Probing τ flavour change with $\mu \to e$

Based on MA, S. Davidson, M. Gorbahn PRD 105, 096040 (2022)

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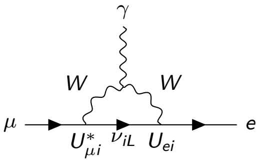
Leptons:

$$\ell_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}, \alpha_R \qquad \text{with } \alpha = e, \mu, \tau \tag{1}$$

The Standard Model with $m_{\nu_\alpha} = 0$ has exact $U(1)_{B/3-L_\alpha}$

...but Neutrino masses break all three symmetries.
Charged Lepton Flavour Violation (CLFV)≡ short-range interaction between the charged leptons that violates LF.



CLFV assuming Dirac neutrino masses is small
$Br(\mu \to e\gamma) \simeq G_F^2 m_\nu^4 \sim 10^{-50}$

Smoking gun signal of New Physics

Experimental searches

The experimental sensitivities on LFV processes are impressive and will improve

$$Br(\mu \to e\gamma) < 4.2 \times 10^{-13} \to 6 \times 10^{-14}$$ A. M. Baldini et al., arXiv:1801.04688

$$Br(\mu \to \bar{e}ee) < 1.0 \times 10^{-12} \to 10^{-16}$$ A. Blondel et al., arXiv:1301.6113

$$Br(\mu A \to eA) < 7 \times 10^{-13} \to 10^{-16} - 10^{-18}$$ R. M. Carey et al., FERMILAB-PROPOSAL-0973

$$Br(\tau \to \bar{e}ee) < 2.7 \times 10^{-8} \to 5 \times 10^{-9}$$ E. Kou et al., arXiv:1808.10567

$$Br(\tau \to l\gamma) < 3.3 \times 10^{-8} \to \sim 10^{-9}$$ B. Aubert et al., PhysRevLett.104.021802

$$Br(B_d^0 \to \mu\tau) < 1.4 \times 10^{-5}$$ Roel Aaij et al., PhysRevLett, 123(21):211801, 2019

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Suppose $\mu \leftrightarrow \tau$ and $\tau \leftrightarrow e$ happen:

$$\begin{array}{ccccc} & & \tau & & \\ & \nearrow & & \searrow & \\ \mu & & \longrightarrow & & e \end{array}$$

$\mu \to e$ can be radiatively generated via $\mu \to \tau \times \tau \to e$

Can $\mu \to e$ teach us something about $\tau \leftrightarrow l$ LFV?

Outline

1. $\mu \to \tau \times \tau \to e$ in EFT
2. Phenomenology
3. Conclusion

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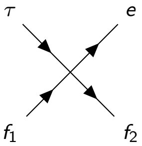
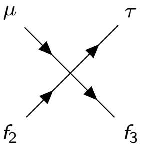
Effective Field Theory (EFT)

We assume that New Physics responsible for LFV is heavy $\Lambda \gtrsim 4$ TeV



$\sim 1/\Lambda^2$



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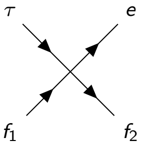

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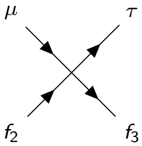

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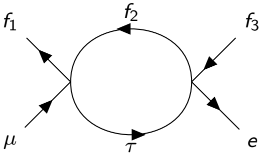
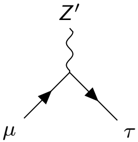


$\sim 1/(16\pi^2\Lambda^4)$

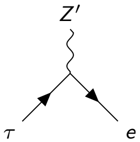
In the EFT $\mu \to \tau \times \tau \to e$ is at dimension eight: $\mu \to e$ observables are sensitive to some $1/\Lambda^4$ contributions MA, S. Davidson, JHEP 08 2021, 2 (2021)

Dimension eight amplitudes might be subleading contributions of $\mu \to \tau \times \tau \to e$

Heavy Z' with $\mu \leftrightarrow \tau$ and $e \leftrightarrow \tau$ couplings

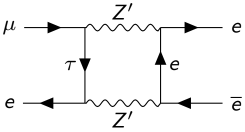


$\sim g_{\tau\mu}$



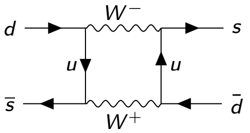
$\sim g_{e\tau}$

Boxes match onto dimension six



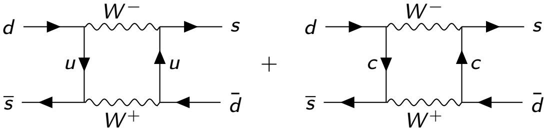
$\propto \frac{g_{e\tau} g_{\tau\mu}}{16\pi^2 M_{Z'}^2}$

$K - \overline{K}$ mixing:



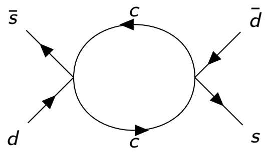
$\sim G_F^2 M_W^2 \sim 1/M_W^2$

In SM the charm and up dimension six contributions cancel (GIM)



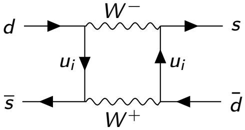
$\sim G_F^2 m_c^2$

The EFT reproduces the log-enhanced $\sim G_F^2 m_c^2$ amplitude



Including the top, GIM applies due to CKM unitarity

$$\sum_{i=u,c,t}$$



$$\sim (V_{cs}^* V_{cd})^2 G_F^2 m_c^2 + (V_{ts}^* V_{td})^2 G_F^2 m_t^2$$

The dimension six $G_F^2 m_t^2$ is suppressed by small mixing with the third generation.

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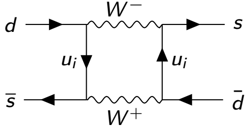

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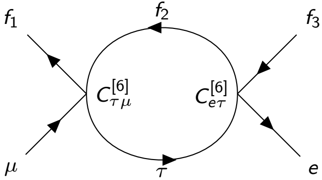
Dimension six $\mu \to \tau \times \tau \to e$ matching is model dependent; we calculate model independent dimension eight contributions

The EFT calculation

Divergent diagrams with two dimension six $\mu \to \tau \times \tau \to e$ insertion renormalize dimension eight $\mu \to e$ operators

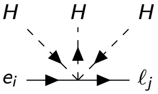


$$C^{[8]}_{e\mu}(E_{exp}) \sim \frac{C^{[6]}_{\tau\mu} C^{[6]}_{e\tau}}{16\pi^2} \log\left(\frac{\Lambda}{E_{exp}}\right) \qquad E_{exp} \equiv \text{exp. scale}$$

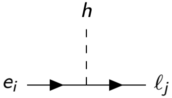
- We only calculate contributions that are (estimated to be) within exp. sensitivities.
- We compute the $(dim6)^2$ one-loop running in the Standard Model EFT ($E > M_W$)
- SMEFT operator list is reduced using the Equations of Motions (EoM)
  EoM get corrections from dimension six operators: need to take it into account when renormalizing EFT up to $1/\Lambda^4$ order.

The EFT calculation

- The dimension eight operators generated in the $\Lambda \to M_W$ running are matched onto low energy contact interactions at $E = M_W$ (EFT with W, Z, h, t integrated out).
- Tree-level matching contributions from two dimension six operator insertion when $\langle H \rangle = v = 174$ GeV:

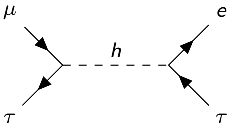


$= \frac{C_{eH}^{ji}}{\Lambda^2} (\bar{\ell}_j H e_i)(H^\dagger H)$



$\propto C_{eH}^{ji} \frac{v^2}{\Lambda^2}$

  Match onto scalar and tensor $\mu e \tau \tau$ operators



- Low energy operators run from M_W to $E_{exp} \longrightarrow$ experiments constrain $C^{[6]}_{\tau\mu} C^{[6]}_{e\tau}$ ($(dim6)^2$ running below M_W is negligible).

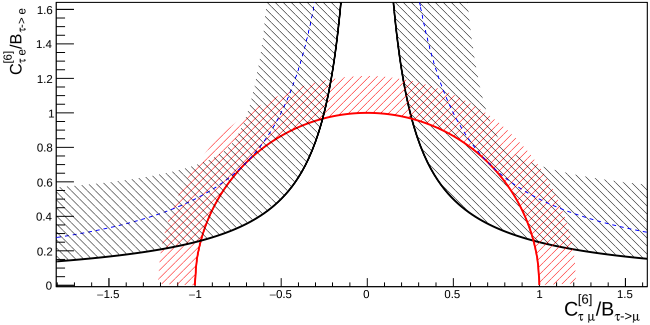
Outline

$\tau \leftrightarrow l$ searches, where $l = e, \mu$, can probe the region outside the ellipse

$$\frac{\left|C^{[6]\tau\mu}\right|^2}{B^2_{\tau\leftrightarrow\mu}} + \frac{\left|C^{[6]e\tau}\right|^2}{B^2_{\tau\leftrightarrow e}} \lesssim 1$$

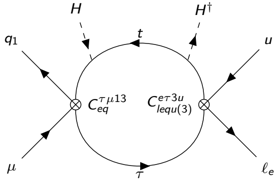
while $\mu \to e$ is sensitive to the parameter space above the hyperbola

$$\left|C^{[6]\tau\mu} C^{[6]e\tau}\right| \lesssim B_{\mu\leftrightarrow e}$$



- Dashed hyperbola: $\frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e} B_{\tau\leftrightarrow\mu}} = \frac{1}{2}$
- Thick hyperbola: $\frac{B_{\mu\leftrightarrow e}}{B_{\tau\leftrightarrow e} B_{\tau\leftrightarrow\mu}} < \frac{1}{2}$

Consider $\mathcal{O}_{eq}^{\tau\mu 13} = 2\sqrt{2}G_F(\bar{\tau}\gamma\mu)(\bar{q}_1\gamma q_3)$ and $\mathcal{O}_{\ell equ}^{(3)e\tau 31} = 2\sqrt{2}G_F(\bar{\ell}_e\sigma\tau)(\bar{q}_3\sigma u)$



The diagram generates $(\overline{e}P_R\mu)(\overline{u}P_R u)$, contributing to $\mu \to e$ conversion in nuclei

$$Br(\mu A \to eA) < 7\times 10^{-13} \to C_{eq}^{\tau\mu 13}\times C_{\ell equ}^{(3)e\tau 31} \lesssim B_{\mu\leftrightarrow e} = 1.5\times 10^{-8}$$

B decays are also sensitive to the above operators

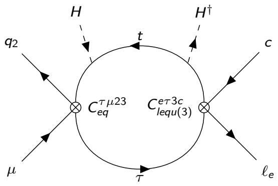
$$Br(B_d^0 \to \mu^\pm\tau^\mp) < 1.4\times 10^{-5} \;\to\; C_{eq}^{\tau\mu 13} \lesssim B_{\tau\leftrightarrow\mu} = 1.5\times 10^{-3}$$

$$Br(B^+ \to \overline{\tau}\nu) = (1.09\pm 0.24)\times 10^{-4} \;\to\; C_{\ell equ}^{(3)e\tau 31} \lesssim B_{\tau\leftrightarrow e} = 1.8\times 10^{-3}$$

$$\frac{B_{\mu\leftrightarrow e}}{(B_{\tau\leftrightarrow e}B_{\tau\leftrightarrow\mu})} \sim (5\times 10^{-3})$$

With future experimental sensitivities this ratio could become ~ 10 times smaller

Same diagram with external charm



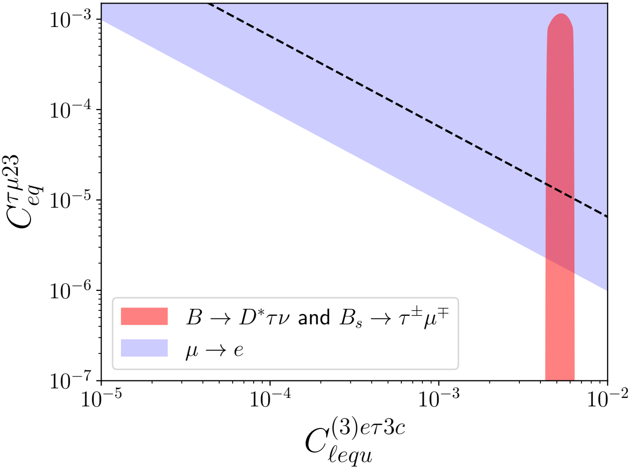
generates a tensor $(\bar{e}\sigma P_R \mu)(\bar{c}\sigma P_R c)$. Tensors with heavy quarks efficiently mix with the dipole, contributing to $\mu \to e\gamma$.

Among the popular B anomalies, the ratio

$$R_{D^*\tau/l} \equiv \frac{Br(B \to D^*\tau\bar{\nu})}{Br(B \to D^* l\bar{\nu})}$$

is observed to be above the SM value $(R^{\rm exp}_{D^*\tau/l} - R^{\rm SM}_{D^*\tau/l}) = 0.052 \pm 0.018$

The LFV tensor can enhance the rate of the numerator and can fit the difference



With a detection of $\tau \leftrightarrow e$, our results can give theoretical guidance on where to search (or not) for $\tau \leftrightarrow \mu$.

Outline

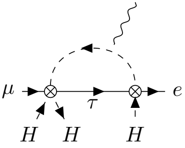
Summary

- $\mu \to e$ impressive experimental improvement is expected in the near future
- The sensitivity on $\mu \to \tau \times \tau \to e$ can compete with direct $\tau \to l$ LFV searches and complementary probe τ flavour changing interactions
- We calculated a subset of unknown RGEs for dimension eight operators in SMEFT
- Our result can relate $\mu \leftrightarrow e$, $\tau \leftrightarrow e$ and $\tau \leftrightarrow \mu$: with a detected $\tau \leftrightarrow e$ ($\tau \leftrightarrow \mu$) signal, non-observation of $\mu \leftrightarrow e$ can suggest the size of some $\tau \leftrightarrow \mu$ ($\tau \leftrightarrow e$) interactions.

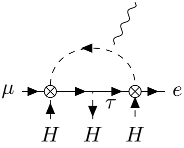
THANK YOU!

BACK-UP

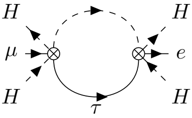
Running diagrams



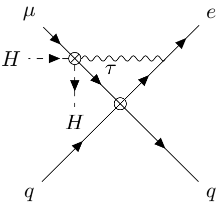
(a) $Y_6 \times P_6 \to D_8$



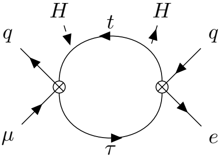
(b) $P_6 \times P_6 \to D_8$



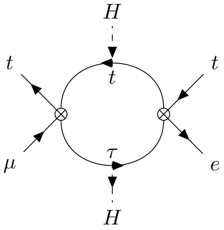
(c) $Y_6 \times Y_6 \to P_8$



(d) $P_6 \times 4f_6 \to 4f_8$

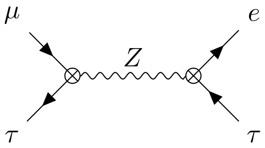


(e) $4f_6 \times 4f_6 \to 4f_8$



(f) $4f_6 \times 4f_6 \to 4f_8$

Matching diagrams



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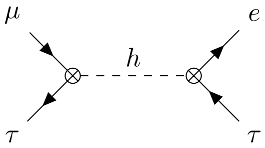
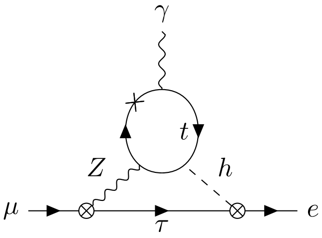
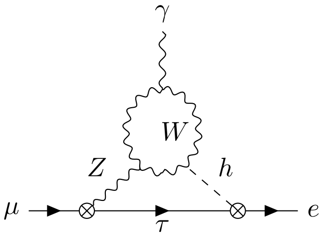


(b)



(c)



(d)

Equations of Motion

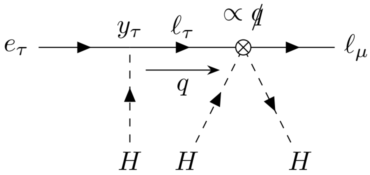
Two operators $\mathcal{O}_1, \mathcal{O}_2$ that differ by an Equation of Motion (EOM) vanishing operator are physically equivalent

$$\mathcal{O}_1 - \mathcal{O}_2 = \mathcal{O}_{EOM} \propto \frac{\delta S}{\delta \phi}$$

because $\mathcal{O}_{EOM}$ has vanishing $S-$matrix elements. For instance, $i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H)$ and $y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H)$ are on-shell equivalent

$$i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) \rightarrow [i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) - y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H)] + y_\tau(\bar{\ell}_\mu H e_\tau)(H^\dagger H)$$

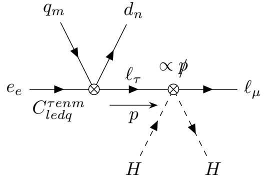
We can understand it diagrammatically:



Suppose that the only dimension six present is $\mathcal{O}_{\ell edq}^{e\tau nm} = (\bar{\ell}_\tau e_e)(\bar{d}_n q_m)$. The operators corrects the EOM, such that

$$i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) \to \left[i(\bar{\ell}_\mu \not{D} \ell_\tau)(H^\dagger H) - y_\tau (\bar{\ell}_\mu H e_\tau)(H^\dagger H) + \frac{C_{\ell edq}^{\tau enm}}{\Lambda_{\rm NP}^2} (\bar{\ell}_\mu e_e)(\bar{d}_n q_m)(H^\dagger H) \right] + y_\tau (\bar{\ell}_\mu H e_\tau)(H^\dagger H) - \frac{C_{\ell edq}^{\tau enm}}{\Lambda_{\rm NP}^2} (\bar{\ell}_\mu e_e)(\bar{d}_n q_m)(H^\dagger H)$$

Diagrammatically:



If a redundant operator is generated via loops, in general:

$$\frac{A}{\Lambda_{\rm NP}^2 \epsilon} \left(\mathcal{O}^{[6]} + \frac{\mathcal{O}^{[8]}}{\Lambda_{\rm NP}^2} - \mathcal{O}_{EOM} \right)$$

where $\mathcal{O}^{[8]}$ is due to the dimension six correction to the EOM.