Causality cuts off Black Hole Hair

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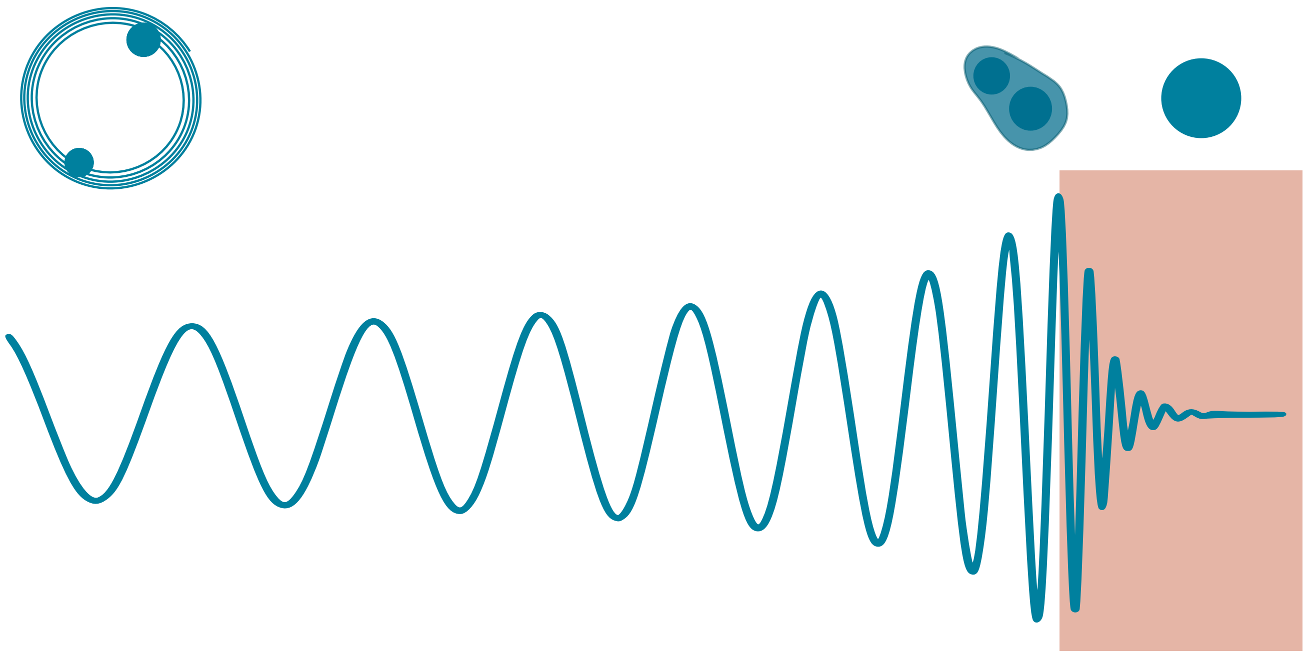
Based on:

FS, J. Serra, L.G. Trombetta, E. Trincherini – 2205.08551

Black hole spectroscopy



Ringdown:

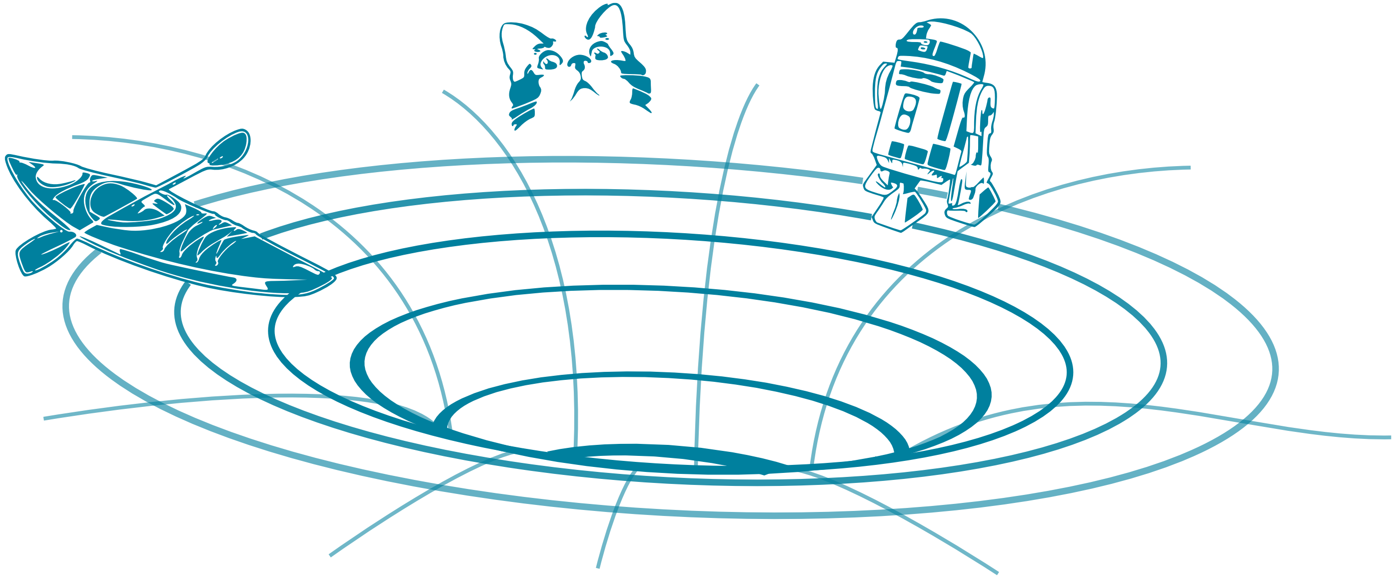
discrete frequencies

↕

Test effects

Beyond GR

Black hole hair



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Mass,
Electric Charge,
Angular momentum

+ hair ?

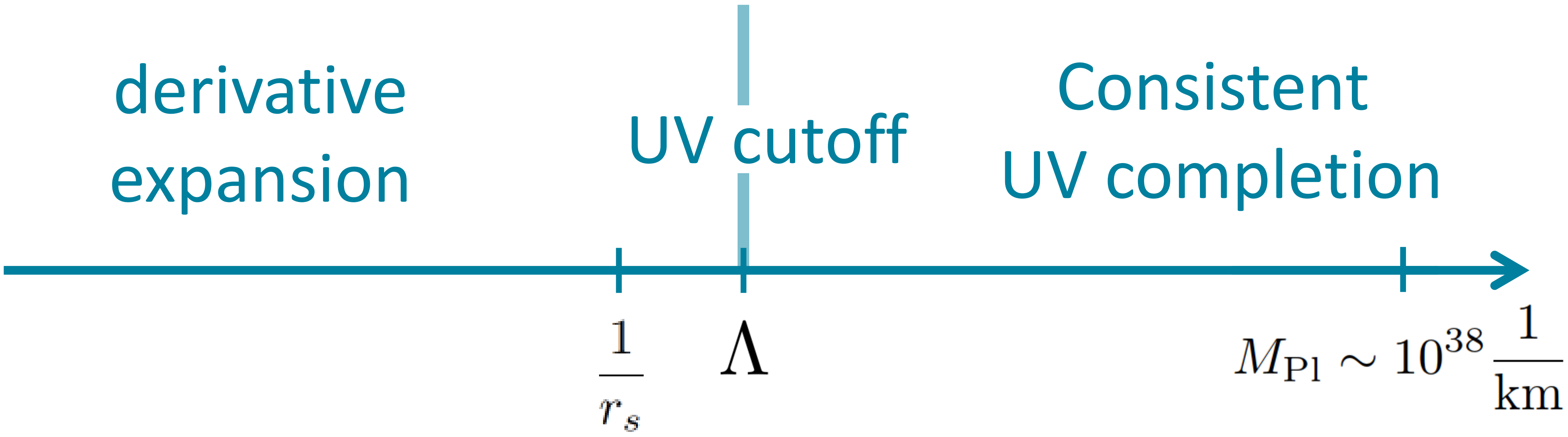
The scalar-tensor paradigm

- Simplest deformation of GR
- Shift symmetry $\phi \to \phi + c$
- The scalar Gauss-Bonnet fairy tale

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 + \alpha M_{\rm Pl}\, \phi \mathcal{R}^2_{\rm GB} \right)$$

$$\mathcal{R}^2_{\rm GB} \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2$$

EFTs Beyond GR



$$\mathcal{L} = \frac{1}{2}\hat{M}_{\rm Pl}^2 R + \frac{\Lambda^4}{g^2}\left[L^{(0)}\left(\frac{\nabla_\mu}{\Lambda}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) + \frac{g^2}{(4\pi)^2}L^{(1)}\left(\frac{\nabla_\mu}{\Lambda}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda^2}, \frac{g\phi}{\Lambda}\right) + \cdots\right]$$

Theoretical constraints on scalar-GB

Causality

- Forbid macroscopic time advances

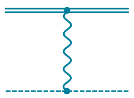
Problem when $\Delta t < 0$ & $|\Delta t| > \dfrac{1}{\omega}$

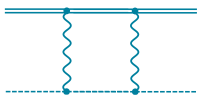
- Eikonal scattering, phase shift matrix

$$m \gg \omega \gg q$$

source mass | probe energy | exchanged momentum

$$\int \frac{d^{D-2}q}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} \left(\text{[diagram 1]} + \text{[diagram 2]} + \cdots \right) \sim e^{i\delta(\omega,\vec{b})} - 1$$

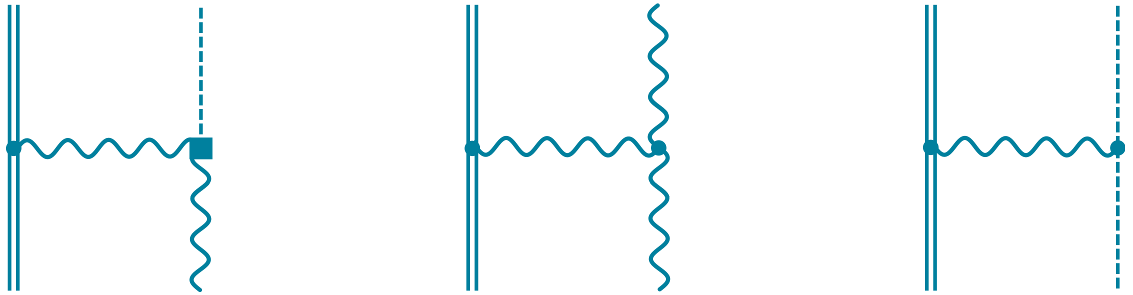




$$\delta(\omega,\vec{b}) = \frac{1}{4m\omega} \int \frac{d^{D-2}q}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} \mathcal{M}(\omega,\vec{q})$$

$$\Delta t = \partial_\omega \delta$$

Causality requires low UV cutoff



$$\delta \simeq 2\omega r_s \begin{pmatrix} C & 0 & A \\ 0 & C & A^* \\ A^* & A & C \end{pmatrix} \quad C = -\frac{1}{2\epsilon} - \frac{\gamma_E}{2} - \log b \quad A = -\frac{\alpha}{(b_1 - ib_2)^2}$$

$$\Delta t_{\pm} = 2r_s \left(\log \frac{b_0}{b} \pm \sqrt{2}\frac{\alpha}{b^2} \right)$$

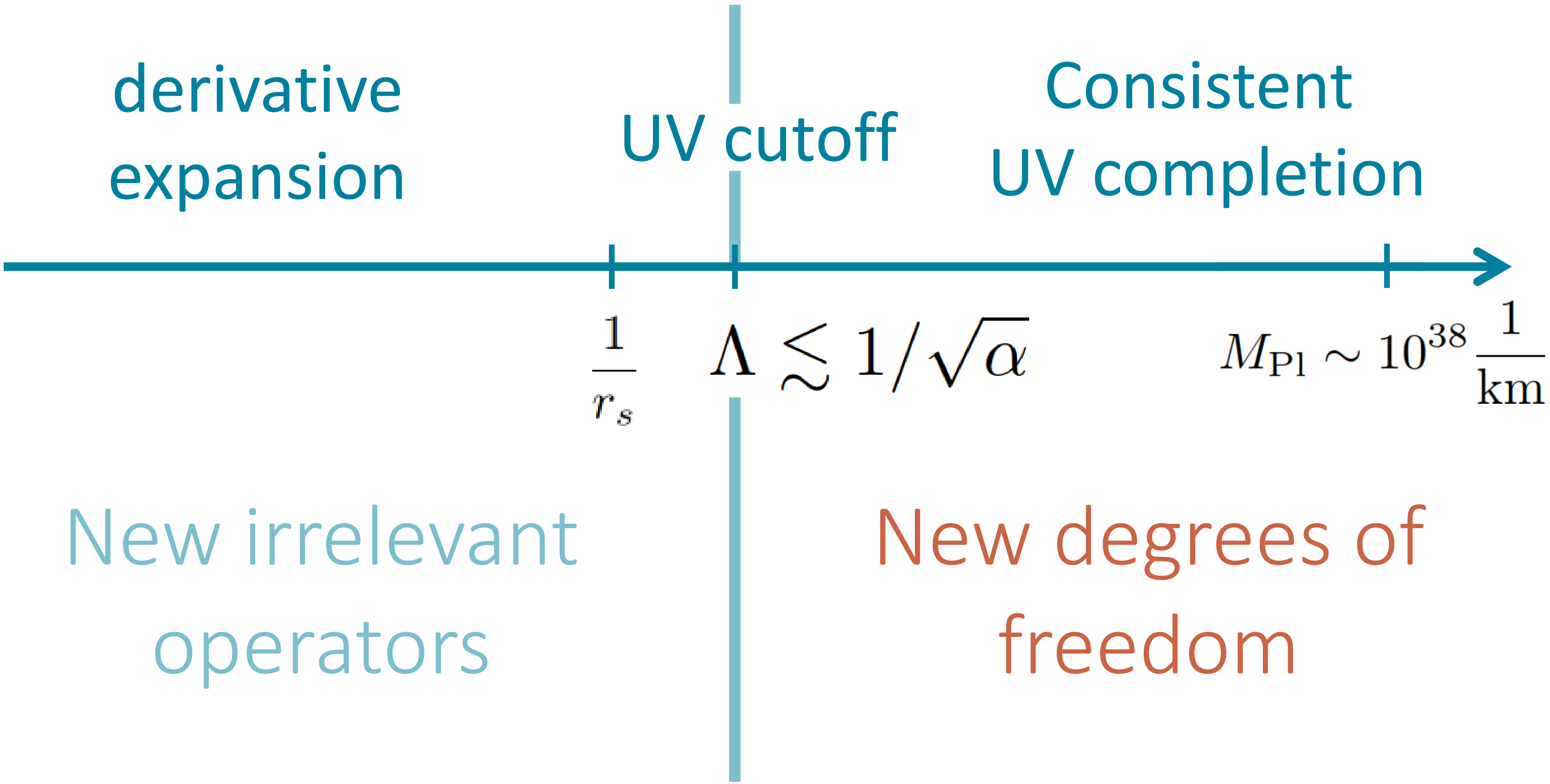
UV completion needed at small impact parameter

Camanho et al.1407.5597

$$\Lambda \lesssim 1/\sqrt{\alpha}$$

Detectable hair when $\sqrt{\alpha} \sim \mathrm{km}$

Consequence of low UV cutoff



Dispersion relations & positivity

Can we derive lower bounds for other operators?

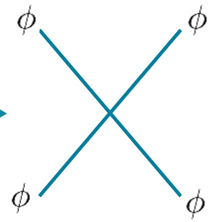
2 → 2 S-matrix:
Unitarity
Analyticity
Boundedness

e.g. Bellazzini et al. 1710.02539

$$\frac{d^2}{ds^2}\mathcal{M}_{\phi\phi\to\phi\phi}(s=0) > \frac{2}{\pi}\int_0^{\Lambda^2}\frac{ds}{s^2}\sigma^{\rm GB}_{\phi\phi\to h^{--}h^{++}} \simeq \left(\frac{\alpha\Lambda^2}{M_{\rm Pl}}\right)^4$$

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \alpha M_{\rm Pl}\,\phi\mathcal{R}^2_{\rm GB} + c_2\frac{(\partial\phi)^4}{4!} + \dots\right)$$

$(\partial\phi)^4$ →



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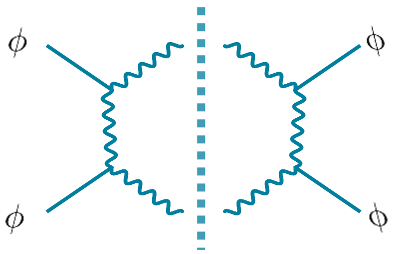
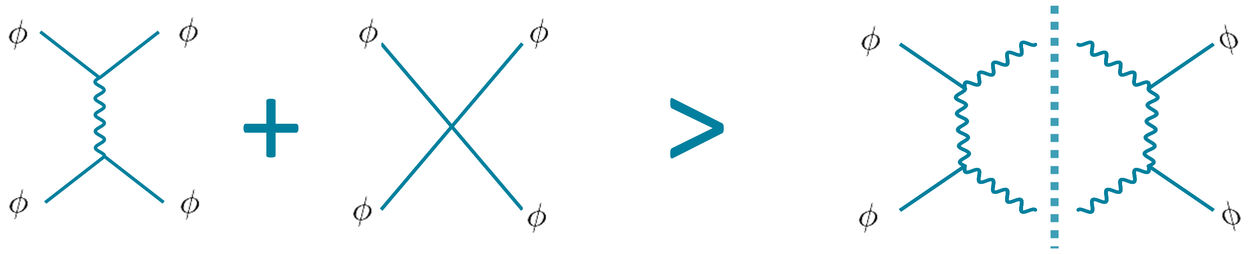


← $\phi\mathcal{R}^2_{GB}$

Dispersion relations & gravity

Gravity: small negativity allowed



$$\left(t \sim -\Lambda^2\right) \qquad \frac{-1}{M_{\rm Pl}^2 t} + c_2 > \left(\frac{\alpha\Lambda^2}{M_{\rm Pl}}\right)^4$$

Caron-Huot et al.
2102.08951

Causality makes positive contributions too small: no bound can be derived

Conclusions

From a fairy tale to a harsh reality:
Causality forces a low cutoff (new dof at low energies!)

Which other theories
beyond GR are affected
by similar issues?

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