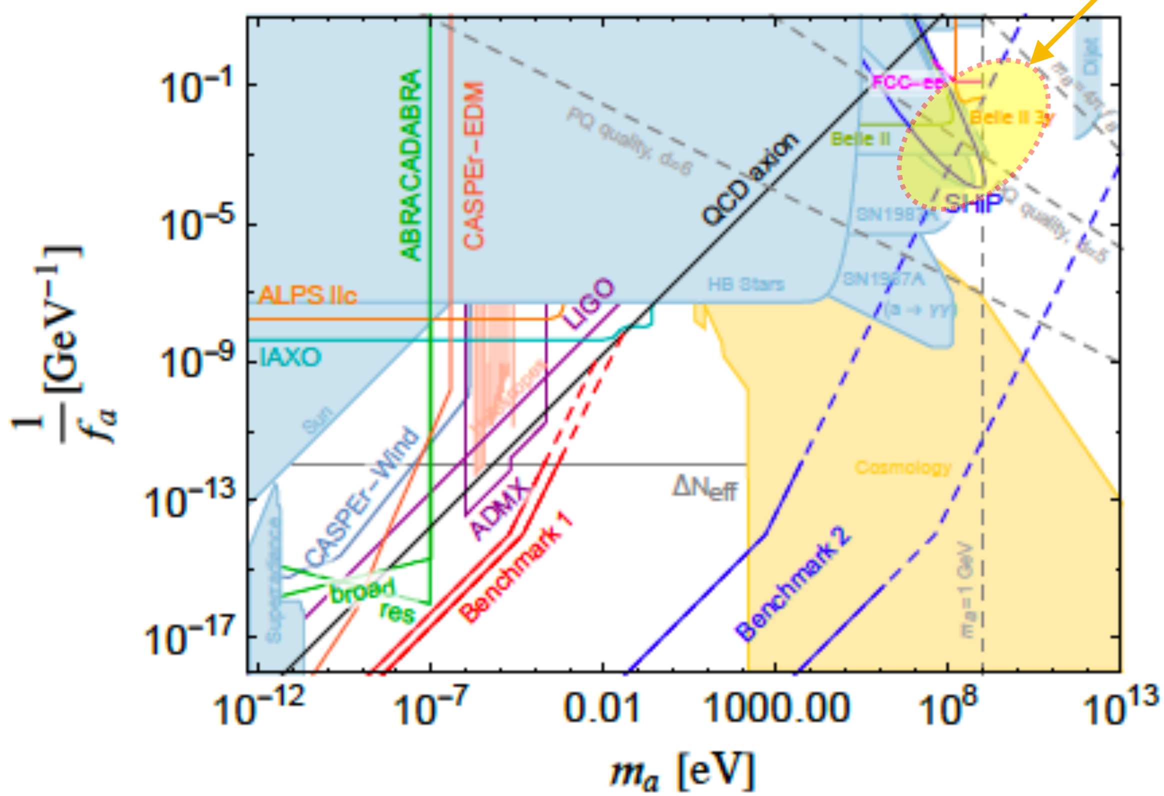
Maximum axion mass enhancement: $\quad m^2_{a,5f} \sim \kappa_f \frac{\Lambda_5^4}{f^2}$



⇨ Small 5D instantons can dominate for $\frac{1}{R} \gtrsim 100\ \text{TeV}$

Heavy Axion Limits

[Agrawal, Howe: 1710.04213]



Favoured by axiogenesis!

[Co, TG, Harigaya, to appear]

Questions/Future Work

- Generalize to z-dependent bulk Higgs VEVs
  — could enhance specific axion-fermion couplings
- Construct 4D dual models with $\Delta \geq 10$
- Dark matter ALPs with axion-fermion couplings?
- Heavy axion axiogenesis → K. Harigaya talk
  — solves CP problem and baryon asymmetry *[Co, TG, Harigaya, to appear]*
- Small instantons in weakly-gauged holographic models

  [TG, Pomarol: 2110.01762] $A_\mu^a(x,z) = 2\eta^a_{\mu\nu}\frac{x_\nu}{x^2}\frac{(x^2+z^2)^2}{x^2\rho^2+(x^2+z^2)^2}$ New "localized" instanton anti-instanton solution!

  — other solutions that give axion mass enhancement?
- ……..

Summary

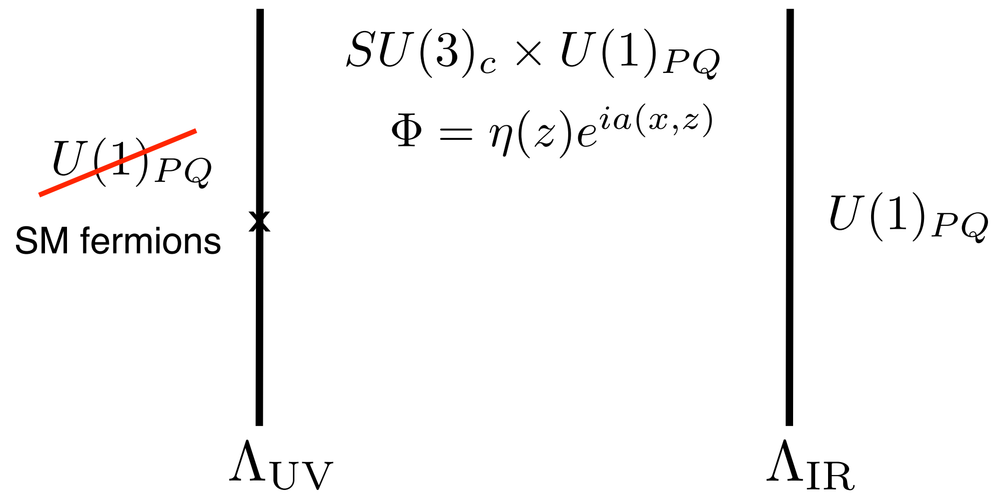
- Axion quality problem can be solved in 5D warped dimension
  - *— dual to 4D dynamical axion with accidental PQ symmetry*

- "Flavored" warped axion
  - *— solves axion quality and explains fermion mass hierarchy*
  - *— predicts flavor-violating axion-fermion couplings*
  - *— light sterile neutrinos*

- 5D small instantons
  - *— can enhance axion mass and not spoil strong CP solution*
  - *— axion mass could be a sensitive probe of UV physics!*

Extra Slides

Axion-Gluon Coupling



Bulk Chern-Simons term: $-\frac{\kappa}{32\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^a_{NP} G^a_{QR}$ ← **generates axion-gluon coupling**

Under 5D gauge transformation: $V_M \to V_M + \partial_M \alpha$

$$\delta S = -\frac{\kappa}{32\pi^2}\left[\int d^4x\, \alpha(x^\mu, z)\, \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}\right]_{z_{UV}}^{z_{IR}}$$

Add IR boundary term: $$\frac{\kappa}{32\pi^2}\int d^4x\, a\, G\tilde{G}\Big|_{z_{IR}}$$

Obtain: $$S_{eff} = \int d^4x \left(\frac{1}{2}a^{(0)}\left(\Box - m_a^2\right)a^{(0)} + \frac{g_s^2}{32\pi^2 F_a}a^{(0)}G\tilde{G}\right)$$

where $$F_a \simeq \frac{1}{\kappa}\frac{\sigma_0}{\sqrt{\Delta-1}}z_{IR}^{-1}$$

$$\Rightarrow \quad (m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})}\left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^{\Delta}\left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4}F_a^2$$

(suppression for $F_a \ll \Lambda_{UV}$ and $\Delta > 4$)

Numerical results

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

$kz_{\rm IR} = 10^{10}, g_5^2 k = 1$

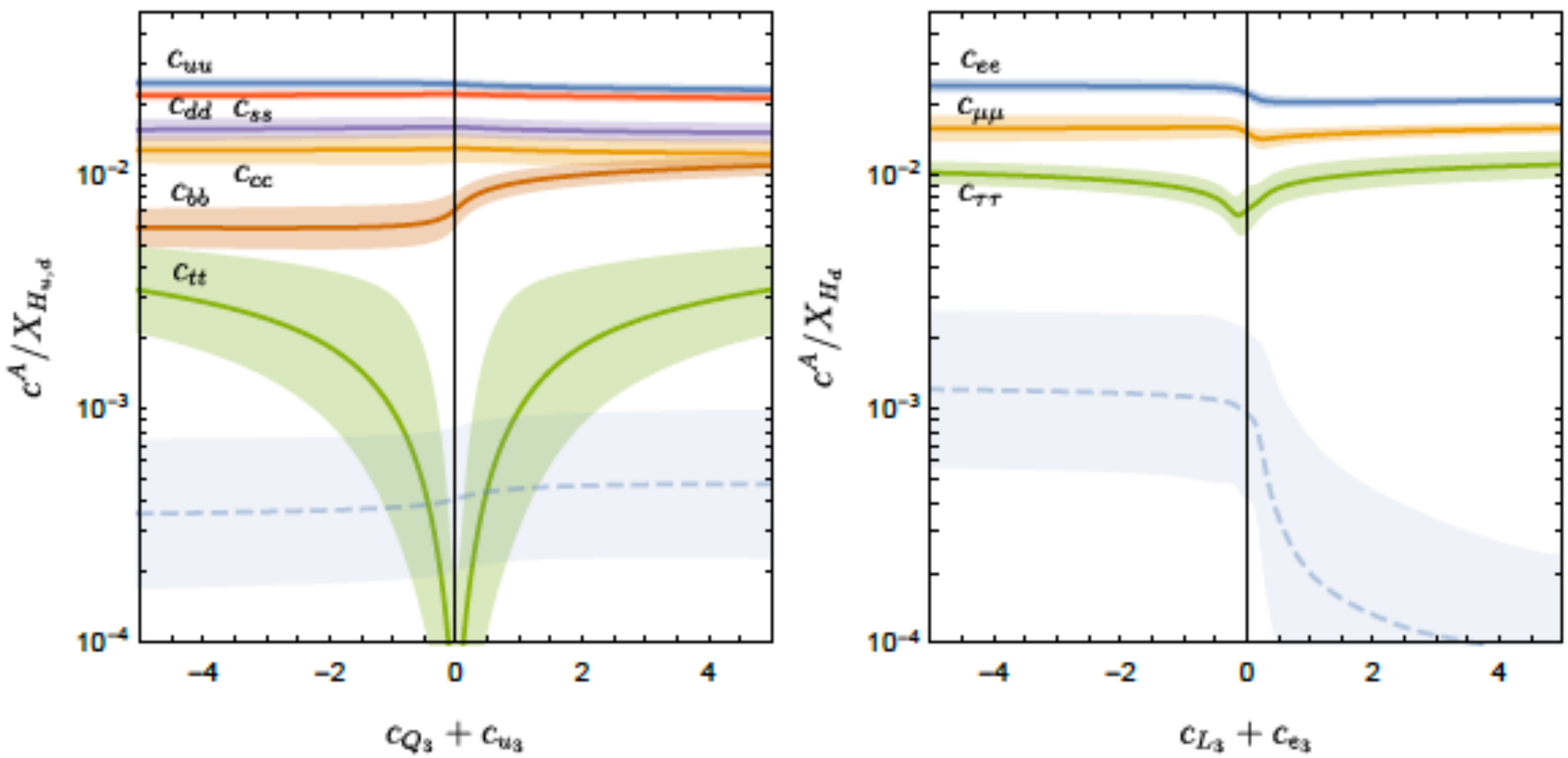
$\Delta = 10 \quad \sigma_0 = 3$

$F_a \simeq 10^9$ GeV.

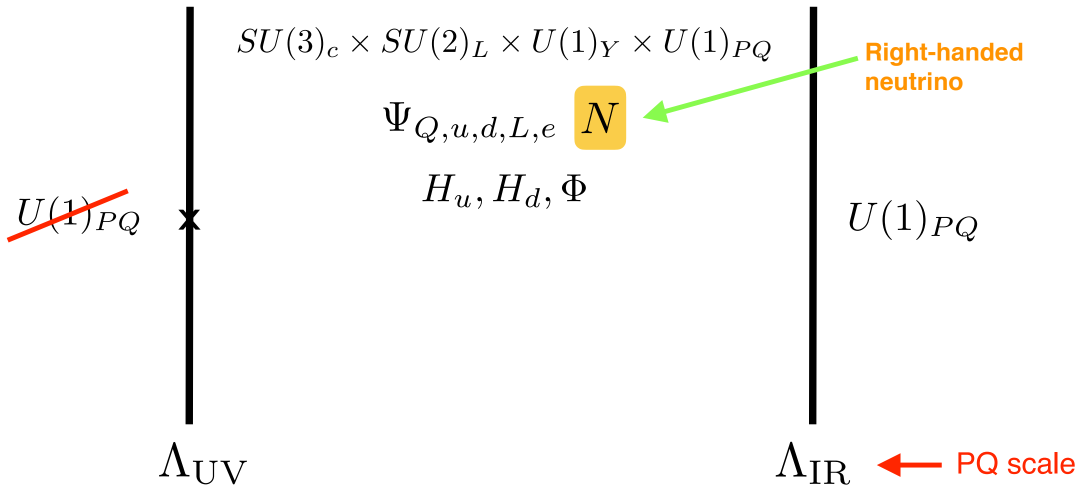
Scan over $y^{(5)}_{u,d,e} \sim 1$

Flavor-preserving couplings:

[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]



Neutrino-Axion Model [Cox, TG, Nguyen: 2107.14018]



Bulk Yukawa coupling: $$\frac{1}{\sqrt{k}}\left(y^{(5)}_{\nu,ij}\overline{L_i}N_jH_u + y^{(5)}_{e,ij}\overline{L_i}E_jH_d + \text{h.c.}\right)$$ ← PQ charge of N forbids bulk Majorana terms

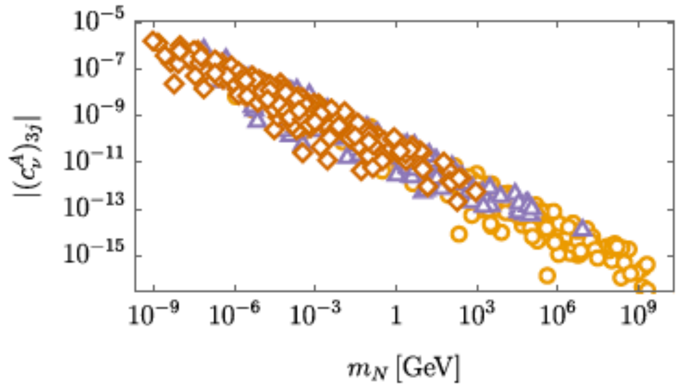
UV boundary: $$\frac{1}{2}\left(b_{N,ij}\overline{N^c_i}N_j + \frac{y^{(5)}_{N,ij}}{k^{3/2}}\Phi\overline{N^c_i}N_j + \text{h.c.}\right)$$

Predictions:

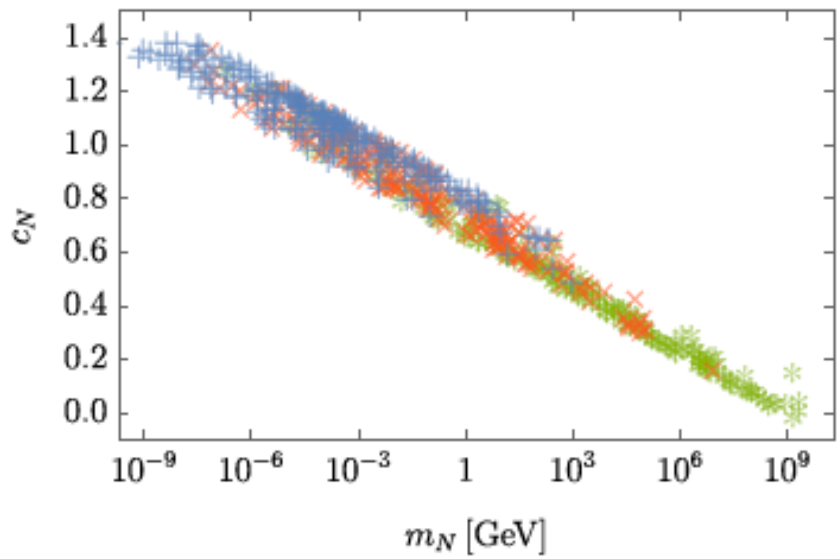
Parameters chosen to explain axion quality, neutrino mass differences and PMNS angles

$\sigma_0 = 0.1, \lambda = 0.1, \Delta = 10, \tan\beta = 3, kz_{\rm IR} = 10^7$ ⇨ $F_a \simeq 8.12 \times 10^9$ GeV

✦ Axion-neutrino couplings



✦ Light sterile neutrinos!



⇨ $1 \text{ eV} \lesssim m_N \lesssim 10^9 \text{ GeV}$

Higher dimension terms:

$$\Delta S_5 = -\frac{1}{4g_5^2}\int d^4x \int_0^L dy\, \frac{c_6}{\Lambda_5^2}\, \mathrm{Tr}\, G_{MN} \Box G^{MN}$$

$$\Rightarrow \quad S_{\rm eff} = \frac{2\pi}{\alpha_s} + \frac{3\pi}{\alpha_s}\frac{c_6}{(\Lambda_5\rho)^2} - 3\xi(R/\rho)\frac{R}{\rho} + \ldots$$

Higher dimension contribution

Extremum:
$(c_6 > 0)$

$$\frac{1}{\rho_*} \simeq \frac{3}{c_6}\xi(R/\rho)\left(\frac{g_5^2\Lambda_5}{24\pi^3}\right)\Lambda_5$$

Provided $\frac{g_5^2\Lambda_5}{24\pi^3} \ll 1$ $\Rightarrow$ $\rho_* \gg \frac{1}{\Lambda_5}$

i.e. instantons of size near UV cutoff (Λ_5) are suppressed