Status of the SM prediction for the muon g-2

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OUTLINE

- Introduction

- Theory aspects: a_μ (and a little bit of a_e) in the SM

- QED contributions

- Weak contributions

- Strong interactions

- SM prediction: the White Paper

- Conclusion and a look into the (near) future

INTRODUCTION

On April 7, 2021, the FNAL-E989 experiment released the first result of a measurement of the anomalous magnetic moment of the muon a_μ, based on the Run-1 data collected during Spring 2018

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

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The uncertainties of both measurements are dominated by statistical errors

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The world-average value

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when compared to the SM predicted value

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leads to a discrepancy at the level of 4.2σ

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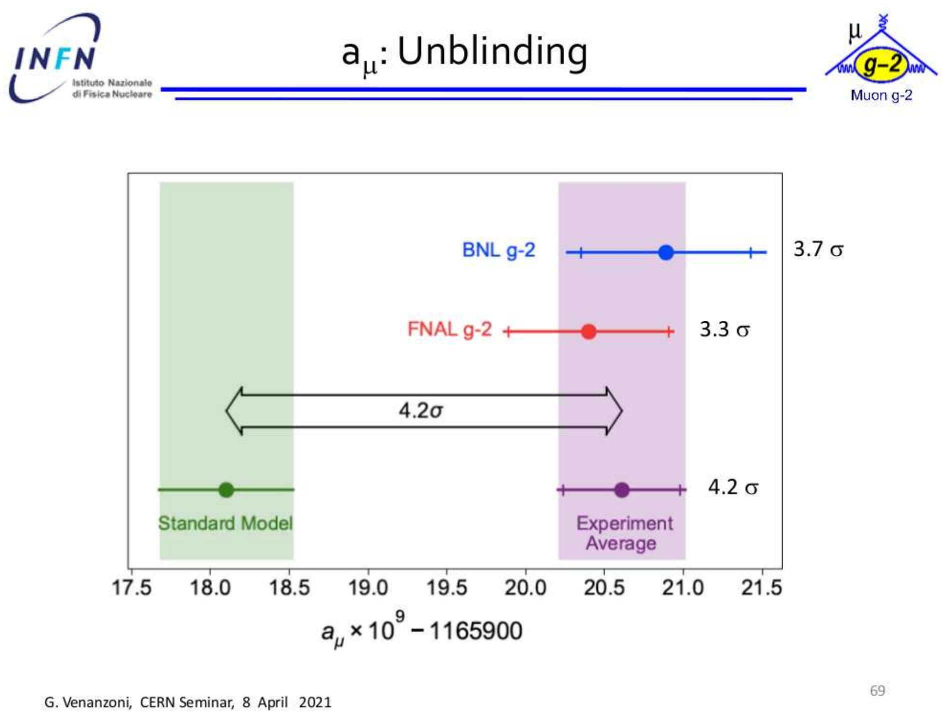

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- How is $a_\mu^{\rm exp}$ obtained?

- If the discrepancy is real, what explains it?

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- What goes into a_μ^{SM}, how is it obtained?

$\longrightarrow$ the topic of this talk

- How is a_μ^{exp} obtained?

$\longrightarrow$ addressed in the publications of FNAL-E989

- If the discrepancy is real, what explains it?

$\longrightarrow$ hundreds of papers on arXiv since April 7, 2021

$\longrightarrow$ talks by R. Dermisek, X. Lou, G. Grilli di Cortona, W. Ke, M. Ramirez-Quezada,...

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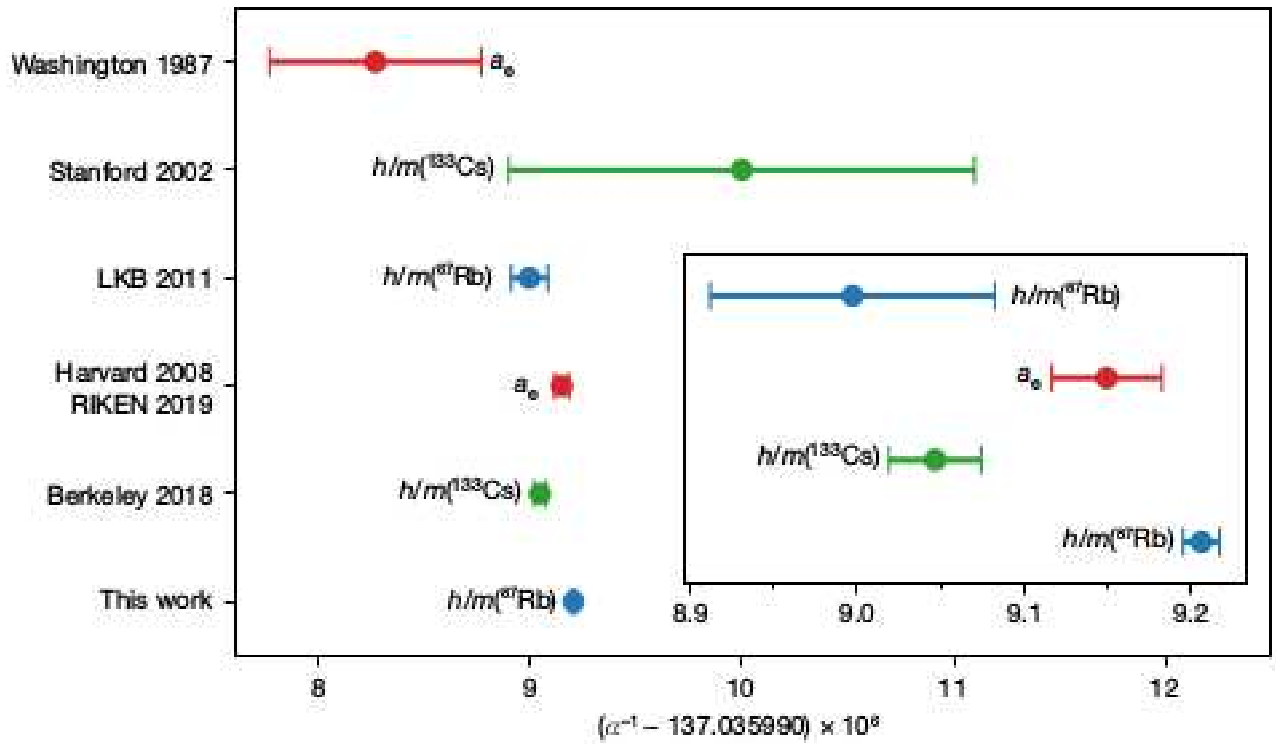
$\longrightarrow$ possibility to test the SM with a_e! (?)



$$a_e^{\text{exp}} - a_e^{\text{SM}}(Rb20) = 4.8(3.0) \cdot 10^{-13} \quad [1.6\sigma]$$

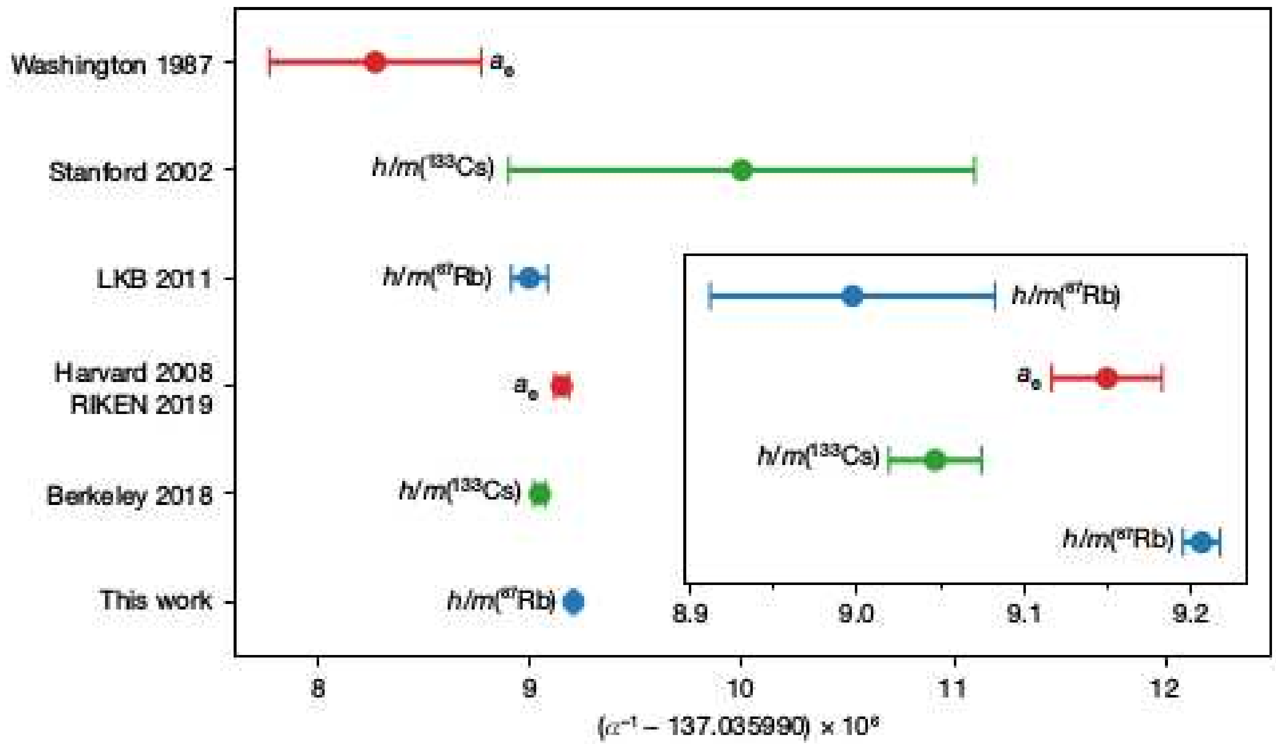
P. Cladé, Moriond EW 2021

Compare to previous values

$$a_e^{\text{exp}} - a_e^{\text{SM}}(Rb11) = -1.31(77) \cdot 10^{-12} \quad [-1.7\sigma]$$

$$a_e^{\text{exp}} - a_e^{\text{SM}}(Cs18) = -0.88(36) \cdot 10^{-12} \quad [-2.4\sigma]$$

T. Aoyama, T. Kinoshita, M. Nio, Atoms 7, 28 (2019)



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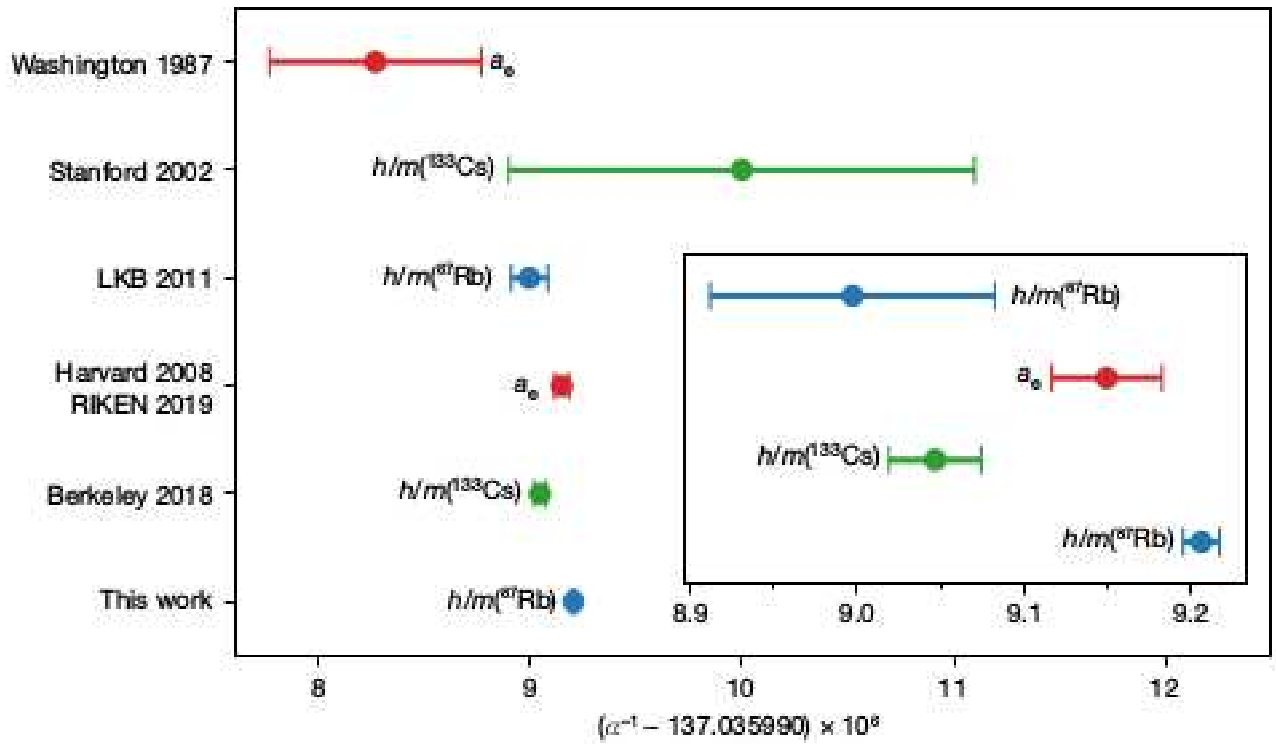

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$\longrightarrow$ need to understand discrepancy between $\alpha(Cs18)$ and $\alpha(Rb20)$, but also between $\alpha(Rb11)$ and $\alpha(Rb20)$

$\longrightarrow$ particularly important in view of the possiblity to improve the accuracy on $a_e^{\rm exp}$ by an order of magnitude!

G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

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The difference between the two values

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reflects the effects of the LFUV sector of the SM

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makes the measurement of a_e very different from the one of a_μ!

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arXiv:0707.2496, arXiv:0807.2366

arXiv:1601.07987, arXiv:1803.00501, arXiv:1810.06699, arXiv:1908.05180, arXiv:2002.05503, arXiv:2111.10378

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$\longrightarrow$ theory is well ahead of experiment: $a_\tau^{\text{SM}} = 117717.1(3.9) \cdot 10^{-8}$ [42 ppm]

S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)

S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)

A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 101, 1 (2020)

Theory aspects

One wants to probe the response of a charged lepton to an external (and static) electromagnetic field

$$\langle \ell; p'|\mathcal{J}_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p', p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\Big[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\Big]\mathrm{u}(p)$$

(uses only the conservation of the electromanetic current $\mathcal{J}_\rho$, $k_\mu \equiv p'_\mu - p_\mu$)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/q_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\; G_M(k^2) = F_1(k^2) + F_2(k^2)$$

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops $\longrightarrow$ calculable!

Before the advent of QFT, this issue was described by the Dirac equation with the minimal coupling precription

$$i\hbar\,\frac{\partial\psi}{\partial t} \;=\; \left[c\boldsymbol{\alpha}\cdot\left(-i\hbar\boldsymbol{\nabla}-\frac{q_\ell}{c}\boldsymbol{\mathcal{A}}\right)+\beta m_\ell c^2+q_\ell\mathcal{A}_0\right]\psi$$

In the non relativistic limit, this reduces to the Pauli equation for the two-component spinor φ describing the large components of the Dirac spinor ψ,

$$i\hbar\,\frac{\partial\varphi}{\partial t} \;=\; \Bigg[\frac{(-i\hbar\boldsymbol{\nabla}-(q_\ell/c)\boldsymbol{\mathcal{A}})^2}{2m_\ell}\;-\;\underbrace{\frac{q_\ell\hbar}{2m_\ell c}\,\boldsymbol{\sigma}\cdot \mathsf{B}}_{\boldsymbol{\mu_\ell}\cdot\mathsf{B}}+q_\ell\mathcal{A}_0\Bigg]\varphi$$

with $\boldsymbol{\mu_\ell} \;=\; g_\ell\,\left(\frac{q_\ell}{2m_\ell c}\right)\,\mathbf{S},\;\mathbf{S}=\hbar\,\frac{\boldsymbol{\sigma}}{2},$ i.e. $g_\ell^{\rm Dirac}=2$

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At tree level in the SM, $g_\ell=g_\ell^{\rm Dirac}\equiv 2$.

The *anomalous* magnetic moment is induced at loop level:

$$a_\ell \equiv \frac{g_\ell-g_\ell^{\rm Dirac}}{g_\ell^{\rm Dirac}}=\frac{g_\ell-2}{2}\;(\equiv F_2(0))$$

a_ℓ probes all the degrees of freedom of the standard model,

and possibly beyond...

Considering SM contributions only, one has, by order of importance

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

a_μ^{QED}: loops with only photons and leptons

a_μ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons

a_μ^{weak}: loops with also contributions from the electroweak sector

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For a full and detailed account [up to June 15, 2020], see the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Theory I: QED (a_e and a_μ)

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\mathsf{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

— Massive fermions with $m_{\ell'} \gg m_\ell$ contribute to a_ℓ through powers of $m_\ell^2/m_{\ell'}^2$, times logarithms (*)

— Light degrees of freedom with $m_{\ell'} \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m_{\ell'}^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

(*) also applies to BSM physics to the extent that it is decoupling!

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

— Massive fermions with $m_{\ell'} \gg m_\ell$ contribute to a_ℓ through powers of $m_\ell^2/m_{\ell'}^2$, times logarithms (*) $\longrightarrow$ for a_e the $A_1^{(2n)}$ matter

— Light degrees of freedom with $m_{\ell'} \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m_{\ell'}^2)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

$\longrightarrow$ for a_μ $A_2^{(2n)}(m_\ell/m_{\ell'})$ and $A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$ matter (for $m_{\ell'} = m_e$)

(*) also applies to BSM physics to the extent that it is decoupling!

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision of $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$

order $(\alpha/\pi)^4$: 891 diagrams

order $(\alpha/\pi)^4$: 891 diagrams

$A_1^{(8)}$ has also been evaluated! (a_e)

S. Laporta, Phys. Lett. B 772, 232 (2017)

$$A_1^{(8)} = -1.912\,245\,764\,926\,445\,574\,152\,647\,167\,439\,830\,054\,060\,873\,390\,658\,725\,345\,171\,329\ldots$$

Good agreement with earlier numerical evaluations

$$A_1^{(8)} = -1.912\,98(84)$$

T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

Mass-dependent contributions (a_μ)

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

Automated generation of diagrams, systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373)\cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118)\cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28)\cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

Agreement at the level of accuracy required by present and future experiments for a_μ

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

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No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ (a_e) is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

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$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops

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No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ (a_e) is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops

$\longrightarrow$ semi-analytical evaluation by S. Laporta?

QED contributions : loops with only photons and leptons
$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.733(159)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$$\Delta C_e^{(10)}\cdot(\alpha/\pi)^5 \sim 0.15\cdot 10^{-13} \qquad \Delta a_e^{\text{exp}} = 2.8\cdot 10^{-13}$$

$[\Delta C_e^{(8)}\cdot(\alpha/\pi)^4$ was $\sim 0.2\cdot 10^{-13}$ before Laporta's calculation$]$

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 41 \cdot 10^{-11}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.54 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 0.08 \cdot 10^{-11}$$

- No sign of substantial contribution to a_μ from higher order QED

A few comments about the QED contributions

- $a_\mu^{\mathrm{QED}}(Cs19) = 116\,584\,718.931(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$

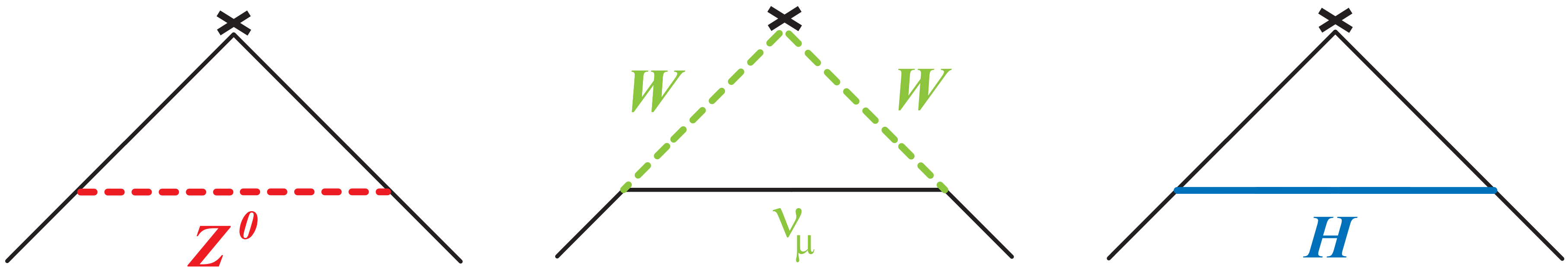
- $$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}}(Cs19) = 7342(41) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

Contributions from weak interactions

- Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} \left(1 - 4\sin^2\theta_W\right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2} \right) + \mathcal{O}\left(\frac{m_\mu^2}{M_H^2} \log \frac{M_H^2}{m_\mu^2} \right) \right]$$

$$= 194.8 \cdot 10^{-11}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\cdot\left[-5.96\ln\frac{M_W^2}{m_\mu^2}+0.19\right] = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left(\frac{\alpha}{\pi}\right)\cdot(-79.3)$$

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

$$a_\mu^{\text{weak}} = (154 \pm 1)\cdot 10^{-11}$$
$$a_e^{\text{weak}} = (0.0297 \pm 0.0005)\cdot 10^{-12}$$

Updated a few years ago: $a_\mu^{\text{weak}} = (153.6 \pm 1.0)\cdot 10^{-11}$

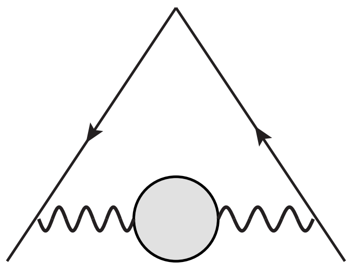
C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

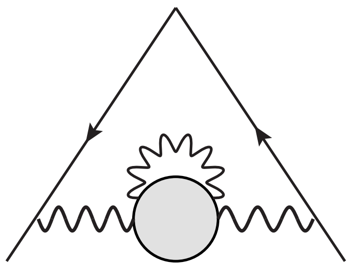
Recent numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0)\cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

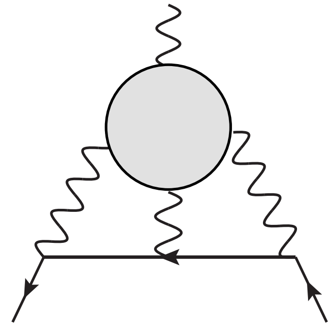
Contributions from strong interactions

- hadronic vacuum polarization





- (virtual) hadronic light-by-light (HLxL



$\longrightarrow$ non-perturbative regime of QCD

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K(s) R_{\text{had}}(s) \qquad K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

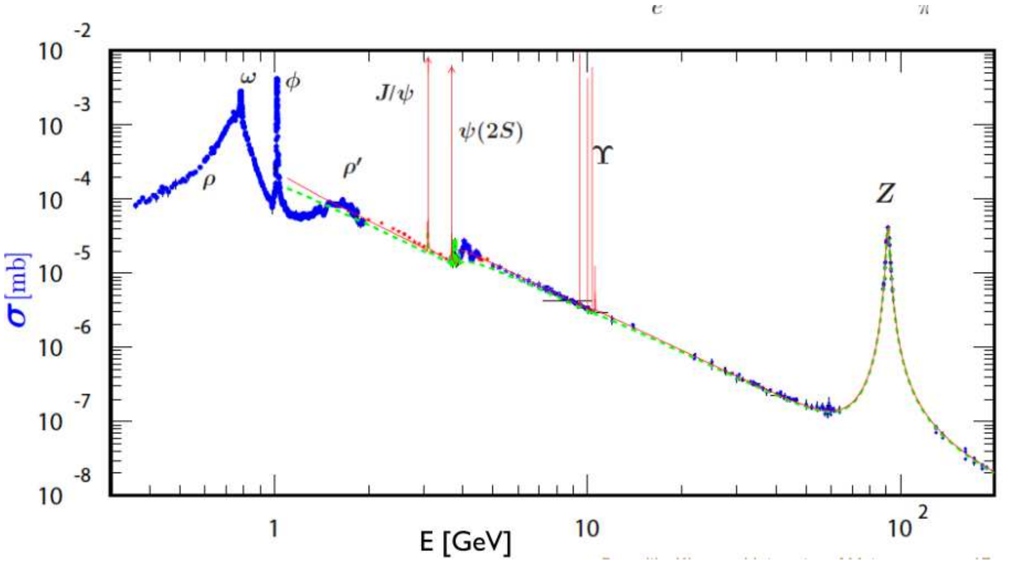
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R_{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

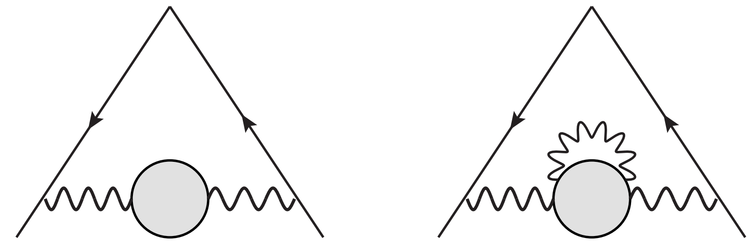
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic vacuum polarization

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$694.0(4.0)$	M. Davier et al., Eur. Phys. J. C 80, 341 (2020); Err. C 80, 410 (2020)
$692.78(2.42)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$, e^+e^-

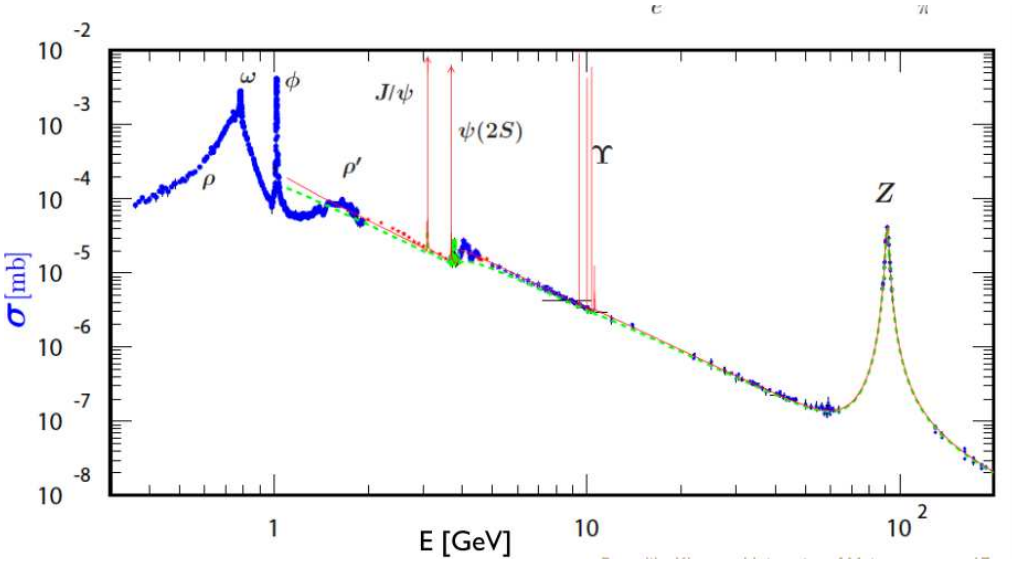
$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$-9.83(4)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$, e^+e^-

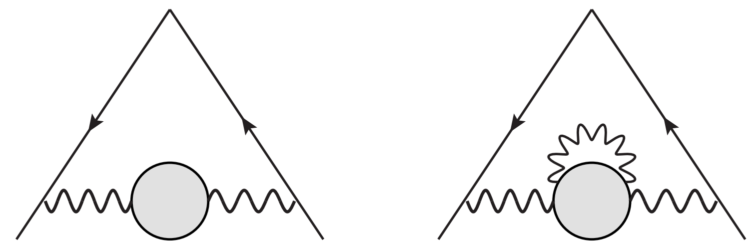
$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- More recently: lattice results

A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)

C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]

E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]

D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)

T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)

S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)

M. Della Morte *et al.*, JHEP 10, 020 (2017)

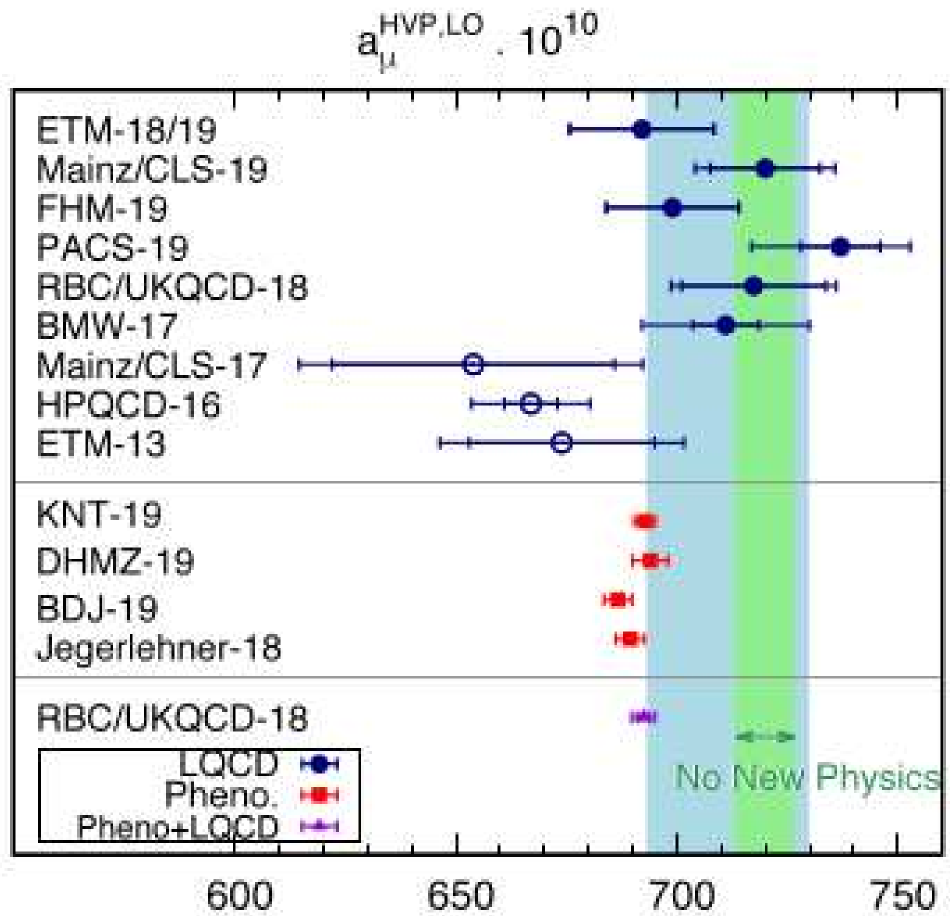
$\longrightarrow$ for BMW result, see later

White Paper summary

- Data evaluation:

$$a_\mu^{\rm HVP;LO} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\rm HVP;NLO} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\rm HVP;NNLO} = 12.4(1)\cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\rm HVP;LO} = 7043(150)\cdot 10^{-11}$

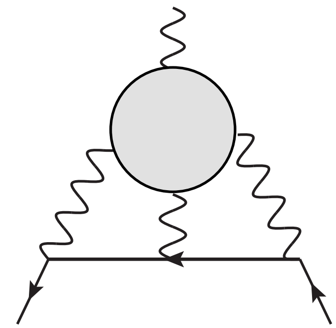


Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

- Not related, as a whole, to an experimental observable...

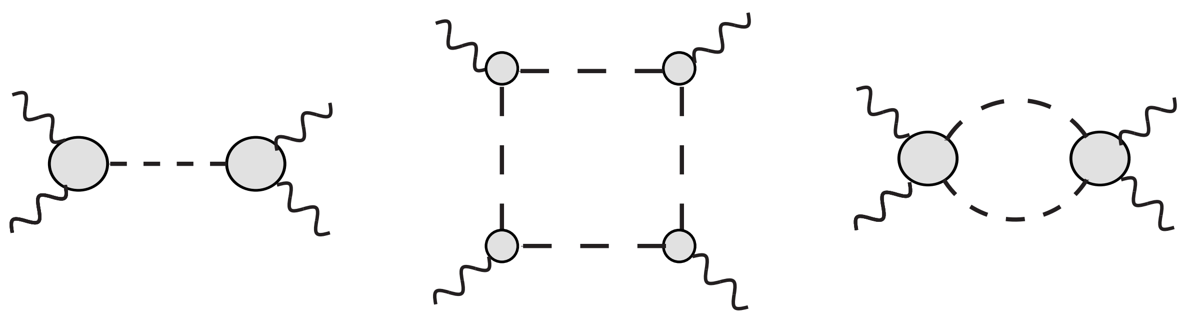
? $\longrightarrow$



- Involves the fourth-rank vacuum polarization tensor

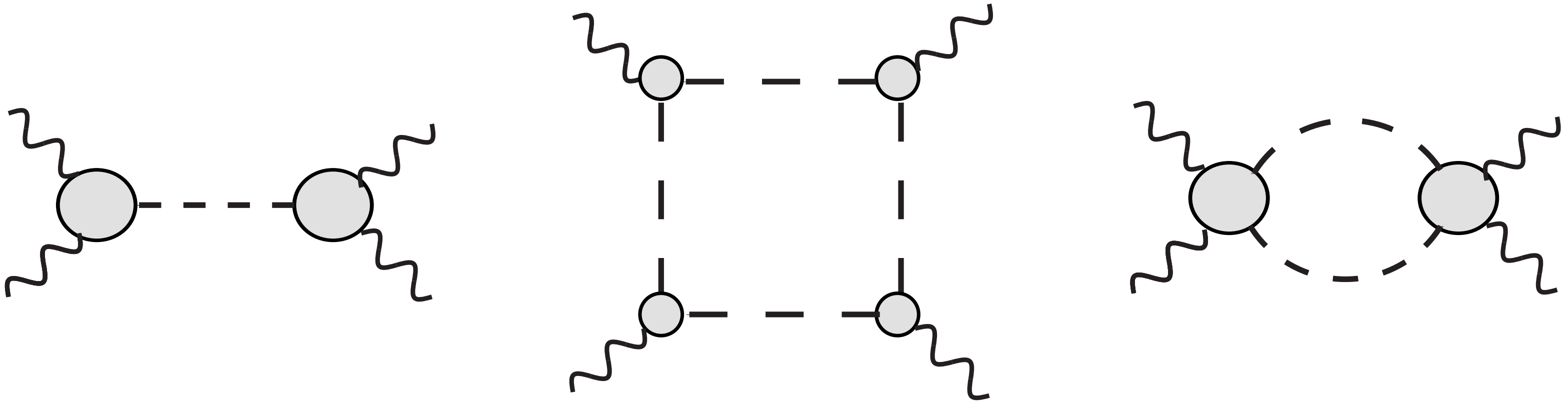
$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) \quad q_1 + q_2 + q_3 + q_4 = 0$$

- Many individual contributions have been identified...



Hadronic light-by-light

- More recently: dispersive approach for $\Pi_{\mu\nu\rho\sigma}$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Need input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

Need to match to QCD short-distance behaviour

K. Melnikov, A. Vainshtein, Phys.Rev.D 70, 113006 (2004)

J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett. B 798, 134994 (2019)

J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, JHEP 10, 203 (2020); JHEP 04, 240 (2021)

Hadronic light-by-light

- Lattice QCD calculations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

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White Paper summary

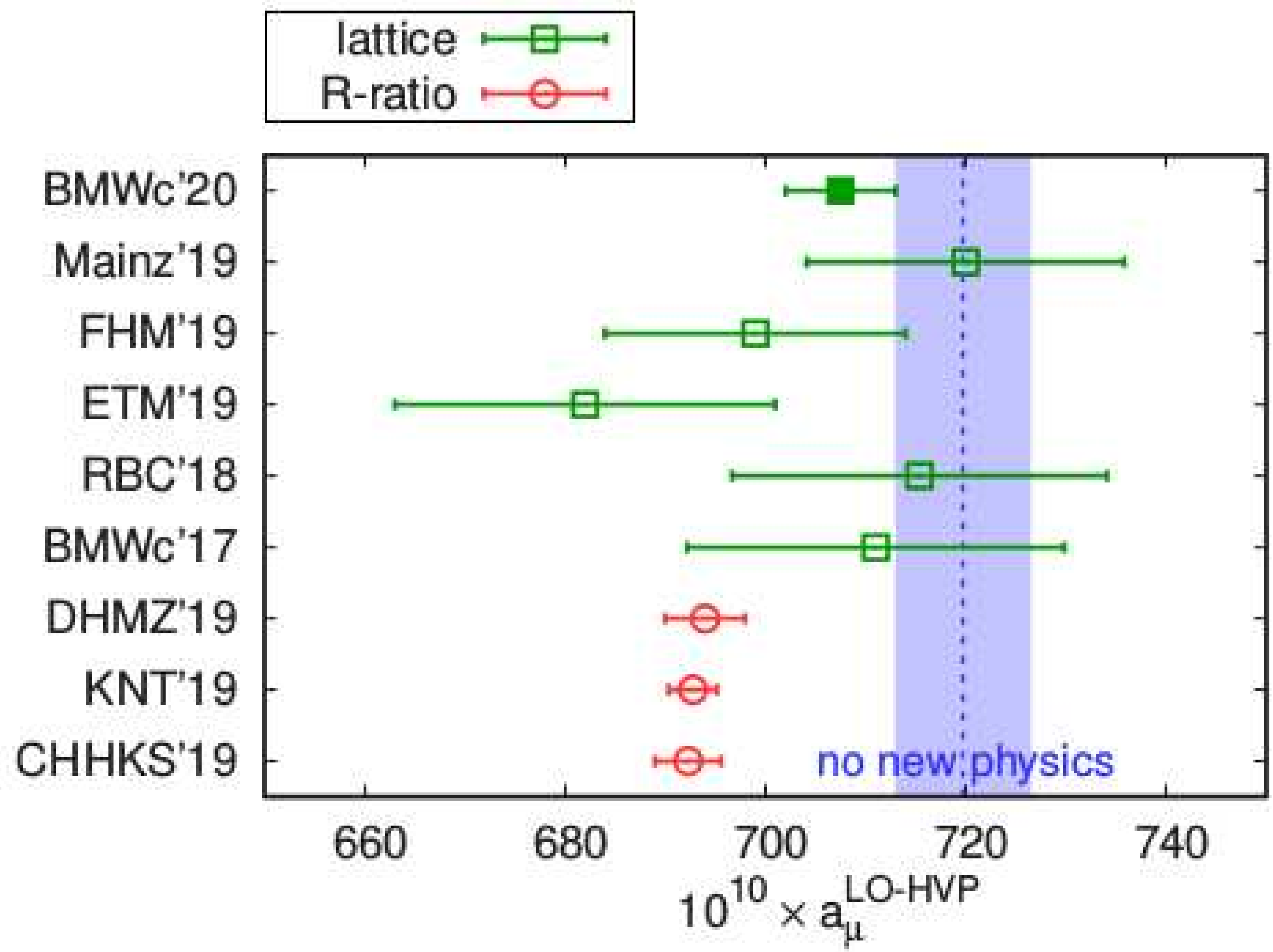
$$a_\mu^{\mathrm{HLxL}} = 92(19) \cdot 10^{-11}$$

Post-WP results

- New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021) [arXiv:2002.12347]



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Requires independent confirmation

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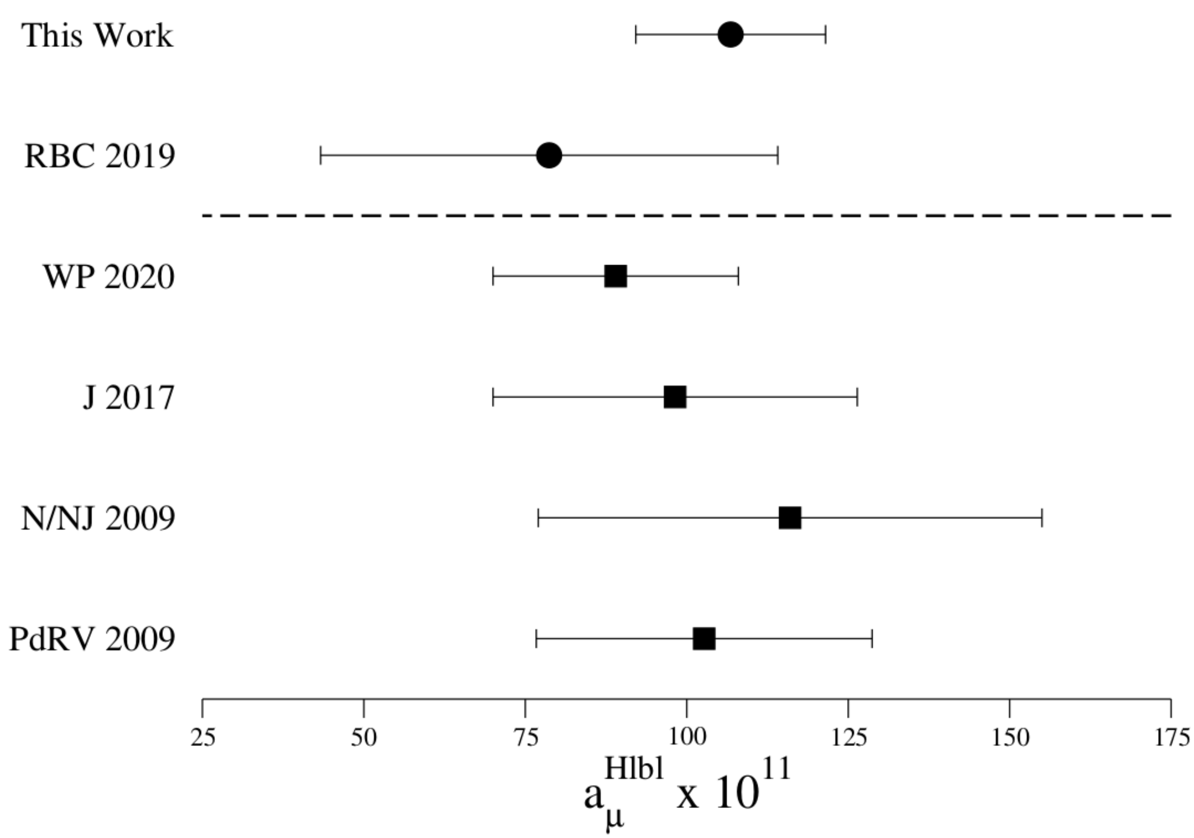
E.-H. Chao et al., arXiv:2104.02632



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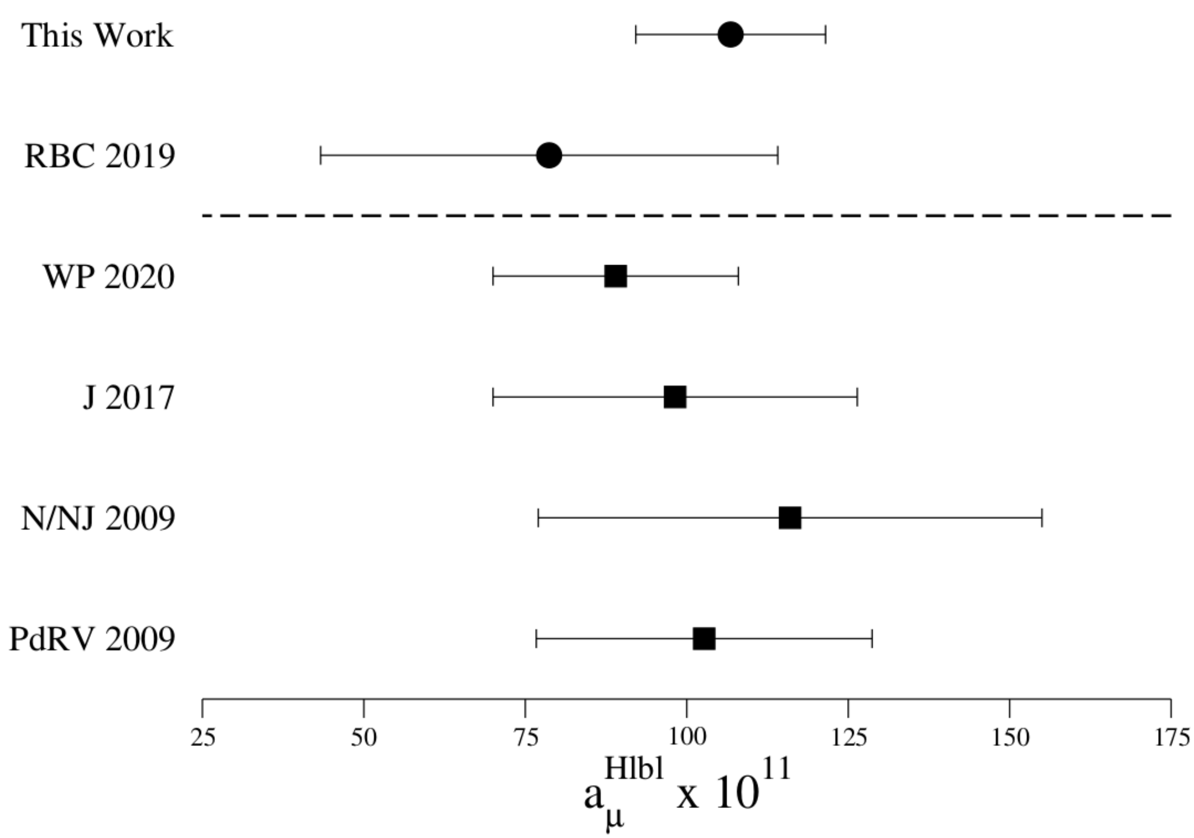

$\sim$10% accuracy goal seems within reach

Conclusion and outlook

- FNAL-E989 seems to work fine, BNL-E821 result confirmed

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

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- Theory situation (as to June 2020) described in detail in the WP

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$$a_{\mu}^{\mathrm{exp;WA}} - a_{\mu}^{\mathrm{SM}} = 251(59) \cdot 10^{-11} \quad [4.2\sigma]$$

- No obvious explanation within the SM

$$a_{\mu}^{\text{exp;WA}} - a_{\mu}^{\text{SM}} \sim \begin{cases} a_{\mu}^{\text{QED}}(\alpha^4) \\ 60 \cdot a_{\mu}^{\text{QED}}(\alpha^5) \\ 5 \cdot a_{\mu}^{\text{weak(2)}} \\ 3 \cdot a_{\mu}^{\text{HLxL}} \end{cases}$$

- Could the problem lie in the determination of HVP using $e^+e^- \to \mathrm{hadron}$ data, as suggested by the latest result from lattice QCD?

$$a_\mu^{\mathrm{HVP;LO\ data}} = 6931(40)\cdot 10^{-11} \quad \mathrm{vs} \quad a_\mu^{\mathrm{HVP;LO\ BMWc}} = 7075(55)\cdot 10^{-11}$$

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- Too early for a definite statement, BMWc result needs confirmation

- Possibility to measure HVP in the space-like region from μe scattering?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{HVP} - \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{had}(-\frac{x^2}{1-x} m_\mu^2)$

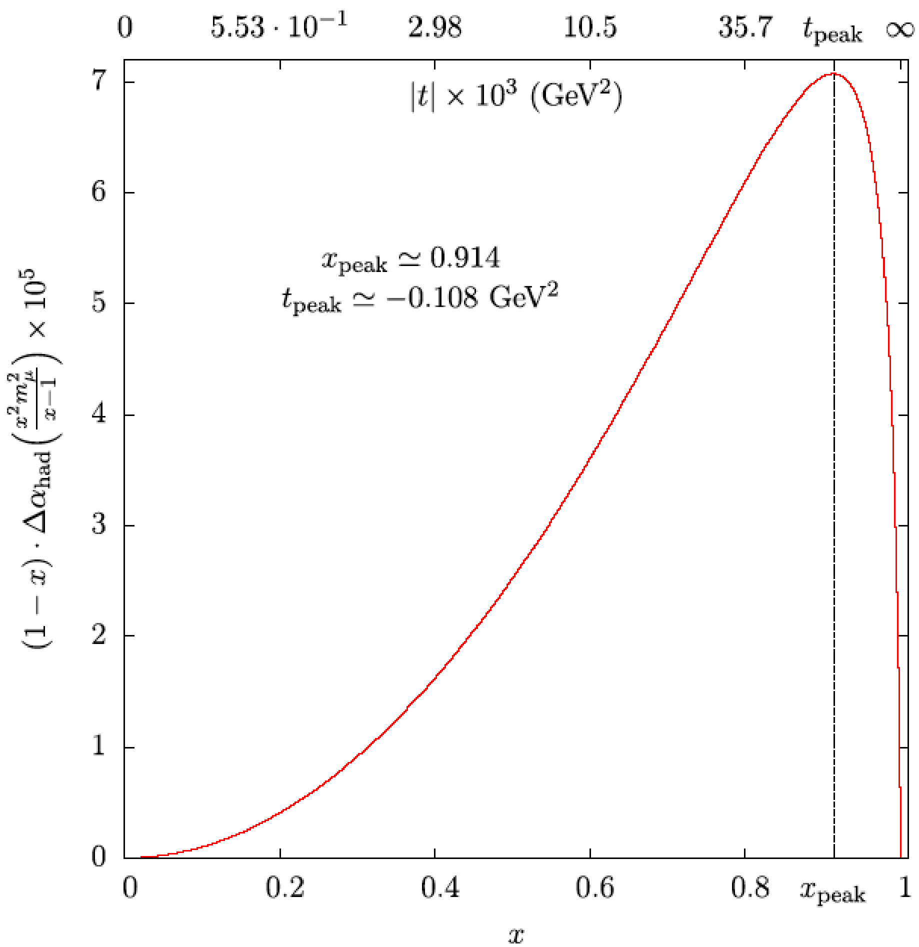
$t = \frac{x^2 m_\mu^2}{x-1} \quad 0 \leq -t < +\infty \quad 0 \leq x < 1$

- a_μ^{HVP} given by the integral

- measurement of $\Delta\alpha_{had}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 μ/s (CERN)



$\longrightarrow$ challenging (systematics)

$\longrightarrow$ inclusive measurement (more like lattice QCD)

$\longrightarrow$ MUonE coll. LoI CERN-LHCC-2017-009/CMS-TDR-014

$\longrightarrow$ test run (proof of concept, assessment of systematics,...) scheduled for the end of 2021

$\longrightarrow$ project starting in 2022, running time during LHC-Run3

- could the experiment be wrong?

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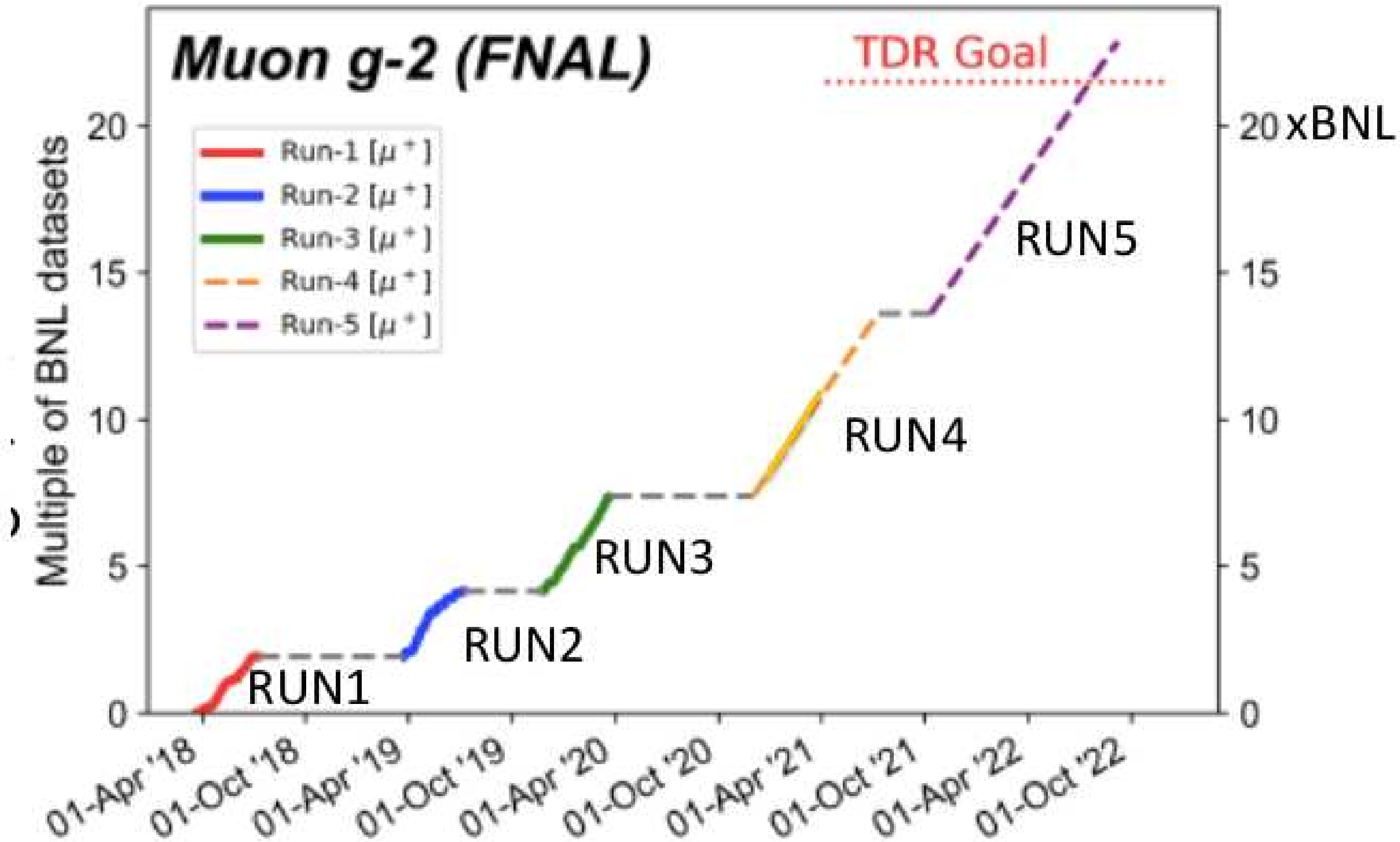
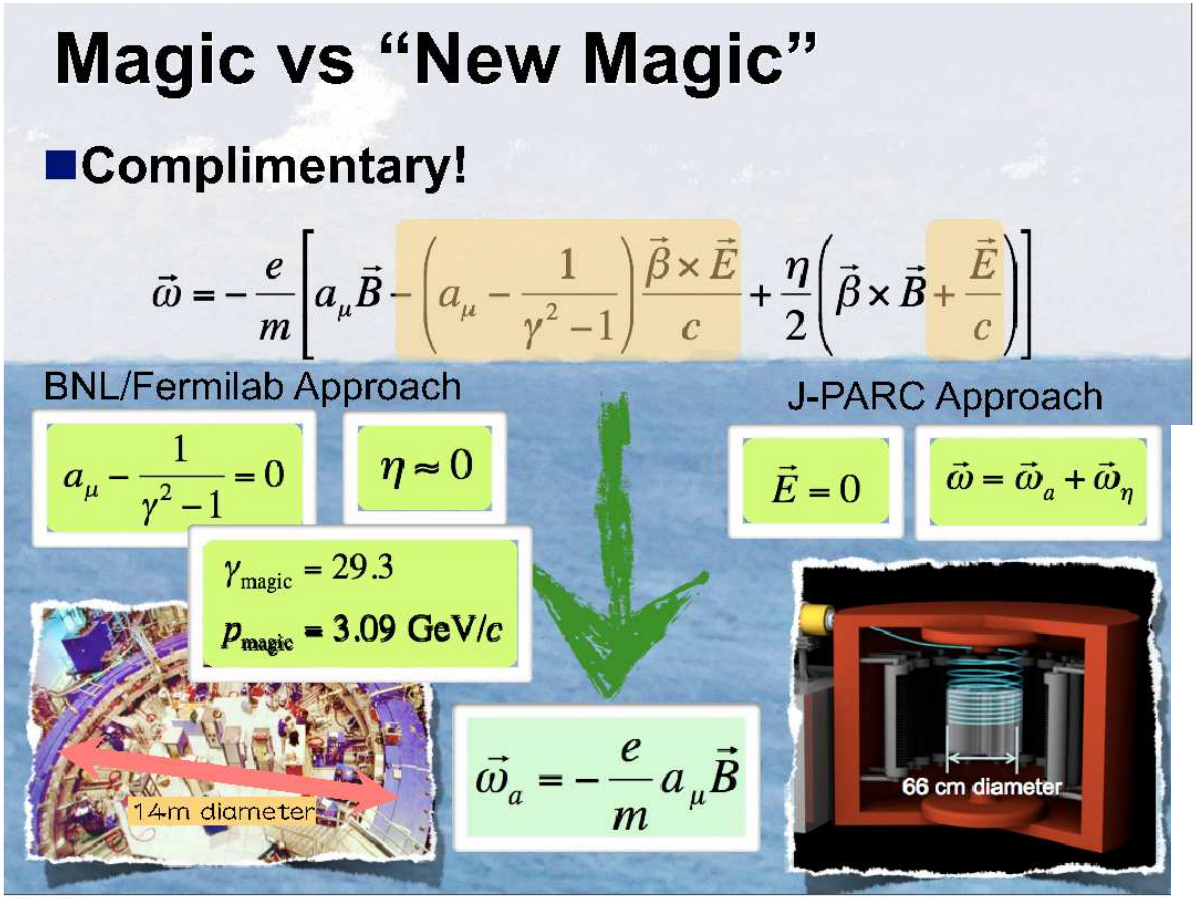
$\longrightarrow$ only part of the data collected so far has been analysed

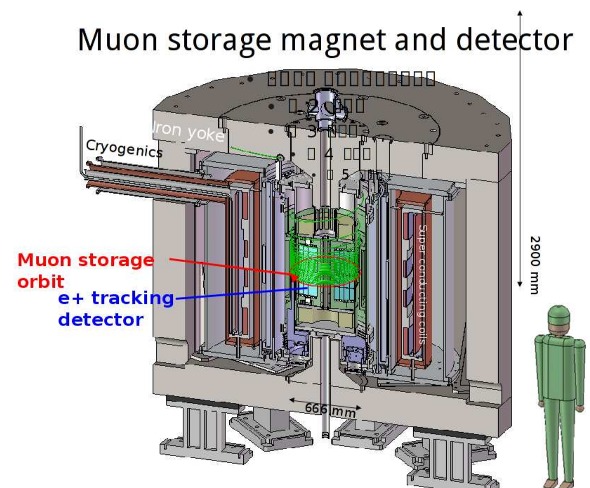
$\longrightarrow$ more will be taken, to reach the accuracy goal of ~ 0.14ppm



- could the experiment be wrong? $\longrightarrow$ project to measure a_μ at J-PARC (E34)



N. Saito, LPNHE Workshop Paris, May 2012



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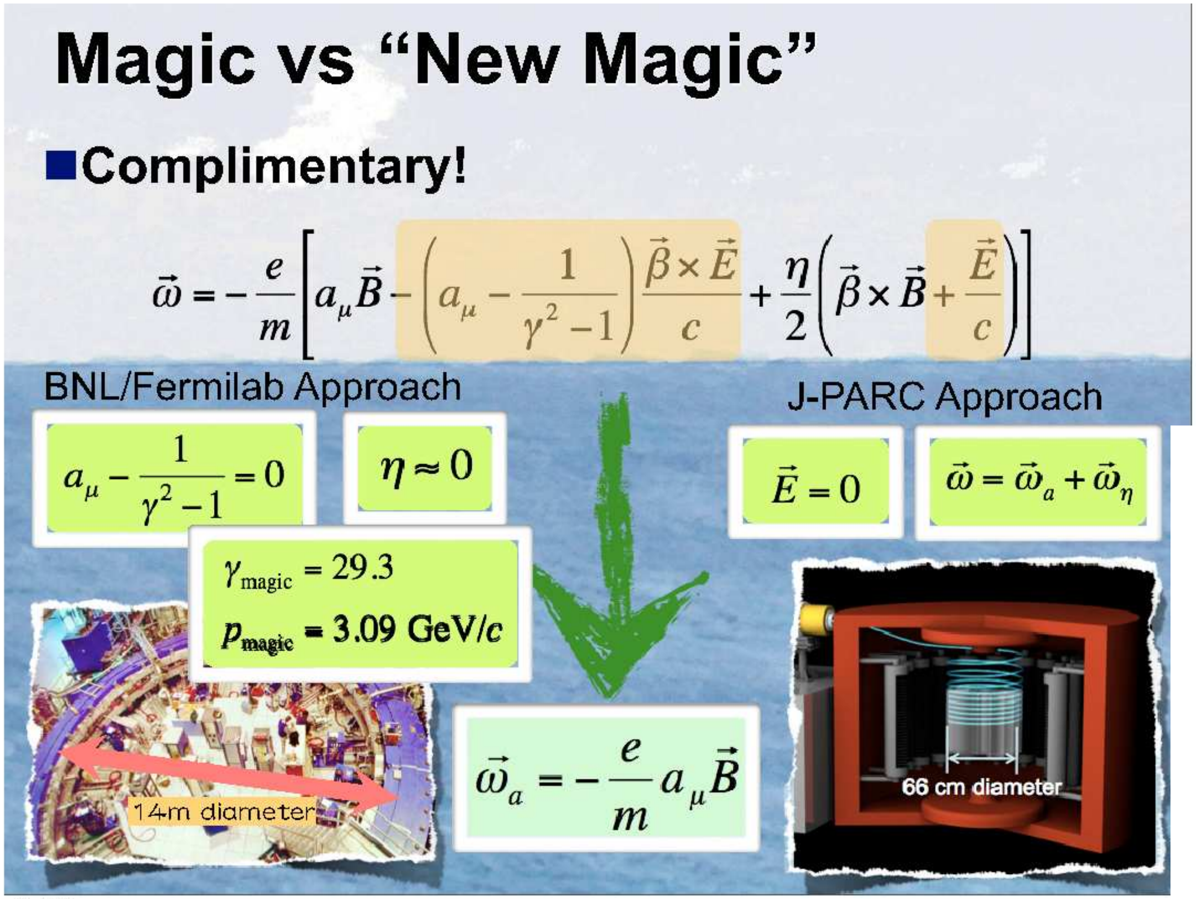

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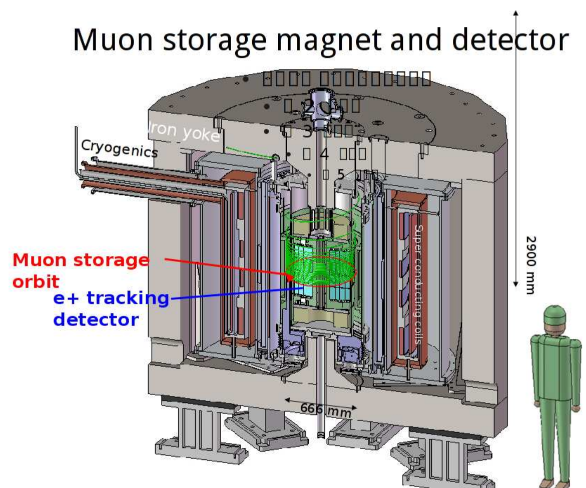

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$\longrightarrow$ completely different set-up (uses slow muons)

$\longrightarrow$ never been tested before

$\longrightarrow$ data taking might start in 2025, accuracy goal 0.45ppm (~ 2 years of running)

- Testing the SM with a_e?

G. F. Giudice, P. Paradisi, M. Passera, JHEP 1211, 113 (2012)

$\longrightarrow$ a_e one of the most precisely measured observable in particle physics

$\longrightarrow$ accuracy goal: from 0.24ppb to 0.02ppb (vs. 0.14ppm for a_μ)

$\longrightarrow$ need to determine the fine structure constant at the same level of accuracy! (at least)

Thanks for your attention!