Black Hole Perturbations in Modified Gravity

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Based on collaborations with D. Langlois and H. Roussille

Plan of the talk

1. Motivations : Era of Gravitational Waves

- Ringdown Phase of Black Hole Binaries : Possibility to see deviations from GR ?

2. Modified Gravity : Scalar-Tensor Theories

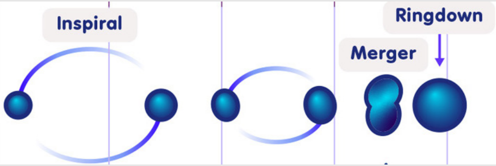
- Adding one more scalar degree of freedom in addition to gravitational modes
- Lagrangian and Disformal transformations of the metric

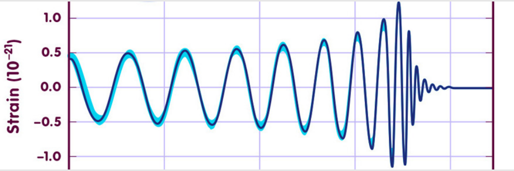
3. Modified Black Holes and Perturbations

- New (non-rotating) Hairy Black Hole solutions
- Perturbations and effective metric

1. Motivations

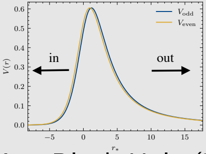
The era of Gravitational Wave Astronomy [LIGO-Virgo]

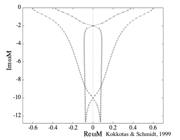




The Ringdown phase is the simplest and fully understood in General Relativity

For a Schwarzschild Black Hole, (odd and even) perturbations described in terms of $\psi(r) Y_{\ell m}(\theta, \varphi) e^{-i\omega t}$ which satisfies a Schrödinger-like equation $\psi'' + [\omega^2 - V(r)]\psi = 0$





→ Similar results for a Kerr Black Hole (Teukolsky equation)

Ringdown : a window onto Strong Gravity

We want to test General Relativity in the strong gravity regime

$\longrightarrow$ Considering Modified Theories of Gravity

$\longrightarrow$ Predict and/or Constrain deviations from General Relativity

Problem : Most of the "nice" features of General Relativity are lost !

- Modified Gravity is not unique. What type of modifications shall we consider ?
- Black Hole solutions are no more unique ? How to find them ?
- Dynamics of perturbations more involved because of the presence of extra degrees of freedom

$\longrightarrow$ Here, we focus on scalar-tensor theories and develop methods to understand systematically (extract universal features) perturbations about black holes...

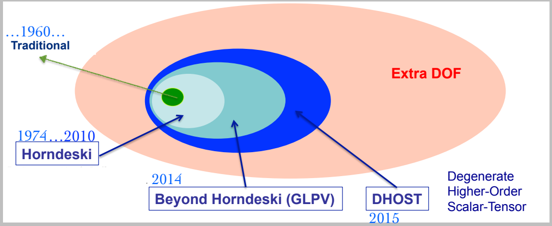
2. Scalar-Tensor Theories

Beyond General Relativity : Relaxing hypothesis of Lovelock Theorem

→ Lovelock : A massless spin 2 field (with Diff-invariance) in 4 dimensions is uniquely described by GR with a cosmological constant

→ Relaxing one hypothesis leads to a huge landscape a modified gravity theories !

Classification of most general scalar-tensor theories : DHOST theories



Action $S[g_{\mu\nu}, \phi]$ involves higher derivatives $\nabla_\mu \nabla_\nu \phi$ but only one scalar dof propaging in addition to the tensor modes.

→ Long history with a renewal of ST theories in '00 due to the problem of dark energy

A few words on Horndeski theories : the cornerstone !

Horndeski theories [rediscovered by Deffayet et al. after Nicolis et al.]

The most general $S[g_{\mu\nu}, \phi]$ whose E.o.M. are second order

$$L[g_{\mu\nu}, \phi] = F(\phi, X)\, R + P(\phi, X) + Q(\phi, X)\Box\phi + 2F_X(\phi_{\mu\nu}\phi^{\mu\nu} - \Box\phi^2) + \cdots$$

With $X = \phi_\mu \phi^\mu$, $\phi_\mu = \nabla_\mu \phi$ and $\phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi$.

Important Properties of Horndeski theories

→ EFT for dark energy (motivated by brane cosmology scenarii) where ϕ is dark energy

→ Metric not uniquely defined due to disformal transformations

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} \equiv C(\phi, X) g_{\mu\nu} + D(\phi, X)\phi_\mu \phi_\nu \,.$$

→ Possibility of non-minimal couplings to matter : $L_{\rm mat}[\psi] = \tilde{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + \cdots$

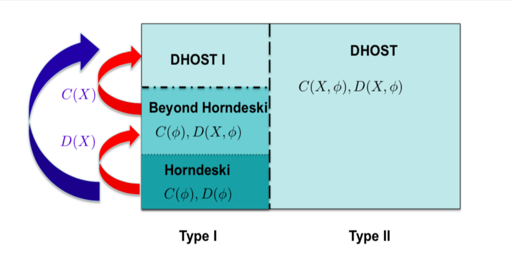
Case of Quadratic DHOST Theories

The DHOST action $S[g_{\mu\nu},\phi]$ whose E.o.M. are not necessarily second order

$$L[g_{\mu\nu},\phi] = F(\phi,X)\,R + P(\phi,X) + Q(\phi,X)\Box\phi + \sum_{i=1}^{5} A_i(\phi,X) L_i$$

$$L_1 = \phi_{\mu\nu}\phi^{\mu\nu}\,,\quad L_2 = \Box\phi^2\,,\quad L_3 = \phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi\,,\quad L_4 = (\phi^\mu\phi_{\mu\nu})^2\,,\quad L_5 = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

With degeneracy conditions relating the functions A_i and $F \Longrightarrow$ 2 classes of theories.



Geometric Formulation of quadratic DHOST theories

Type I are physically viable (with no gradient instabilities, nor ghosts)

- Disformally related to Horndeski theories : $S[g_{\mu\nu}, \phi] = S_H[\tilde{g}_{\mu\nu}, \phi]$
- DHOST theories are not equivalent to Horndeski theories in the presence of matter

Geometric Formulation

$\rightarrow$ Let Σ_ϕ be the hypersurface of constant ϕ (ϕ plays the role of time when $X < 0$)
$\rightarrow$ Then, Type I theories are disformally related to the simple action

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} \, {}^4R + \lambda(\phi, X) \, {}^3R \right)$$

$\rightarrow$ ${}^3R$ is the 3-dimensional Ricci scalar on Σ_ϕ.

3. Black Hole Perturbations

DHOST Theories as EFT of Hairy Black Holes

Evading the No-Hair Theorem [Babichev-Charmousis]

New static and spherically symmetric black holes with (for shift-symmetric theories)

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + C(r)d\Omega^2\,, \qquad \phi(t,r) = qt + \psi(r)\,.$$

A few analytic solutions :

- Stealth solutions with $A(r) = B(r) = 1 - 2M/r$, $C(r) = r^2$ and $X = \text{cst}$
- BCL solution with $A(r) = B(r) = (1 - r_+/r)(1 + r_-/r)$ and $C(r) = r^2$
- $D \to 4$ Gauss-Bonnet solution with

$$A(r) = B(r) = 1 - \frac{2M(r)}{r}\,, \quad M(r) = \frac{2M}{1+\sqrt{1+8\alpha M/r^3}}\,, \quad C(r) = r^2$$

Dynamics of Linear Perturbations

Modified Einstein equations at linear order

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\,, \qquad \phi = \bar{\phi} + \delta\phi$$

- Axial perturbations : $\delta\phi = 0$ while $h_{\mu\nu}^{\rm axial}$ depend on $\chi = \psi(r)e^{-i\omega t}Y_{\ell m}(\theta,\varphi)$

$$-\frac{d^2\psi}{dr^2} + V(r)\psi = \frac{\omega^2}{c(r)^2}\psi$$

- Polar perturbations : $\delta\phi$ and $h_{\mu\nu}^{\rm polar}$ are coupled with no explicit decoupling

Effective metric of axial perturbations

Dynamics of $h_{\mu\nu}^{\rm axial}$ about $\bar{g}_{\mu\nu}$ in DHOST are equivalent to those in GR with $\tilde{g}_{\mu\nu}$

$$\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\psi - m_{\rm eff}^2\psi = 0\,.$$

On the effective metric of axial perturbations

Physical interpretation : coupling between the metric and the scalar field

→ (Minimally coupled) photons and Gravitons do not see the same space-time
→ Photons are sensitive to the background metric $\overline{g}_{\mu\nu}$ while gravitons evolve in $\tilde{g}_{\mu\nu}$
→ Consequence of the interactions between gravitons and the scalar field (as if gravitons were evolving in a medium)

Disformal transformations

In quadratic DHOST theories, $\tilde{g}_{\mu\nu}$ is disformally related to $\overline{g}_{\mu\nu}$ by

$$\tilde{g}_{\mu\nu} = \sqrt{F(F - XA_1)}\left(\overline{g}_{\mu\nu} + \frac{A_1}{F - XA_1}\overline{\phi_\mu}\,\overline{\phi_\nu}\right)$$

Where the functions are evaluated on the background solution.

Examples of effective metrics

The effective metric can be very different from the background metric

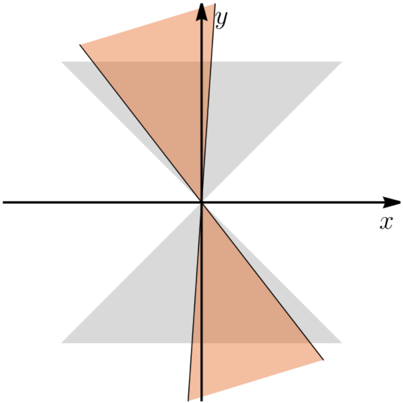
- Stealth solutions : $\tilde{g}_{\mu\nu}$ is still a black hole with a different horizon

$$R_s = 2GM/c_\gamma^2\,, \quad R_g = 2GM/c_g^2\,, \qquad c_g < c_\gamma$$

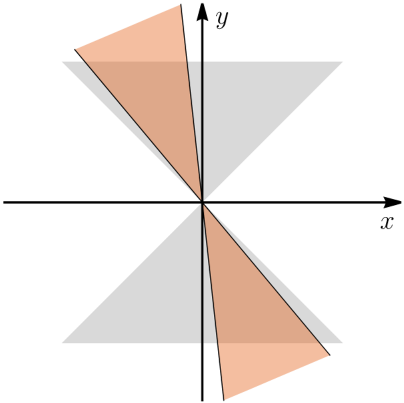
- BCL solution : $\tilde{g}_{\mu\nu}$ is a black hole with the same horizon
- $D \to 4$ Gauss-Bonnet solution : $\tilde{g}_{\mu\nu}$ is a naked singularity

$\Longrightarrow$ Eventual strong physical pathologies and/or instabilities.

Case of the stealth solution : causal structures are compatible



(a) Lightcones for $r > r_g$.



(b) Lightcones for $r_s < r < r_g$.

No instabilities between the two horizons following 1803.11444 [Babichev et al]

4. Conclusion

Deviations from GR in the Strong Gravity regime : background/perturbations

DHOST Theories as EFT of gravity in the Strong Gravity regime

- Background solutions : Hairy Black Holes or Exotic compact objects
- Axial perturbations : effective metric different from background metric
- Polar perturbations : interactions between the graviton and the scalar
  $\rightarrow$ not easy to handle and to decouple
  $\rightarrow$ needs new techniques to study the coupled dynamics : asymptotic analysis

Going further....

▷ Compute Quasi-Normal Modes and Deviations from General Relativity

▷ Constrain theories from the systematic study of pathologies and instabilities