Gravitational Causality

the Eikonal Approximation, and the Photon Self-Stress

2108.05896 [B. Bellazzini, GI, M. Lewandowski, F. Sgarlata]

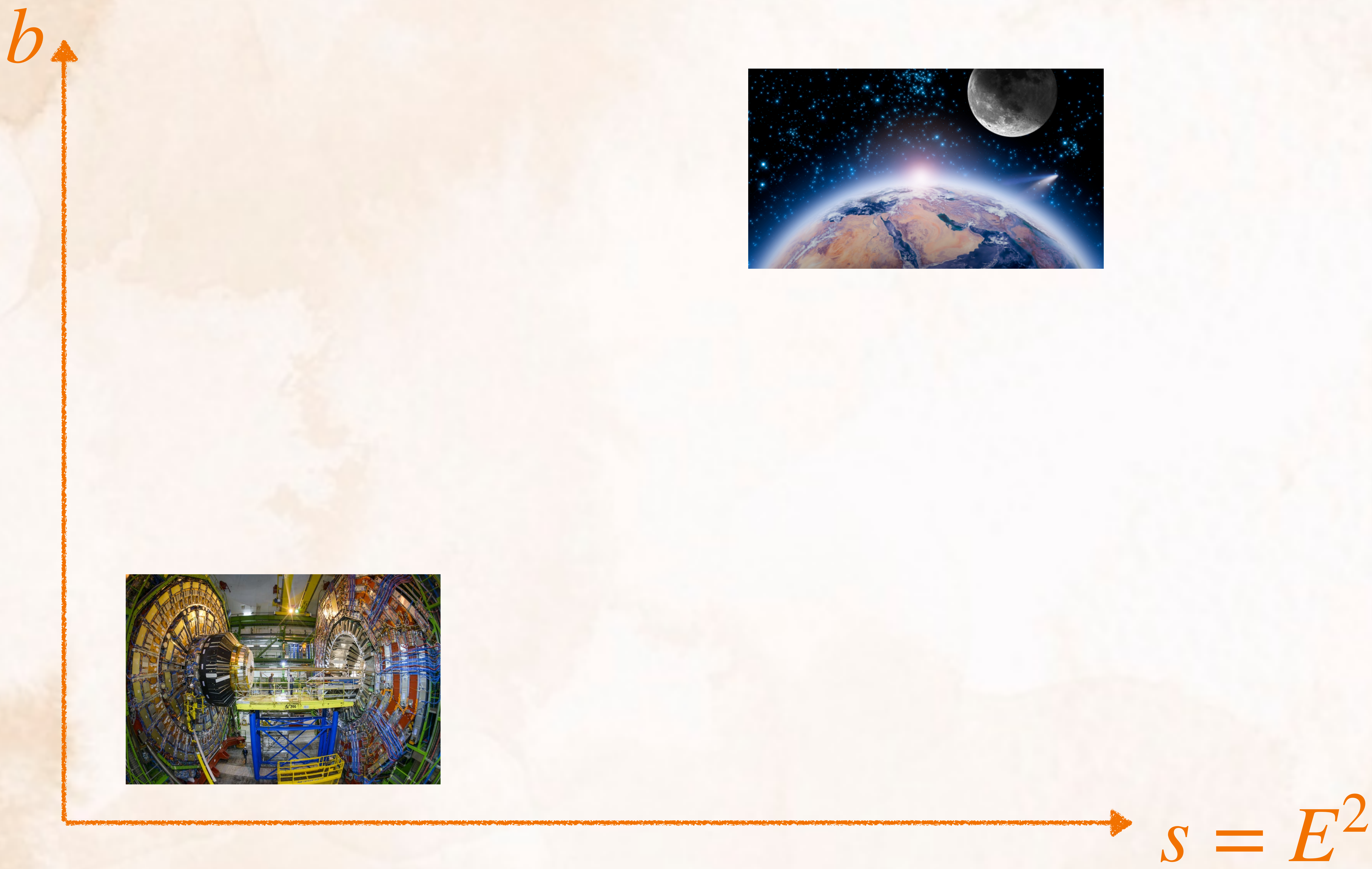
Giulia Isabella

Planck 2022

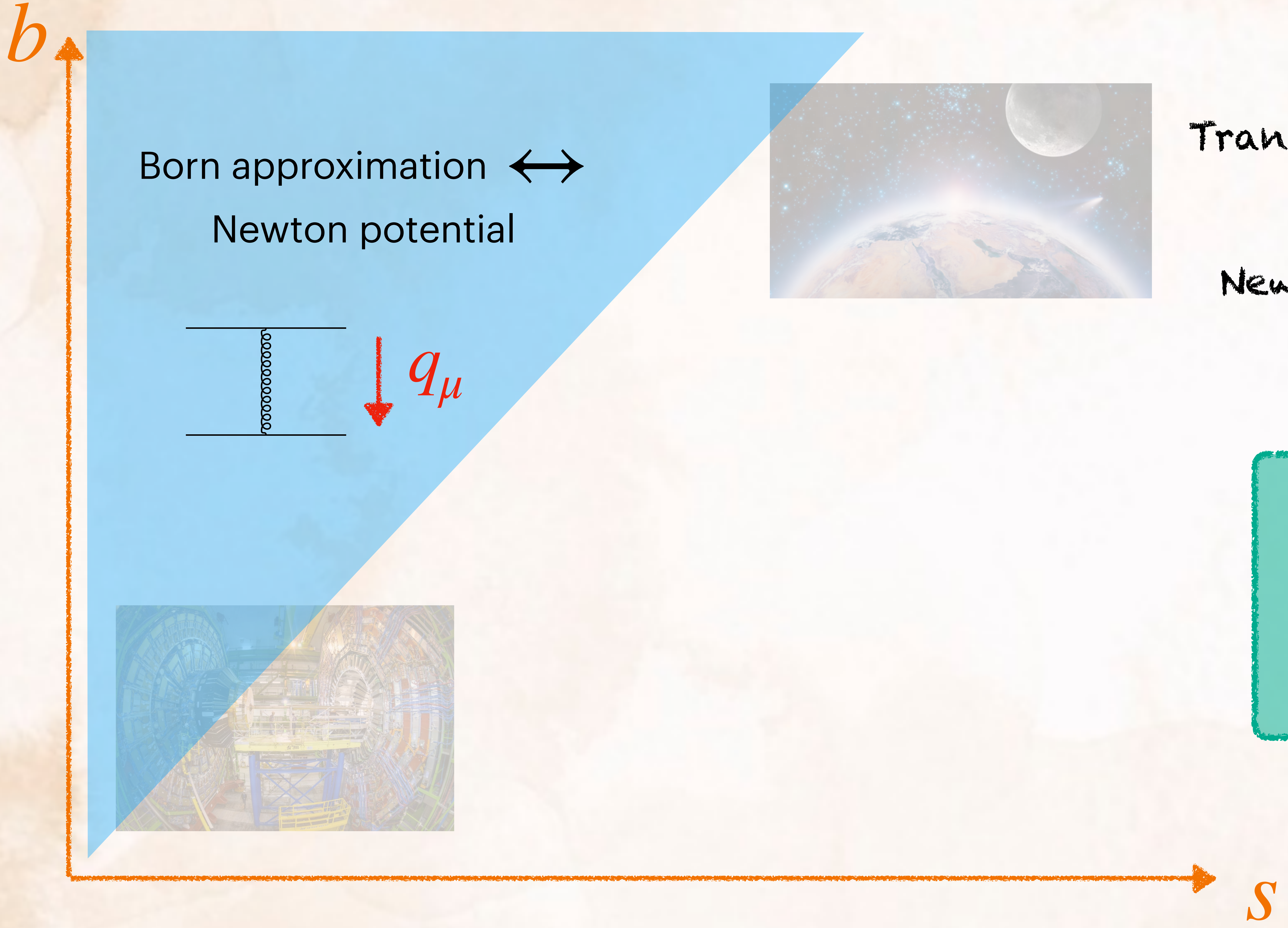
2nd June 2022

What do we know about the gravitational S-Matrix?

Validity of gravitational amplitudes

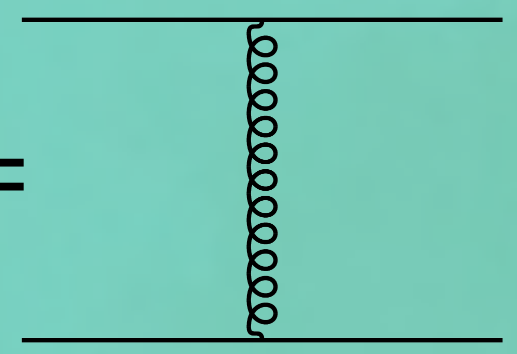


Validity of gravitational amplitudes

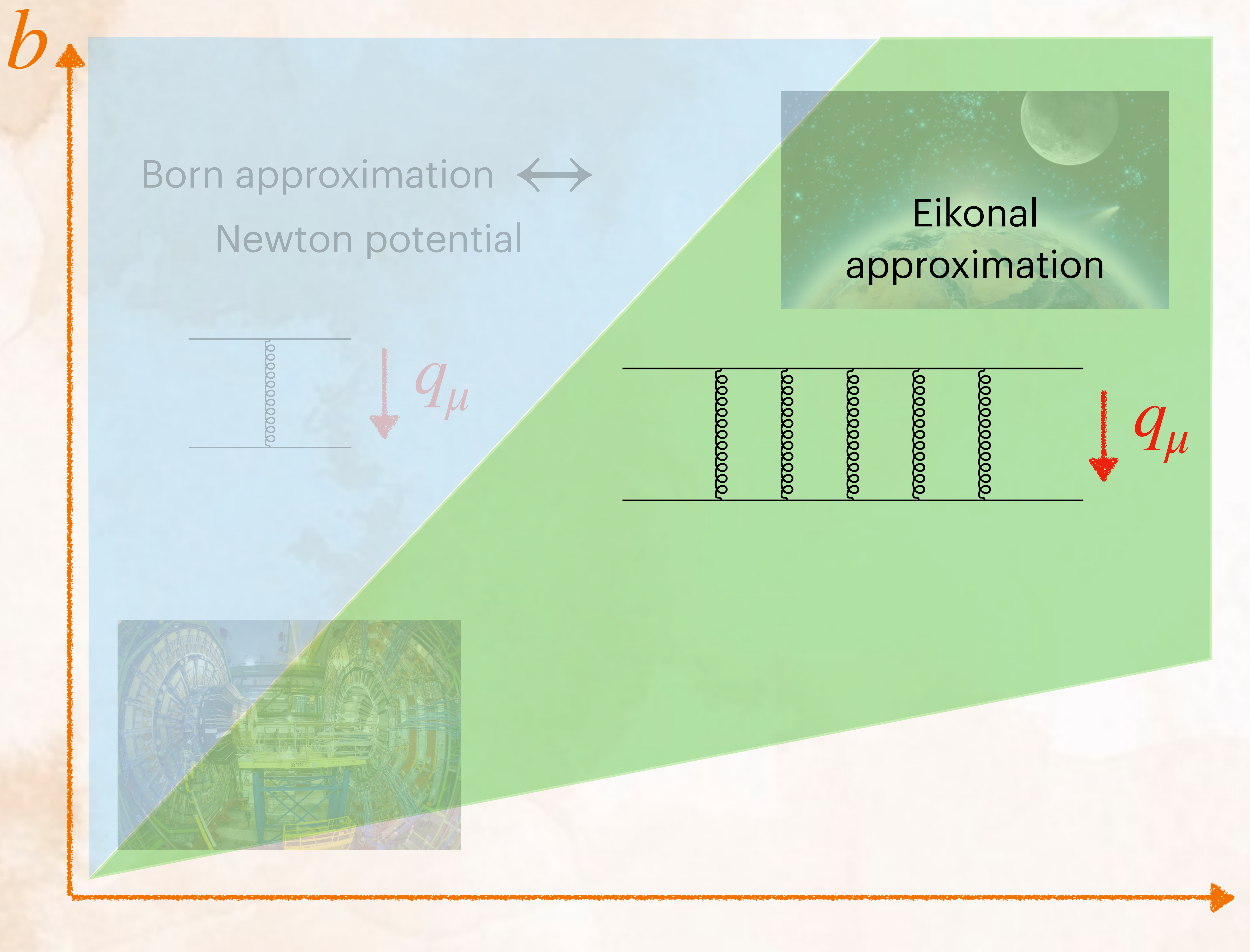


Transferred momentum : $t = q^2$

Newton's constant : $G = \frac{1}{8\pi M_{Pl}^2}$

$\mathcal{A}_{tree}(s,t) =$  $= 8\pi \frac{Gs^2}{t}$

Validity of gravitational amplitudes



$GE_{CM}^2 = Gs \gg 1$ ⟺ Transplanckian scattering

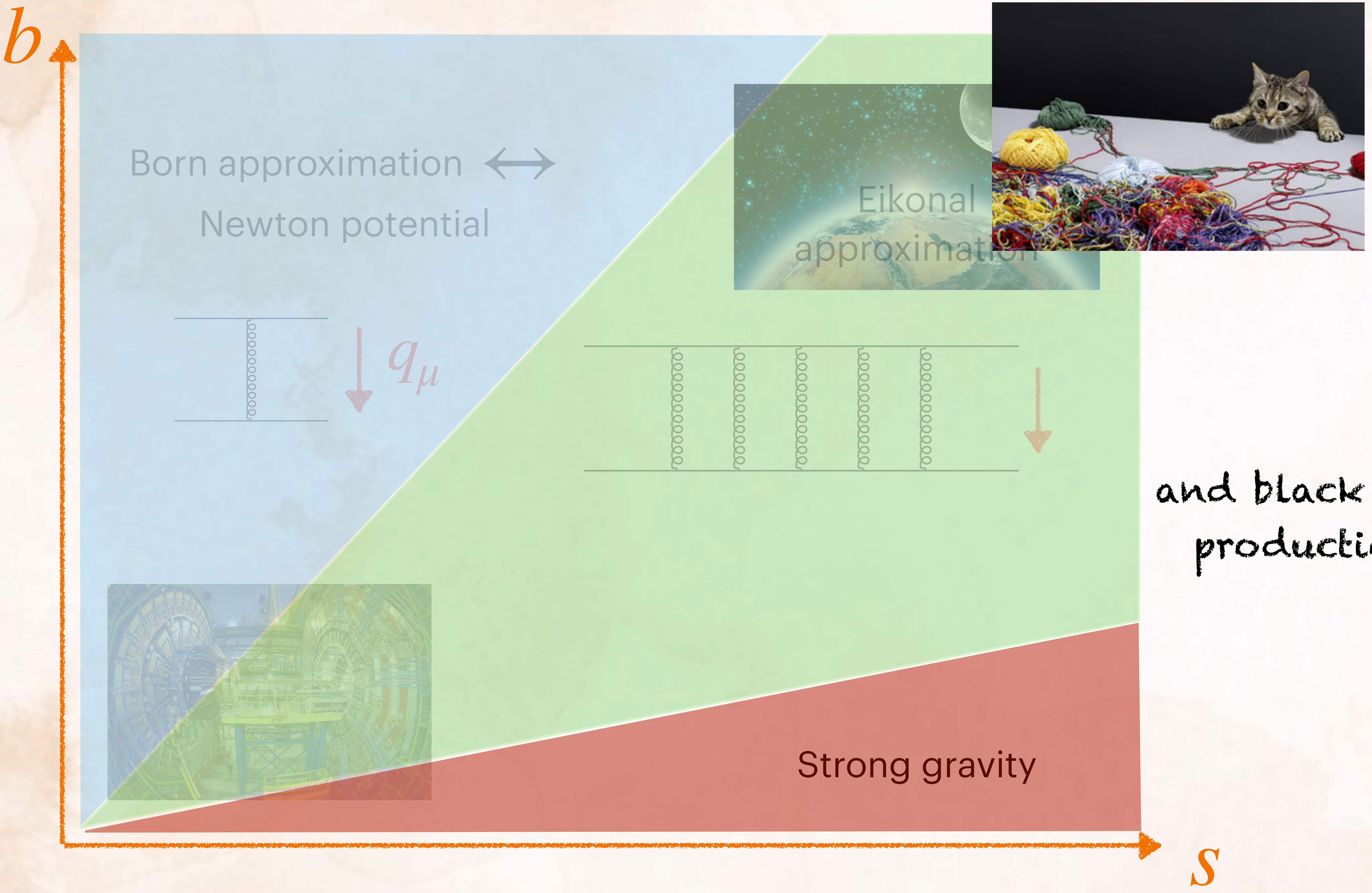
Large impact parameter b

⟺ Forward scattering $\frac{t}{s} \ll 1$

$$\mathcal{A}_{eik}(s,t) = \frac{1}{4s}\int \left[e^{2i\delta(s,b)} - 1\right] e^{-i\vec{b}\cdot\vec{q}} d^2b$$

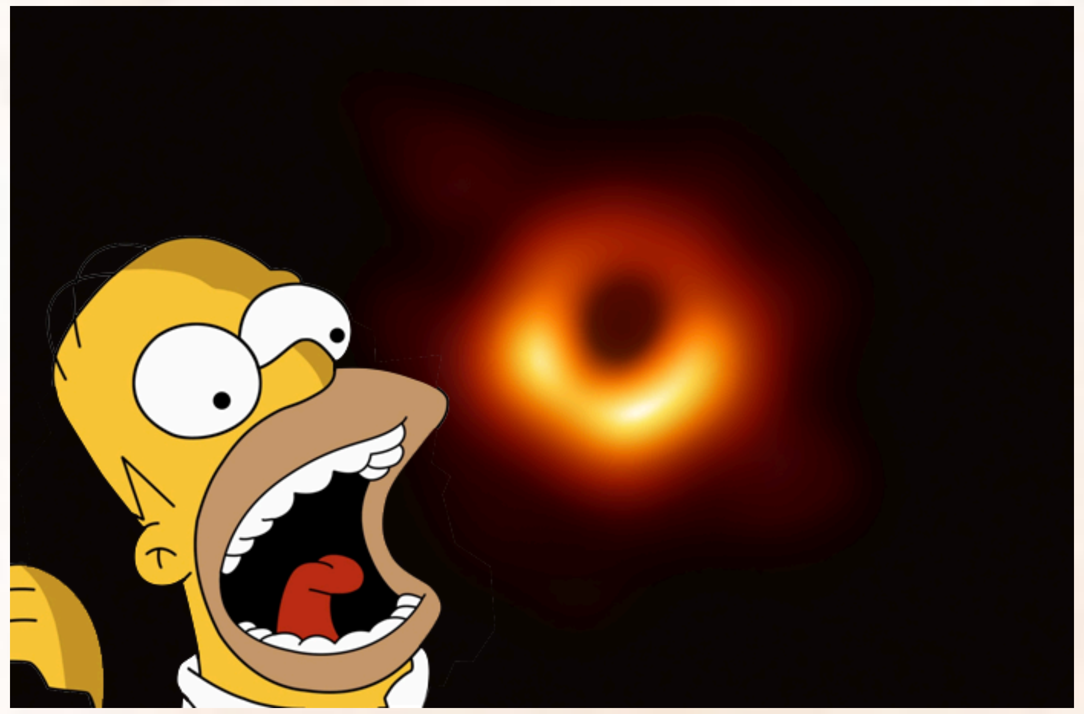
Amati, Ciafaloni, Veneziano
90's

Validity of gravitational amplitudes



New degrees of freedom (string theory, ...)

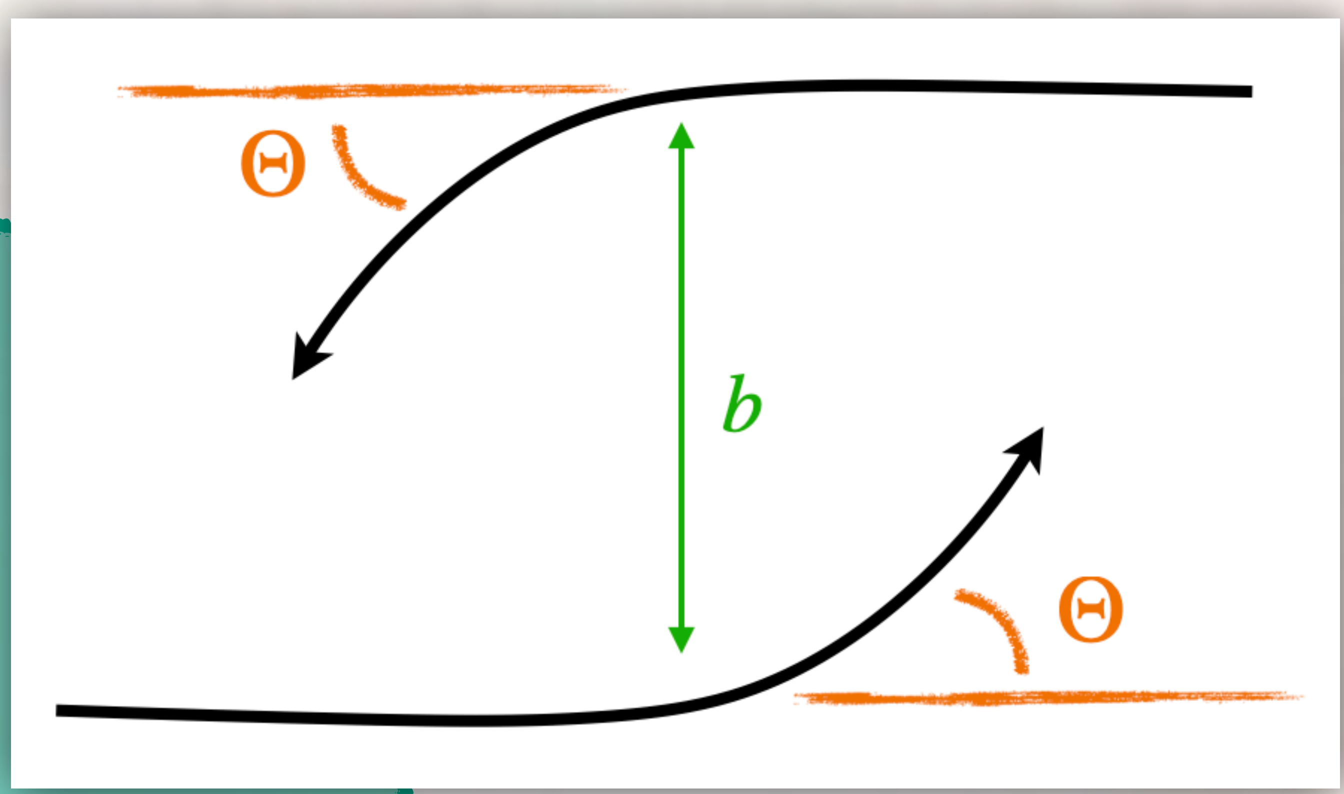
and black hole production.

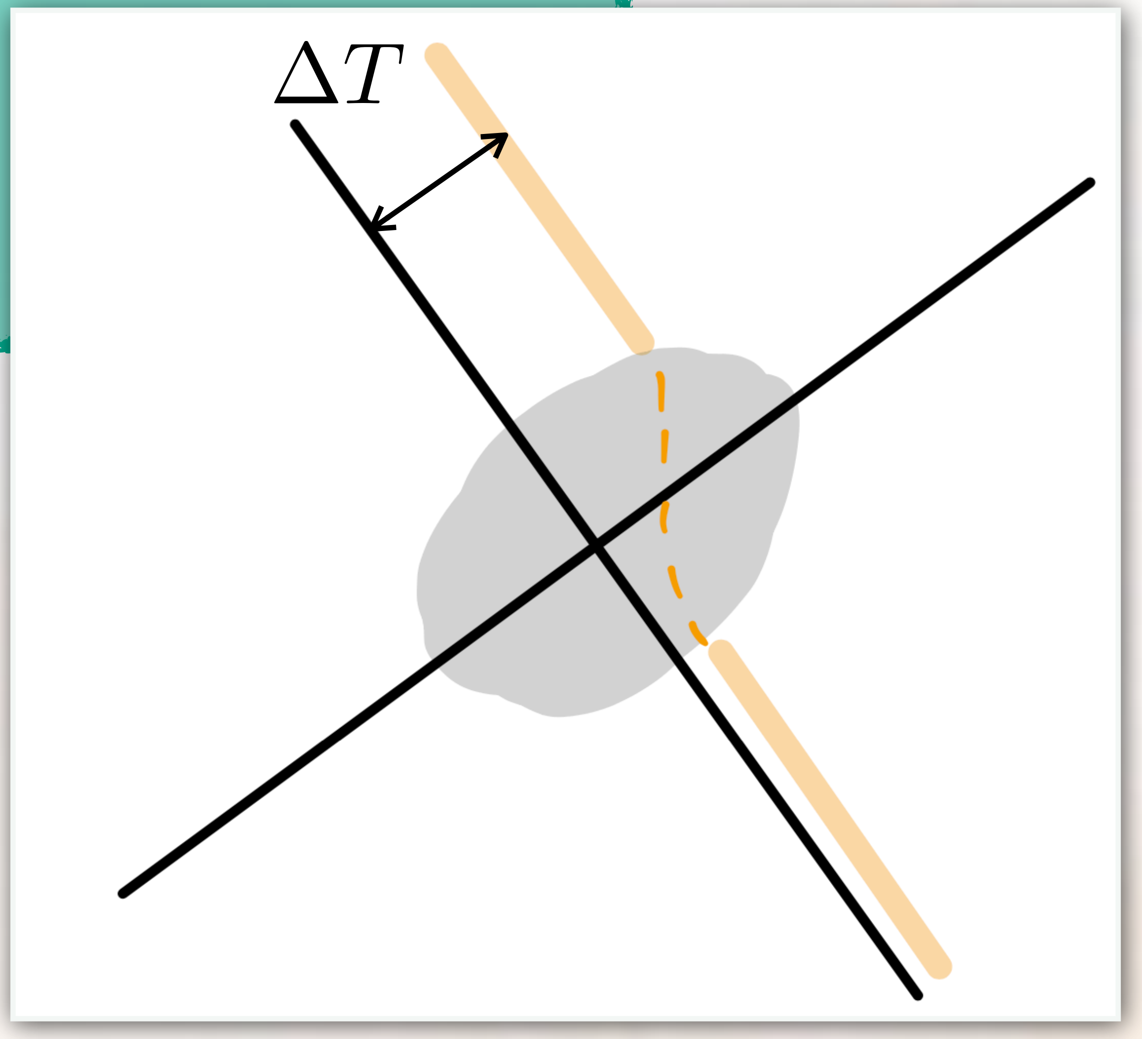


Why do we like the phase shift?

$$\vec{\Theta} = \frac{1}{E}\frac{\partial\delta(b,s)}{\partial\vec{b}}$$

$$\Delta T = \frac{\partial\delta(b,s)}{\partial E}$$





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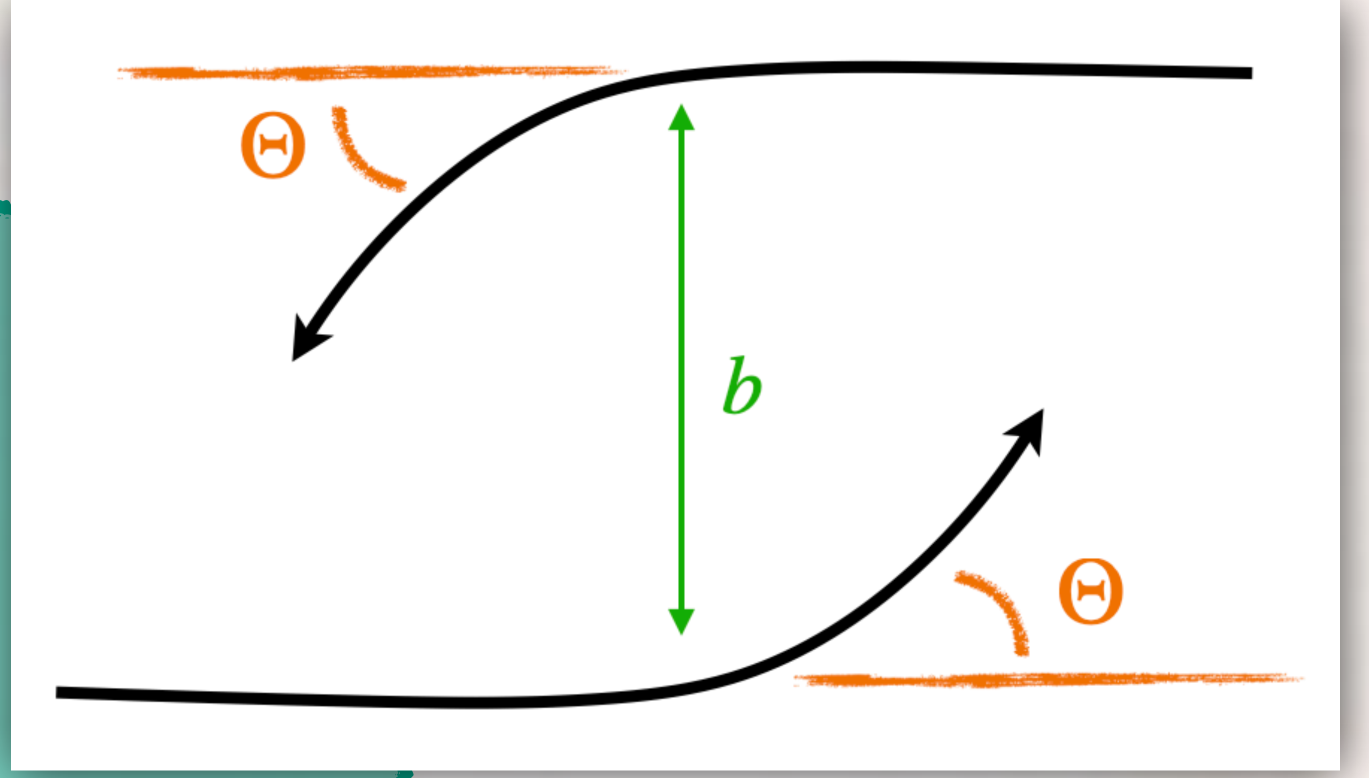

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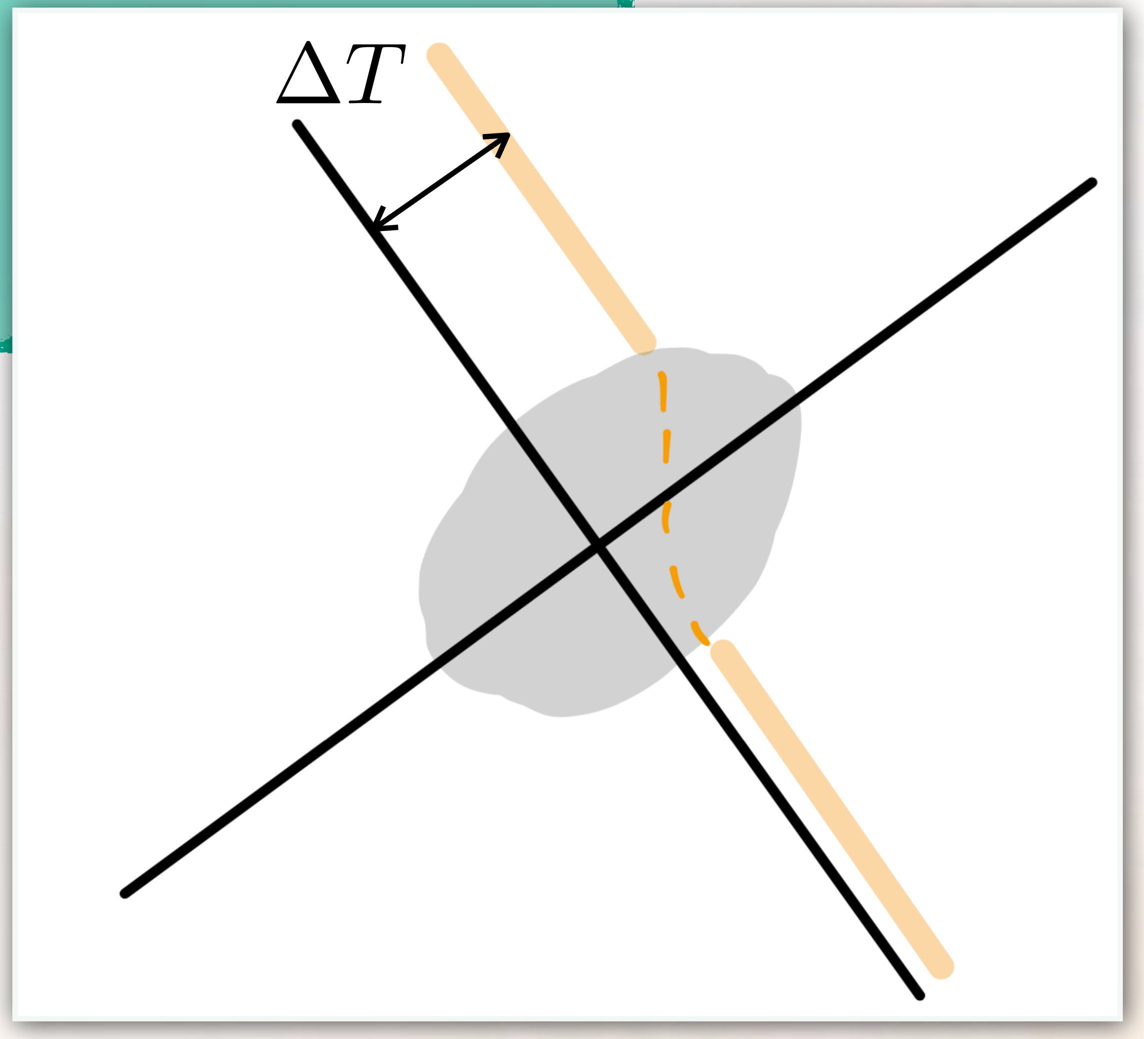


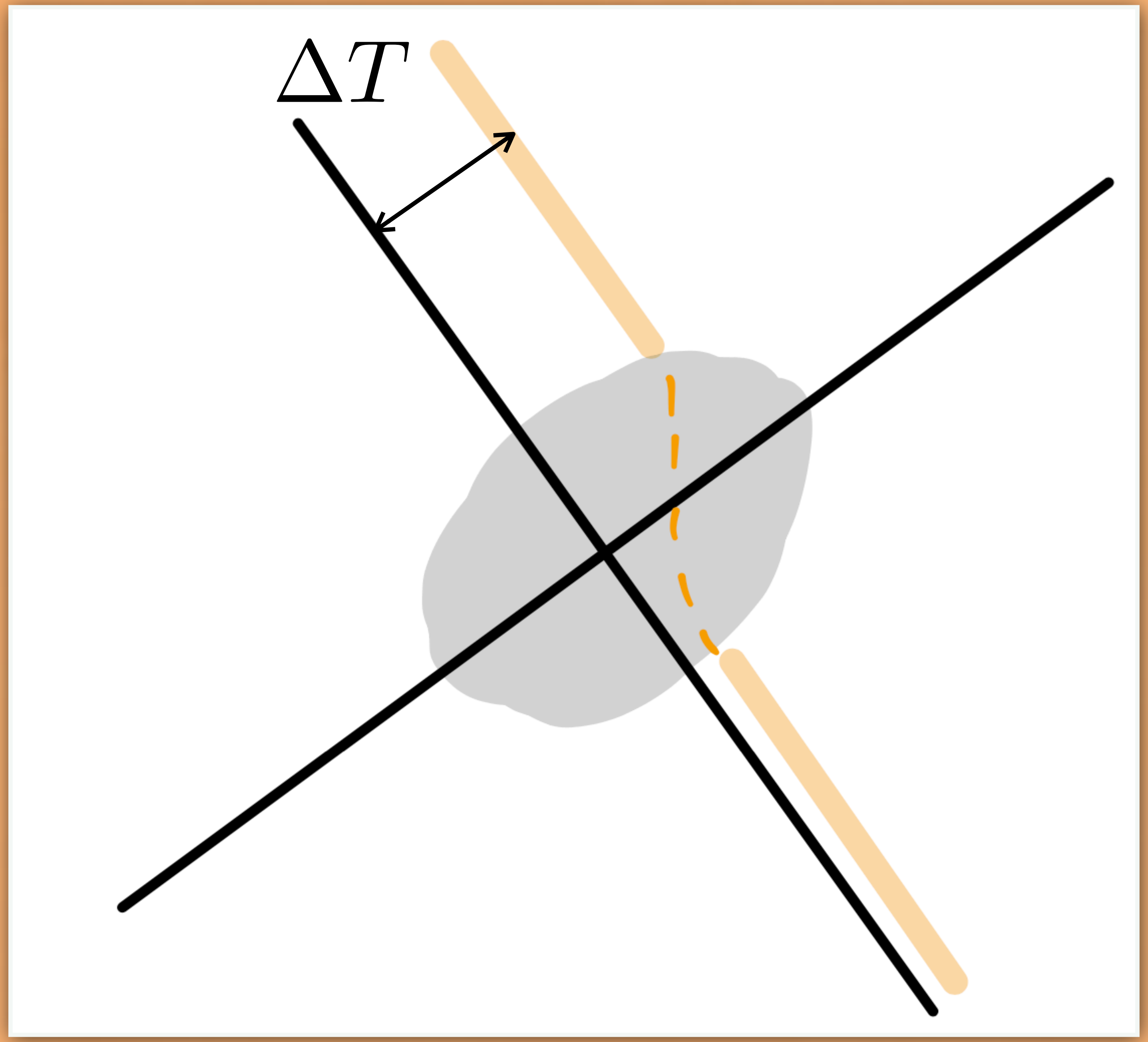
Shapiro time-delay

$$\Delta T_{shap} = \frac{\partial \delta_0(b,s)}{\partial E} = -\frac{E}{4\pi M_{Pl}^2}\log b/b_{IR}$$

Gravitational Causality

And the photon self-stress

Asymptotic Causality



Wald & Gao
gr-qc/0007021

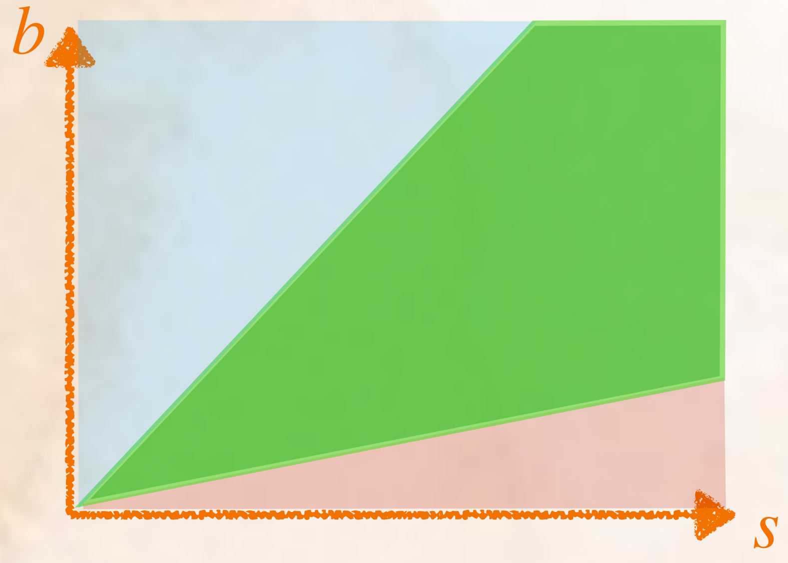
$$\Delta T = \frac{\partial \delta(s, \vec{b})}{\partial E} \geq 0$$

For all species

Resolvability

$$|\Delta T| \gg 1/E \quad \rightarrow \quad \delta \gg 1$$

Transplanckian scattering



A tale of scales

Hierarchy in the transplanckian regime

Kinematic scales

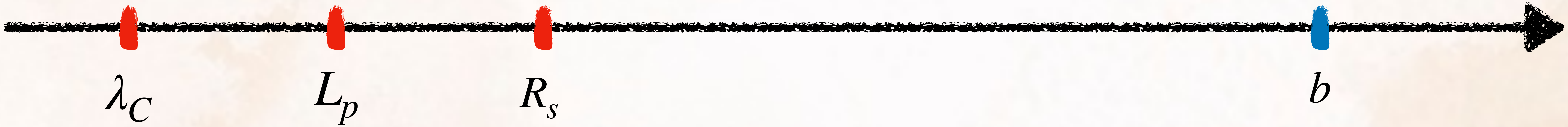
$$\lambda_C = \frac{1}{\sqrt{s}} \quad , b$$

Planck length

$$L_p \sim \frac{1}{M_{Pl}}$$

Schwarzschild Radius

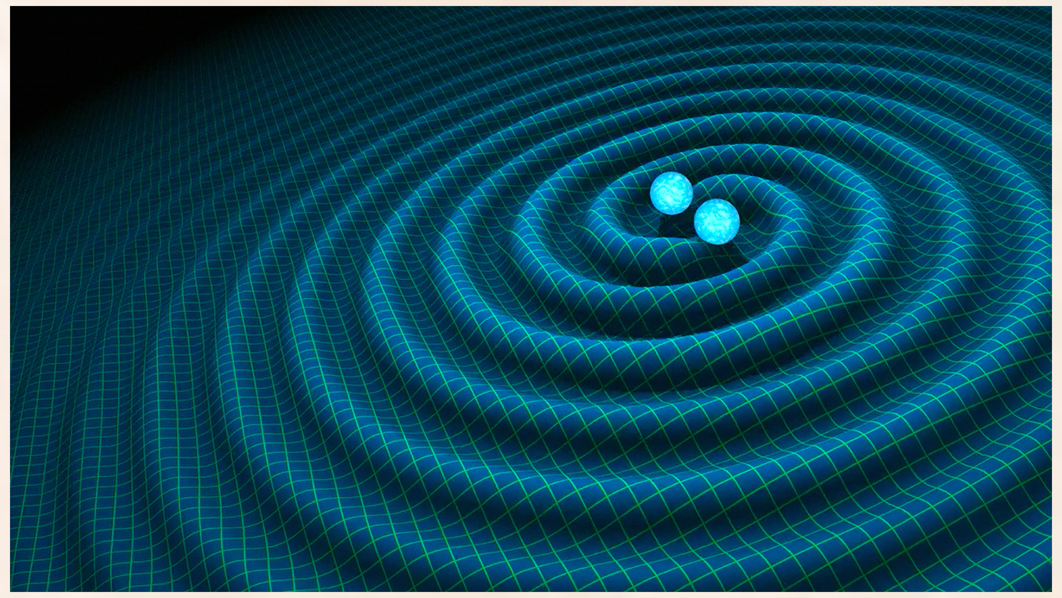
$$R_s = 2G\sqrt{s} = \frac{\sqrt{s}}{M_{Pl}} L_p$$

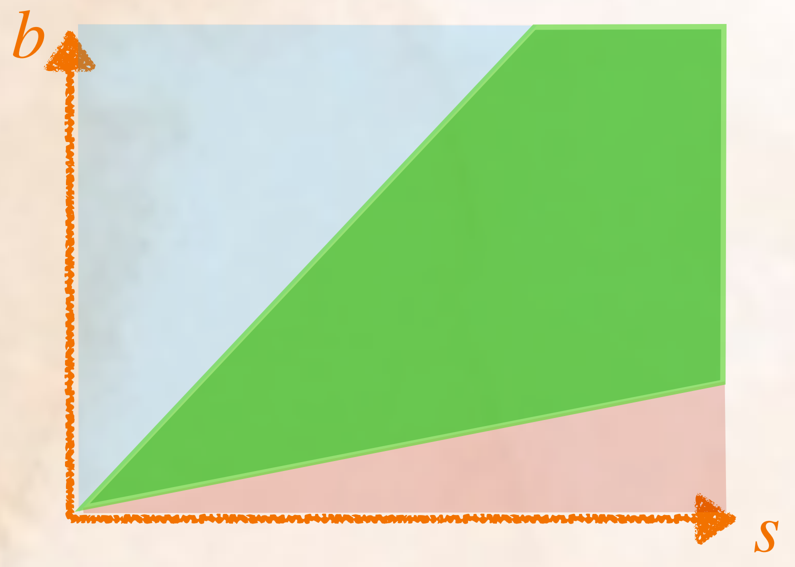


$$S_{eik}(s, \overrightarrow{b}) = e^{2i(\delta_0(s,\overrightarrow{b})+\delta_1(s,\overrightarrow{b})+\delta_2(s,\overrightarrow{b})+\ldots)}$$

Dimensionless small parameters

$$\left(\frac{R_s}{b}\right)^n \quad \left(\frac{L_p}{b}\right)^n$$





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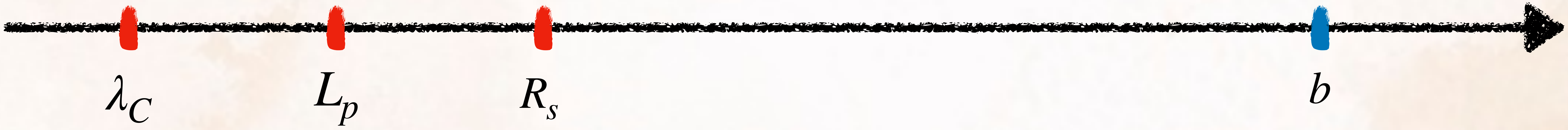

$$S_{eik}(s, \vec{b}) = e^{2i(\delta_0(s,\vec{b})+\delta_1(s,\vec{b})+\delta_2(s,\vec{b})+\ldots)}$$

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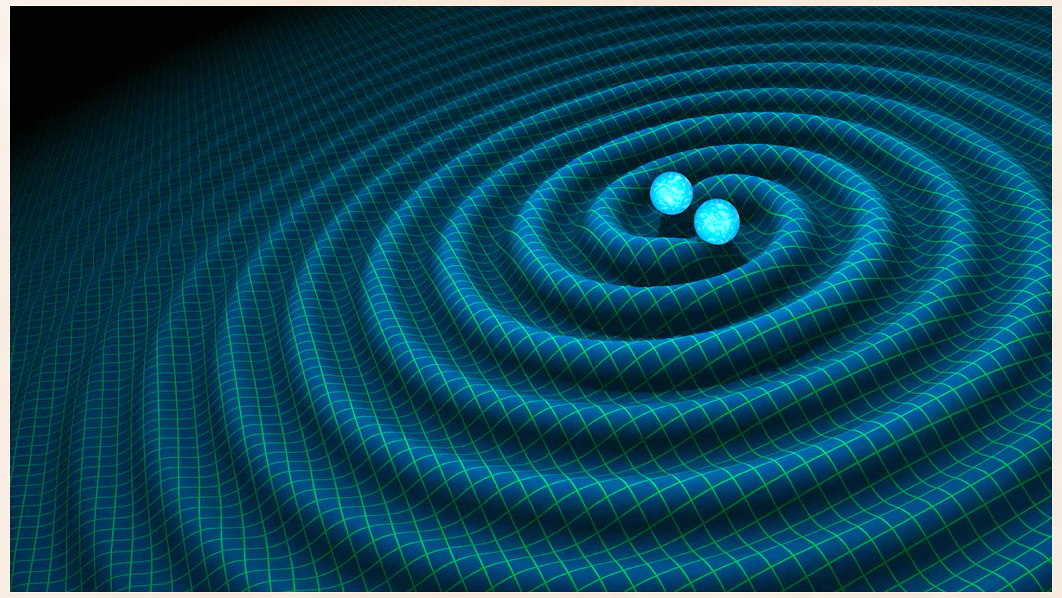
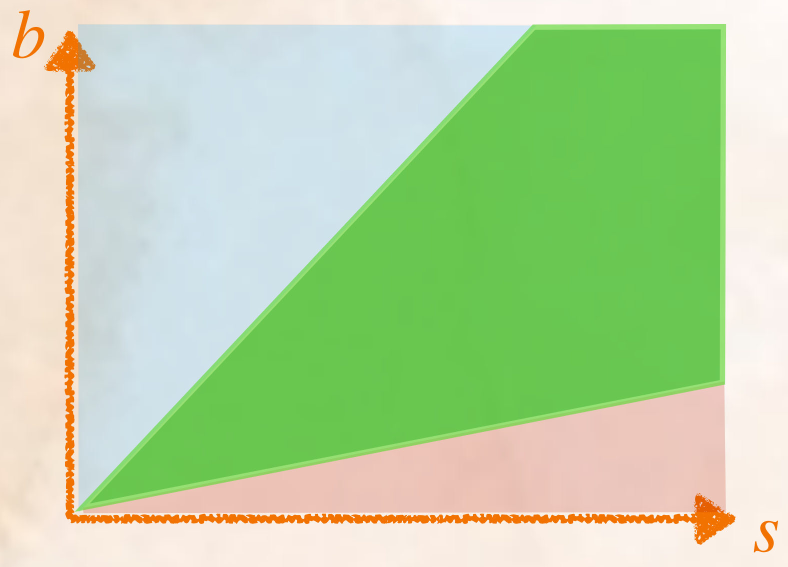


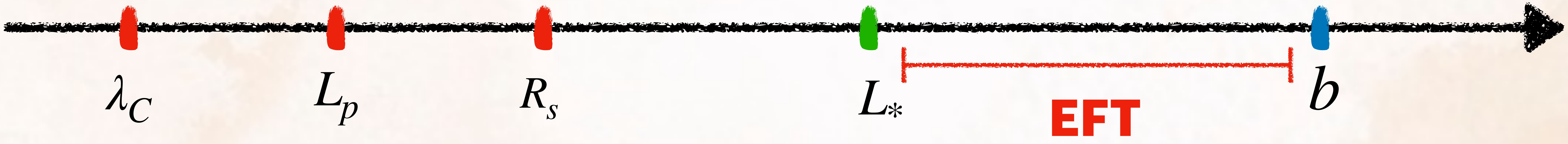


A tale of scales

What about new physics?

New d.o.f.

$$L_* \sim \frac{1}{M_*}$$



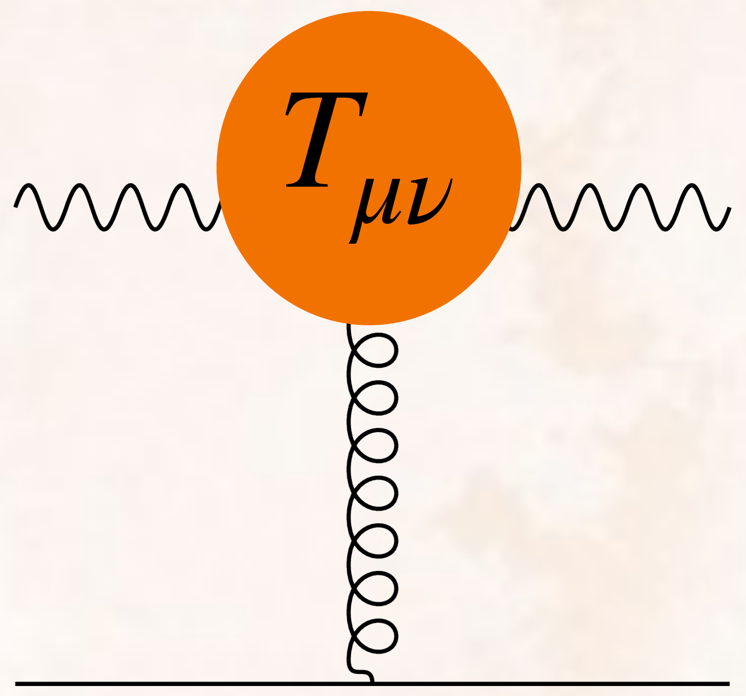
$$S_{eik}(s, \vec{b}) = e^{2i(\delta_0(s,\vec{b}) + \delta_1(s,\vec{b}) + \delta_2(s,\vec{b}) + \ldots)}$$

Dimensionless small parameters

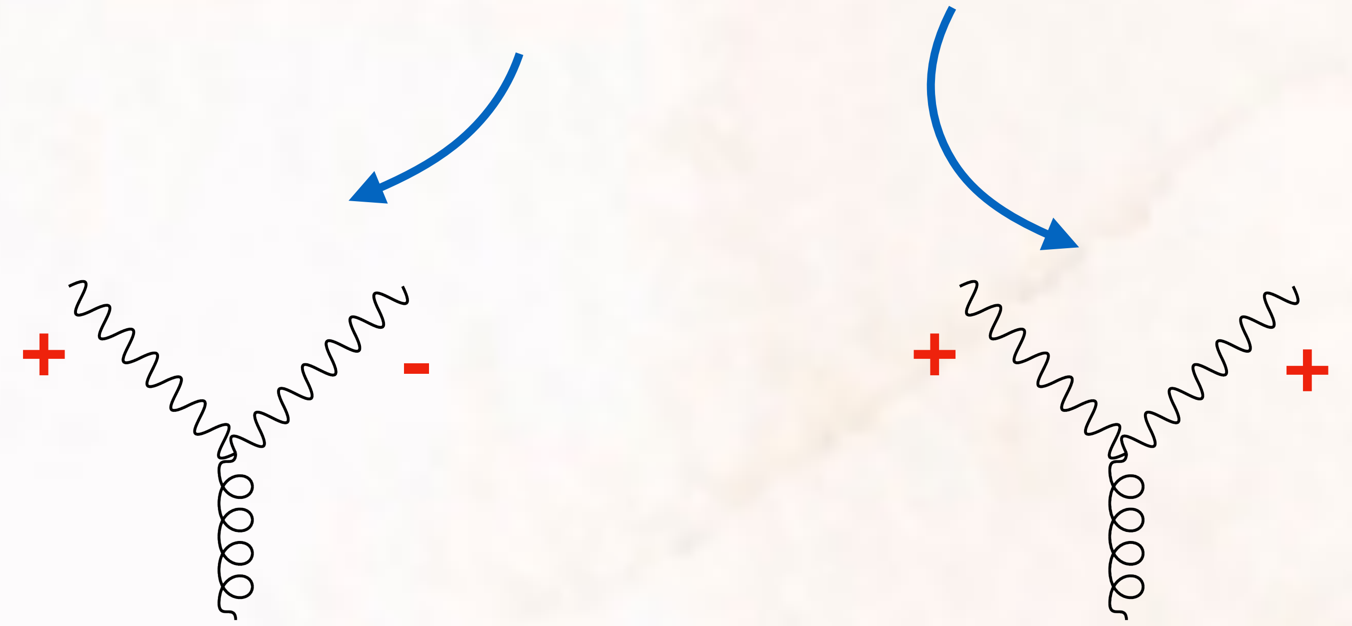
$$\left(\frac{R_s}{b}\right)^n \quad \left(\frac{L_p}{b}\right)^n \quad \left(\frac{L_*}{b}\right)^n$$

New physics effects

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597



$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}^2 + \frac{\alpha}{\Lambda^2}F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}\right]$$



Helicity preserving

Helicity flipping

New physics effects

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597

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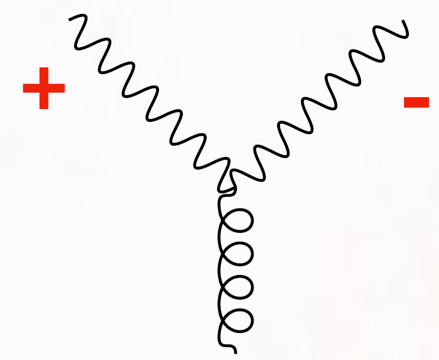
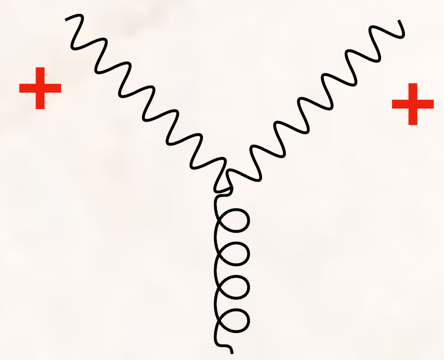
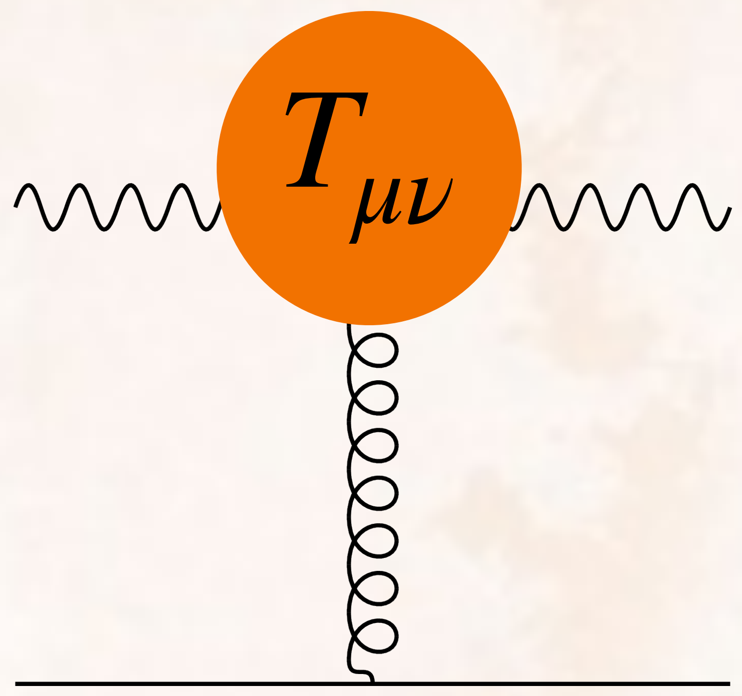






+ -   + +

$$\mathcal{M}_{eik} = \frac{s^2}{M_{pl}^2\, t}\begin{pmatrix} 1 & -\frac{4\alpha q_+^2}{\Lambda^2} \\ -\frac{4\alpha q_-^2}{\Lambda^2} & 1 \end{pmatrix}$$

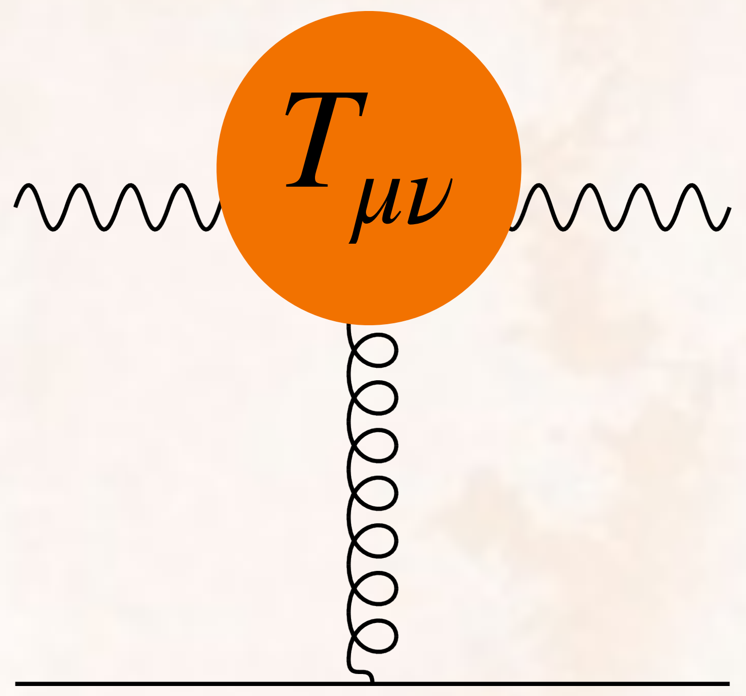
- -   - +

$$q_\pm \propto q_1 \pm i q_2$$

New physics effects

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597

$$\delta(s, \vec{b}) = \frac{1}{4s}\int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \mathcal{M}_{eik}(s, \vec{q}_\perp)$$



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$q_\pm \propto q_1 \pm i q_2$

$$\delta_\pm(s, \vec{b}) = Gs\left(-\log\frac{b}{b_{IR}} \pm \frac{\alpha}{m^2 b^2}\right)$$

It exists an impact parameter b such that asymptotic causality is violated*

$$\Delta T = \frac{\partial \delta(s, \vec{b})}{\partial E} \geq 0$$

What have we learnt?

Asymptotic causality correctly predicts the breakdown of the theory

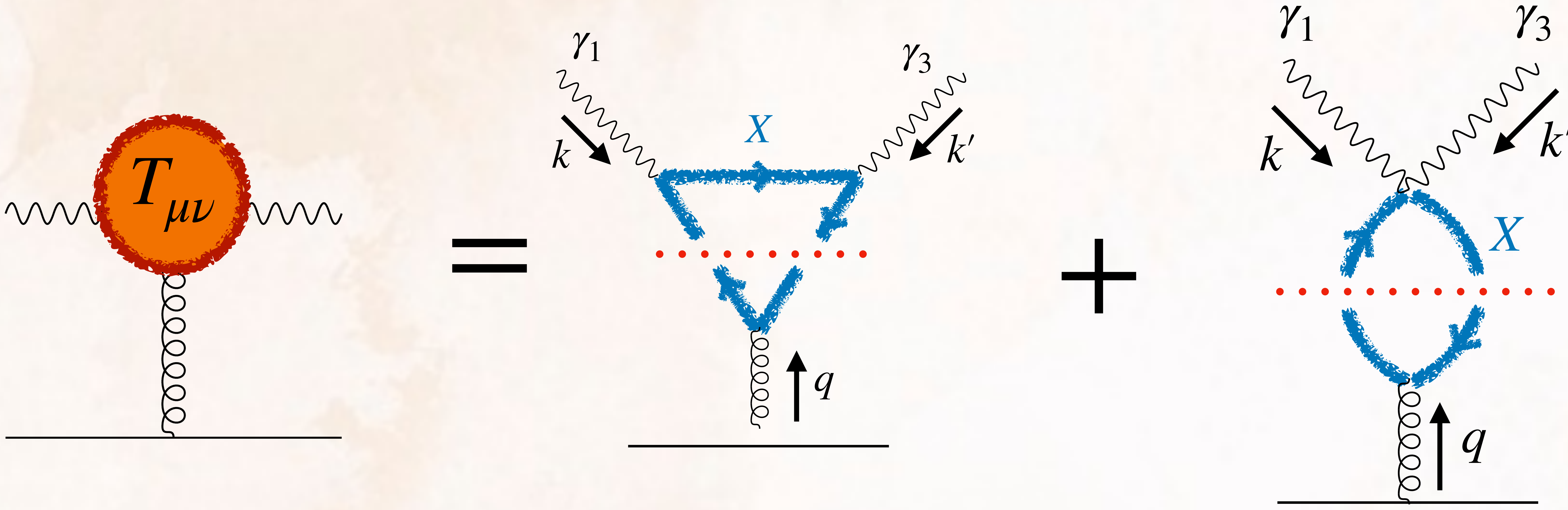
→ EFT

(1407.5597) shows that tree-level solution to causality issue requires tower of higher spin

How things change with loops ?

A loop solution to causality violation ?

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}^2 - |D_\mu\phi|^2 + \ldots\right]$$

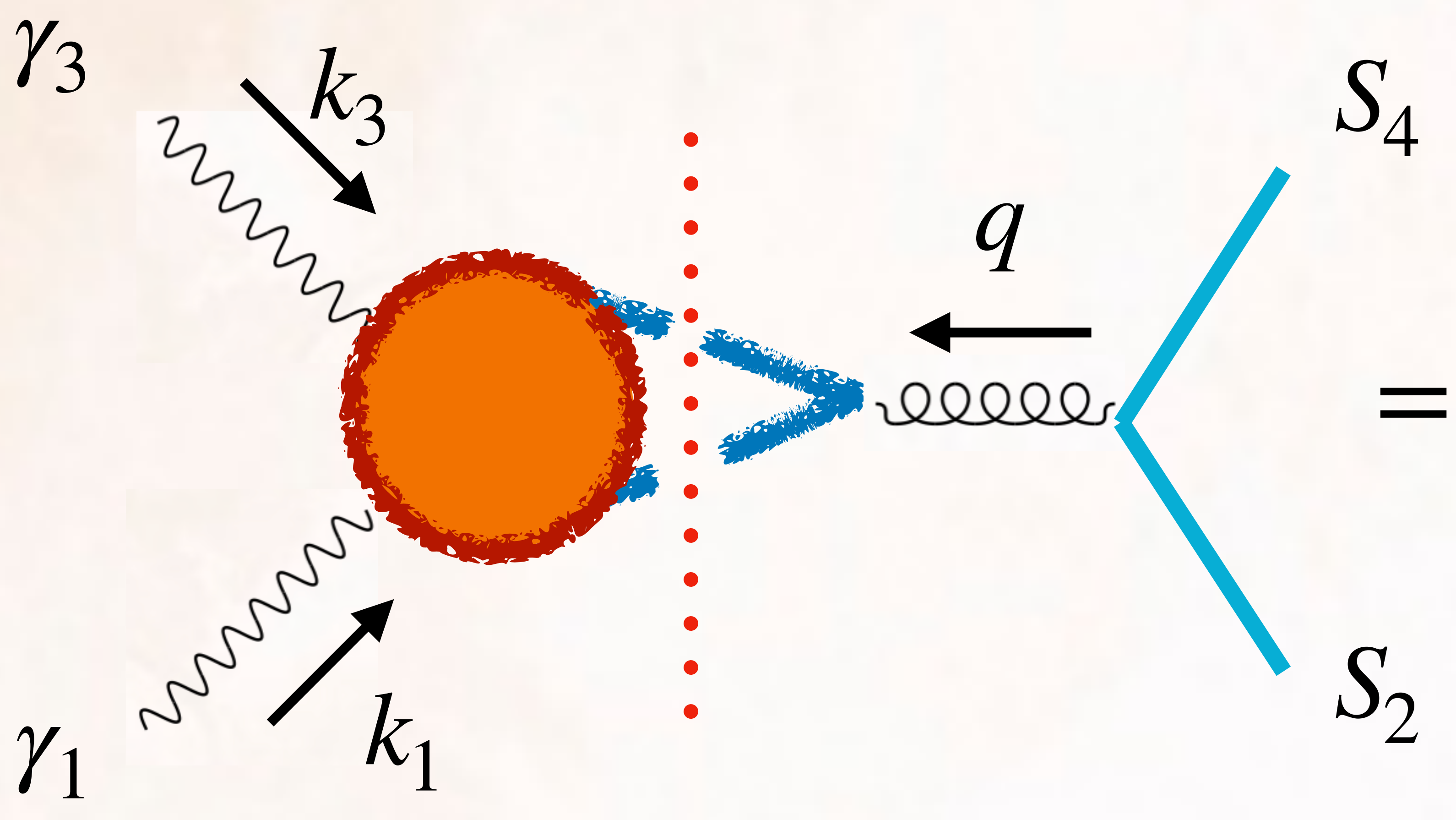


$$\rightarrow \mathcal{M}_{eik} = \frac{s^2}{M_{pl}^2 t}\begin{pmatrix} F_1(t) & -4q_+^2F_3(t) \\ -4q_-^2F_3(t) & F_1(t) \end{pmatrix}$$

$q_\pm \propto q_1 \pm iq_2$

Details of the calculation

$$\mathrm{Disc}\mathscr{A}(\gamma_1\gamma_3 \to SS) =$$



$$= i\int d\Pi_{56}\mathscr{A}(\gamma_1\gamma_3 \to 56)\mathscr{A}(56 \to S_2S_4)$$

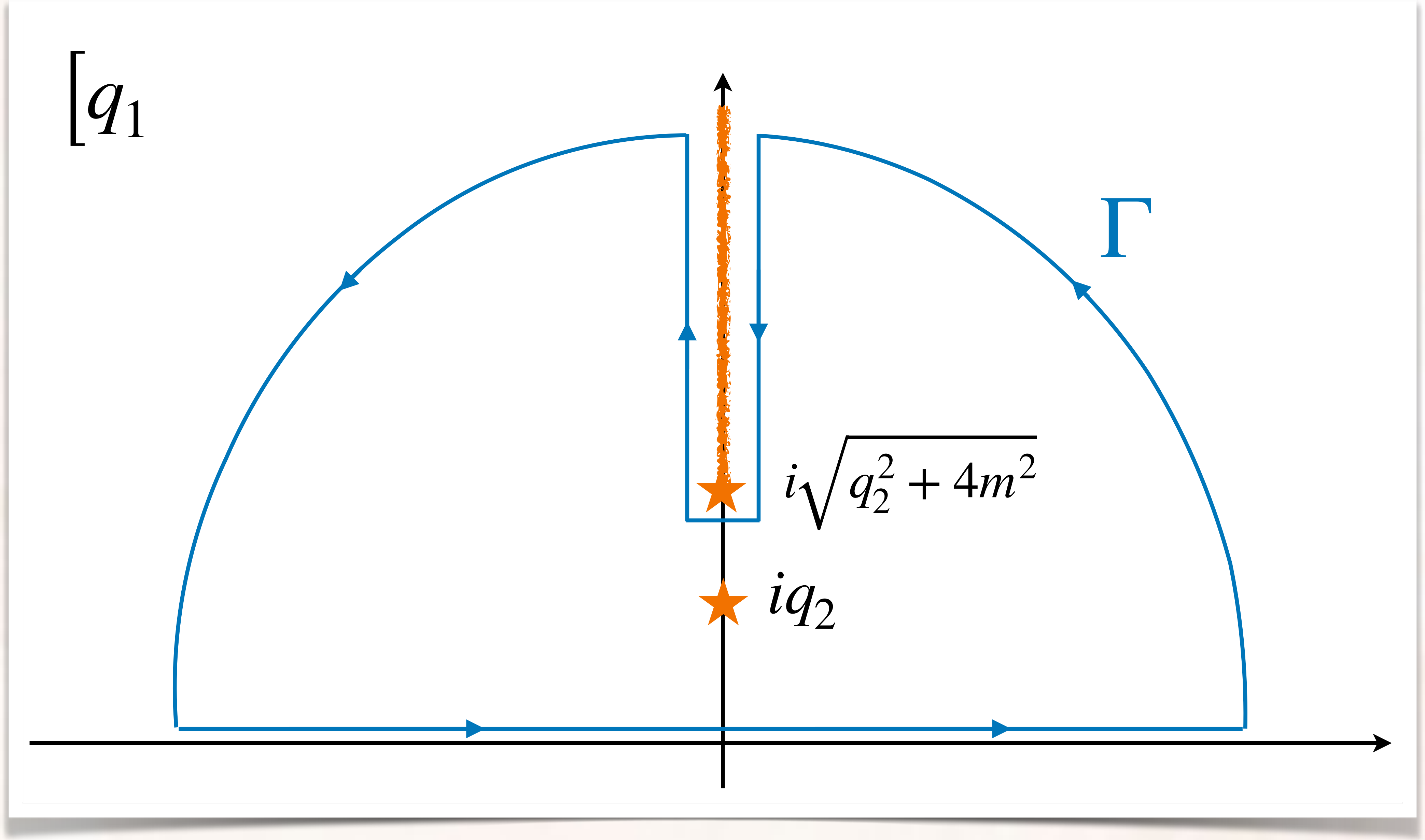
	$\mathcal{M}(1_\gamma^- 3_\gamma^+ 5_X 6_{\bar{X}})$	$\mathcal{M}(1_\gamma^- 3_\gamma^- 5_X 6_{\bar{X}})$	$\mathcal{M}(5_X 6_{\bar{X}} 2_S 4_S)$
ϕ	$\frac{g^2\langle 1(k_5-k_6)3]^2}{2(s_{15}-m^2)(s_{16}-m^2)}$	$\frac{2g^2m^2\langle 13\rangle^2}{(s_{15}-m^2)(s_{16}-m^2)}$	$\frac{(s_{25}-m^2)(s_{45}-m^2)}{m_{\text{Pl}}^2 s_{24}} - \xi_\phi \frac{s_{24}}{6m_{\text{Pl}}^2}$
ψ	$\frac{g^2\langle 1(k_6-k_5)3]}{(s_{15}-m^2)(s_{16}-m^2)}(\langle \mathbf{15}\rangle[\mathbf{36}] + \langle \mathbf{16}\rangle[\mathbf{35}])$	$\frac{2g^2m\langle 13\rangle^2\langle \mathbf{65}\rangle}{(s_{15}-m^2)(s_{16}-m^2)}$	$\frac{s_{25}-s_{45}}{4m_{\text{Pl}}^2 s_{24}}(\langle \mathbf{6}(k_2-k_4)\mathbf{5}] + \langle \mathbf{5}(k_2-k_4)\mathbf{6}])$
W	$\frac{2g^2}{(s_{15}-m^2)(s_{16}-m^2)}(\langle \mathbf{15}\rangle[\mathbf{36}] + \langle \mathbf{16}\rangle[\mathbf{35}])^2$	$\frac{2g^2\langle 13\rangle^2\langle \mathbf{65}\rangle^2}{(s_{15}-m^2)(s_{16}-m^2)}$	$\frac{-1}{4m_{\text{Pl}}^2 s_{24}}(\langle \mathbf{6}(k_2-k_4)\mathbf{5}] + \langle \mathbf{5}(k_2-k_4)\mathbf{6}])^2$

Computation of the phase shift

$$\delta_\pm(b,s) = \frac{s}{4M_{Pl}^2}\left[-\frac{1}{2\pi}\left(F_1(0)\log\frac{b}{b_{IR}} \mp \frac{8}{b^2}F_3(0)\right) + \frac{i}{(2\pi)^2}\int_{4m^2}^{\infty}\frac{dt}{t}\left(\mathrm{Disc}F_1(t)K_0(b\sqrt{t}) \pm 4t\,\mathrm{Disc}F_3(t)K_2(b\sqrt{t})\right)\right]$$

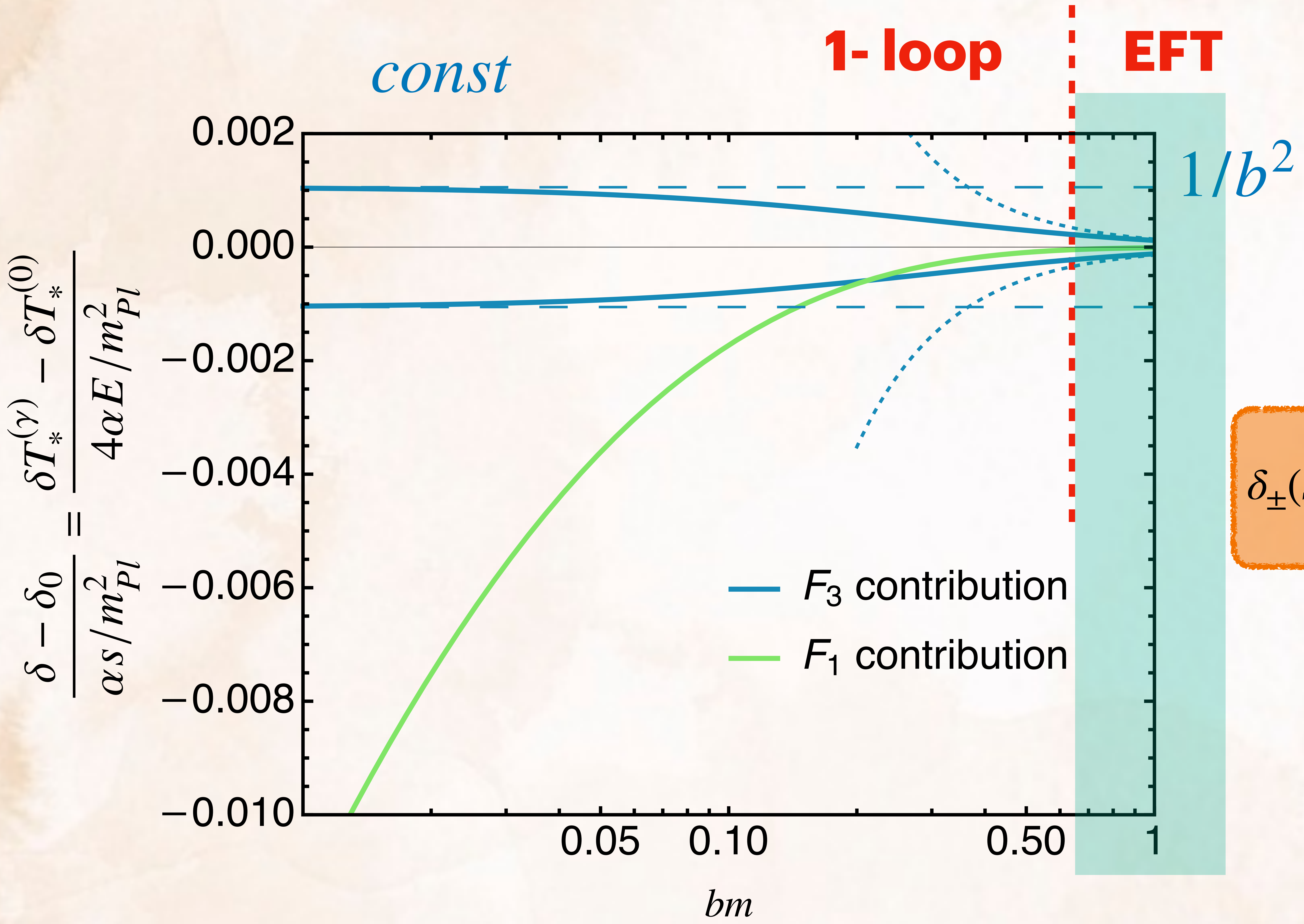
EFT **loop**

$$\delta(s,\vec{b}) = \frac{1}{4s}\int\frac{d^2q_\perp}{(2\pi)^2}e^{-i\vec{q}_\perp\cdot\vec{b}}\mathcal{M}_{eik}(s,\vec{q}_\perp)$$



A loop solution to causality?

Scalars and Electrons



$$b \gg 1/m$$

$$\delta_\pm(s, \vec{b}) = Gs\left(-\log\frac{b}{b_{IR}} \pm \frac{\alpha}{m^2 b^2}\right)$$

F_3

A loop solution to causality?

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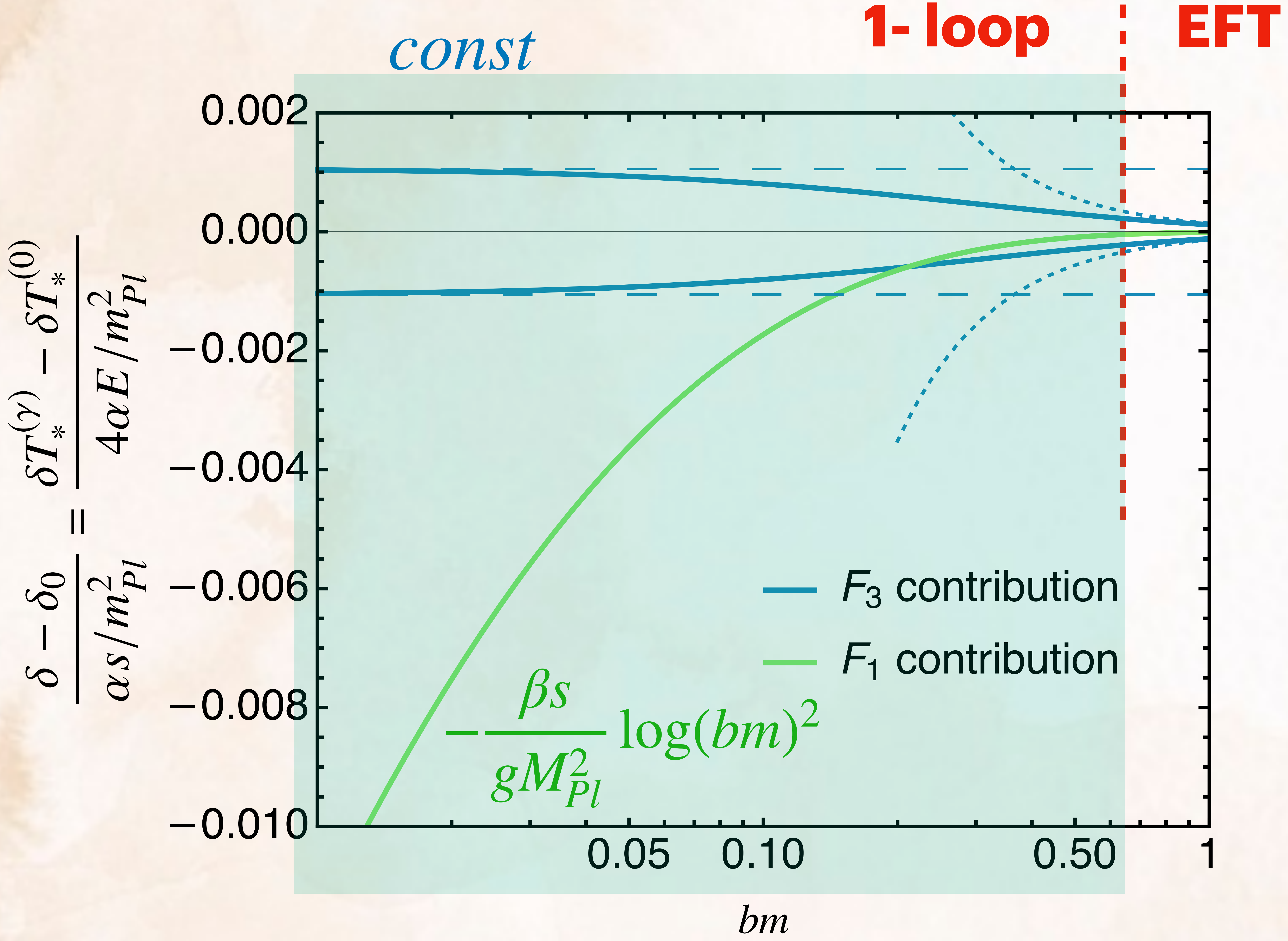


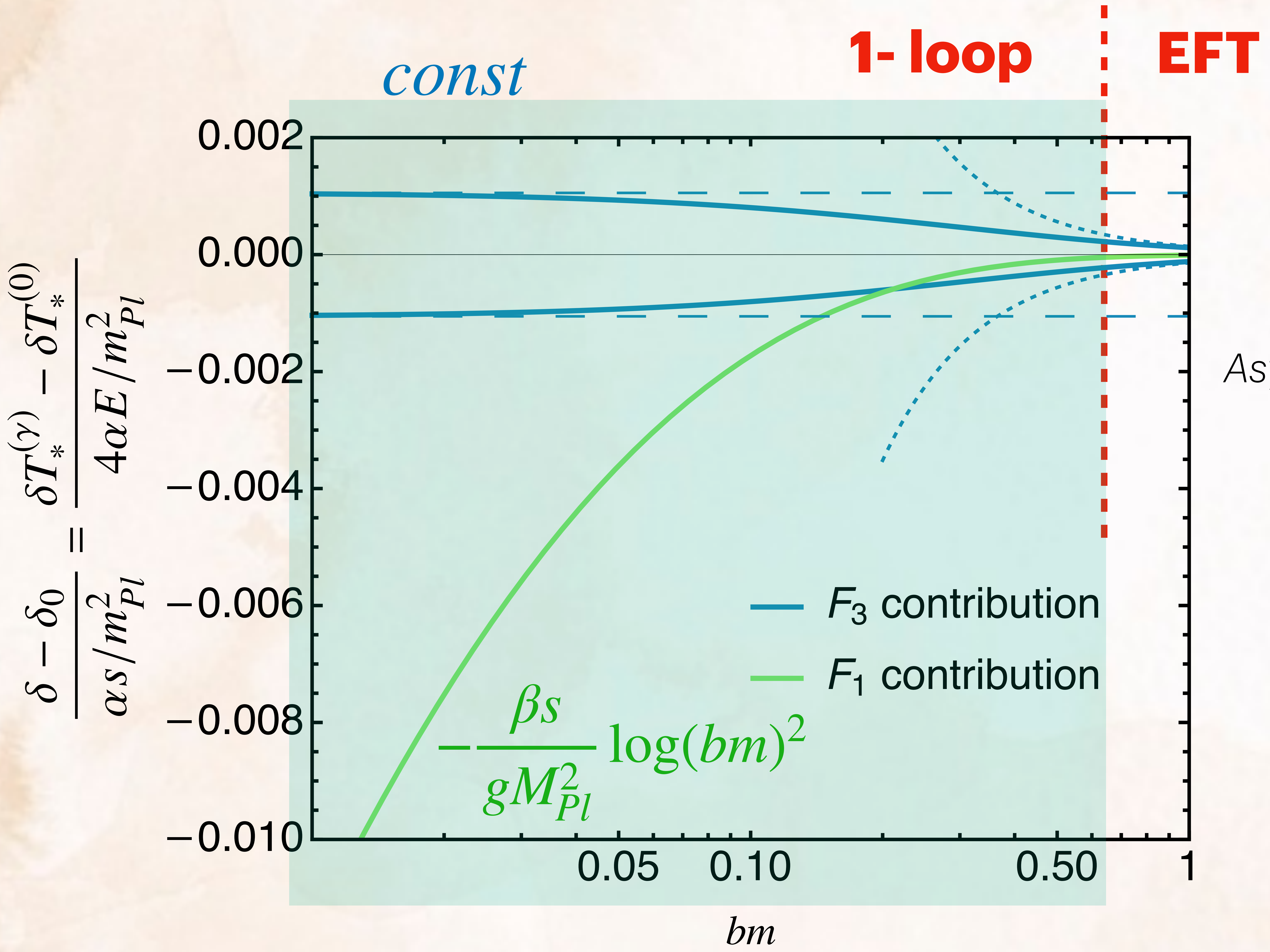
$$b \ll 1/m$$

$$\delta_\pm(s, \vec{b}) = -Gs \log \frac{b}{b_{IR}} - \frac{s\beta}{8\pi g M_{Pl}^2} \log bm^2$$

F_1

A loop solution to causality?

Scalars and Electrons



Asymptotic causality is only violated at

$$b_L \sim \frac{1}{m} e^{-g/\beta}$$

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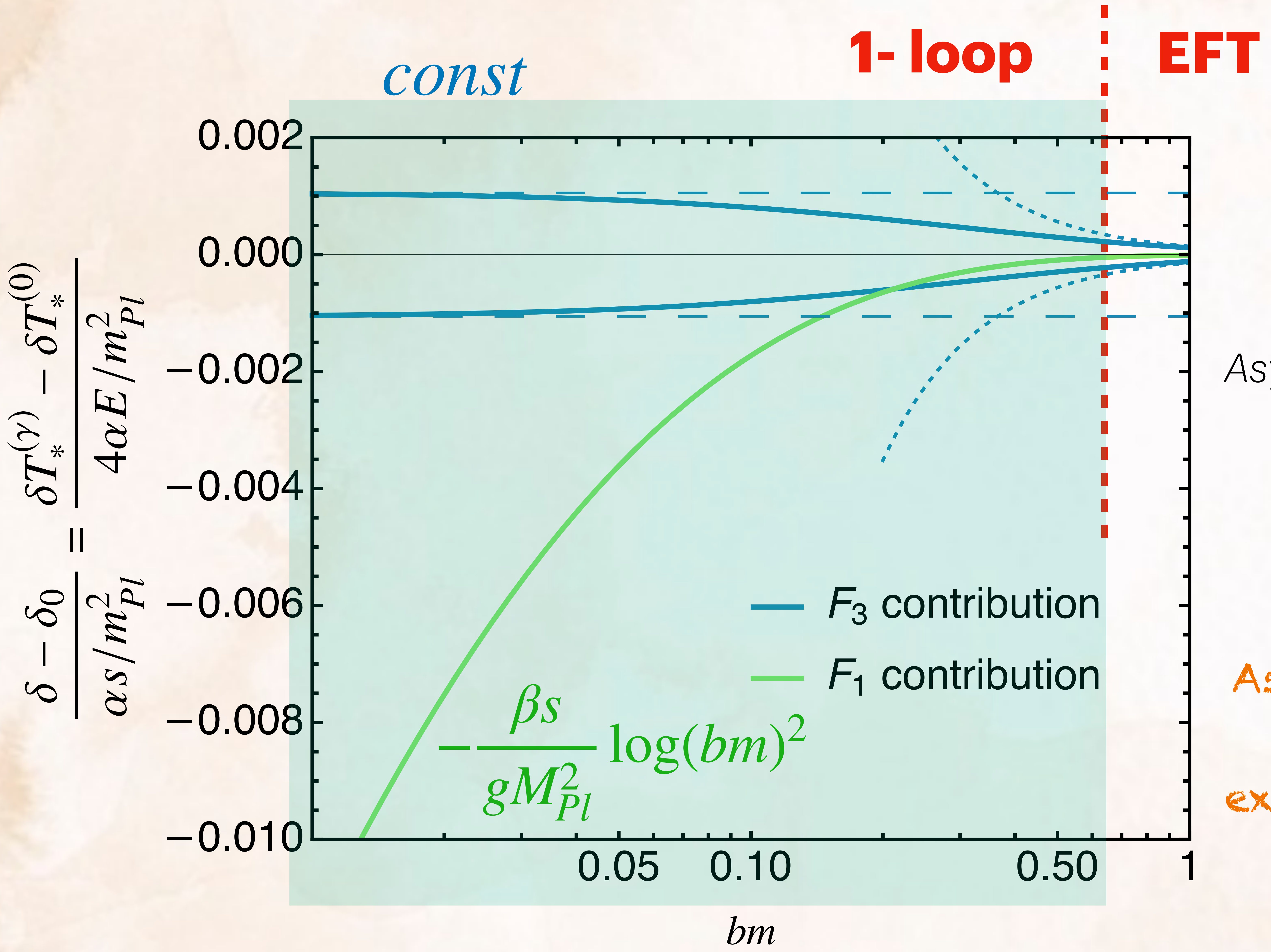

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Asymptotic causality predicts the existence of the Landau pole!

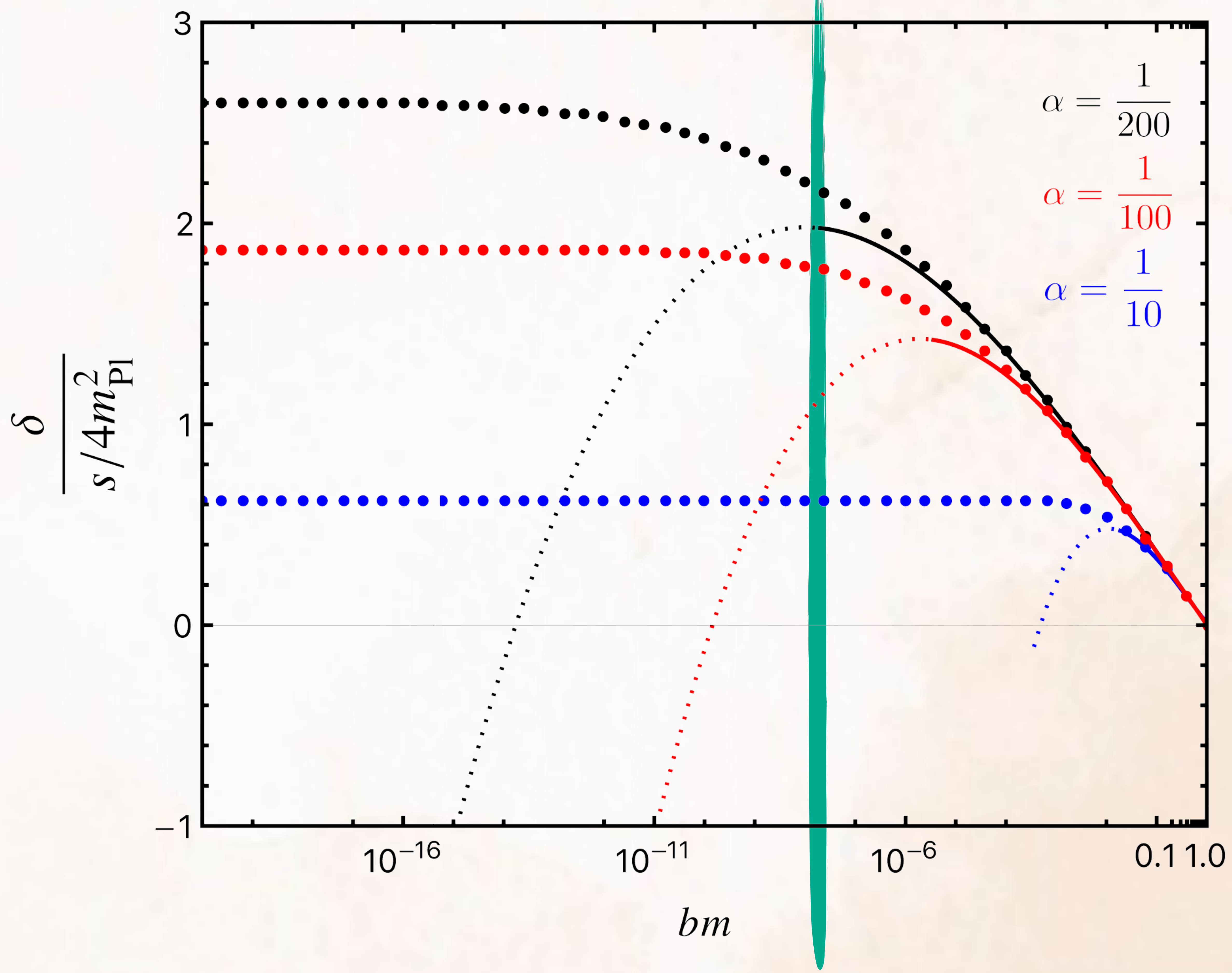
The strange case of the W

X	$\mathrm{Disc}\, F_1(t \gg m^2)$
ϕ	$\frac{i\alpha}{6}$
ψ	$\frac{2i\alpha}{3}$
W	$-\frac{i\alpha}{2}\left(7 - 4\log\frac{t}{m^2}\right)$

Large double logs related to IR divergence in the $m_W \to 0$ limit

$$F_1(t)^{\text{Sudakov}} \to e^{F_1(t)-1}$$

Sudakov resummation



$e^{-\sqrt{2\pi/\alpha}}$

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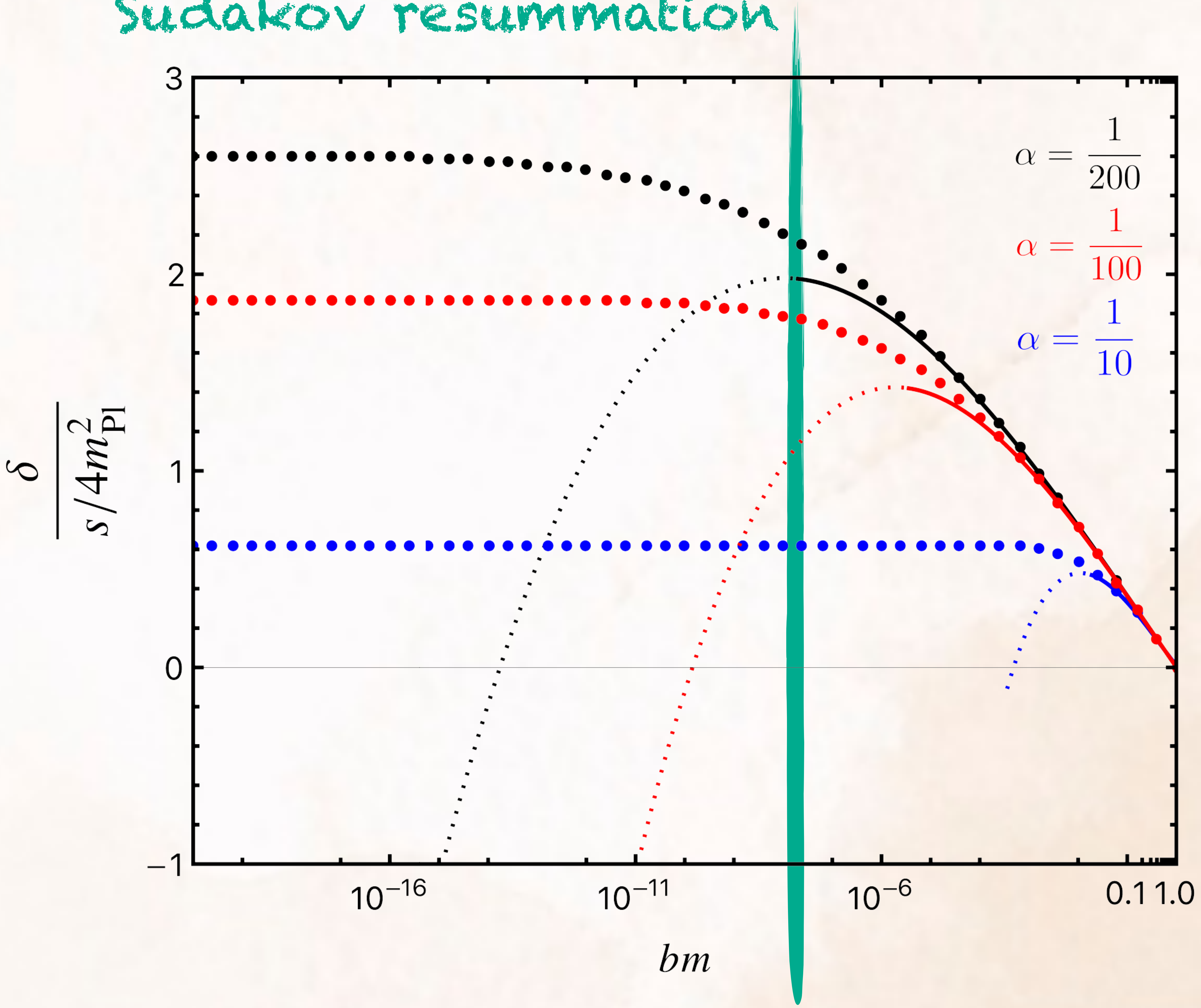
No violation of asymptotic causality

No Landau pole in asymptotic free theories

Sudakov resummation



$e^{-\sqrt{2\pi/\alpha}}$

Conclusion

Transplanckian Scattering

- Transplanckian physics at large impact parameter is captured by the Eikonal regime

Asymptotic Causality

Seems to be a robust definition

- Correctly detects the breakdown of the theory both at tree and loop-level

- It does not predict the details of the UV completion

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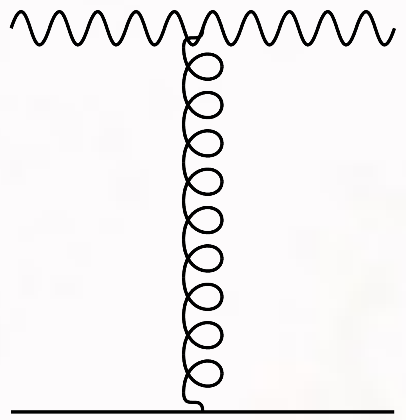
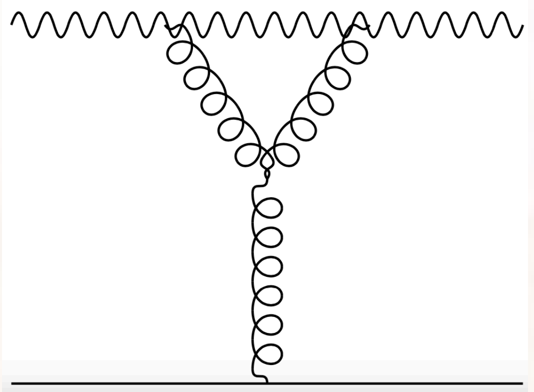
Backup

Extracting the phase shift

in pure gravity

$$1 - e^{2i\delta(s,\vec{b})} = \frac{1}{4s}\int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \mathcal{M}_{eik}\left(\frac{t}{s} \ll 1\right)$$

$\mathcal{O}(G)$: $\quad 2\delta_0(s,b) = -\,Gs\log\frac{b}{b_{IR}} =$ $FT[$  $] = FT[8\pi\frac{Gs^2}{t}]$ 1PM

$\mathcal{O}(G^2)$: $\quad 4i\delta_0(s,b)^2 + 2\delta_1(s,b) =$ $FT[$  $+\dots\ ]$ 2PM

⋮

Infrared/Bulk causality

Exploring other causality definitions

Bulk Causality

Graviton propagation determines relevant light cone

$$\Delta T^{g} - \Delta T^{\gamma} > 0$$

Violated for electrons and scalars for $b < 1/m$

Infrared causality

De Rham, Tolley, etc

$$\Delta T^{net} = \Delta T^{GR} + \Delta T^{EFT}$$

$$\Delta T^{GR} = \lim_{\Lambda \to \infty} \Delta T^{net}$$

$$\Rightarrow \quad \Delta T^{EFT} > 0$$

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Infrared = Bulk causality

Exploring other causality definitions

$$\Lambda \sim m e^{-g/\beta}$$

$$\Lambda \to \infty$$

$m \to \infty$ ⇒ $\Delta T^{EFT} = \pm \dfrac{\alpha}{m^2 b^2 M_{Pl}^2} = \Delta T^{\gamma} - \Delta T^{g}$

$\alpha \to 0$ ⇒ $\Delta T^{EFT} = -\dfrac{\alpha}{M_{Pl}^2} \log bm^2 = \Delta T^{\gamma} - \Delta T^{g} < 0$

$$\Delta T^{net} = \Delta T^{GR} + \Delta T^{EFT}$$

$$\Delta T^{GR} = \lim_{\Lambda \to \infty} \Delta T^{net}$$

$$\Delta T^{EFT} > 0$$

ΔT^{EFT} for $b \gg 1/m$

is never resolvable within the EFT

Infrared = Bulk causality

Exploring other causality definitions

$$\Lambda \sim m e^{-g/\beta}$$

$$\Lambda \to \infty$$

$m \to \infty \Rightarrow \Delta T^{EFT} = \pm \dfrac{\alpha}{m^2 b^2 M_{Pl}^2} = \Delta T^{\gamma} - \Delta T^{g}$

$\alpha \to 0 \Rightarrow \Delta T^{EFT} = -\dfrac{\alpha}{M_{Pl}^2} \log b m^2 = \Delta T^{\gamma} - \Delta T^{g} < 0$

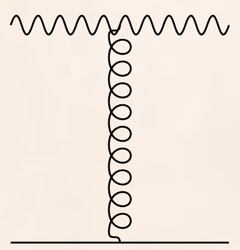
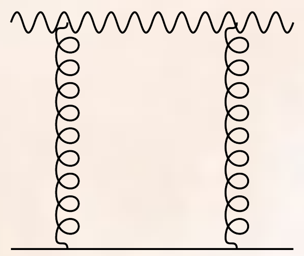
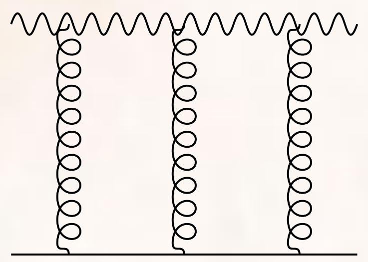
$$\Delta T^{net} = \Delta T^{GR} + \Delta T^{EFT}$$

$$\Delta T^{GR} = \lim_{\Lambda \to \infty} \Delta T^{net}$$

$$\Delta T^{EFT} > 0$$

Bulk causality is the same as IR causality in our example. Both are violated at $b < 1/m$

Exponentiation in the eikonal

$\mathscr{A}_{eik}(s,t) =$  $+$  $+$  $+ \; \ldots$

$$= \frac{1}{2s}\mathscr{M}_0 \otimes \mathscr{M}_0(\vec{Q}) + \ldots = \frac{1}{2s}\sum \frac{1}{n!}\mathscr{M}_0 \otimes \mathscr{M}_0 \ldots \otimes \mathscr{M}_0(\vec{Q})$$

$$\mathscr{A}_{eik}(s,\vec{b}) = e^{2i\delta_0(s,\vec{b})}$$

Born approximation

Phase-shift $\delta_0(s,\vec{b}) = \frac{1}{4s}\int \mathscr{M}_0\, e^{i\vec{Q}\cdot\vec{b}} = -\frac{s}{8\pi M_{Pl}^2}\log\left(\frac{b}{b_{IR}}\right)$

$$\Delta T = 2\frac{\partial\delta(s,\vec{b})}{\partial E}$$

Beta function

Form factors control the coupling's running

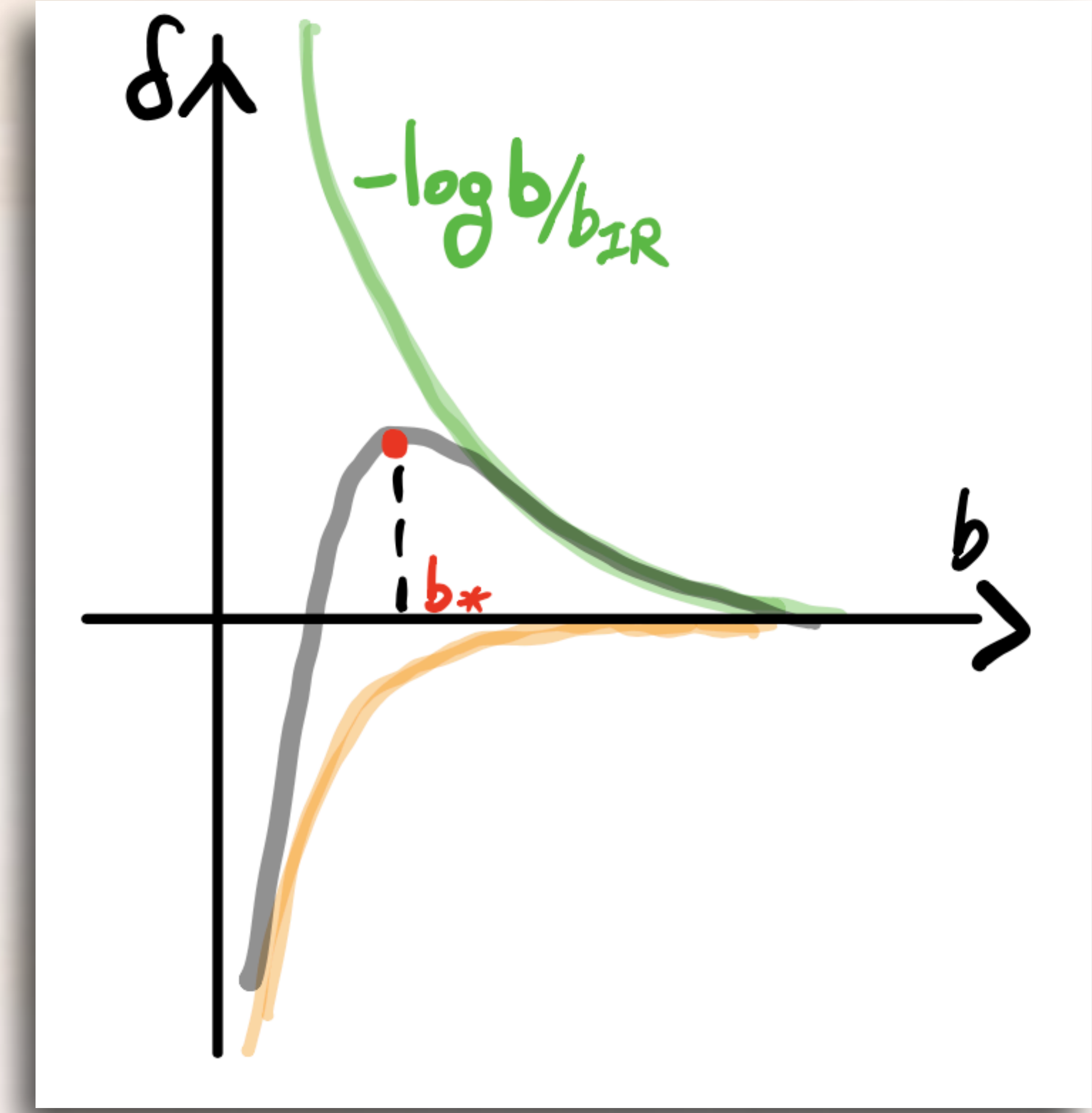
$$S \supset \int d^4x \sqrt{-g} \left(M_{Pl}^2 R - \frac{1}{g^2(\mu)} F_{\mu\nu} F^{\mu\nu} + \dots \right)$$

$$\supset \int d^4x \, h_{\mu\nu} \frac{1}{g^2(\mu)} < 0 | T^{\mu\nu} | \gamma\gamma >$$

$$\beta = -\frac{g}{2} \frac{d}{d \log \mu} F_1(t = -\mu^2) \Big|_{m \ll \mu} = \lim_{t/m^2 \to \infty} \frac{g}{\pi} \frac{\text{Disc} F_1(t)}{2i}$$

IR cutoff

Form factors control the coupling's running



b^* is where gravity becomes repulsive, and it is independent of b_{IR}