Heterotic strings on $T^3/\mathbb{Z}_2$, Nikulin Involutions and M-theory

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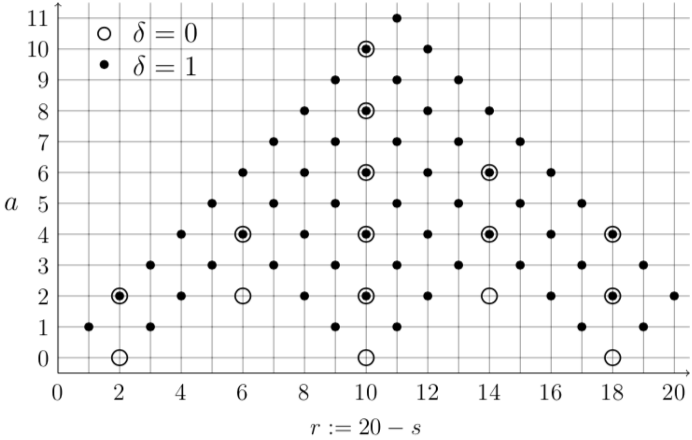
M theory compactified on $K3$ gives a 7-dim. theory with 16 supercharges. The rank of the gauge group is 19+3 and this comes from the 3-form field expanded in terms of the 19 anti-self dual and 3 selfdual 2-forms on K3. Membranes wrapped on the two cycles of $K3$ are charged under these gauge fields and these charges span a lattice $\Gamma_{19,3}$ which is even and self-dual with Lorentzian signature $(19, 3)$.

Heterotic dual of this theory is obtained by compactifying on T^3, and combining it with the $E_8 \times E_8$ one gets the charge lattice $\Gamma_{19,3}$ which is even and self-dual with Lorentzian signature $(19, 3)$.

We will consider non-supersymmetric orbifolds of Heterotic string T^3 obtained by reflection on two of the three right moving directions (the fermionic string sector) and s of the 19 left moving directions. We would expect these models to be dual to M theory on K_3 modded by Z_2 involutions that acts on the membrane charge lattice in an analogous fashion.

Such involutions have been classified by Nikulin in terms of the sublattice I of $\Gamma_{19,3}$ which is invariant under the involution. This classification is given by the triples (r, a, δ) where r is the dim of of I which has signature $(r-1, 1)$, the number a is defined by $[I^*/I] = 2^a$ and finally δ is 1 if all the vectors in I^* have integer length squares and otherwise $\delta = 0$.

We denote by N the sublattice of $\Gamma_{19,3}$ which is orthogonal to I (and therefore has dim. = 22-r and has signature $(20-r, 2)$, the s defined above is related to r via $s = 20 - r$).



Points (r, a, δ) determining all 75 invariant lattices of signature $(r-1, 1)$ which are embedded primitively in the K3 lattice $\Gamma_{(19,3)}$.

Flat Connections on T^3/Z_2

Before proceeding further let us take a simple example of a freely acting Z_2 orbifold of T^3 with $SU(2)$ flat connections.

In standard Euclidean coordinates (x_1, x_2, x_3), we can describe the generators of the fundamental group of T^3/Z_2 as the three commuting translations of T^3, namely, g_1, g_2, g_3 and the generator g_θ which is order two on T^3. We take the translations to act as

$$g_i : x_i \longrightarrow x_i + 1$$

and the fourth generator acts as

$$g_\theta : (x_1, x_2, x_3) \longrightarrow (-x_1, -x_2, x_3 + \frac{1}{2})$$

Flat Connections on T^3/Z_2

The fundamental group can be described abstractly as having four generators subject to the relations

$$\begin{aligned} g_i g_j &= g_j g_i, \quad \forall i,j = 1,2,3, \\ g_\theta g_1 g_\theta^{-1} &= g_1^{-1}, \\ g_\theta g_2 g_\theta^{-1} &= g_2^{-1}, \\ g_\theta g_3 g_\theta^{-1} &= g_3, \\ g_\theta^2 &= g_3 \end{aligned}$$

A flat connection on the heterotic $E_8 \times E_8$ or $Spin(32)/Z_2$ gauge bundle is specified by a set of four Wilson lines, one for each generator, obeying these relations. In other words we look for homomorphisms from $\pi_1(M^3)$ to the gauge group. As we will see, there are different classes of solutions.

Higgs branch solutions

$$g_1 = e^{i\phi_1\sigma_3}, \quad g_2 = e^{i\phi_2\sigma_3}, \quad g_\theta = i\sigma_2, \quad g_3 = -\mathbf{1}$$

Note that g_θ in these solutions obeys $g_\theta^4 = 1$.

Clearly such solutions generalise to higher rank subgroups since we have, up to a discrete factor, that $SU(2)^{16} \subset E_8 \times E_8$ (or $Spin(32)/Z_2$). Hence we can embed the above solution into any of the sixteen $SU(2)$ factors.

These solutions have a moduli space which is the moduli space of flat $SU(2)$-connections on T^2. Hence the low energy field theory will contain two light scalars, which will naturally form a complex scalar field.

Higgs branch solutions

Notice that at the origin of the moduli space,when $\phi_{1,2} = 0$, there is an $SO(2)$ subgroup of $SU(2)$ which commutes with the flat connection; these are the $SU(2)$ matrices with real entries. Therefore the 7d theory has an enhanced $SO(2)$ gauge symmetry at that point, broken for generic values of the ϕ_i.

The low energy effective theory is an $SO(2)$ gauge theory coupled to a complex field in the fundamental representation of $SO(2)$. The potential for this theory arises from the reduction of $SU(2)$ Yang-Mills on the 3-manifold. The generic vacuum expectation values for these charged scalars which minimise the potential break $SO(2)$ completely, leaving behind two massless scalars without a potential. These are identified with the ϕ_i.

Coulomb branch solutions

Another family of solutions, which is identity connected, are the following:

$$g_1 = g_2 = \mathbf{1}, \quad g_\theta = e^{i\phi_3\sigma_3}, \quad g_3 = g_\theta^2$$

Clearly, we can extend it to any gauge group, if one takes g_θ to be any element of the maximal torus of the full gauge group, these solutions break the gauge symmetry down to the maximal torus generically. In this case the low energy theory in seven dimensions will have an $E_8 \times E_8$ gauge symmetry with a real adjoint scalar field. Diagonalising the field minimises the potential and hence we have a sixteen dimensional moduli space of vacua with $E_8 \times E_8$ unbroken at the origin.

We refer to these solutions as Coulomb branch vacua since the gauge group is the maximal torus at generic points. Notice that, at the origin of the Higgs branch solution above, the solution is gauge equivalent to a particular Coulomb branch solution. Hence these two types of branches of moduli space intersect there.

Asymmetric orbifolds of Heterotic theory on T^3

Now we do not necessarily restrict to freely acting orbifolds.

1) On the right movers, orbifold group element g acts as rotation by π on a two dimensional plane. Thus g^2 acts as -1 on the space-time fermions. This means that actually the orbifold group becomes Z_4.

2) In general we may also include some shift v in the definition of g, which acts on the states $|P>$ carrying lattice momentum $P \in I$ as $g|P>= e^{2\pi i v.P}|P>$.

Asymmetric orbifolds of Heterotic theory on T^3

3) More generally, when the lattice vector $P \in \Gamma_{19,3}$ has components along both I and N directions labelled as $P = (P_N, P_I)$ then g acts as

$$g|P_N, P_I >= f(P_N)e^{2\pi i v.P_I)}| - P_N, P_I >$$

This implies that

$$g^2|P_N, P_I >= f(P_N)f(-P_N)e^{4\pi i v.P_I)}|P_N, P_I >$$

We will assume that $f(P_N)$ satisfy, $f(P_N)f(-P_N) = e^{2\pi i P_N^2}$.

4) **1-loop Modular invariance condition**

$$2v^2 + \frac{s}{4} = \mathrm{Integer}$$

Asymmetric orbifolds of Heterotic theory on T^3

5) This way, we were able get all the Nikulin points except for 2 points $(r, a, \delta) = (1, 1, 1)$ and $(2, 2, 1)$ because in these cases I is too small and it does not admit v satisfying the modular invariance condition. On the M theory side, v would imply that under the involution, membrane states pick up some well defined phases. It will be nice to understand what forces these phases.

6) All the remaining points can be connected to each other by moving in the classical Higgs and Coulomb branch . On the M-theory these transitions will be non-perturbative because it happens when a membrane state becomes massless and acquires vev i.e. is a topology changing transition.

Asymmetric orbifolds of Heterotic theory on T^3

7) Most of the Nikulin points do not admit spin-structure, therefore the KK modes of 11-dim SUGRA will only give bosons. On the heterotic side there are fermions, but they are all charged under $\Gamma_{19,3}$. On the M-theory side they would therefore correspond to Membrane states.

8) On the heterotic side, there are tachyons coming from the twisted sectors. But there are regions in the moduli space where they become massive. However quantum effective potential could drive the system to the region where tachyons appear (**JHEP** 04 (2021) 026).

Thank you!