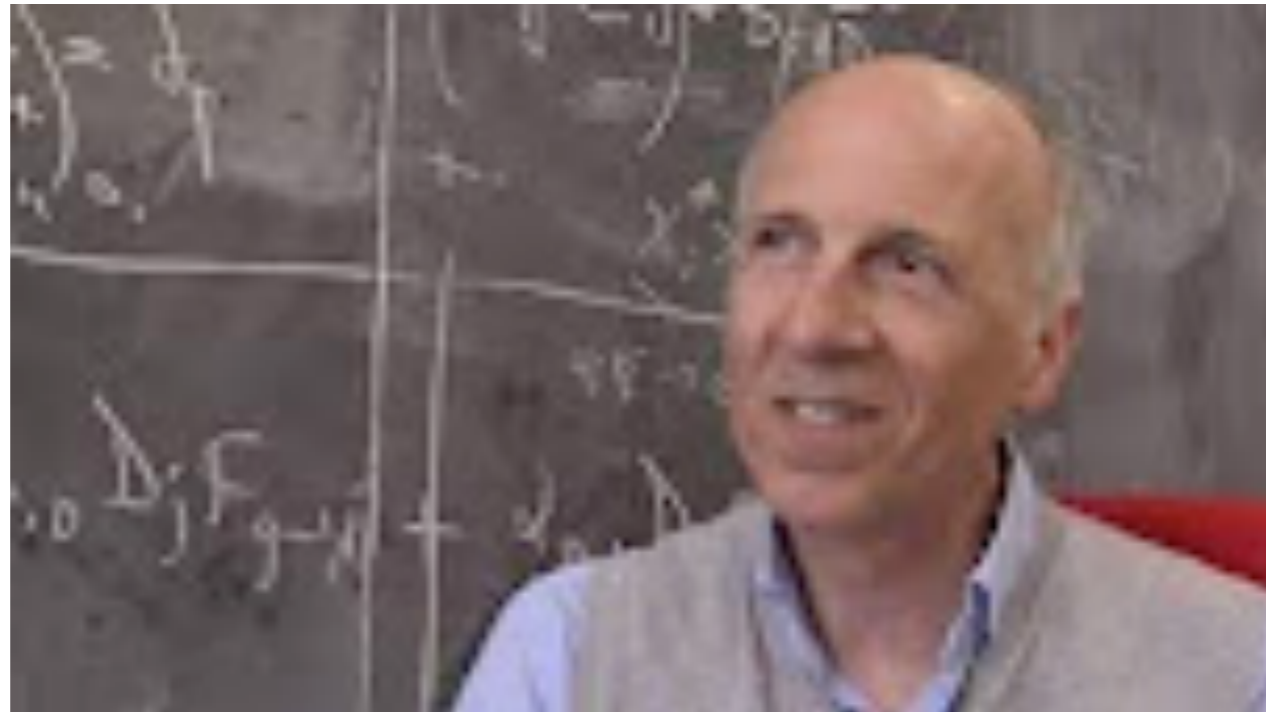
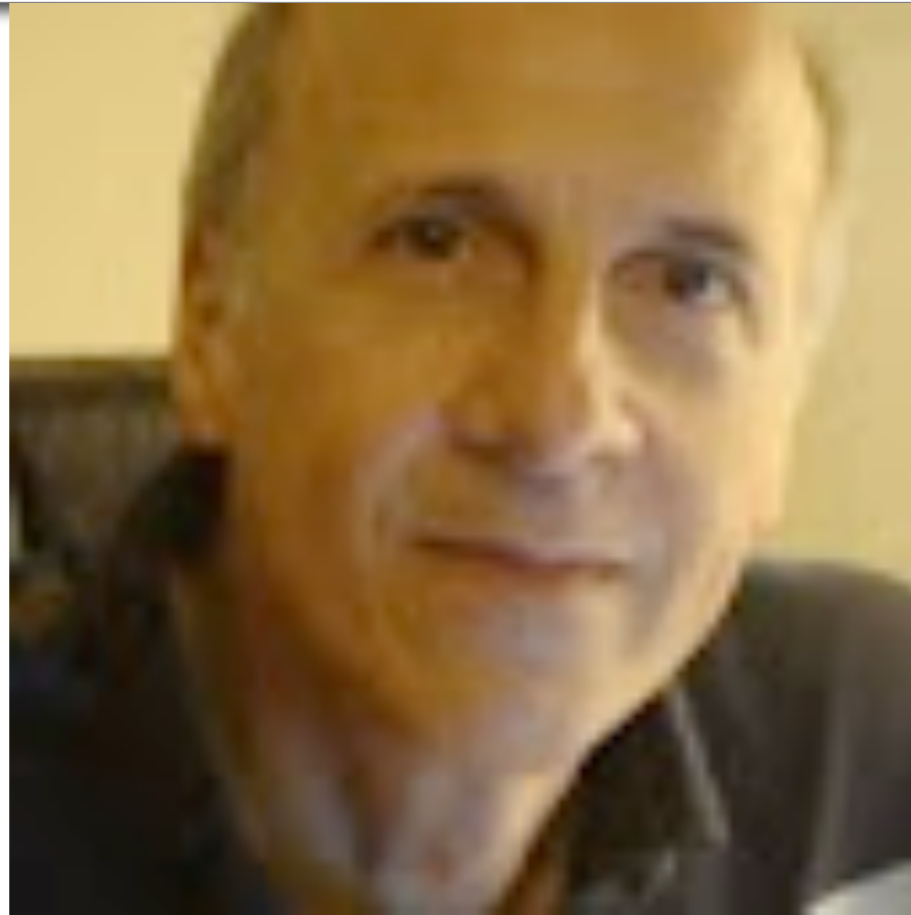


# HAPPY BIRTHDAY IGNATIOS



# My friend



**We met for the first time in the early 1990 (Ioannina?), around 30 years ago**

**We collaborated mainly in the period between 1992-2016**

**Ignatios immersed me into the world of extra dimensions**

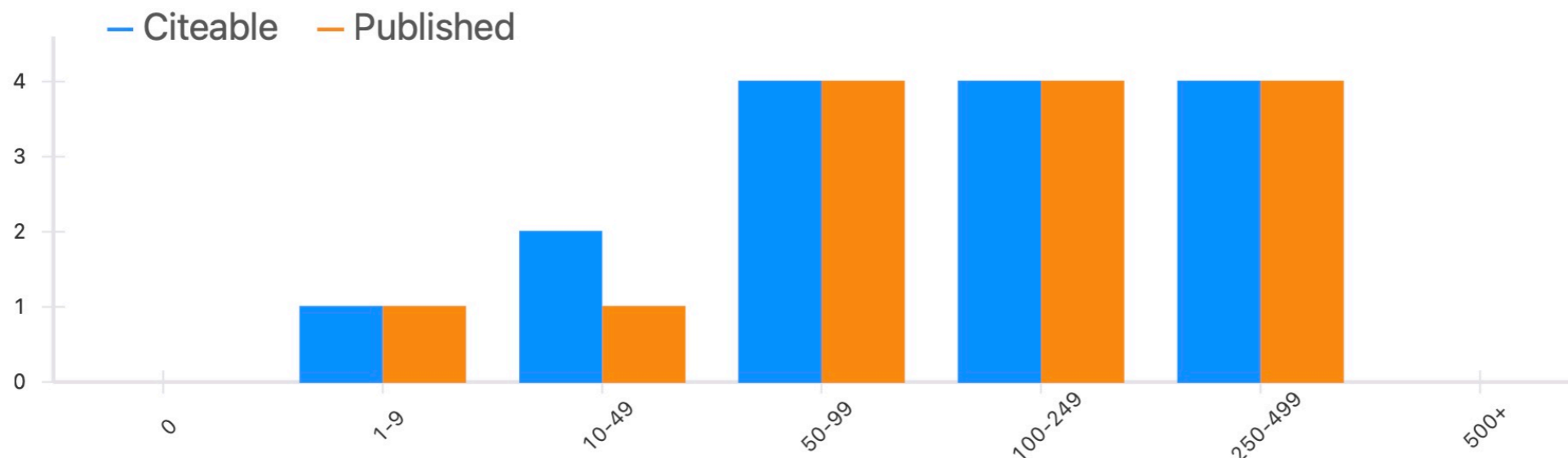
**Thank you Ignatios, I learned a lot from you**

# My collaborator

## Citation Summary

Exclude self-citations [?](#)

	Citeable <a href="#">?</a>	Published <a href="#">?</a>
Papers	15	14
Citations	2,241	2,221
h-index <a href="#">?</a>	13	12
Citations/paper (avg)	149.4	158.6



# CONTINUUM SPECTRA FROM WARPED DIMENSIONS

Mariano Quirós

**Institute of High Energy Physics (IFAE)**



**Planck 2022**

**24th International Conference from the Planck Scale to  
the Electroweak Scale**

**May 30-June 3, 2022 , Old Ecole Polytechnique, Paris**

# A warped solution to the hierarchy problem

- A warped extra dimension was proposed in 1999 by Randall and Sundrum (RS)

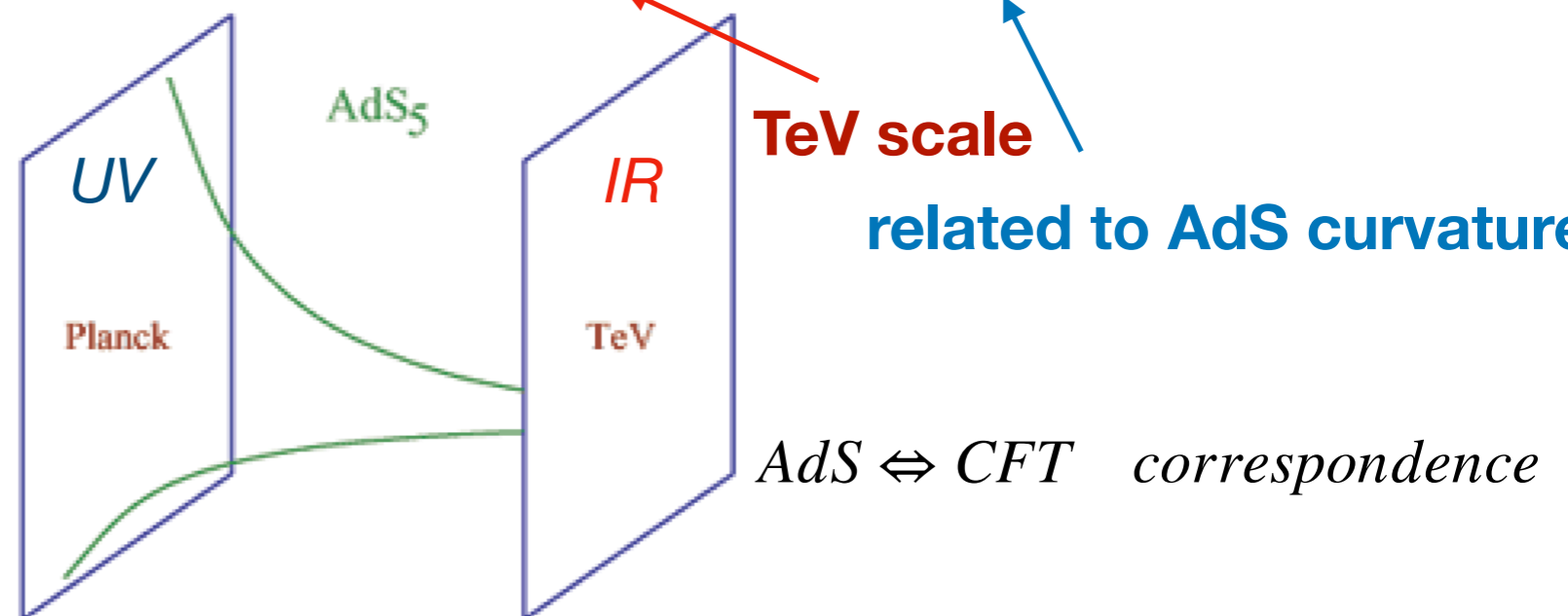
*L. Randall, R. Sundrum, 9905221*

- It was based on an  $AdS_5$  space with line element

$$ds^2 = e^{-2A} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad A = ky \quad \text{RS}$$

$$\rho \simeq e^{-A(y_1)} k, \quad k \sim M_{Pl}, \quad A(y_1) \sim 35$$

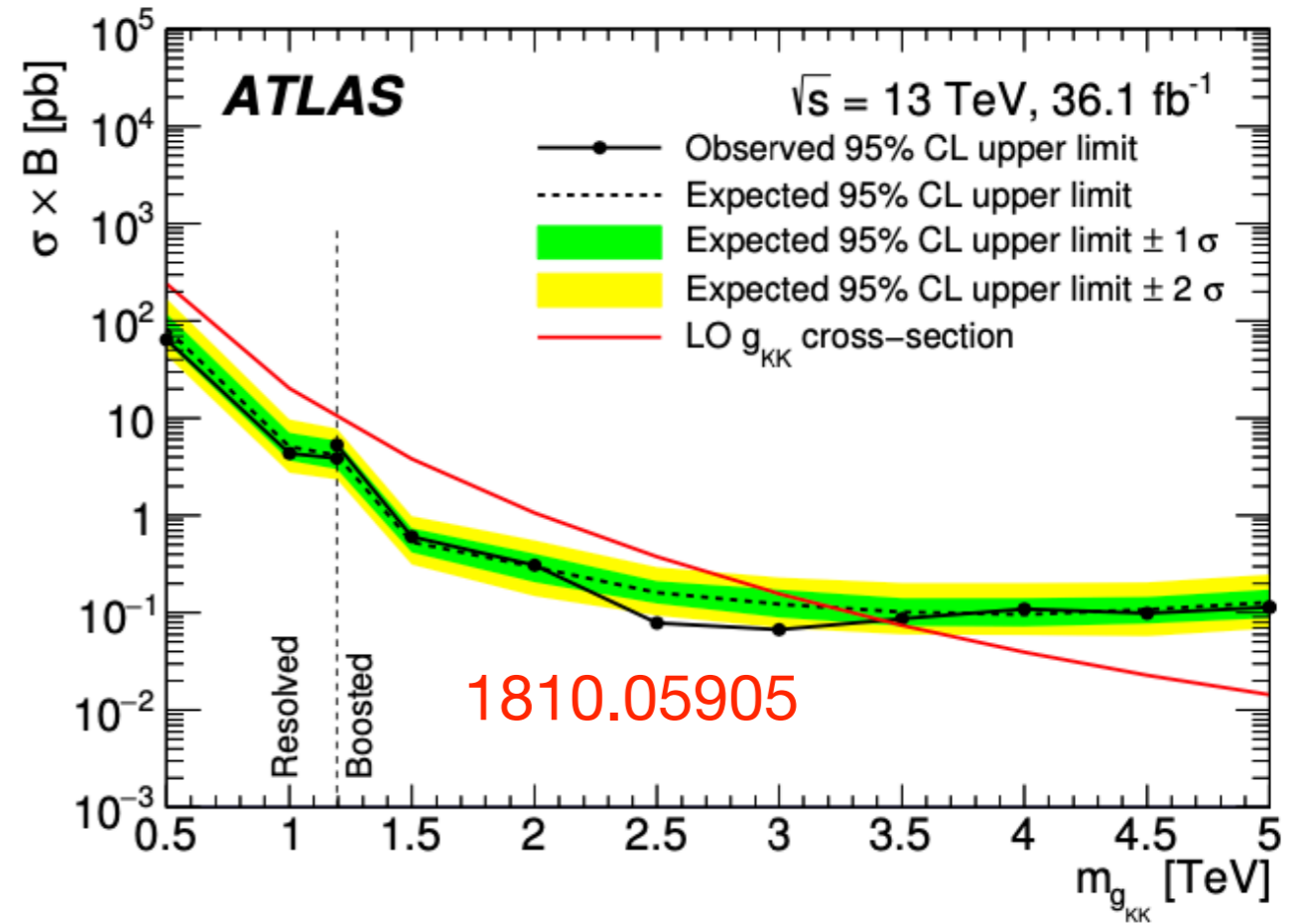
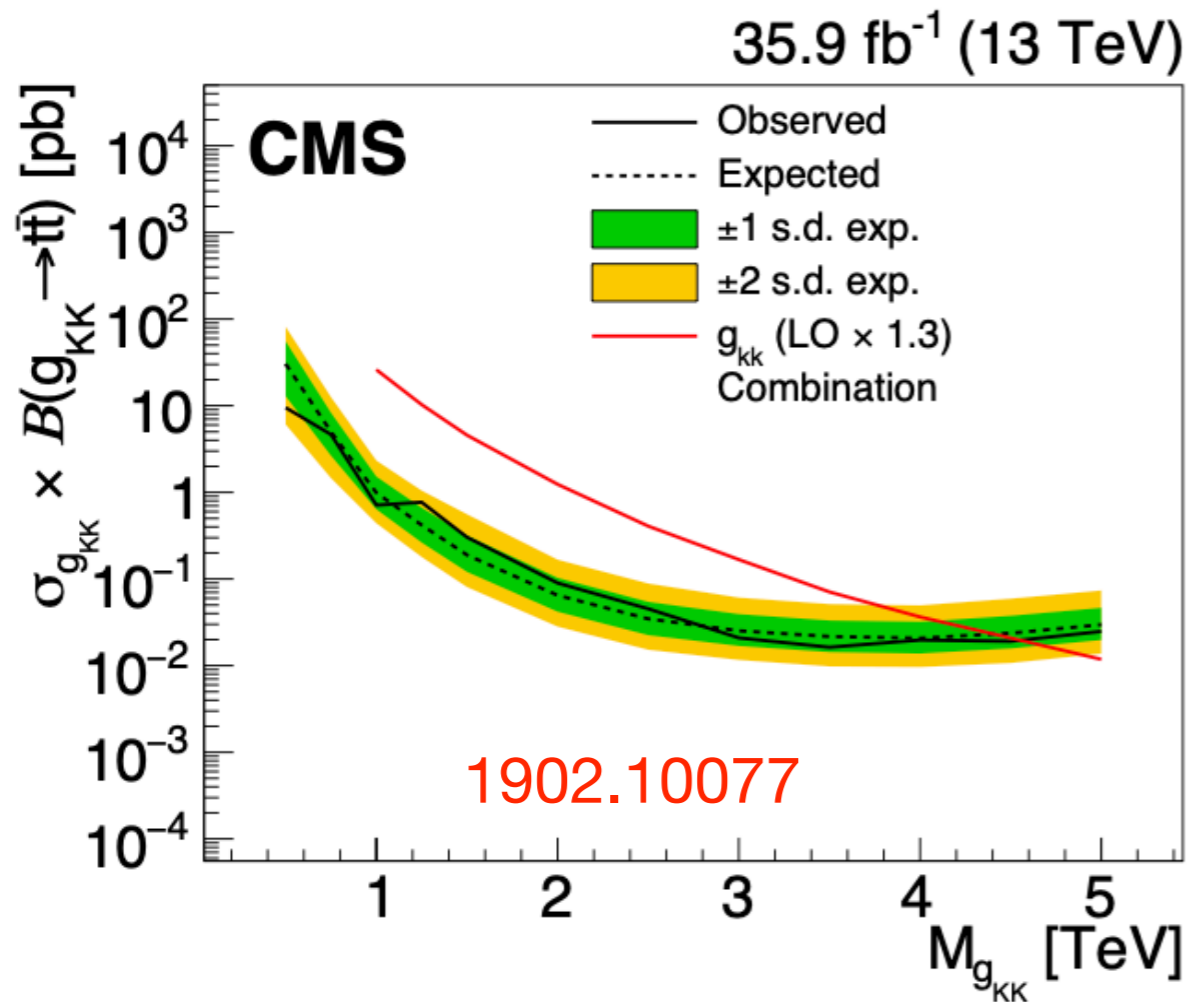
- With two branes at  $(0, y_1)$



- The Higgs is localized toward the **IR** brane (**composite**):
- Heavy (light) fermions are localized toward the **IR** (**UV**) brane: **composite** (**elementary**)
- Zero mode gauge bosons are flat
- **KK resonances (TeV)** are localized toward the **IR** brane (**composite**)

# Collider challenges: $t\bar{t}$ production

- The LHC data are putting severe bound on the mass of the lightest KK resonances, e.g. for KK gluons:



$$g_{g_{\mu}^{(0)}qq} = -0.2g_s, \quad g_{g_{\mu}^{(0)}t_L t_L} = g_s, \quad \Gamma_{g_{\mu}^{(0)}} = 30\%$$

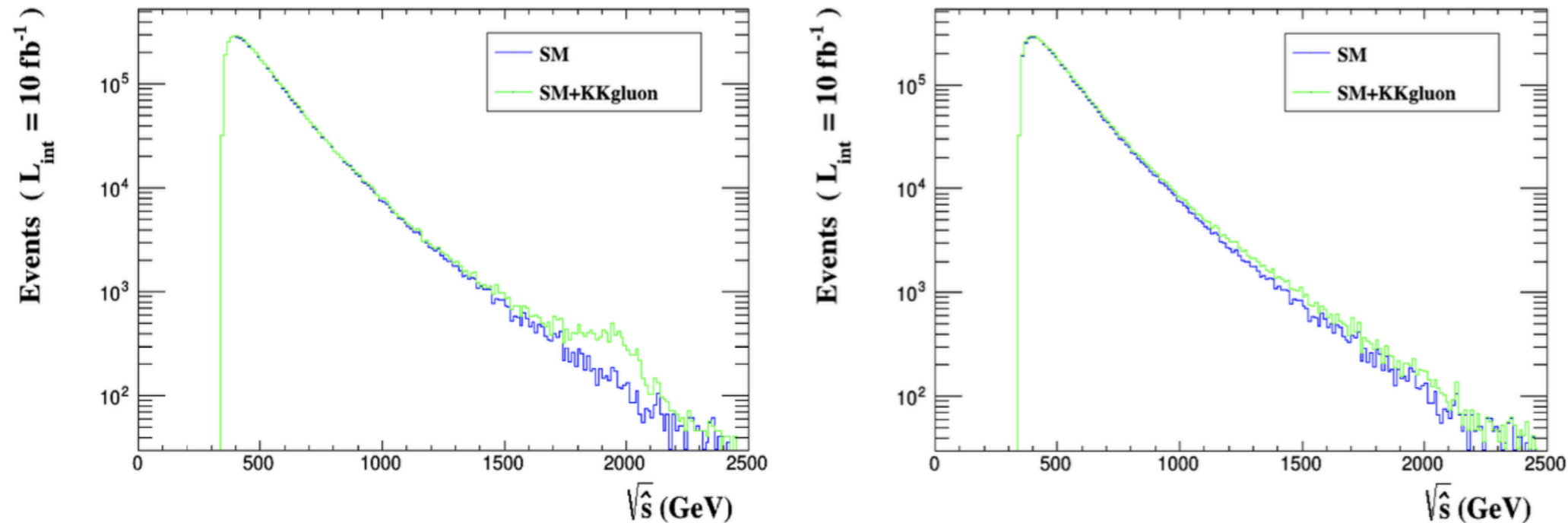
- These limits point toward the possibility that nature might have chosen values of  $\rho \gg TeV$ , or that we have to reformulate our model. Two ways out:
- i) Way out 1: Strongly coupled (gluon) resonances are not narrow, but they are **broad resonances**: Bumps flatten in detection!
- ii) Way out 2: Resonances are a **gapped continuum**: no bumps at all!

# Broad resonances

- If some fermions ( $t_R$ ) are localized toward the IR brane, where KK resonances  $g_\mu^{(0)}$  are localized, the latter can be very broad and difficult to detect [R. Escribano et al., 2102.11241](#)

$$\Gamma/M = 0.1, \quad \Gamma = 200 \text{ GeV}$$

$$\Gamma/M = 0.8 \quad \Gamma = 1.6 \text{ TeV}$$



$$\sigma(pp \rightarrow t\bar{t})$$

$$M=2 \text{ TeV}$$



# Gapped continuum KK resonances

- Another possible “explanation” for elusiveness: KK states is a TeV gapped continuum of states instead of isolated particles  
[C. Csaki et al., 1811.06019](#)  
[G. Giudice et al., 1711.08437](#)
- A theory in that direction is the [clockwork mechanism](#), or its 5D version, where [TeV is the fundamental scale](#), and Planck is a derived scale. The KK modes have a TeV mass gap and a (quasi continuum) spacing of 30 GeV. Similar to Linear Dilaton scenarios, dual to Little String theories (where hierarchy problem is solved by the string theory)

[I. Antoniadis et al., 1102.4043](#)

- The class of 5D models we have considered here share some properties

$$A(y) = \overset{\text{UV AdS}}{ky} - \overset{\text{IR deformation}}{\frac{1}{\nu^2}} \log(1 - y/y_s)$$

1. They reproduce RS in the UV and therefore they can explain **conventionally** the hierarchy with a **fundamental** Planck scale and a **warped** TeV scale
2. For  $\nu > 1$  they yield discrete KK spectra with TeV spacing
3. For  $\nu < 1$  they yield ungapped continuum spectra similar to **unparticles**
4. For critical  $\nu = 1$  they yield gapped continuum spectra

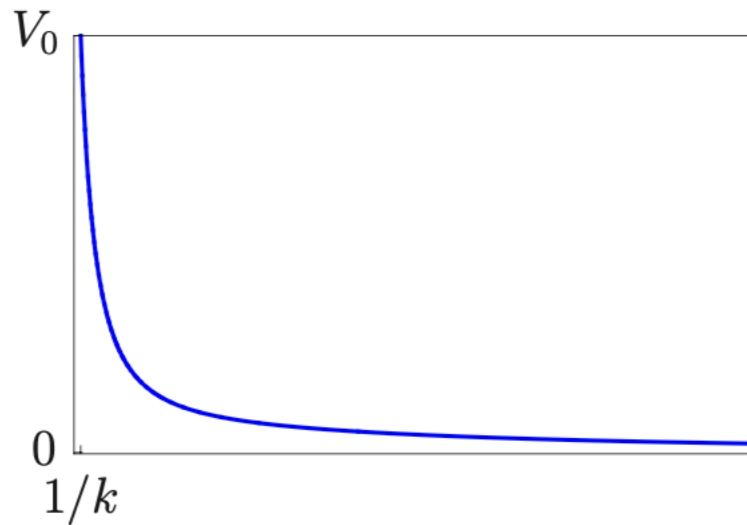
H. Georgi, 0703260

A. Falkowski et al., 0806.1737

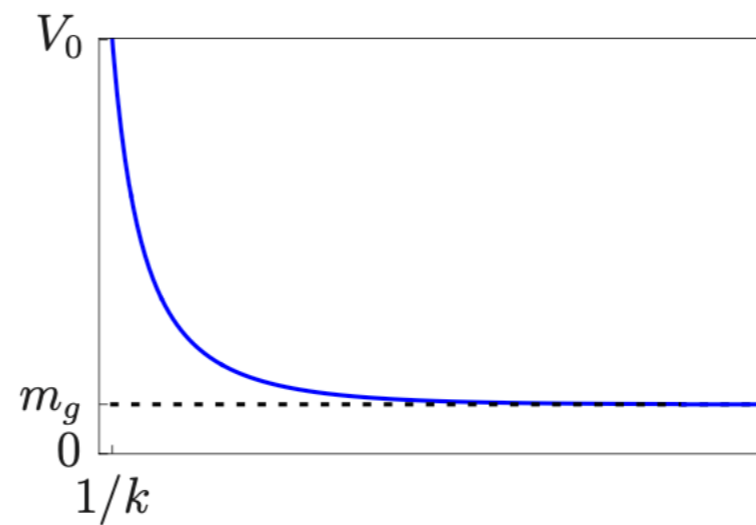
J.A. Cabrer et al., 0907.5361

# Schrödinger-like potential (for the graviton)

*for different values of  $\nu$*

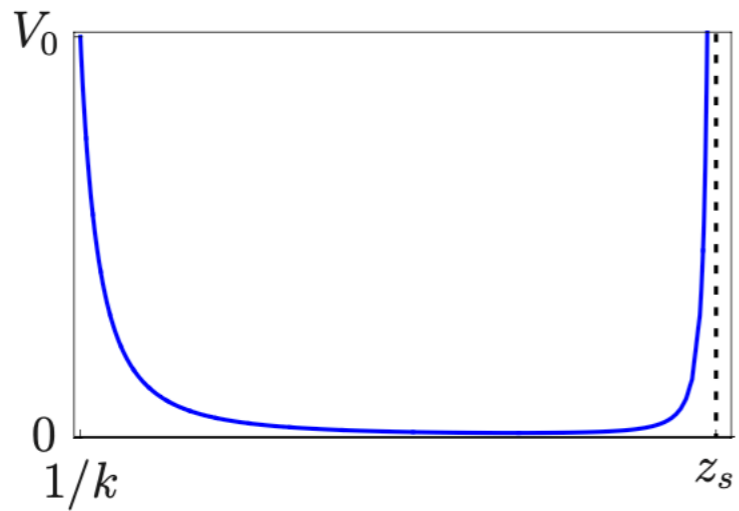


$z$   
(a)  $0 < \nu < 1$

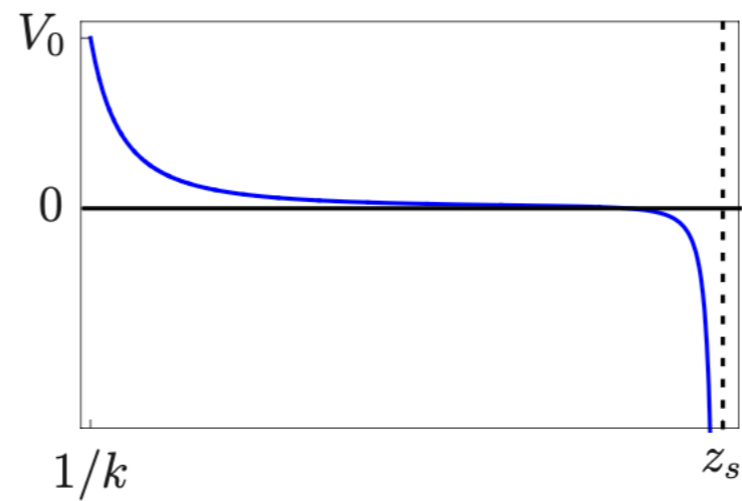


$z$   
(b)  $\nu = 1$

J.A. Cabrer et al., 0907.5361



$z$   
(c)  $1 < \nu < \sqrt{5/2}$



$z$   
(d)  $\nu > \sqrt{5/2}$

# The critical case $\nu = 1$ : continuum with mass gap

- Their Green functions generalize from **particle propagator** with isolated poles

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathcal{P} \frac{1}{p^2 - m^2} + i\pi \delta(p^2 - m^2)$$

- ... to Green functions with an **isolated pole** (the zero mode) and a **continuum of states**, instead of a discrete sum of KK modes, with a mass gap  $m$

$$G_A(p^2, m^2) = \text{Re}G_A(p^2, m^2) + i \text{Im}G_A(p^2, m^2)\theta(p^2 - m^2)$$

- This is the behavior of gapped unparticles where the gap was usually produced by EW breaking
- Here the gap is TeV, and is linked to the solution of the hierarchy problem
- The extra dimension has **UV boundary** ( $y=0$ ), **IR brane** ( $y = y_1$ ) where Higgs lives, and **singularity** ( $y = y_s > y_1$ )
- The mass gap is different for different bulk fields

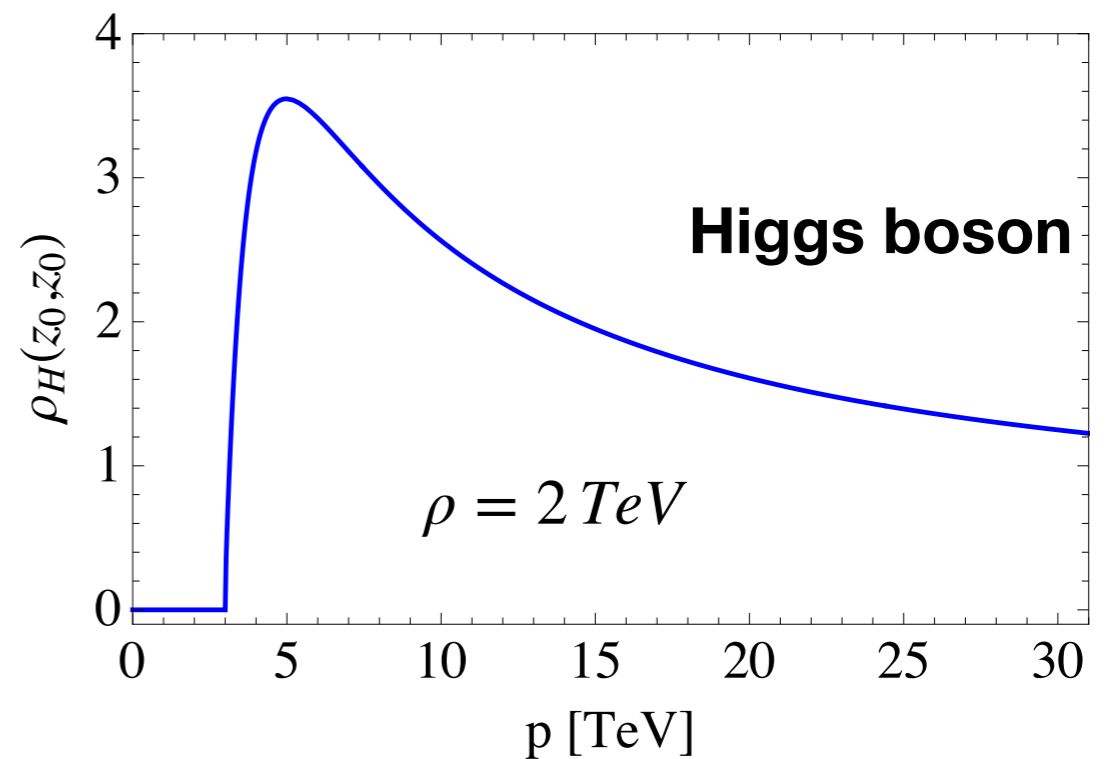
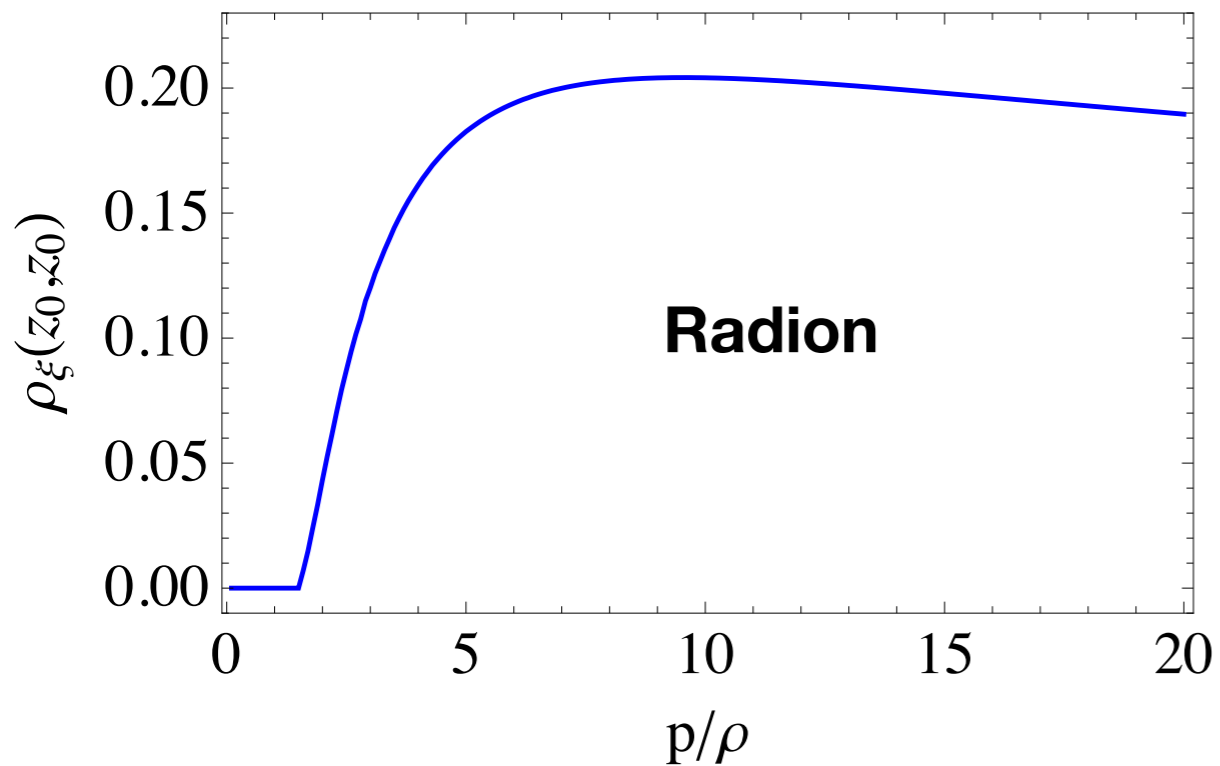
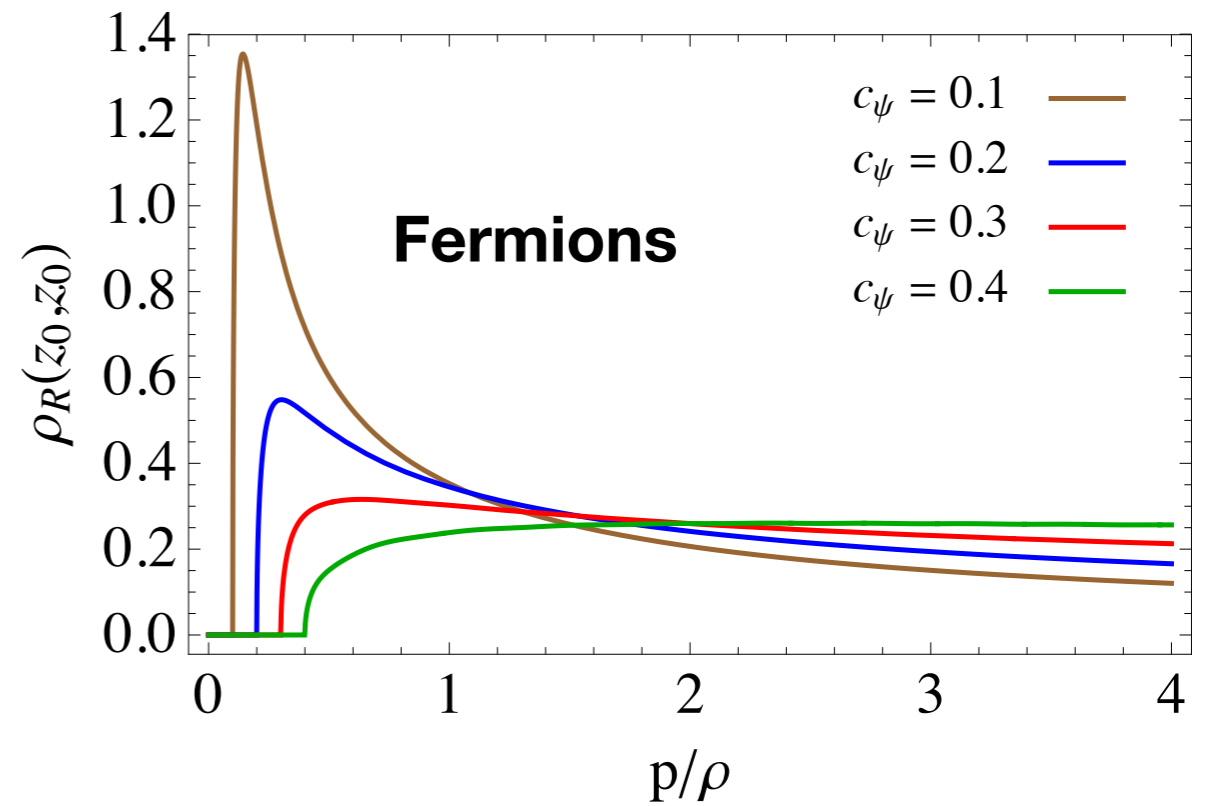
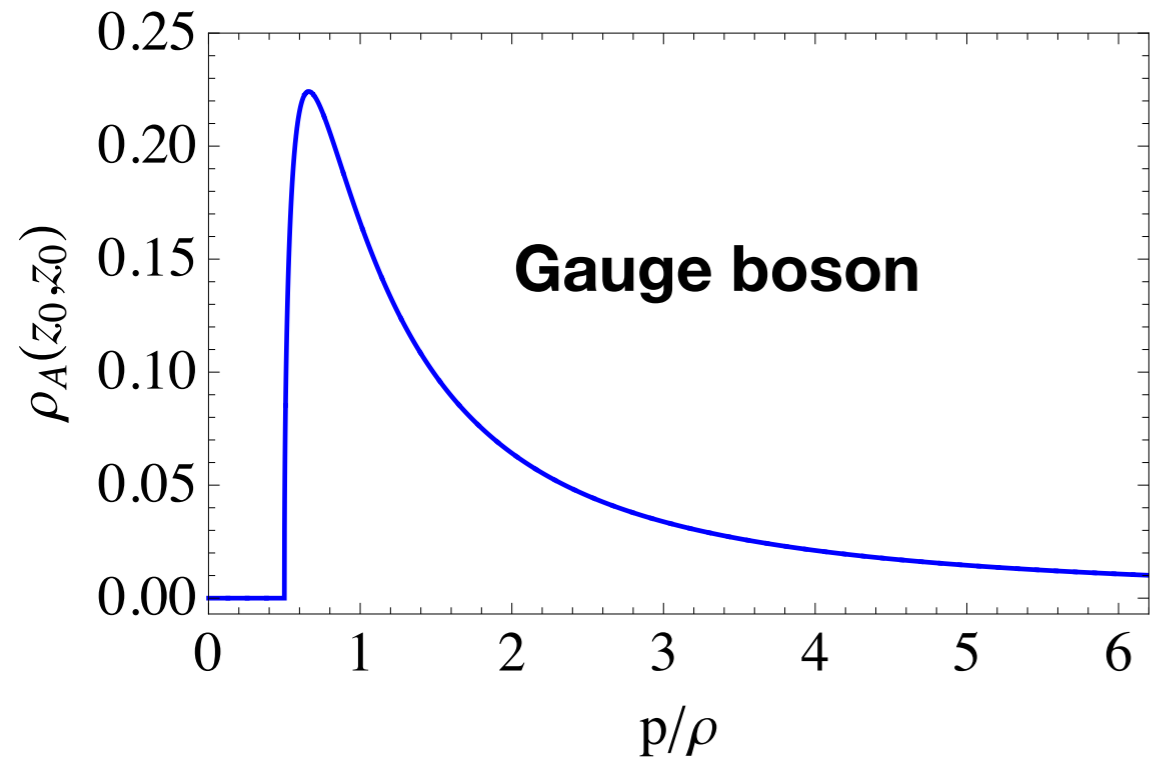
- The mass gap for the different states

field	gauge boson	fermion	graviton radion Higgs
mass gap	$\frac{1}{2}\rho$	$ c_f \rho$	$\frac{3}{2}\rho$

The continuum spectrum shows up in the holographic **spectral density functions**

$$p \equiv \sqrt{s}, \quad m_g = \text{mass gap}$$

E. Megías and MQ, 1905.07364

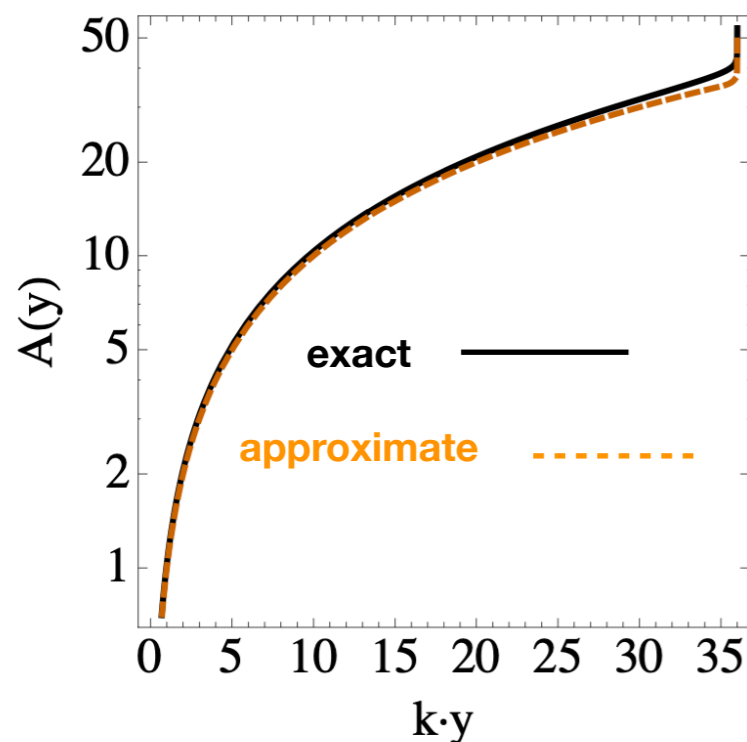


# Analytical approximation on Green's functions

- We can get analytical insight into the structure of Green's function by approximating the exact metric (in proper coordinates)  $A_{exact}(y) = ky - \log(1 - y/y_s)$

- By an approximate one (in conformal coordinates)

$$A_{app}(z) = \log kz \Theta(z_1 - z) + [\log kz_1 - \rho(z - z_1)] \Theta(z - z_1)$$



RS

LDM

$$\text{Condition: } ky_s - ky_1 = 1$$

*E. Megías, M.Q., 2106.09598*

# One can explore the structure of Green's functions in the complex s-plane

- Green's functions for (massless) gauge bosons  $A_\mu$  with  $y$  and/or  $y' < y_1$

$$G_A(y, y'; p) \propto \frac{1}{\Phi(p)}$$

$$\Phi(p) = \mathcal{K} \cdot J_+(p/\rho) - Y_+(p/\rho)$$

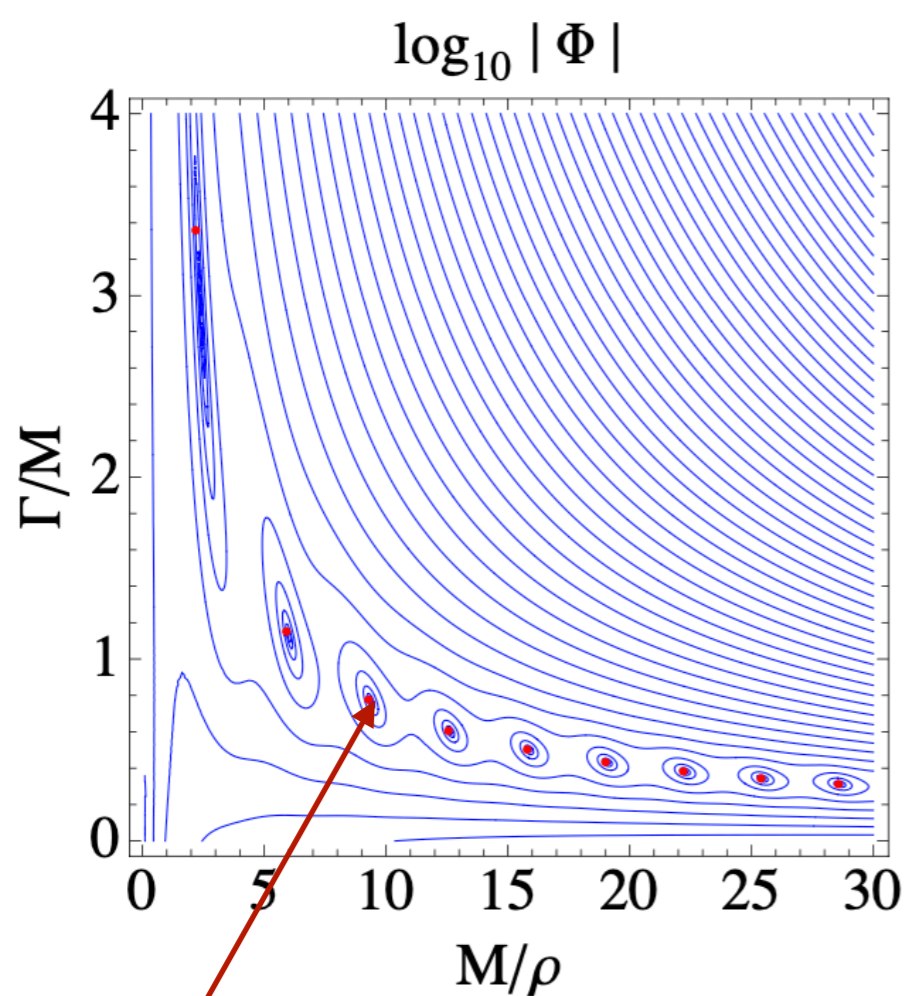
$$\mathcal{K} \equiv \frac{2}{\pi} (\gamma_E - \log(2) + \log(p/\rho) - ky_1)$$

$$J_\pm(p/\rho) = 2\frac{p}{\rho} J_0(p/\rho) + \Delta_A^\pm J_1(p/\rho), \quad Y_\pm(p/\rho) = 2\frac{p}{\rho} Y_0(p/\rho) + \Delta_A^\pm Y_1(p/\rho)$$

$$\Delta_A^\pm \equiv \pm\delta_A - 1, \quad \delta_A = \sqrt{1 - 4p^2/\rho^2}$$



- The zeros of  $\Phi(p)$  are poles of  $G_A(y, y'; p)$
- We have looked for resonances by looking for poles in the second (unphysical) Riemann sheet of the complex  $s$ -plane
- Unexpectedly we have found a tower of broad resonances located in the unphysical Riemann sheet



*n*-th branch of Lambert function

$$\frac{s}{\rho^2} = -\mathcal{W}_n \left[ \pm \frac{1}{4}(1+i) \right]^2, \quad n = -1, -2, -3, \dots$$

*No poles found in the first (physical) Riemann sheet*

**Poles interpreted as resonances !!!**

**Poles**

$$(M/\rho, r) = (2.42, 2.87), (6.03, 1.12), (9.37, 0.768), (12.64, 0.601), (15.87, 0.500), \dots$$

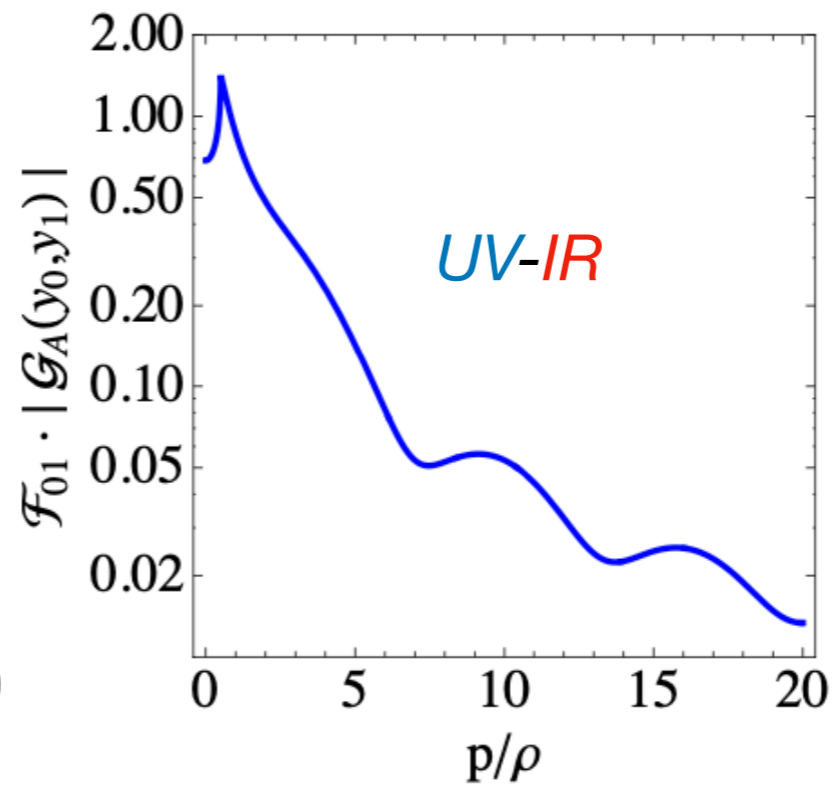
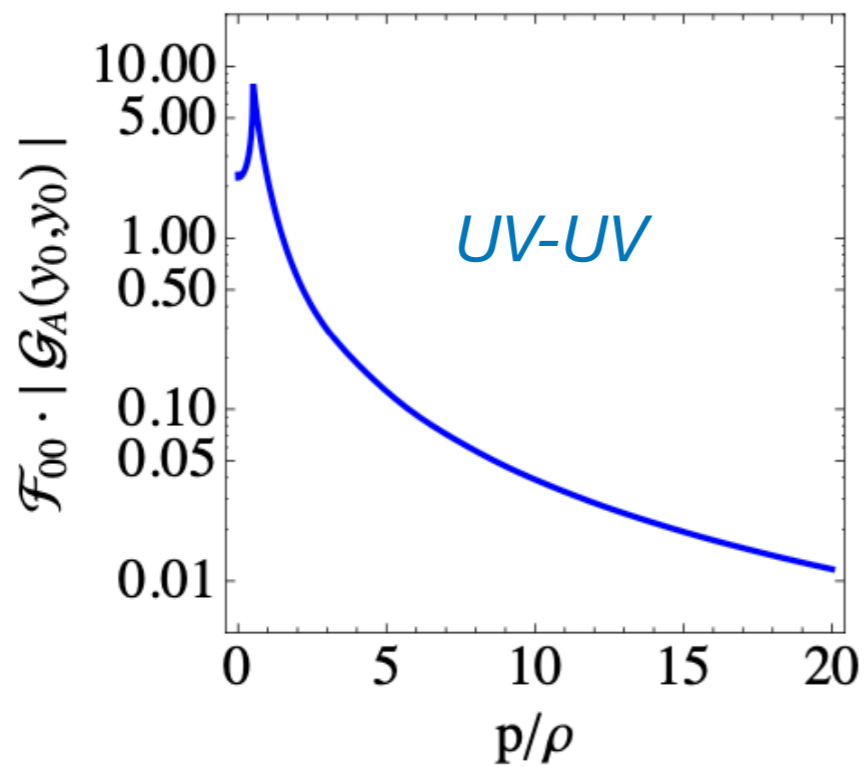
- Green's functions with  $y$  and  $y' > y_1$  are decomposed as

- $G_A(y, y'; p) = G_{res}(y, y'; p) + G_{un}(y, y'; p)$

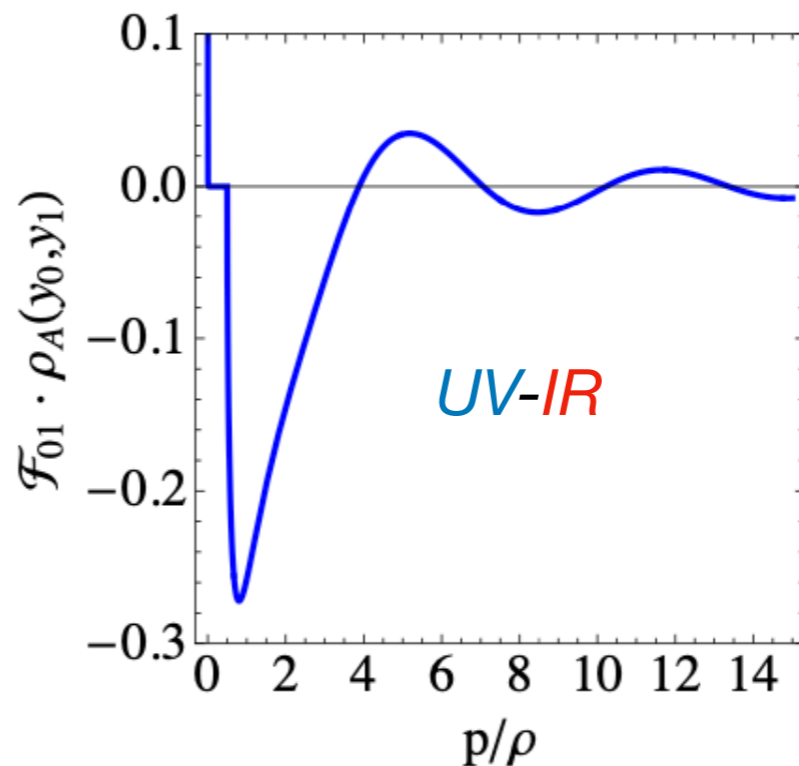
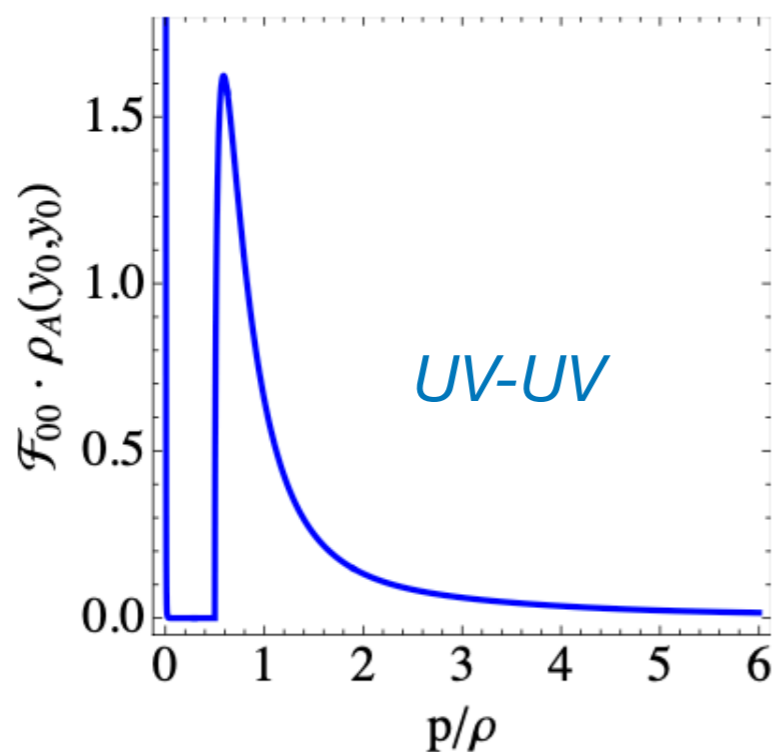
- where  $G_{res}(y, y'; p) \propto \frac{1}{\Phi(p)}$  ← *poles*

- and  $G_{un}(y, y'; p) \propto \frac{1}{\delta_A(p)}$  which correspond to gapped unparticles with  $d_U = 3/2$

# Some Green and spectral functions



$$\mathcal{F}_{00} = \frac{\rho^2}{k} (ky_s)^2$$



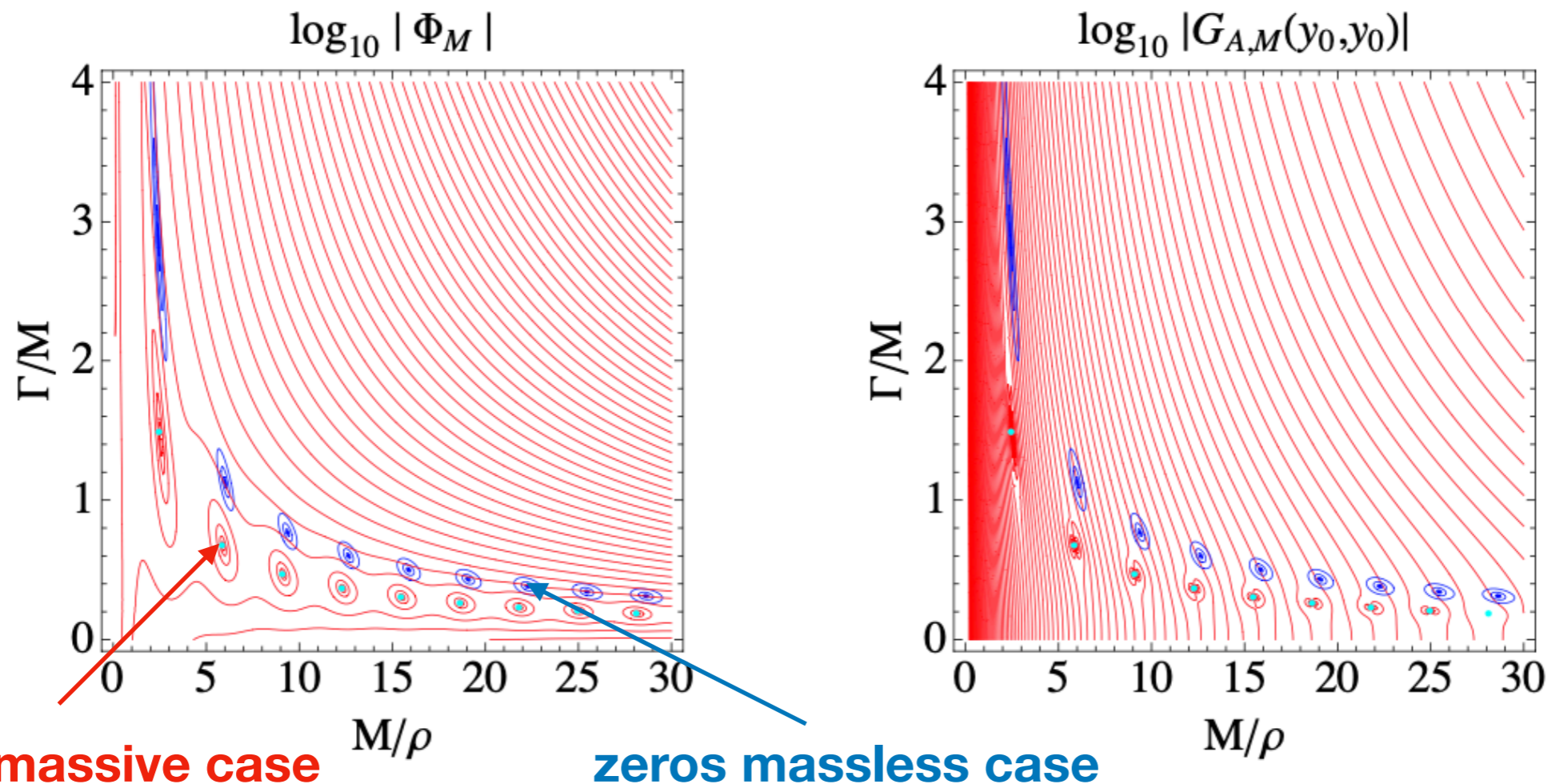
$$\mathcal{F}_{01} = \frac{\rho^2}{k} (ky_s)$$

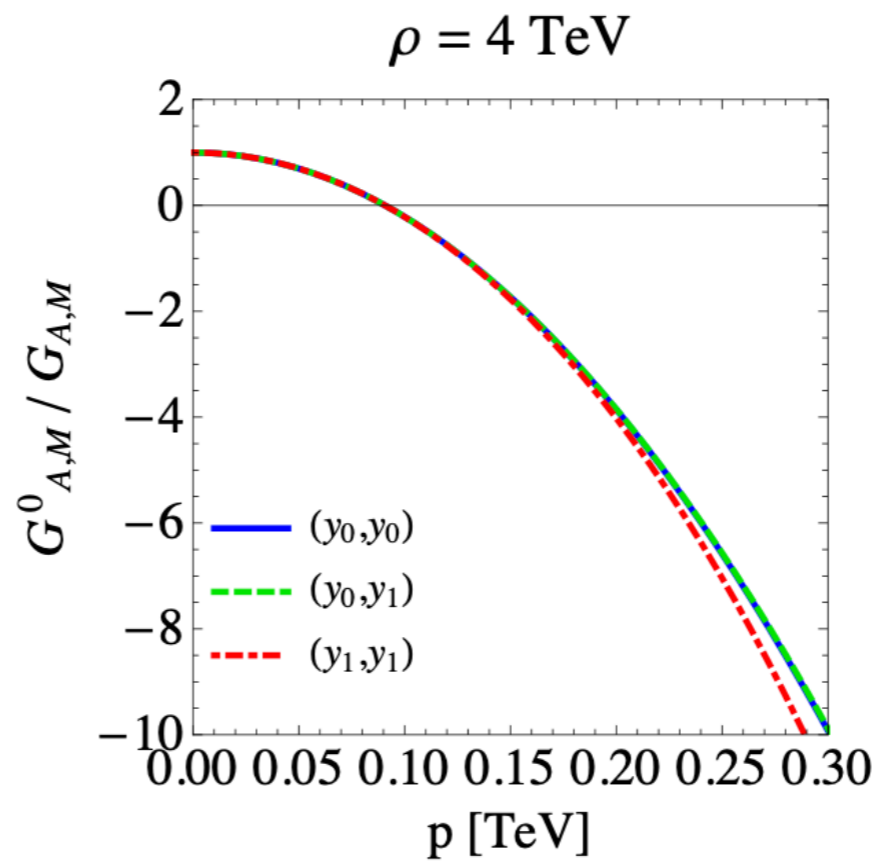
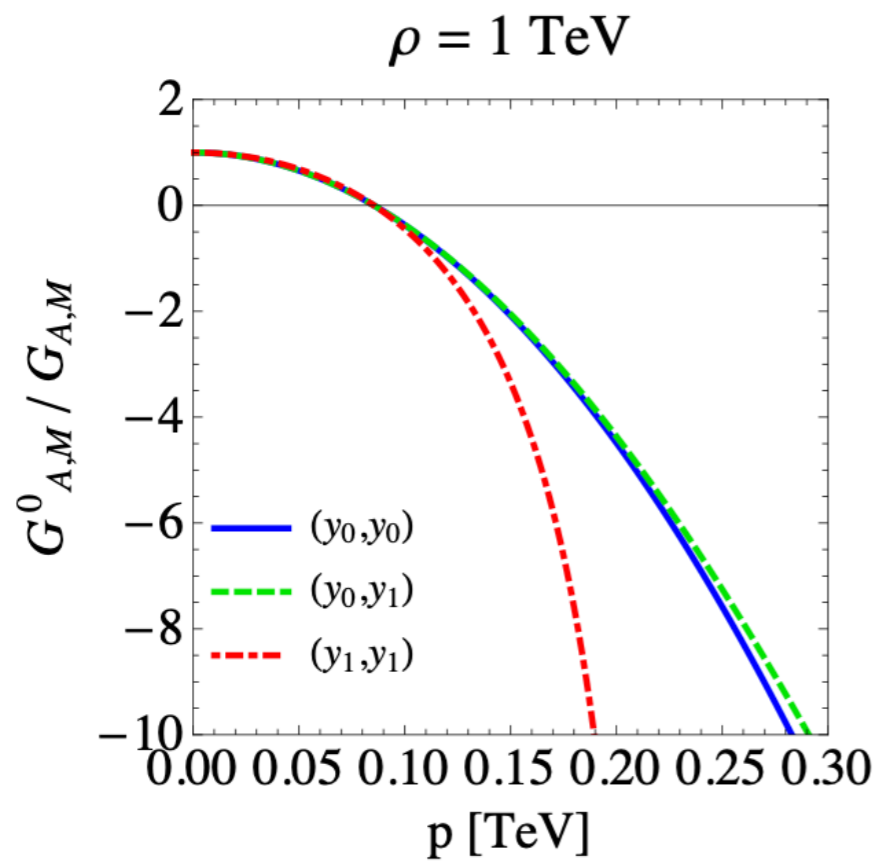
*Massless gauge bosons*

$$\rho_A(y, y'; p) = \frac{1}{y_s} \delta(s) + \dots$$

# For massive Gauge bosons $W_\mu, Z_\mu$

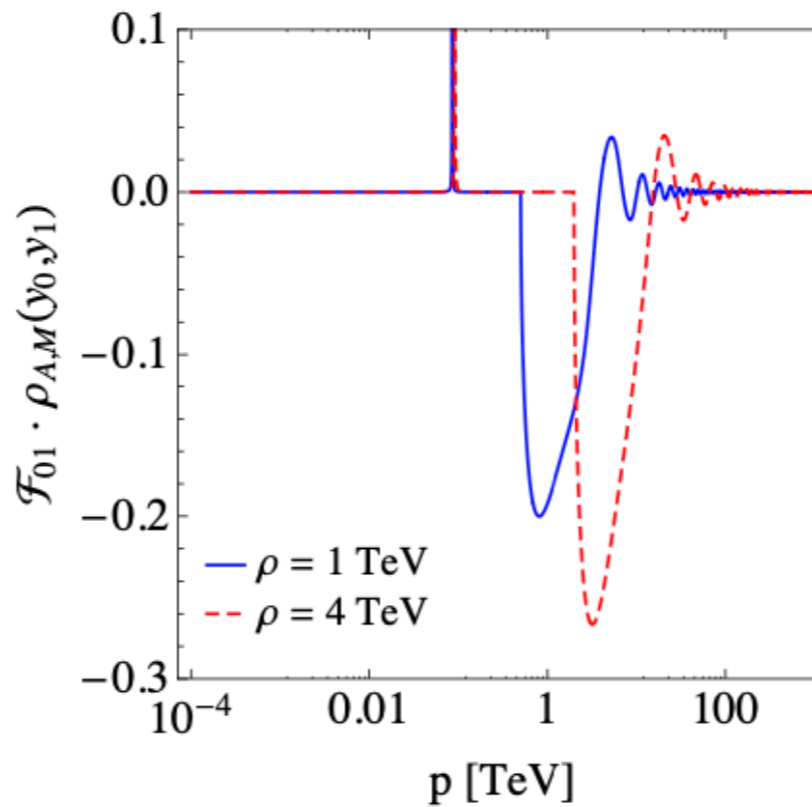
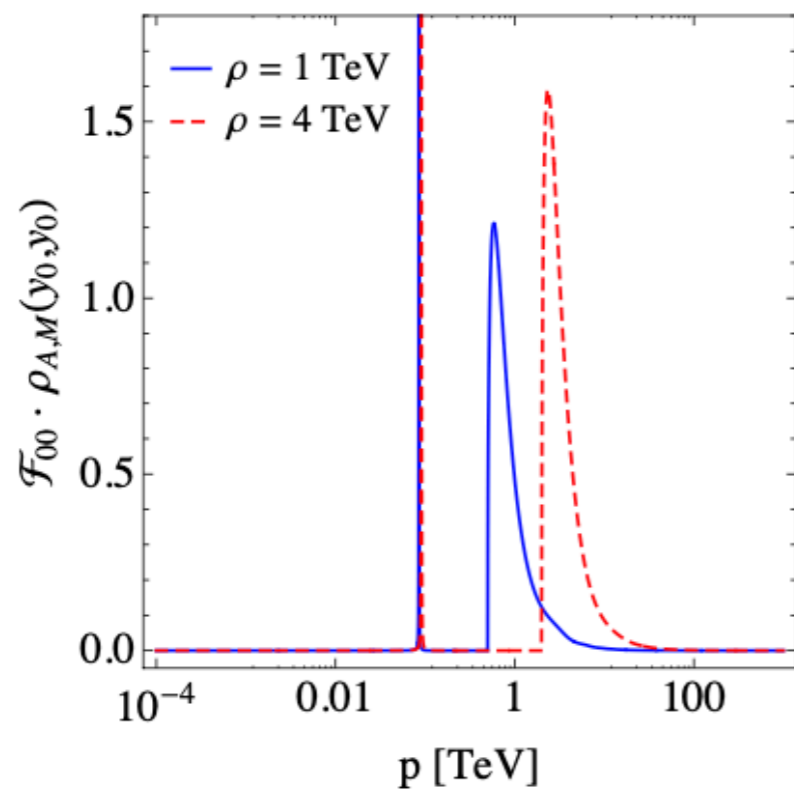
- Similar results follow by replacing the function  $\Phi(p)$  by  $\Phi_M(p)$  after replacing  $\Delta_A^\pm \rightarrow \Delta_A^\pm + 2ky_s(m_A/\rho)^2$





$$G^0_{A,M} = \lim_{p \rightarrow 0} G_{A,M}(y, y', p)$$

$$m_Z = 0.091 \text{ TeV}$$



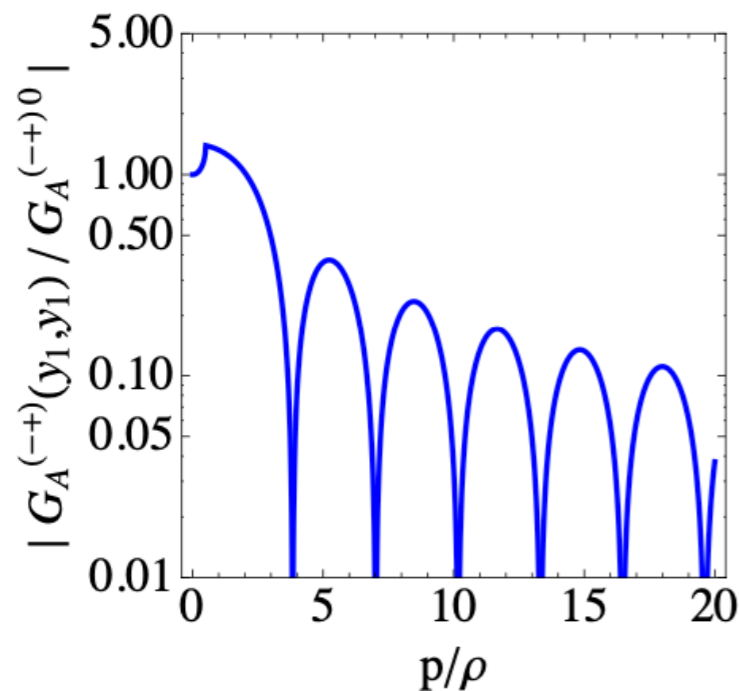
$$\rho_A(y, y'; p) = \frac{1}{y_s} \delta(s - m_A^2) + \dots$$

- Notice that while  $\rho_A(y_0, y_0)$  is positive definite (as it is 4D spectral function in the UV brane),  $\rho_A(y_0, y_1)$  **is not** that challenges the physical interpretation of the spectral function in 4D QFT
- This apparent contradiction can be resolved by noticing that  $\rho_A(y, y')$  can be considered as the matrix element  $(y, y')$  of an operator  $(\hat{\rho}_A)_{y'}^y \equiv \rho_A(y, y')$
- This operator acts on the infinite dimensional space parametrized by the coordinate  $y$
- The matrix  $\hat{\rho}_A$  turns out to have a factorizable form, i.e.
 
$$(\hat{\rho}_A)_{y'}^y = \rho_y \rho_{y'}, \quad \rho_y \equiv \sqrt{(\hat{\rho}_A)_y^y}$$
- All eigenvalues of  $\hat{\rho}_A$  are zero except one which is given by
 
$$\lambda(p) \equiv \text{tr } \hat{\rho}_A = \int dy \rho_A(y, y; p) \geq 0$$

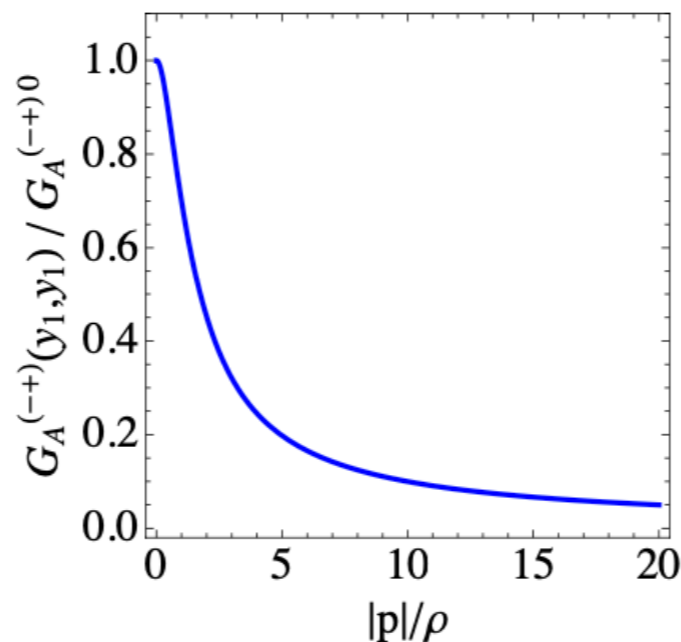
# For gauge bosons with Dirichlet boundary conditions: i.e. $SU(2)_R$

- There are no zero modes, only the continuum and broad resonances
- The Green's functions satisfy  $G^{(-,+)}(y_0, y'; p) = 0$ , which follows from the Dirichlet boundary condition  $G^{(-,+)}(y_0) = 0$

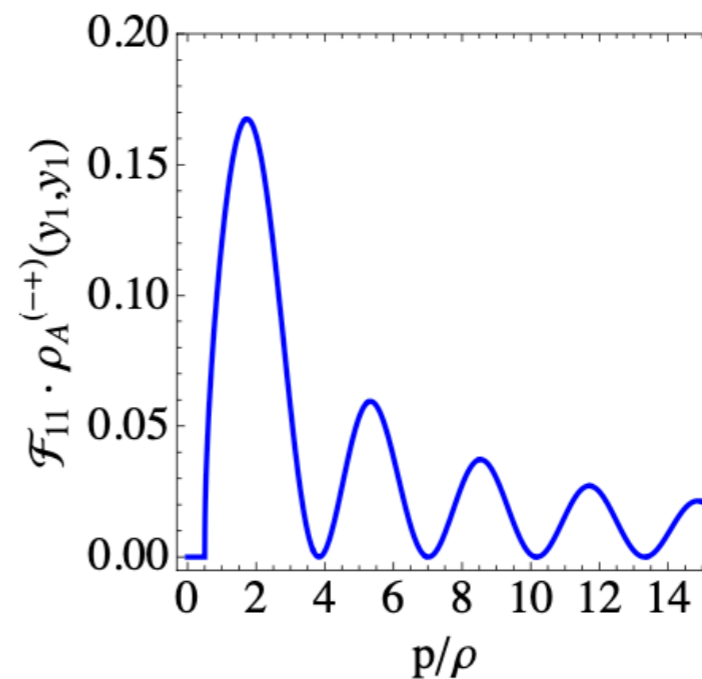
$$p^2 > 0$$

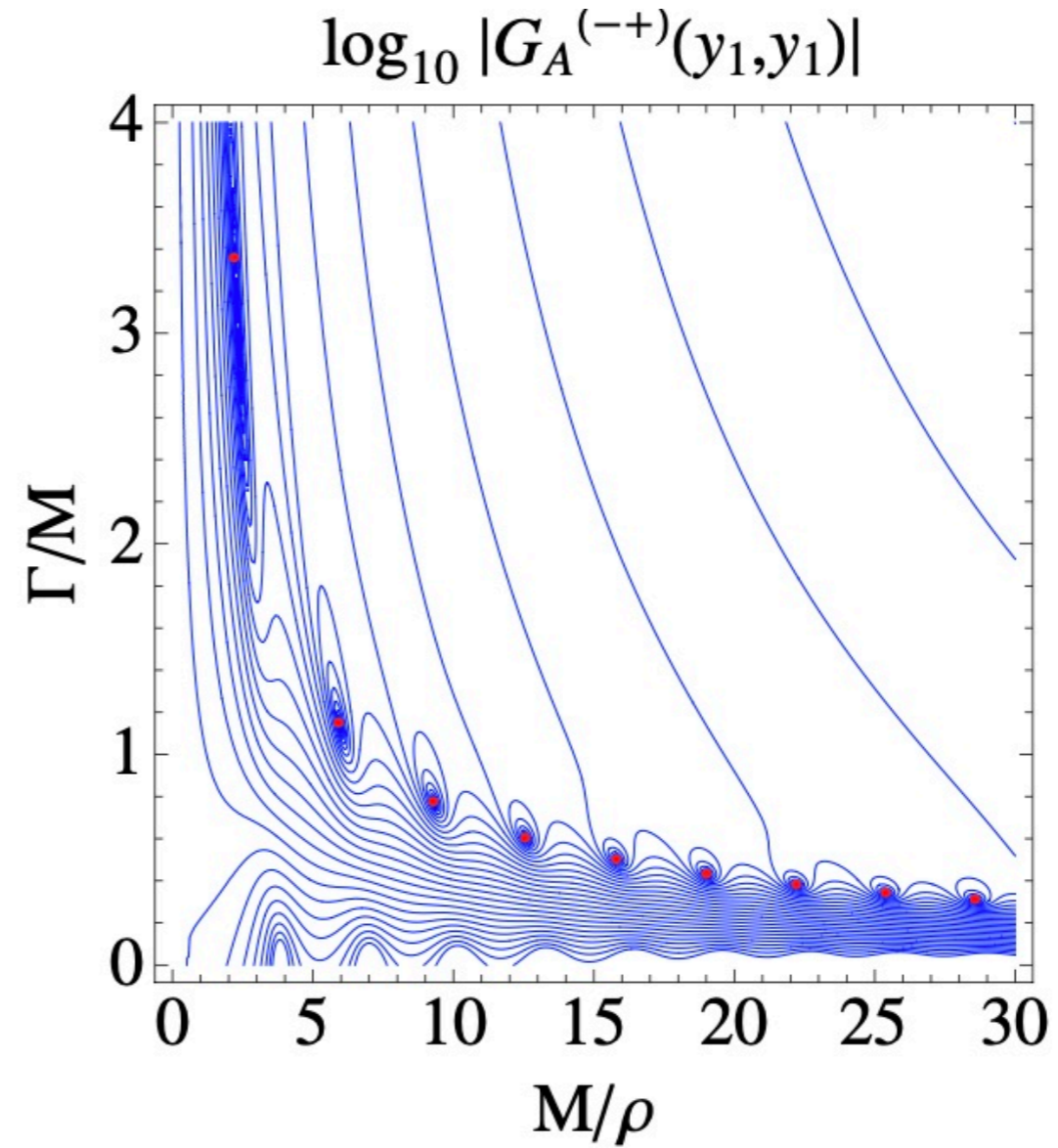
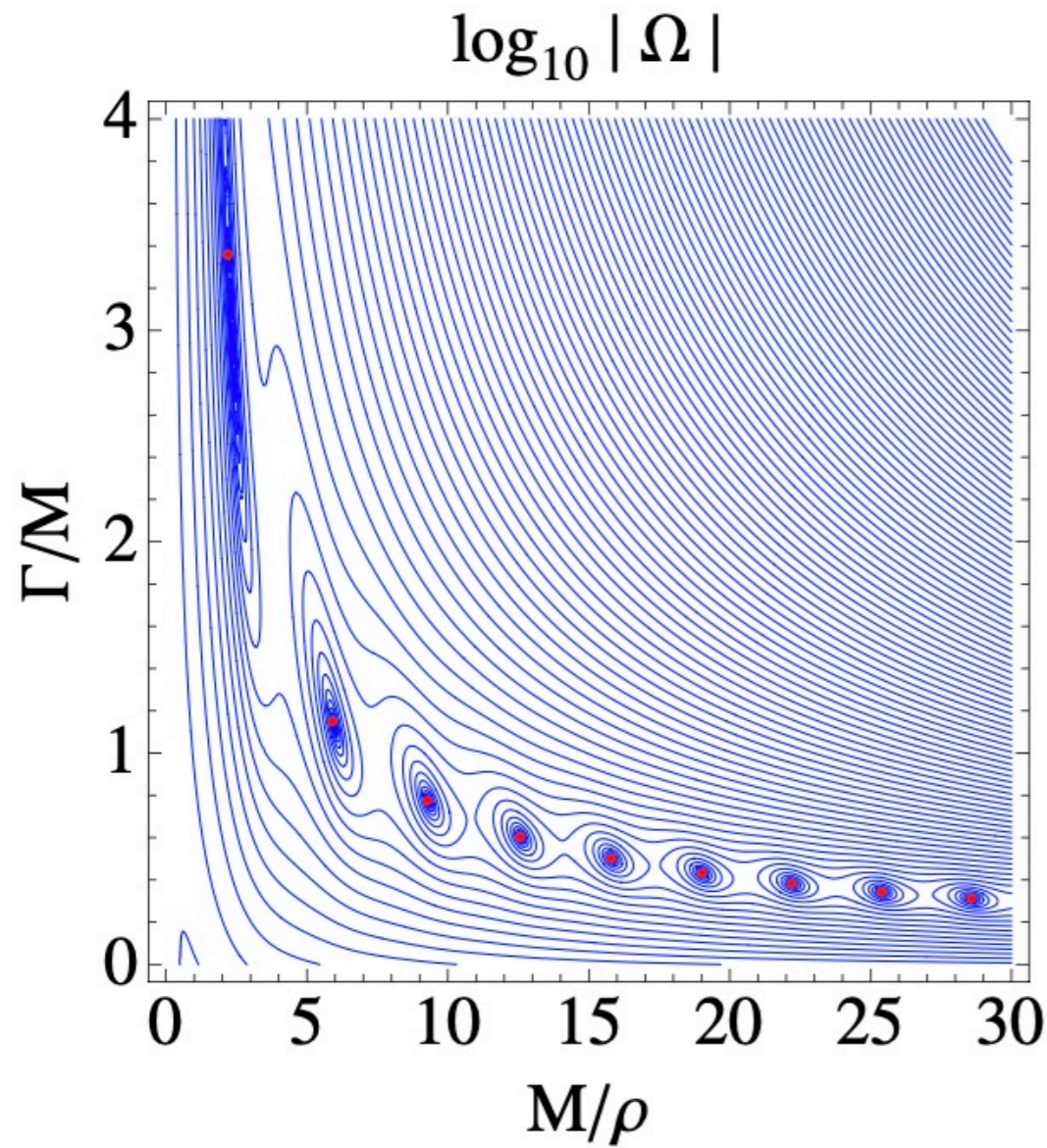


$$p^2 < 0$$



$$G^{(-,+)}(0) = -\frac{k}{2\rho^2}$$





**There appear poles in the second Riemann sheet with  
a width  $\Gamma$  corresponding to resonances with  $n \geq 1$**

**Location of poles similar to that of massless gauge bosons**



# Phenomenology of continuum KK modes

- The mass gap is different for different states

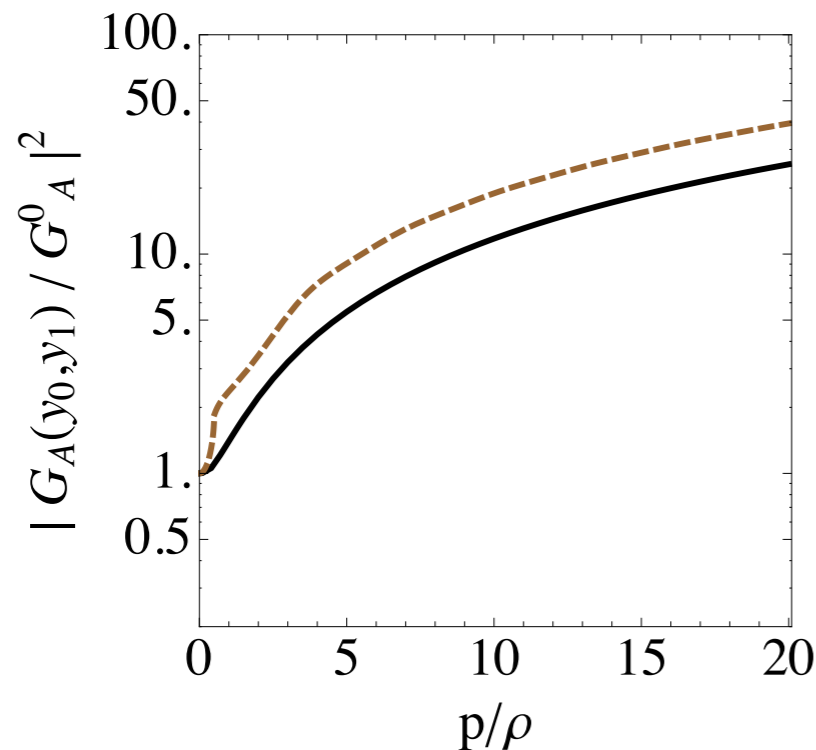
field	gauge boson	fermion	graviton radion Higgs
mass gap	$\frac{1}{2}\rho$	$ c_f \rho$	$\frac{3}{2}\rho$

*As for light fermions  $c_f > 1/2$ , the easiest produced continuum is for gauge bosons*

# Phenomenological applications

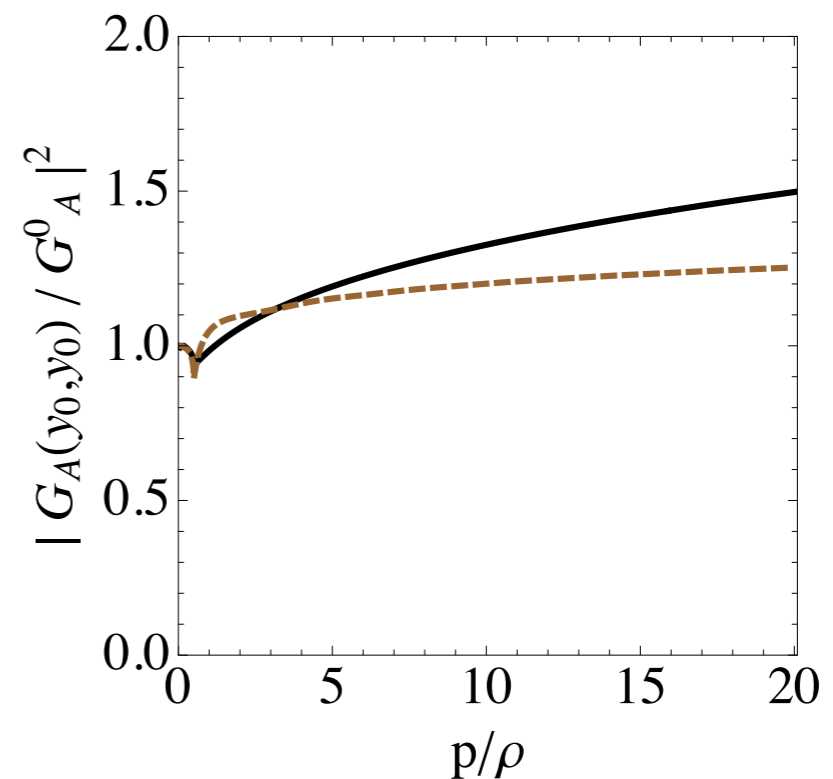
- A set of broad KK resonances or a continuum of unparticles can be produced at the LHC
- In the production of fermions from DY processes via gluon KK continuum, there is just an increase in the cross section:

$$\hat{\sigma}/\hat{\sigma}_{SM}(q\bar{q} \rightarrow f_{IR}\bar{f}_{IR})$$



**Strong increase with energy**

$$\hat{\sigma}/\hat{\sigma}_{SM}(q\bar{q} \rightarrow f_{UV}\bar{f}_{UV})$$



**Little increase with energy**

$$p \equiv \sqrt{\hat{s}}$$

# Conclusions

- **Warped extra dimension** is an interesting solution the **hierarchy problem** (dual to CFT,...)
- One exploring possibility to solve the elusiveness of signals at LHC is that KK modes are **very broad**, triggered by **fermions** very localized toward the IR brane (e.g.  $t_R$ )
- Another (most intriguing) possibility is a **continuum of KK** states (related to CFT, unparticles,..)
- The 5D model with a continuum of KK states is pretty unique: it corresponds to a critical value of the metric parameters

- It is a 5D modelization of a 4D conformal theory with a mass gap
- Green's functions behave like those of unparticles: i.e. they have an **imaginary part not corresponding to particle creation!**
- In general, Green's functions can be decomposed as **unparticle component + Green function with broad resonances**: i.e. with poles in the second Riemann sheet of  $s$
- The location of masses is very similar to that in the RS model, while the width is very broad for the first resonances while for heavy resonances it is very narrow
- Confronting this theory with LHC experimental data is in progress...
- There are also plenty of theoretical problems to be understood: unitarity of the theory, spectral densities,...

*E. Megías, M. Pérez-Victoria, M.Q., work in progress*

**Thank you**

**Ignatios, I wish you the best  
in your professional  
and personal life**