Covariant Quantization of Quadratic Gravity

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Outline

- Motivations

- Theoretical background
  - LSZ operator formalism
  - BRST quantization

- Application to quadratic (conformal) gravity
  - Second order formulation
  - Quantization
  - A new take on the ghost problem

Motivations

- Establish a rigorous quantization of QG from the operator perspective
  - QG is renormalizable [Stelle], but there are serious problems
  - Rewrite QG in the familiar language of non-Abelian gauge theory
  - Covariant operator formalism makes studying off-shell quantities (e.g. correlation functions) much more transparent
  - Fourth order theories resist this description due to their “hidden” dofs

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- Ghost problem: fourth order theories have problems with unitarity
  - It may be possible to overcome the classical Ostrogradky instability with quantum physics [Donoghue, Menezes]
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- Make available well-known techniques for studying quantum corrections (renormalization, anomalies, etc.)

LSZ operator formalism

Interacting fields behave as free fields in the asymptotic limit

$$\Phi(x) \to \begin{cases} \Phi^{\text{in}}(x)\,, & x^0 \to -\infty \\ \Phi^{\text{out}}(x)\,, & x^0 \to +\infty \end{cases}$$

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Asymptotic fields decompose as a sum of (oscillators)*(plane wave functions)

$$\Phi^{\text{as}}(x) = \sum_{\boldsymbol{p}} \Big(\Phi_f^{\text{as}}(\boldsymbol{p}) f_{\boldsymbol{p}}(x) + \Phi_g^{\text{as}}(\boldsymbol{p}) g_{\boldsymbol{p}}(x) + \Phi_h^{\text{as}}(\boldsymbol{p}) h_{\boldsymbol{p}}(x) + \cdots + (\text{h.c.}) \Big)$$

$$\Box\, f_{\boldsymbol{p}}(x) = \Box^2 g_{\boldsymbol{p}}(x) = \Box^3 h_{\boldsymbol{p}}(x) = \cdots = 0$$

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Inverting the decomposition defines oscillators in terms of their *interacting* field

$$\Phi^{\rm as}_f(\boldsymbol{p}) = \lim_{x^0 \to \pm\infty} \left[i \int {\rm d}^3\boldsymbol{x} \Big(f^*_{\boldsymbol{p}}(x) \overleftrightarrow{\partial}_0 + g^*_{\boldsymbol{p}}(x) \overleftrightarrow{\partial}_0 \Box + h^*_{\boldsymbol{p}}(x) \overleftrightarrow{\partial}_0 \Box^2 + \cdots \Big) \Phi(x) \right]$$

Needed for the LSZ reduction formula for the S-matrix ⟶ optical theorem and unitarity

BRST quantization and physical states

[Nakanishi, Ojima 1990]

BRST theory introduces new fields to account for the redundant dofs in gauge theories

Nakanishi-Lautrup (NL) bosons: $B_a(x)$

(Lagrange multipliers to enforce gauge conditions)

Faddeev-Popov ghosts and anti-ghosts: $C^a(x)\,,\ \bar{C}_a(x)$

(cancel unphysical contributions to loops, establish BRST symmetry)

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Cancellation is made precise by *global* BRST symmetry, generated by the charge operator $\mathcal{Q}$

$$\delta_\epsilon \phi^A = \epsilon \sum_a \left(\delta_{\xi^a}\phi^A\right)\Big|_{\xi^a = C^a} \qquad \delta_\epsilon B^a = 0$$

$$\delta_\epsilon C^a = 0 \quad \text{(free theory)} \qquad \delta_\epsilon \bar{C}^a = \epsilon\, i B^a$$

$$S_{\mathrm{T}} = S_{\mathrm{cl}} - \mathcal{Q}\left(\bar{C}_a \chi^a\right) = S_{\mathrm{cl}} + S_{\mathrm{gf}} + S_{\mathrm{FP}}$$

classical action   gauge conditions   gauge fixing & ghost actions

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classical action gauge conditions gauge fixing & ghost actions

Kugo-Ojima “quartet mechanism” classifies all states into physical singlets or unphysical quartets

$\mathcal{V}$ (full Fock space)

$$\mathcal{V}_{\mathrm{tr}} \ni \left\{\, |a_i\rangle\,;\ \ \mathcal{Q}\,|a_i\rangle = 0 \right\}$$

BRST singlet states with no parents (transverse parts of ϕ^A)

$$\mathcal{V}_{\mathrm{q}} \ni \left\{ \begin{array}{ll} (|\pi_0\rangle\,,\,|\delta_1\rangle); & |\delta_1\rangle = \mathcal{Q}\,|\pi_0\rangle \neq 0 \\ (|\pi_{-1}\rangle\,,\,|\delta_0\rangle); & |\delta_0\rangle = \mathcal{Q}\,|\pi_{-1}\rangle \neq 0 \end{array} \right\}$$

matching pairs of parent-daughter doublets

Quadratic (conformal) gravity at second order

General (scale-invariant) Riemann^2 action simplified by dropping Gauss-Bonnet invariant (T.D.)

$$S_{\mathrm{QG}} = \int \mathrm{d}^4x\sqrt{-g}\Big(aR_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} + bR_{\alpha\beta}R^{\alpha\beta} + cR^2\Big) \rightarrow \frac{2}{\alpha_g^2}\int \mathrm{d}^4x\sqrt{-g}\Big[\underbrace{2\Big(R_{\alpha\beta}R^{\alpha\beta} - \frac{1}{3}R^2\Big)}_{\simeq C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}} + \beta R^2\Big]$$

Setting β = 0 gives *local* conformal symmetry

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Reduce from fourth to second order in derivatives with auxiliary field

$$S_{\mathrm{H}} = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{2}{\alpha_g}G_{\alpha\beta}H^{\alpha\beta} - \frac{1}{4}\left(H_{\alpha\beta}H^{\alpha\beta} - H_{\alpha}{}^{\alpha}H_{\beta}{}^{\beta}\right)\right)$$

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Introduce Stückelberg vector field to make all constraints first class

$$H_{\alpha\beta} \;\to\; H_{\alpha\beta} + \left(\nabla_\alpha A_\beta + \nabla_\beta A_\alpha\right)$$

$$S_{\mathrm{SOCG}} = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{2}{\alpha_g}G_{\alpha\beta}H^{\alpha\beta} - \frac{1}{4}\left(H_{\alpha\beta}H^{\alpha\beta} - H_\alpha{}^\alpha H_\beta{}^\beta\right) - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + R_{\alpha\beta}A^\alpha A^\beta - H_{\alpha\beta}\nabla^\beta A^\alpha + H_\alpha{}^\alpha\nabla_\beta A^\beta\right)$$

Total non-linear action

24 fields – 2*(4 + 4 + 1) gauge symmetries = 6 dofs ✓

Our action is diffeomorphism, “Stückelberg diffeomorphism”, and Weyl invariant

$\delta_\xi g_{\alpha\beta} = \alpha_g \mathcal{L}_\xi g_{\alpha\beta} = \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha$	$\delta_\zeta g_{\alpha\beta} = 0$	$\delta_\omega g_{\alpha\beta} = \alpha_g \, \omega g_{\alpha\beta}$
$\delta_\xi H_{\alpha\beta} = \alpha_g \mathcal{L}_\xi H_{\alpha\beta}$	$\delta_\zeta H_{\alpha\beta} = \nabla_\alpha \zeta_\beta + \nabla_\beta \zeta_\alpha$	$\delta_\omega H_{\alpha\beta} = 4\nabla_\beta \nabla_\alpha \omega + \alpha_g \left(2A_{(\alpha} \nabla_{\beta)} \omega - g_{\alpha\beta} A_\gamma \nabla^\gamma \omega\right)$
$\delta_\xi A_\alpha = \alpha_g \mathcal{L}_\xi A_\alpha$	$\delta_\zeta A_\alpha = -\zeta_\alpha$	$\delta_\omega A_\alpha = 0$

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$$\delta_\xi A_\alpha = \alpha_g \mathcal{L}_\xi A_\alpha \qquad \delta_\zeta A_\alpha = -\zeta_\alpha \qquad \delta_\omega A_\alpha = 0$$

Choose gauge conditions and introduce sets of NL fields, ghosts, and anti-ghosts

de Donder (standard GR)

$$\partial_\beta\left(\sqrt{-g} g^{\alpha\beta}\right) = 0$$

$$\{b_\alpha,\, c^\alpha,\, \bar{c}_\alpha\} \;\rightarrow\; S_{\mathrm{gf}\xi} + S_{\mathrm{FP}\xi}$$

“de Donder”

$$\nabla_\beta H_\alpha{}^\beta - \frac{1}{2}\nabla_\alpha H_\beta{}^\beta = 0$$

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Feynman (w.r.t. *A*)

$$H_\alpha{}^\alpha + 2\nabla_\alpha A^\alpha + B = 0$$

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Assemble the total non-linear (BRST-invariant) action

$$S_{\mathrm{T}} = S_{\mathrm{SOCG}} + S_{\mathrm{gf}\xi} + S_{\mathrm{gf}\zeta} + S_{\mathrm{gf}\omega} + S_{\mathrm{FP}\xi} + S_{\mathrm{FP}\zeta} + S_{\mathrm{FP}\omega}$$

Total free action

Linearize around Minkowski and drop *O*(α_g) interaction terms

$$S_{\mathrm{T}}\Big|_{g_{\alpha\beta}\to\eta_{\alpha\beta}+\alpha_g h_{\alpha\beta}} = S_0 + S_{\mathrm{int}} \qquad \text{where} \qquad S_{\mathrm{int}} = \mathcal{O}(\alpha_g)$$

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$$S_0 = \int \mathrm{d}^4x \bigg(4H^{\alpha\beta}\mathcal{E}_{\alpha\beta\gamma\delta}h^{\gamma\delta} - \frac{1}{4}\left(H_{\alpha\beta}H^{\alpha\beta} - H_{\alpha}{}^{\alpha}H_{\beta}{}^{\beta}\right) - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - H_{\alpha\beta}\partial^{\alpha}A^{\beta} + H_{\alpha}{}^{\alpha}\partial_{\beta}A^{\beta}$$

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$$\begin{aligned} S_0 = \int \mathrm{d}^4x \Bigg(& 4H^{\alpha\beta}\mathcal{E}_{\alpha\beta\gamma\delta}h^{\gamma\delta} - \frac{1}{4}\left(H_{\alpha\beta}H^{\alpha\beta} - H_{\alpha}{}^{\alpha}H_{\beta}{}^{\beta}\right) - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - H_{\alpha\beta}\partial^{\alpha}A^{\beta} + H_{\alpha}{}^{\alpha}\partial_{\beta}A^{\beta} \\ & + \underbrace{b_{\alpha}\left(\partial_{\beta}h^{\alpha\beta} - \frac{1}{2}\partial^{\alpha}h_{\beta}{}^{\beta}\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\xi}} + \underbrace{B_{\alpha}\left(\partial_{\beta}H^{\alpha\beta} - \frac{1}{2}\partial^{\alpha}H_{\beta}{}^{\beta}\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\zeta}} + \underbrace{\frac{1}{8}B\left(2H_{\alpha}{}^{\alpha} + 4\partial_{\alpha}A^{\alpha} + B\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\omega}} \end{aligned}$$

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$$+ \underbrace{b_\alpha\left(\partial_\beta h^{\alpha\beta} - \frac{1}{2}\partial^\alpha h_\beta{}^\beta\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\xi}} + \underbrace{B_\alpha\left(\partial_\beta H^{\alpha\beta} - \frac{1}{2}\partial^\alpha H_\beta{}^\beta\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\zeta}} + \underbrace{\frac{1}{8}B\left(2H_\alpha{}^\alpha + 4\partial_\alpha A^\alpha + B\right)}_{S^{(\mathrm{lin})}_{\mathrm{gf}\omega}}$$

$$+ \underbrace{i\bar{c}_\alpha\left(\Box\, c^\alpha - \partial^\alpha C\right)}_{S^{(\mathrm{lin})}_{\mathrm{FP}\xi}} + \underbrace{i\bar{C}_\alpha\left(\Box\, C^\alpha + 2\Box\partial^\alpha C\right)}_{S^{(\mathrm{lin})}_{\mathrm{FP}\zeta}} + \underbrace{i\bar{C}\,\Box\, C}_{S^{(\mathrm{lin})}_{\mathrm{FP}\omega}} \Bigg)$$

Propagators

Propagators are the components of the inverse of the Hessian matrix

$$-i\,\langle 0|\, T\Phi_A\Phi_B\,|0\rangle = \Omega^{-1}_{AB}(p) \qquad \text{where} \qquad \Omega^{AB}(p) = \int \mathrm{d}^4x \frac{\delta^2 S_0}{\delta\Phi_A(x)\delta\Phi_B(y)}\, e^{-ip(x-y)}$$

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Important (non-zero) classical field propagators:

$$-\,i \langle 0| T h^{\alpha\beta} H^{\gamma\delta} |0\rangle = \frac{1}{2p^4}\left(\eta^{\alpha\gamma}p^\beta p^\delta + \eta^{\alpha\delta}p^\beta p^\gamma + \eta^{\beta\gamma}p^\alpha p^\delta + \eta^{\beta\delta}p^\alpha p^\gamma\right) - \frac{1}{2p^2}\left(\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\gamma} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right)$$

H has a double pole

$$-\,i \langle 0| T h^{\alpha\beta} h^{\gamma\delta} |0\rangle = -\frac{1}{2p^2}\Big(-\,i \langle 0| T h^{\alpha\beta} H^{\gamma\delta} |0\rangle\Big)$$

h has a triple pole

$$-\,i \langle 0| T A^\alpha A^\beta |0\rangle = -\frac{\eta^{\alpha\beta}}{p^2}$$

A has just a simple pole

High energy behavior is nice, but there is a complicated pole structure in the spin-2 sector

Oscillator decomposition

Make ansatz for each field based on pole structure of its propagators

$$h^{\alpha\beta}(x) = h_f^{\alpha\beta}(\boldsymbol{p}) f_{\boldsymbol{p}}(x) + h_g^{\alpha\beta}(\boldsymbol{p}) g_{\boldsymbol{p}}(x) + h_h^{\alpha\beta}(\boldsymbol{p}) h_{\boldsymbol{p}}(x) + (\mathrm{h.c.})$$

triple pole, need $f_{\boldsymbol{p}}$, $g_{\boldsymbol{p}}$, and $h_{\boldsymbol{p}}$

$$H^{\alpha\beta}(x) = H_f^{\alpha\beta}(\boldsymbol{p}) f_{\boldsymbol{p}}(x) + H_g^{\alpha\beta}(\boldsymbol{p}) g_{\boldsymbol{p}}(x) + (\mathrm{h.c.})$$

double pole, need $f_{\boldsymbol{p}}$ and $g_{\boldsymbol{p}}$

$$A^{\alpha}(x) = A_f^{\alpha}(\boldsymbol{p}) f_{\boldsymbol{p}}(x) + (\mathrm{h.c.})$$

simple pole, just need $f_{\boldsymbol{p}}$

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triple pole, need f_p, g_p, and h_p

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double pole, need f_p and g_p

$$A^{\alpha}(x) = A_f^{\alpha}(\boldsymbol{p}) f_{\boldsymbol{p}}(x) + (\text{h.c.})$$

simple pole, just need f_p

Write higher pole oscillators in terms of other simple oscillators using EOMs/gauge conditions

$$\Box\, h_{\alpha\beta} - \frac{1}{2}\Big(H_{\alpha\beta} + \partial_\alpha(A_\beta + B_\beta) + \partial_\beta(A_\alpha + B_\alpha)\Big) = 0 \qquad \Box\, H_{\alpha\beta} - \frac{1}{2}\Big(\partial_\alpha b_\beta + \partial_\beta b_\alpha\Big) = 0 \qquad \Box\, A_\alpha = 0$$

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simple pole, just need $f_{\boldsymbol{p}}$

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Oscillator commutators match ($-p^{-2}$) coefficient of the full field propagators

$$\Big[h_{f\,\alpha\beta}(\boldsymbol{p}), H^\dagger_{f\,\gamma\delta}(\boldsymbol{q})\Big] = \frac{1}{2}\big(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma} - \eta_{\alpha\beta}\eta_{\gamma\delta}\big)\delta^3(\boldsymbol{p} - \boldsymbol{q}) \qquad \Big[A_{f\,\alpha}(\boldsymbol{p}), A^\dagger_{f\,\beta}(\boldsymbol{q})\Big] = \eta_{\alpha\beta}\delta^3(\boldsymbol{p} - \boldsymbol{q})$$

Physical states

BRST transformation singles out six obvious invariant combos of oscillator components

$$a_{h,\pm} = \frac{1}{2}\Big(h_{f\,11} - h_{f\,22}\Big) \mp i h_{f\,12} \qquad a_{H,\pm} = \frac{1}{2}\Big(H_{f\,11} - H_{f\,22}\Big) \mp i H_{f\,12} \qquad a_{A,\pm} = \frac{1}{\sqrt{2}}\left(A_{f\,1} - \frac{iH_{f\,13}}{E} \mp i\left(A_{f\,2} - \frac{iH_{f\,23}}{E}\right)\right)$$

$$\big[\mathcal{Q}, a_{h,\pm}\big] = 0 \qquad \big[\mathcal{Q}, a_{H,\pm}\big] = 0 \qquad \big[\mathcal{Q}, a_{A,\pm}\big] = 0$$

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$$\big[\mathcal{Q}, a_{h,\pm}\big] = 0 \qquad \big[\mathcal{Q}, a_{H,\pm}\big] = 0 \qquad \big[\mathcal{Q}, a_{A,\pm}\big] = 0$$

Writing oscillators in terms of these operators, transverse polarizations appear naturally

circular polarization tensors   longitudinal components (quartets)

$$h_{f\,\alpha\beta}(\boldsymbol{p}) = \varepsilon_{+\,\alpha\beta}(\boldsymbol{p})a_{h,+}(\boldsymbol{p}) + \varepsilon_{-\,\alpha\beta}(\boldsymbol{p})a_{h,-}(\boldsymbol{p}) + \cdots + (\text{h.c.})$$

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$$A_{f\,\alpha}(\boldsymbol{p}) = \varepsilon_{+\,\alpha}(\boldsymbol{p})a_{A,+}(\boldsymbol{p}) + \varepsilon_{-\,\alpha}(\boldsymbol{p})a_{A,-}(\boldsymbol{p}) + \cdots + (\text{h.c.})$$

All other independent components are longitudinal and fit into quartets with the NL, ghost, and anti-ghost fields

Physical states

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All other independent components are longitudinal and fit into quartets with the NL, ghost, and anti-ghost fields

Commutators between physical states define the interacting quantum theory

$$\Big[a_{h,\lambda}(\boldsymbol{p}), a^{\dagger}_{H,\lambda'}(\boldsymbol{q})\Big] = \delta_{\lambda\lambda'}\delta^3(\boldsymbol{p} - \boldsymbol{q})$$

Off-diagonal, indefinite inner product metric!

$$\Big[a_{A,\lambda}(\boldsymbol{p}), a^{\dagger}_{A,\lambda'}(\boldsymbol{q})\Big] = \delta_{\lambda\lambda'}\delta^3(\boldsymbol{p} - \boldsymbol{q})$$

Standard inner product metric

Unitarity

A healthy S-matrix is pseudo-unitary, leaves the vacuum invariant, and commutes with the Hamiltonian and BRST charge

$$SS^\dagger = S^\dagger S = \mathbb{1} \qquad S\,|0\rangle = S^\dagger\,|0\rangle = |0\rangle \qquad [\mathcal{H}, S] = [\mathcal{Q}, S] = 0 \longrightarrow$$

spin-1 (definite metric) ✓

spin-2 (indefinite metric) ?

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spin-1 (definite metric) ✓
spin-2 (indefinite metric) **?**

Indefinite metric means only off-diagonal interactions contribute to the S-matrix

$$_h\langle n, \mathrm{out}|m, \mathrm{in}\rangle_h = \;_H\langle n, \mathrm{out}|m, \mathrm{in}\rangle_H = 0 \qquad _h\langle n, \mathrm{out}|m, \mathrm{in}\rangle_H = \;_h\langle n, \mathrm{in}|S|m, \mathrm{in}\rangle_H \neq 0$$

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LSZ reduction formula lets us check the requirement for unitarity explicitly

$$_h\langle \alpha, \mathrm{in}|S^\dagger \mathbb{1} S|\alpha, \mathrm{in}\rangle_H = \sum_{m=0} \int \mathrm{d}^3\boldsymbol{p}_m (-1)^m \, _h\langle \cdots \rangle_H \neq 1$$

(α is some arbitrary initial state)

This $(-1)^m$ is not typical and comes from the indefinite metric – it spoils unitarity!

$$_h\langle \alpha, \mathrm{in}|S^\dagger|m, \mathrm{in}\rangle_H \neq \left(_h\langle m, \mathrm{in}|S|\alpha, \mathrm{in}\rangle_H \right)^\dagger$$

Summary

- We can rewrite fourth order quadratic gravity as a second order theory with first class constraints by introducing some additional fields
  - This rewriting makes QG look remarkably like non-Abelian gauge theory

- This formulation is well-suited for BRST quantization
  - Standard techniques let us identify the subspace of physical transverse states
  - We can construct all the important quantum operators – Hamiltonian, S-matrix, etc.
  - This formalism allows one to study off-shell quantities in quantum QG (e.g. correlation functions) with tools that were previously unavailable

- The ghost problem may be viewed in the context of indefinite metric QFT

Thank you for your attention!

Comparison to quadratic gravity

An additional auxiliary field is needed, but no conformal symmetry means no scalar BRST fields

$$S_\chi = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{2}{\alpha_g}R\chi - \frac{1}{2\beta}\chi^2\right)$$

$$\cancel{\{B,\, C,\, \bar{C}\} \to S_{\mathrm{gf}\omega} + S_{\mathrm{FP}\omega}}$$

Quantum spin-2,1 sectors work out the same, additional spin-0 sector with long. mode of *A* (ϕ)

$${}_\phi\langle\beta;\mathrm{in}|\alpha;\mathrm{in}\rangle_\phi = {}_\chi\langle\beta;\mathrm{in}|\alpha;\mathrm{in}\rangle_\chi = 0$$

$${}_\phi\langle\beta;\mathrm{in}|\alpha;\mathrm{in}\rangle_\chi = \delta_{\alpha\beta}$$

Spin-0 sector shows the same off-diagonal indefinite metric behavior

The quantum Hamiltonian

The Heisenberg equation grants us the Hamiltonian operator

$$[\mathcal{H}, \phi_A(x)] = -i\partial_0\phi_A(x) \qquad \mathcal{H} = \int \mathrm{d}^3\boldsymbol{p} \sum_{\lambda=\pm} \left(E\left(a^\dagger_{h,\lambda}(\boldsymbol{p}) a_{H,\lambda}(\boldsymbol{p}) + a^\dagger_{H,\lambda}(\boldsymbol{p}) a_{h,\lambda}(\boldsymbol{p})\right) + \frac{1}{4E}\left(a^\dagger_{H,\lambda}(\boldsymbol{p}) a_{H,\lambda}(\boldsymbol{p})\right)\right)$$

Spin-2 Hamiltonian has a single one-particle eigenstate, and an atypical two-particle eigenstate

$$\mathcal{H}\,|\boldsymbol{p}, \lambda\rangle = \frac{a^\dagger_{H,\lambda}(\boldsymbol{p})}{4E}\,|0\rangle = E\,|\boldsymbol{p}, \lambda\rangle$$

$$\mathcal{H}\,|\boldsymbol{p}, \boldsymbol{q}, \lambda\rangle = \frac{1}{2}\left(\left(E_{\boldsymbol{p}}/E_{\boldsymbol{q}}\right)^{1/2} a^\dagger_{h,\lambda}(\boldsymbol{p}) a^\dagger_{H,\lambda}(\boldsymbol{q}) - \left(E_{\boldsymbol{q}}/E_{\boldsymbol{p}}\right)^{1/2} a^\dagger_{h,\lambda}(\boldsymbol{q}) a^\dagger_{H,\lambda}(\boldsymbol{p})\right)|0\rangle = \left(E_{\boldsymbol{p}} + E_{\boldsymbol{q}}\right)|\boldsymbol{p}, \boldsymbol{q}, \lambda\rangle$$

From these we build the unit operator, where the troublesome (-1)^m appears

$$\mathbb{1} = \sum_{m,n=0} \sum_{\lambda_m,\zeta_n} \int \mathrm{d}^3\boldsymbol{p}_m \mathrm{d}^3\boldsymbol{q}_m \mathrm{d}^3\boldsymbol{k}_n (-1)^m\,|\boldsymbol{p}_m, \boldsymbol{q}_m, \lambda_m; \boldsymbol{k}_n, \zeta_n\rangle_{H}\;{}_{h}\langle\boldsymbol{p}_m, \boldsymbol{q}_m, \lambda_m; \boldsymbol{k}_n, \zeta_n|$$

The LSZ reduction formula

Inverting the spin-2 oscillator definitions gives their in-out overlaps

$$a^{\text{out}}_{h,\lambda}(\boldsymbol{p}) - a^{\text{in}}_{h,\lambda}(\boldsymbol{p}) = -i\int \mathrm{d}^4x\, \varepsilon^*_{\lambda\,\alpha\beta}(\boldsymbol{p}) g^*_{\boldsymbol{p}}(x)\Box^2 h^{\alpha\beta}(x) = -\frac{i}{2}\int \mathrm{d}^4x\, \varepsilon^*_{\lambda\,\alpha\beta}(\boldsymbol{p}) g^*_{\boldsymbol{p}}(x)\Box H^{\alpha\beta}(x)$$

$$a^{\text{out}\,\dagger}_{H,\lambda}(\boldsymbol{p}) - a^{\text{in}\,\dagger}_{H,\lambda}(\boldsymbol{p}) = i\int \mathrm{d}^4x\, \varepsilon_{\lambda\,\alpha\beta}(\boldsymbol{p}) f_{\boldsymbol{p}}(x)\Box H^{\alpha\beta}(x)$$

In regular gauge theory no dipoles appear here, but in fourth order theories they survive

$$\begin{aligned}
&{}_h\langle \boldsymbol{p}'_{m'}, \boldsymbol{q}'_{m'}, \lambda'_{m'}; \boldsymbol{k}'_{n'}, \zeta'_{n'}; \text{out}|\boldsymbol{p}_m, \boldsymbol{q}_m, \lambda_m; \boldsymbol{k}_n, \zeta_n; \text{in}\rangle_H \\
&\quad = \prod_{k=1}^{m'}\left[-\frac{1}{4}\int \mathrm{d}^4x'_k \mathrm{d}^4y'_k\Big(\left(E_{\boldsymbol{p}'_k}/E_{\boldsymbol{q}'_k}\right)^{1/2}\varepsilon^{*\,\alpha'_k\beta'_k}_{\lambda'_k}(\boldsymbol{p}'_k)\varepsilon^{*\,\gamma'_k\delta'_k}_{\lambda'_k}(\boldsymbol{q}'_k) g^*_{\boldsymbol{p}'_k}(x'_k) f^*_{\boldsymbol{q}'_k}(y'_k) - (\boldsymbol{p}'{}_k \leftrightarrow \boldsymbol{q}'{}_k)\Big)\Box_{x'_k}\Box_{y'_k}\right]\prod_{l=1}^{n'}\left[-\frac{i}{2}\int \mathrm{d}^4z'_l\, \varepsilon^{*\mu'_l\nu'_l}_{\zeta'_l}(\boldsymbol{k}'_l) g^*_{\boldsymbol{k}'_l}(z'_l)\Box_{z'_l}\right] \\
&\quad\quad \prod_{i=1}^{m}\left[-\frac{1}{4}\int \mathrm{d}^4x_i \mathrm{d}^4y_i\Big(\left(E_{\boldsymbol{p}_i}/E_{\boldsymbol{q}_i}\right)^{1/2}\varepsilon_{\lambda_i}{}^{\alpha_i\beta_i}(\boldsymbol{p}_i)\varepsilon_{\lambda_i}{}^{\gamma_i\delta_i}(\boldsymbol{q}_i) g_{\boldsymbol{p}_i}(x_i) f_{\boldsymbol{q}_i}(y_i) - (\boldsymbol{p}_i \leftrightarrow \boldsymbol{q}_i)\Big)\Box_{x_i}\Box_{y_i}\right]\prod_{j=1}^{n}\left[-\,i\int \mathrm{d}^4z_j\, \varepsilon_{\zeta_j}{}^{\mu_j\nu_j}(\boldsymbol{k}_j) f_{\boldsymbol{k}_j}(z_j)\Box_{z_j}\right] \\
&\quad\quad \langle 0|\, T H_{\alpha'_1\beta'_1}(x'_1)\cdots H_{\alpha'_{m'}\beta_{m'}}(x'_{m'}) H_{\gamma'_1\delta'_1}(y'_1)\cdots H_{\gamma'_{m'}\delta'_{m'}}(y'_{m'}) H_{\mu'_1\nu'_1}(z'_1)\cdots H_{\mu'_{n'}\nu'_{n'}}(z'_{n'}) \\
&\quad\quad\quad H_{\alpha_1\beta_1}(x_1)\cdots H_{\alpha_m\beta_m}(x_m) H_{\gamma_1\delta_1}(y_1)\cdots H_{\gamma_m\delta_m}(y_m) H_{\mu_1\nu_1}(z_1)\cdots H_{\mu_n\nu_n}(z_n)\,|0\rangle
\end{aligned}$$

Full propagators

$$\Omega^{-1}_{AB}(p) = \begin{pmatrix} \Omega^{-1}_{\text{boson}} & 0 \\ 0 & \Omega^{-1}_{\text{ghost}} \end{pmatrix}_{AB}$$

$$\Omega^{-1}_{\text{boson}} = \begin{array}{ccccccc}
 & h^{\gamma\delta} & H^{\gamma\delta} & A^{\gamma} & b^{\gamma} & B^{\gamma} & B \\
h^{\mu\nu} & -\frac{G^{\mu\nu\gamma\delta}}{2p^2} & G^{\mu\nu\gamma\delta} & 0 & -\frac{i(\eta^{\nu\gamma}p^{\mu}+\eta^{\mu\gamma}p^{\nu})}{p^2} & 0 & -\frac{2p^{\mu}p^{\nu}}{p^4}-\frac{\eta^{\mu\nu}}{p^2} \\
H^{\mu\nu} & & 0 & 0 & 0 & -\frac{i(\eta^{\nu\gamma}p^{\mu}+\eta^{\mu\gamma}p^{\nu})}{p^2} & 0 \\
A^{\mu} & & & -\frac{\eta^{\mu\gamma}}{p^2} & 0 & \frac{\eta^{\mu\gamma}}{p^2} & -\frac{2ip^{\mu}}{p^2} \\
b^{\mu} & & & & 0 & 0 & 0 \\
B^{\mu} & & (\text{h.c.}) & & & 0 & 0 \\
B & & & & & & 0
\end{array}$$

$$\Omega^{-1}_{\text{ghost}} = \begin{array}{ccccccc}
 & c^{\gamma} & C^{\gamma} & C & \bar{c}^{\gamma} & \bar{C}^{\gamma} & \bar{C} \\
c^{\mu} & 0 & 0 & 0 & -\frac{i\eta^{\mu\gamma}}{p^2} & 0 & -\frac{p^{\mu}}{p^4} \\
C^{\mu} & & 0 & 0 & 0 & -\frac{i\eta^{\mu\gamma}}{p^2} & -\frac{2p^{\mu}}{p^2} \\
C & & & 0 & 0 & 0 & -\frac{i}{p^2} \\
\bar{c}^{\mu} & & & & 0 & 0 & 0 \\
\bar{C}^{\mu} & & (\text{h.c.}) & & & 0 & 0 \\
\bar{C} & & & & & & 0
\end{array}$$

$$G^{\mu\nu\gamma\delta} = \frac{1}{p^2}\left(\frac{1}{2}\eta^{\mu\nu}\eta^{\gamma\delta} - \delta^{\mu\nu\gamma\delta}\right) + \frac{1}{2p^4}\left(\eta^{\mu\gamma}p^{\nu}p^{\delta} + \eta^{\mu\delta}p^{\nu}p^{\gamma} + \eta^{\nu\gamma}p^{\mu}p^{\delta} + \eta^{\nu\delta}p^{\mu}p^{\gamma}\right)$$

More LSZ

$$f_{\boldsymbol{p}}(x) = \frac{1}{\sqrt{2EV}} e^{ipx}$$

$$\Box f_{\boldsymbol{p}}(x) = 0$$

$$g_{\boldsymbol{p}}(x) = -\frac{1}{2\sqrt{2EV}} \left(\frac{1}{2E^2} + \frac{ix^0}{E} \right) e^{ipx}$$

$$\Box^2 g_{\boldsymbol{p}}(x) = 0 \qquad \Box\, g_{\boldsymbol{p}}(x) = f_{\boldsymbol{p}}(x)$$

$$h_{\boldsymbol{p}}(x) = \frac{1}{8\sqrt{2EV}} \left(\frac{5}{4E^4} + \frac{2ix^0}{E^3} - \frac{(x^0)^2}{E^2} \right) e^{ipx}$$

$$\Box^3 h_{\boldsymbol{p}}(x) = 0 \qquad \Box\, h_{\boldsymbol{p}}(x) = g_{\boldsymbol{p}}(x)$$

$$D^{(+)}(x-y) = \sum_{\boldsymbol{p}} f_{\boldsymbol{p}}(x) f^*_{\boldsymbol{p}}(y)$$

$$D = D^{(+)} - D^{(-)}$$

$$E^{(+)}(x-y) = \sum_{\boldsymbol{p}} \left(f_{\boldsymbol{p}}(x) g^*_{\boldsymbol{p}}(y) + g_{\boldsymbol{p}}(x) f^*_{\boldsymbol{p}}(y) \right)$$

$$E = E^{(+)} - E^{(-)}$$

$$F^{(+)}(x-y) = \sum_{\boldsymbol{p}} \left(f_{\boldsymbol{p}}(x) h^*_{\boldsymbol{p}}(y) + h_{\boldsymbol{p}}(x) f^*_{\boldsymbol{p}}(y) + g_{\boldsymbol{p}}(x) g^*_{\boldsymbol{p}}(y) \right)$$

$$F = F^{(+)} - F^{(-)}$$

$$D_F(x) = \theta(x^0) D^{(+)}(x) + \theta(-x^0) D^{(-)}(x) = -i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ipx}}{p^2 - i\epsilon}$$

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