IGNATIOS

D=4?

SUSY?

SUPERSTRINGS?

IgnatiosFest

Planck 2022

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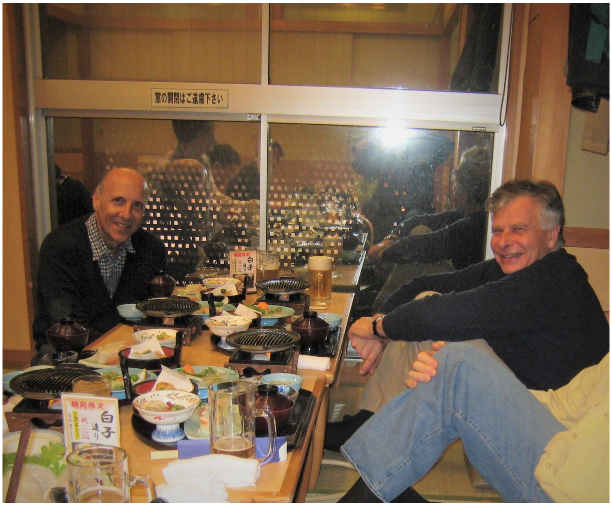
Paris

We met in 1979
at Cargèse Summer School

Since then 34 papers together
& we are most frequent collaborators

Topics range from
topological string amplitudes
to 750 GeV diphoton excess

But most important...
we are best friends



Going back to 1979:

At that time... we worked on QCD...

& nobody was asking $D = ?$

because you just open your eyes

and see $D = 4$

... and they are still there...

SUSY & SUPERSTRINGS were
rather esoteric branches of
mathematical physics

because there was no
physical (experimental) reason
to consider them seriously

... and that didn't change, either

But in the meantime,

D became a parameter – as the central charge of world-sheet CFT

arguably, $D = 10$ is the preferred value

Over the last 35 years, there has been extensive discussion on the shape and size of "extra" dimensions. In particular

Ignatios pioneered the concept of large extra dimensions

But very few ideas why there are noncompact $D = 4$?

1. Brandenberger - Vafa string gas cosmology
   $D = 1 \nearrow D = 4$
2. T & Veneziano
   $D = 10 \searrow D = 4$ compactification
   driven by infrared divergences

What's special about $D = 4$?

4 is max D with IR divergences

$D=4$: 0 · probability of 1 γ (bremsstrahlung)
$D>4$: finite probability of 1 γ soft theorems

What about SUSY?

Here again, exact SUSY is preferred by strings
$D=4$ SUSY $\longleftrightarrow$ world-sheet SUSY

Summary: $D \neq 4$ & SUSY
are "natural" in string theory

Celestial holography:

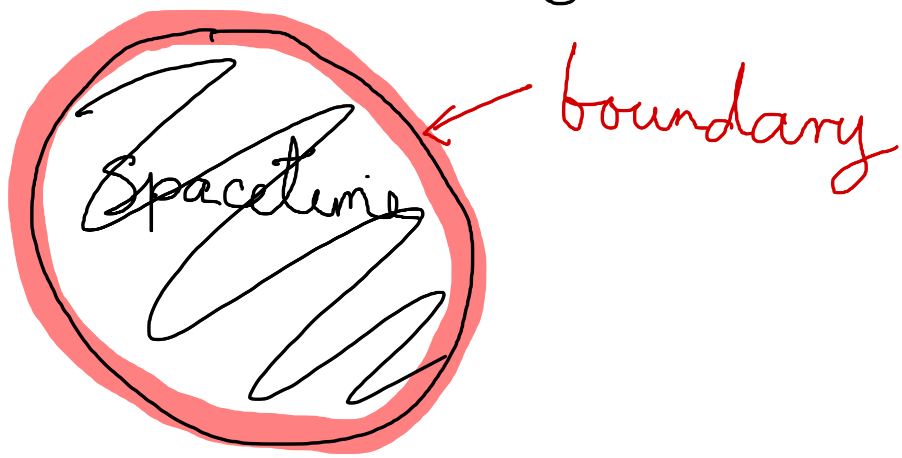
$D=2$ hologram of $D=4$ world makes

$D \neq 4$ & SUSY
unnatural

If invented in 1979, nobody would worry about $D \neq 4$ and SUSY

... but with holography, we had to wait until 1990's ('t Hooft, Maldacena, ...)

The idea of holography:



4D physics $\longleftrightarrow$ (simpler) physics on lower-dimensional "boundary"

What is the boundary of flat (Minkowski) or asymptotically flat (our Universe) spacetime?

You can see it on a clear night

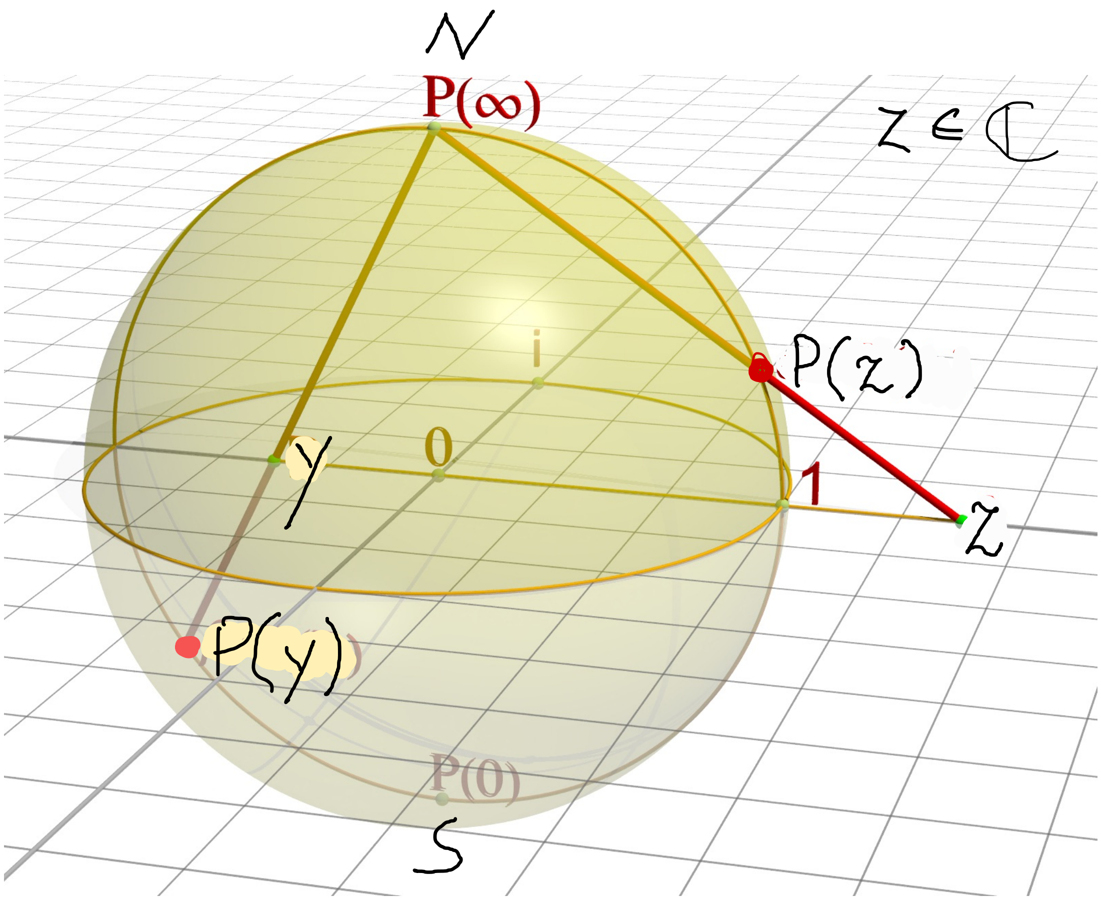
CELESTIAL SPHERE

CS_2

You can get there by following a light ray back to the past at "null infinity"

Direction of light ray $\equiv$ point on CS_2

$\equiv$ point z on complex plane



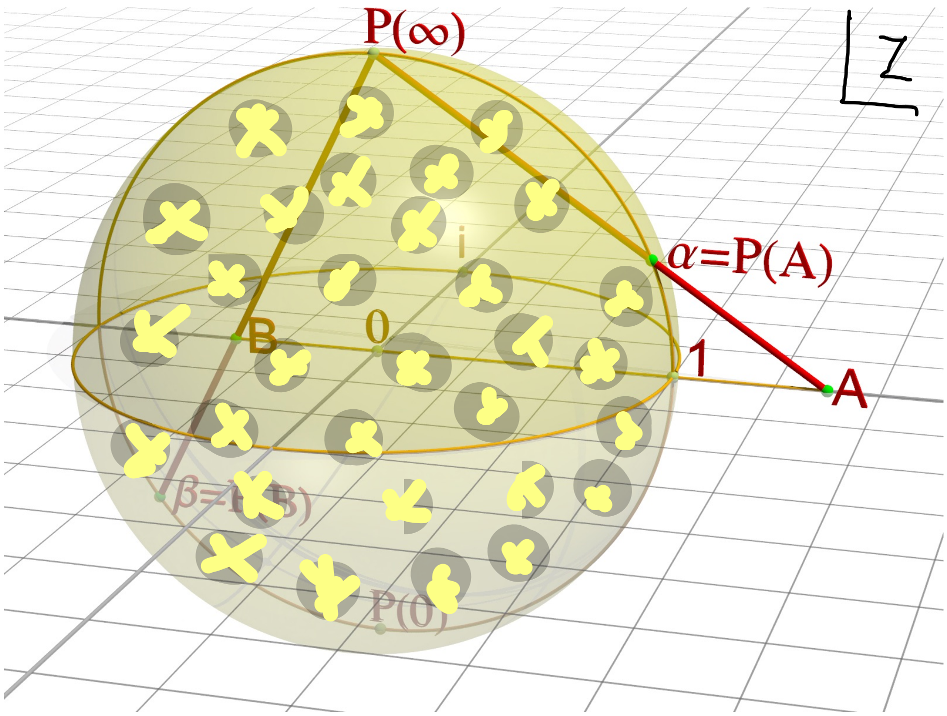
Stereographic projection

$$CS_2 \longrightarrow \mathbb{C}$$

$$p^2 = 0 \Rightarrow p^\mu = \omega q^\mu$$

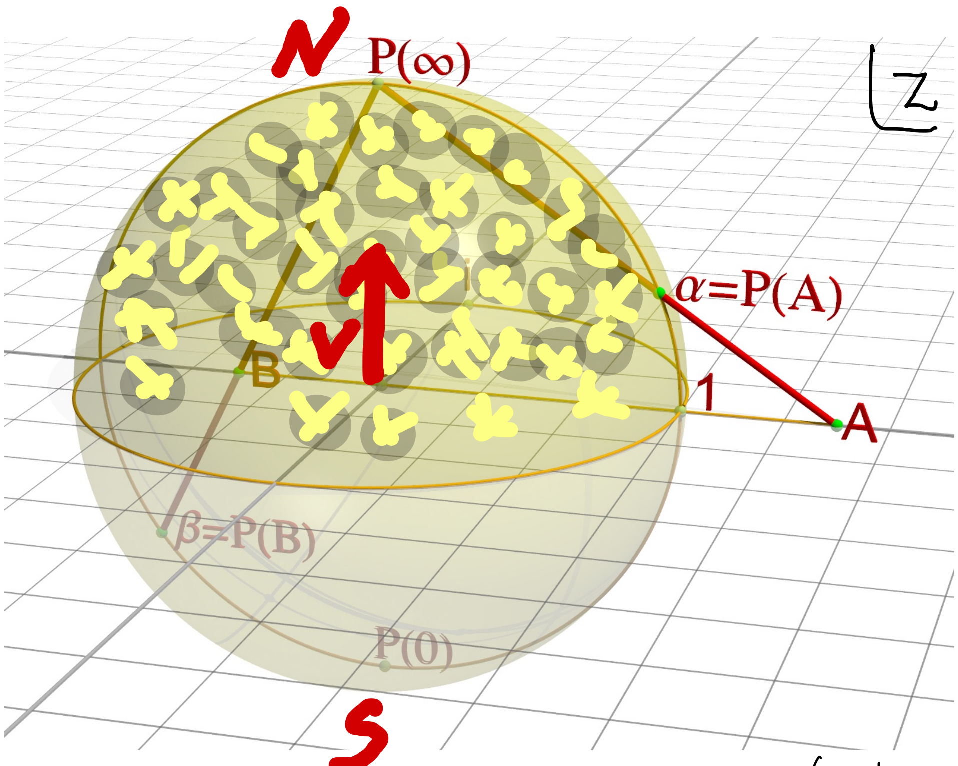
$$q^\mu = \frac{1}{2}\left(1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2\right)$$

z specifies direction of p^μ



This is your night sky

How does it look like from $v \sim c$ Millenium Falcon?



Stars "pile up" towards North Pole $z = \infty$

$\equiv$ dragged away from South Pole $z = 0$

rapidity →

$$z \to e^{\eta} z$$

Lorentz boost
$= 2D$ dilation (scale transf)

In general:

Lorentz transformation $SO(1,3)$

$= 2D$ conformal symmetry $SL(2,\mathbb{C})$

$$z \to \frac{az+b}{cz+d} \qquad \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1$$

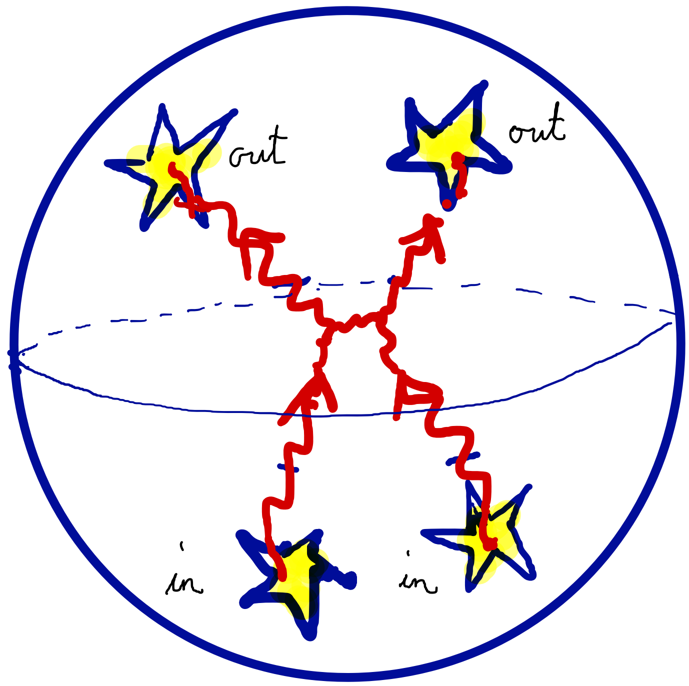
Physics in $D=4$ asympt. flat spacetime encoded in 2D CFT on CS_2

$\to$ use power of $\mathbb{C}$

$(D=4) : (D \neq 4)$

$1 : 0$

Suppose that we want to describe 4D scattering processes in terms of amplitudes on CS_2:



need to encode info not only about directions z but also about energies ω,

in $SL(2,\mathbb{C})$ invariant way?

Solution : replace plane waves characterized by $p^\mu(\omega, z)$ by conformal wave packets characterized by Δ, z

Mellin dual

dimension

Conformal wave-packets

Mellin transf.

$$\Psi(x;p) = e^{\pm i p x}$$

$$\longrightarrow \varphi_\Delta(x; z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i \omega q x}$$

p

$$\sim \frac{1}{(xq)^\Delta} \qquad h = \bar{h} = \frac{\Delta}{2}$$

normalizable only if

ω traded for λ

$$\Delta = 1 + i\lambda$$

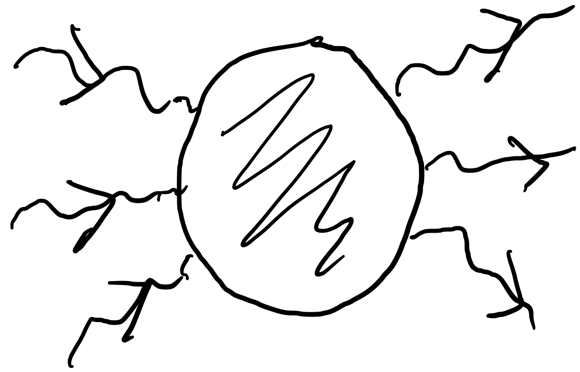
principal conf. series

Pasterski & Shao

We want to understand scattering amplitudes as two-dimensional conformal correlators

$$\langle \text{in} | \text{out} \rangle \to \langle \mathcal{O}_1(z_1) \ldots \mathcal{O}_N(z_N) \rangle$$

Amplitudes : $\langle \text{in} | \text{out} \rangle$



gauge bosons

gravitons

$m = 0$

YM, Einstein's GR

$$\langle p_1, h_1, a_1 \ldots | \ldots p_n, h_n, a_n \rangle \equiv \mathcal{A}(p, h, a)$$

$$= (2\pi)^4 i \, \delta^{(4)}\left(\sum p_{out} - \sum p_{in}\right) \mathcal{M}$$

Feynman, recursions, scattering eqs, ...

$$\mathcal{A}(z,\omega,\dots) \to \tilde{\mathcal{A}}(\Delta, z, \dots)$$

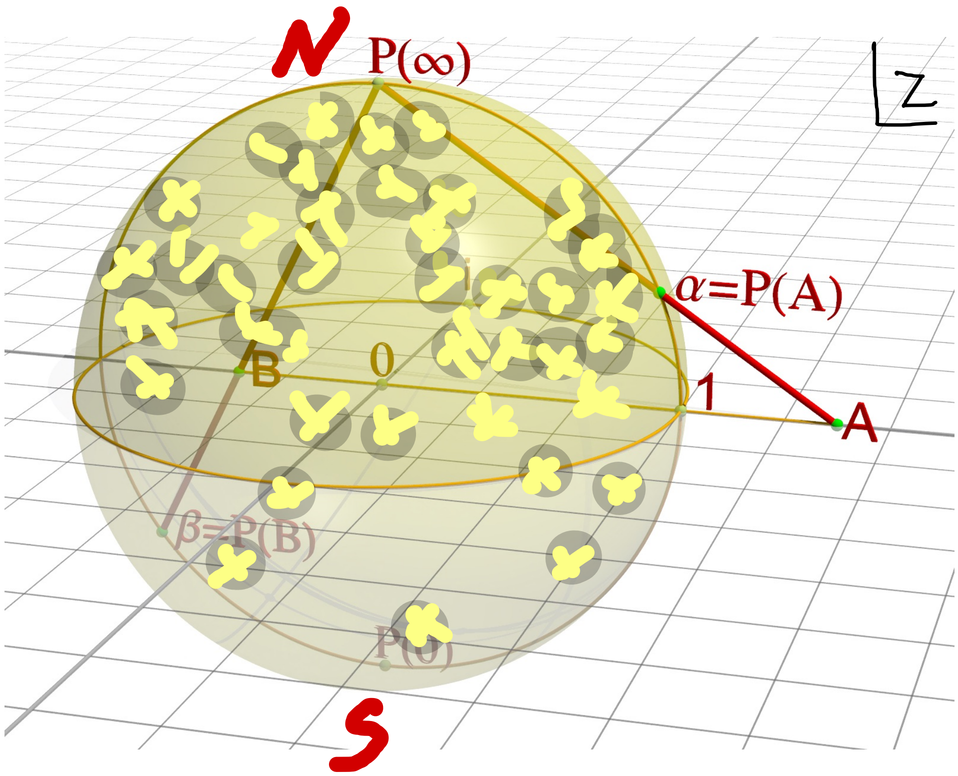
$$\tilde{\mathcal{A}} = \int d\omega_1 \omega_1^{i\lambda_1} \cdots d\omega_n \omega_n^{i\lambda_n} \delta^{(4)}(\omega_1 q_1 + \dots \omega_n q_n) \mathcal{M}(\omega_i, z_i, \bar{z}_i)$$

$$\langle \mathcal{O}_{\Delta_1}(z_1) \dots \mathcal{O}_{\Delta_N}(z_N) \rangle \equiv \tilde{\mathcal{A}}(\Delta_1, z_1; \dots \Delta_N, z_N)$$

correlation functions of conformal wave packets (primary fields)

$$\boxed{\begin{array}{l}\text{CELESTIAL AMPLITUDES} \\ \equiv \text{AMPLITUDES IN CONFORMAL BASIS}\end{array}}$$

In 2013, Andy Strominger showed that such correlators are "hiding" a symmetry much larger than the original Lorentz symmetry...



all possible (smooth) deformations

$\equiv$ diffeomorphisms of CS_2

$$z \longrightarrow f(z)$$

2D "local" conformal transformations

generated by infinite-dimensional

Virasoro algebra

$\equiv$ BMS superrotations

The goal is to construct

Celestial Conformal Field Theory (CCFT)

in 2D

that describes 4D

elementary particle physics,

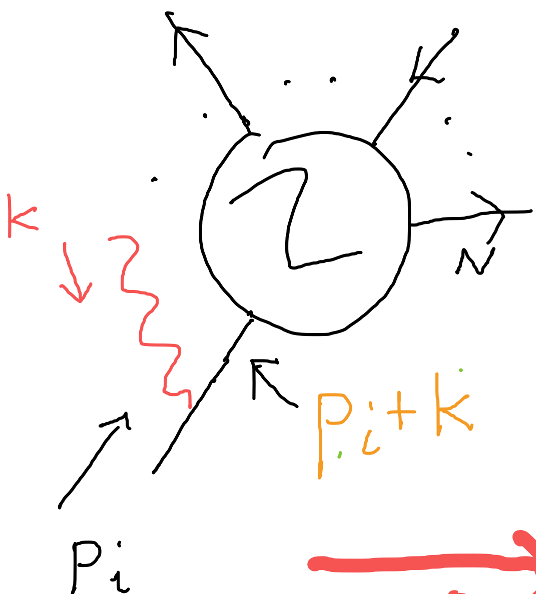
hopefully beyond the standard model

(gauge-gravity unification?)

By now a famous example of 4D — 2D dictionary:

Soft theorems $(D=4)$

simplest case: scalar QED

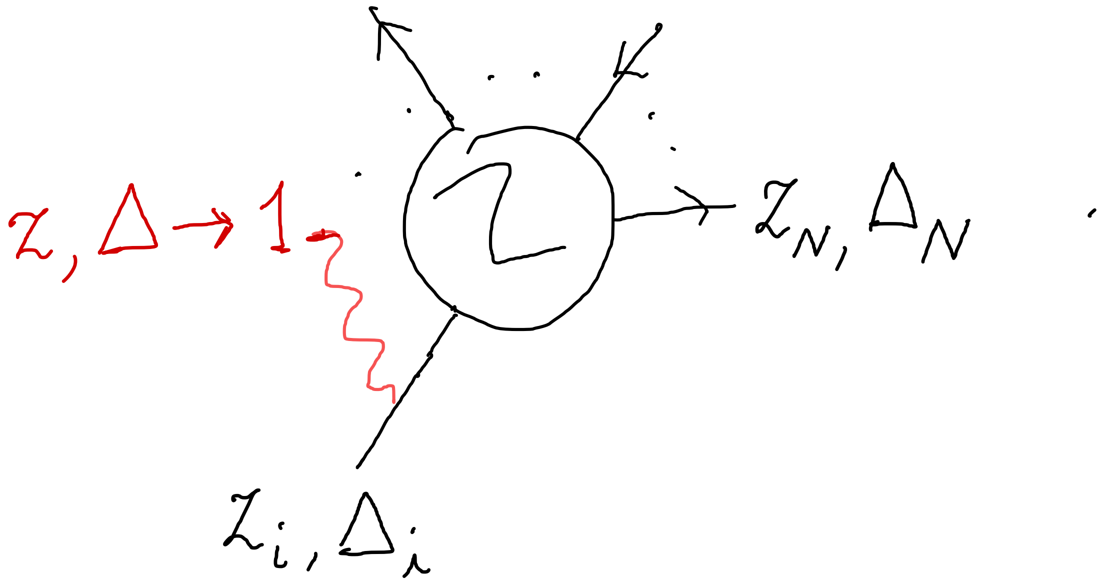


$$\mathcal{M}(k, p_1, \ldots p_N)$$

$$\xrightarrow{k\to 0} \left(\sum_{i=1}^{N} Q_i \frac{\varepsilon p_i}{k p_i}\right)\mathcal{M}(p_1, \ldots p_N)$$

leading singularity $\sim \frac{1}{\omega}$

universal soft factor $\sim \frac{1}{\omega}$



$$\tilde{\mathcal{A}}(\underbrace{z, \Delta}_{\text{photon}} \to 1;\ \underbrace{z_1, \Delta_1 \ldots\ldots z_N, \Delta_N}_{\text{charged scalars}})$$

$$\to \underbrace{\frac{1}{\Delta - 1}}_{\text{instead of } \frac{1}{\omega}} \sum_{i=1}^{N} \frac{Q_i}{z - z_i} \tilde{\mathcal{A}}(z_1, \Delta_1, \ldots z_N, \Delta_N)$$

This is a Ward identity of 2D current

$\to$ soft photon $\equiv$ "Goldstone boson" of asymptotic gauge symmetry

$(D=4) : (D \neq 4)$

$2 : 0$

What about SUSY ?

$$\{Q, \bar{Q}\} \sim P$$

$$D=4 : P_\mu \sim \omega\left(1+z\bar{z},\ z+\bar{z},\ -i(z-\bar{z}),\ 1-z\bar{z}\right)$$

(super) translations are non-holomorphic

NO 2D SUSY in Super $BMS_{D=4}$

(NO SUSY) : (SUSY)

1 : 0

Still, long way towards

CCFT

Many things to be done...

next workshop starts June 21

at CMSA/BHI/Harvard

Thank you, Ignatios
for being a wonderful
friend & collaborator

and for inspiring all of us
with your ground-breaking work!

Happy Birthday!