Cosmological Stasis

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In collaboration with

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Based on Phys.Rev.D 105 (2022) 2, 023530 [arXiv:2111.04753]

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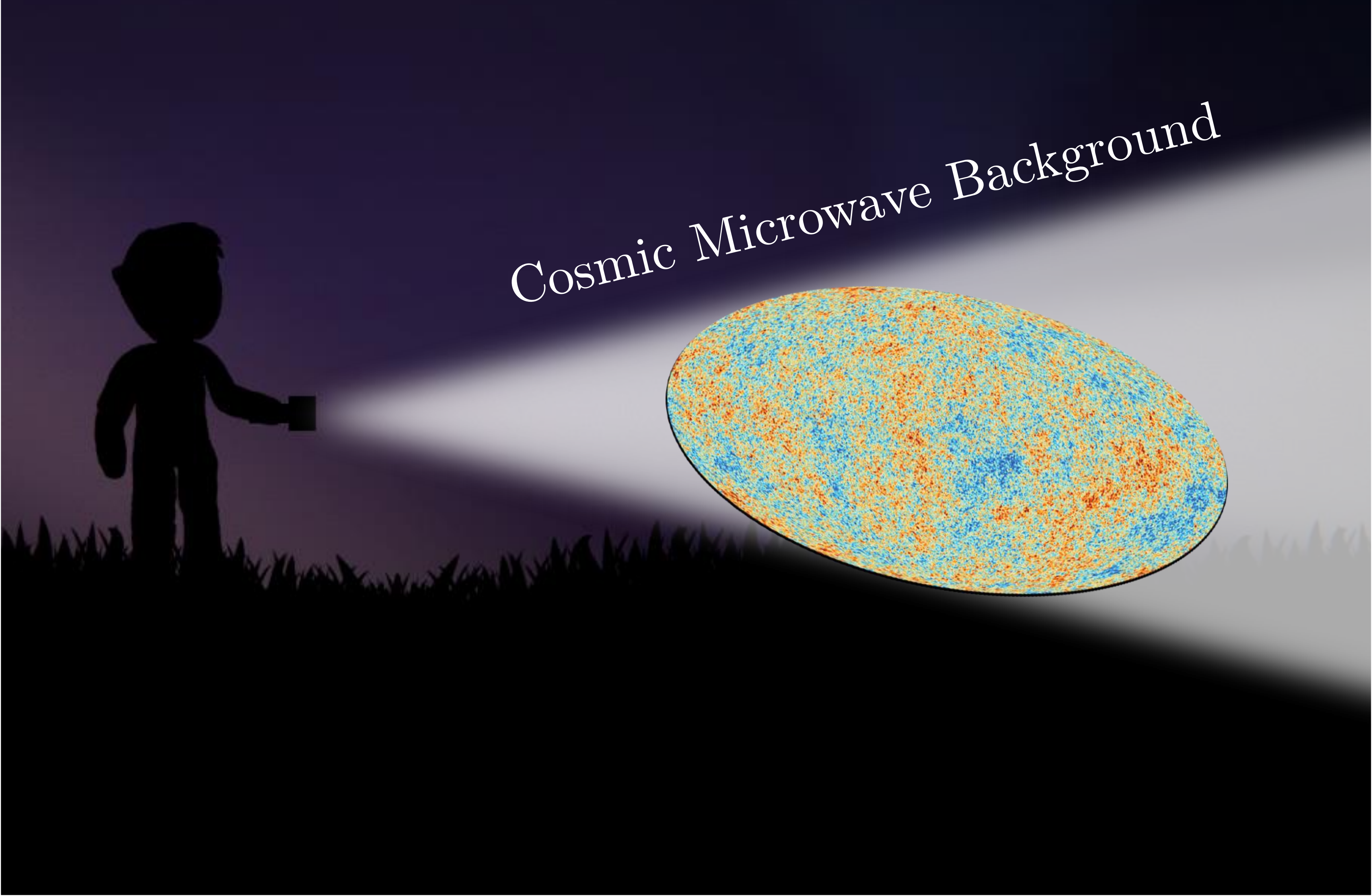
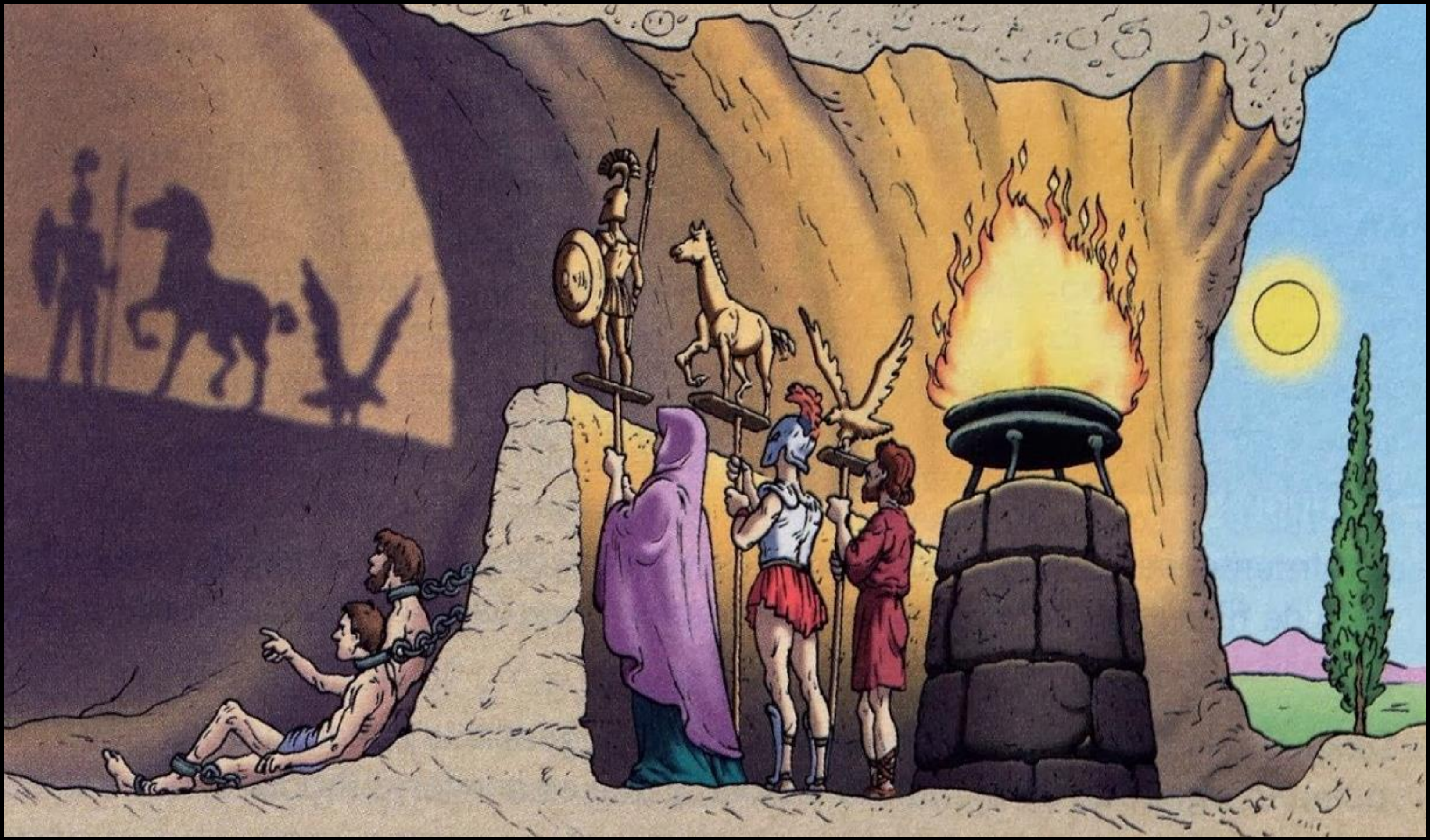
Durham University

IP3

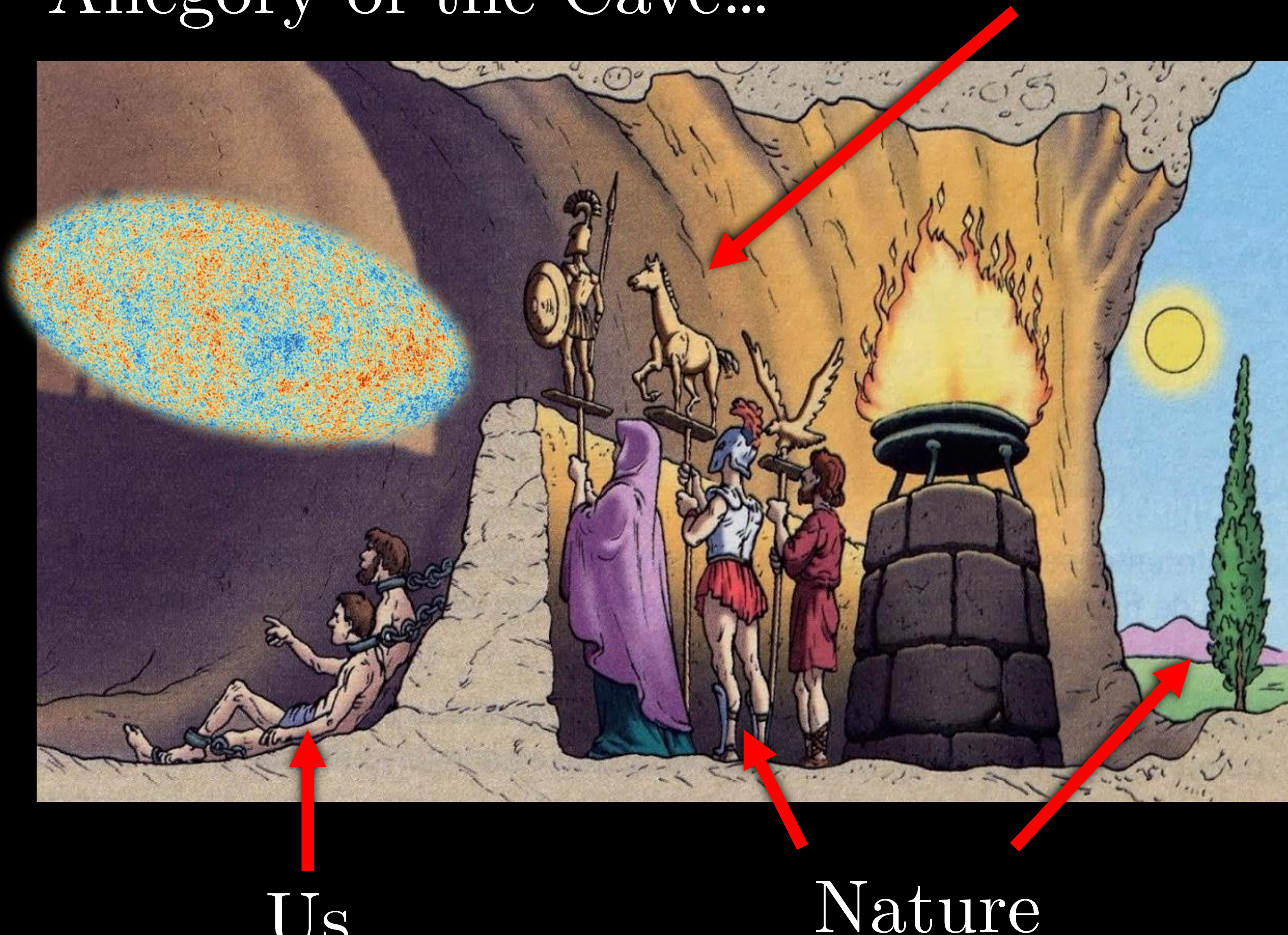
Looking Back



Allegory of the Cave...

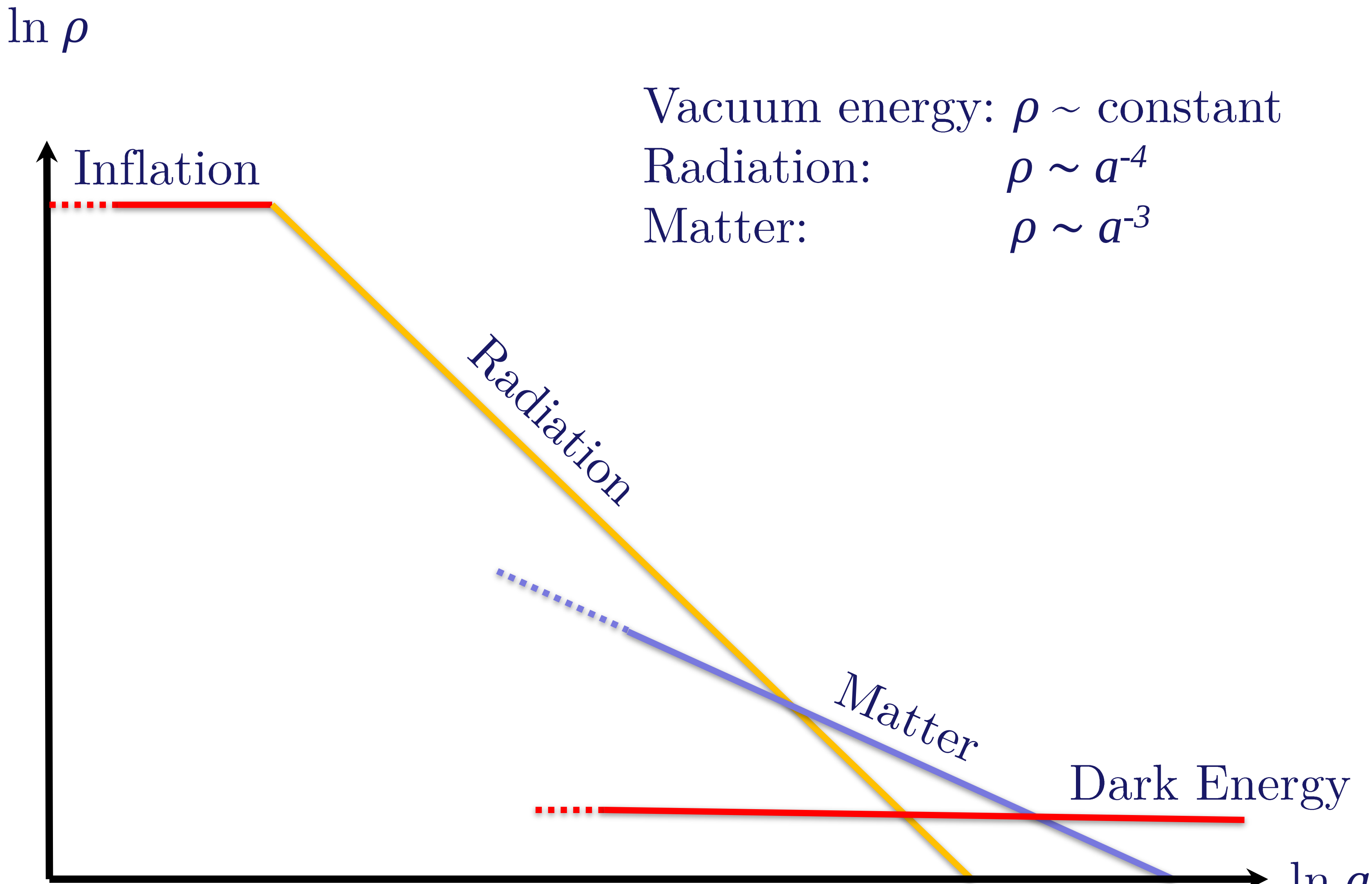


Allegory of the Cave...

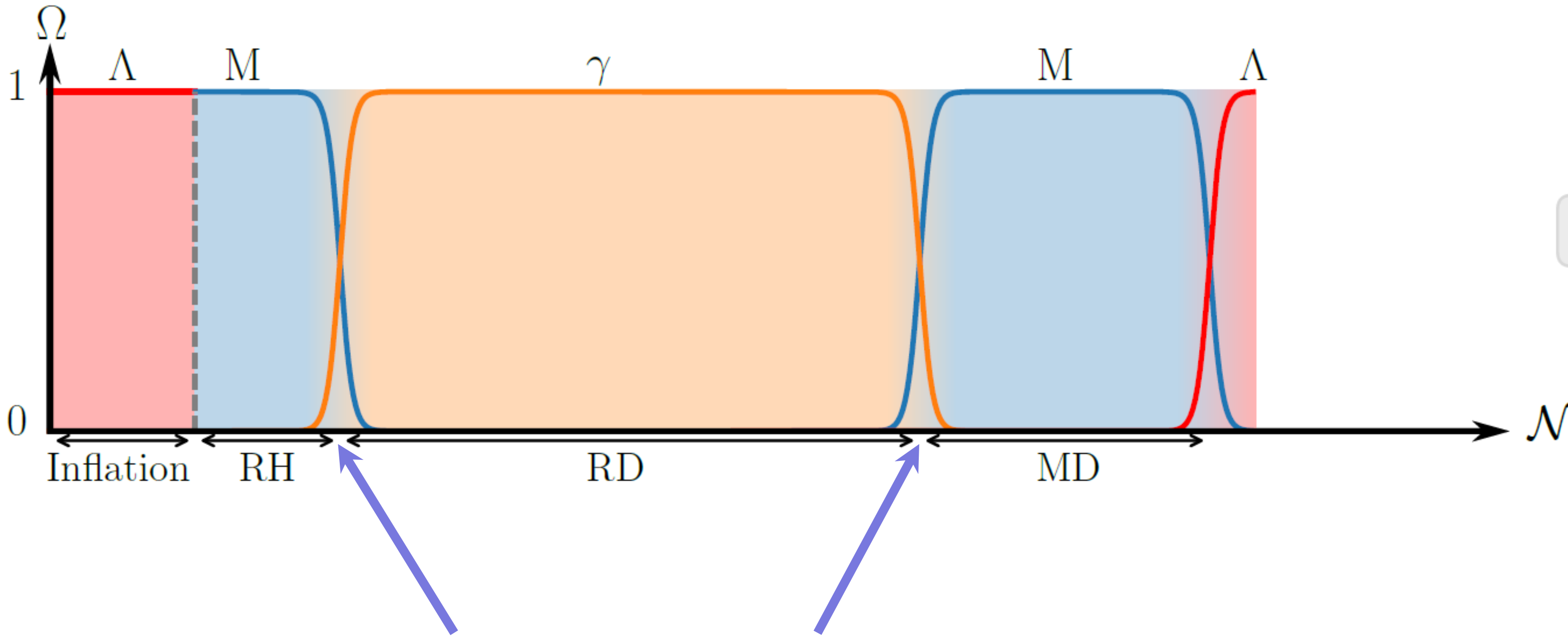


The ΛCDM Ideology

Vacuum energy: $\rho \sim$ constant
Radiation: $\rho \sim a^{-4}$
Matter: $\rho \sim a^{-3}$



The ΛCDM Ideology

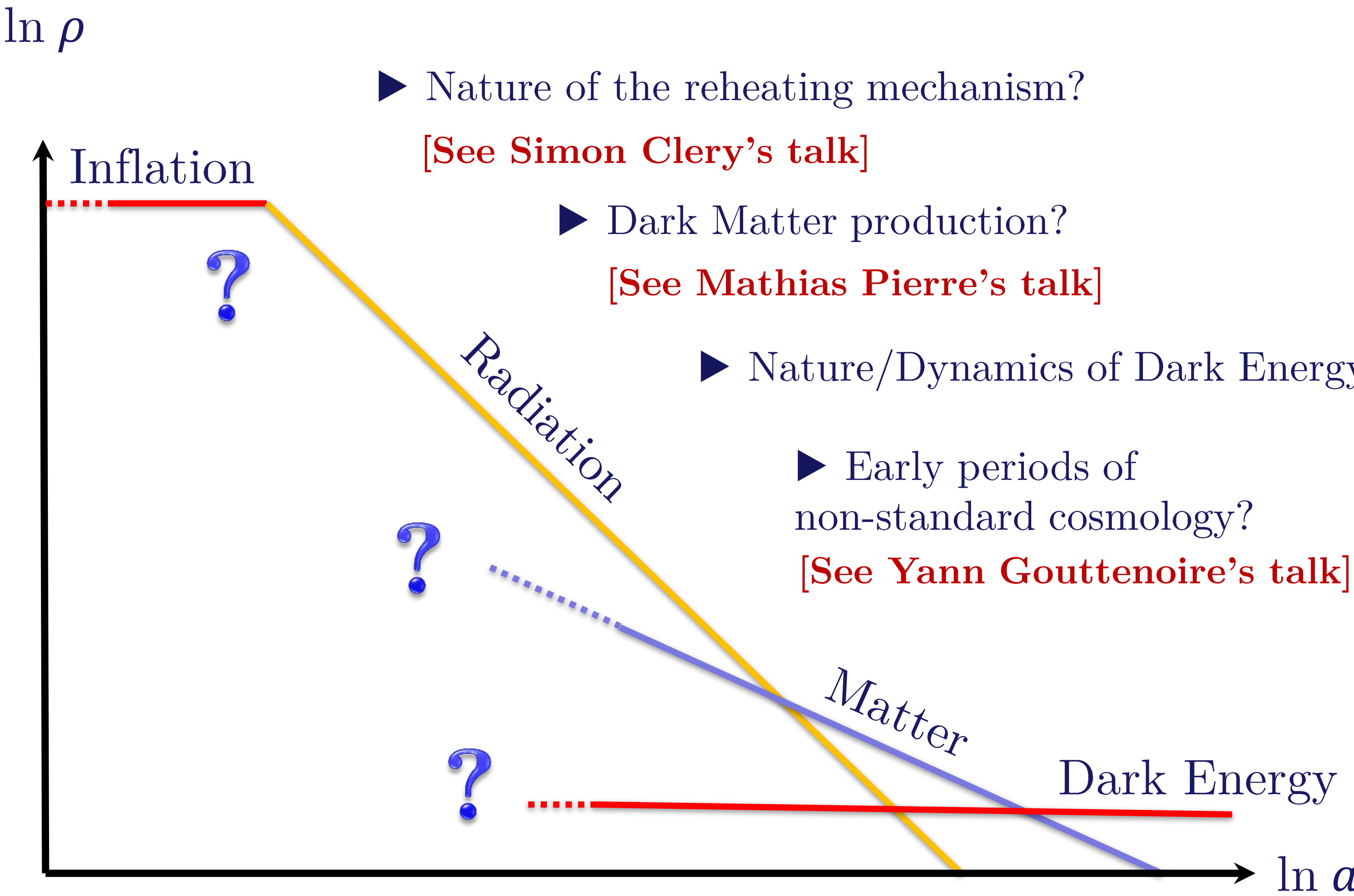


ΛCDM

Matter-Radiation Equality (MRE)

In the ΛCDM model, **MRE is localized in time**. The Universe HAS to *choose* between Matter Domination (MD) and Radiation Domination (MD).

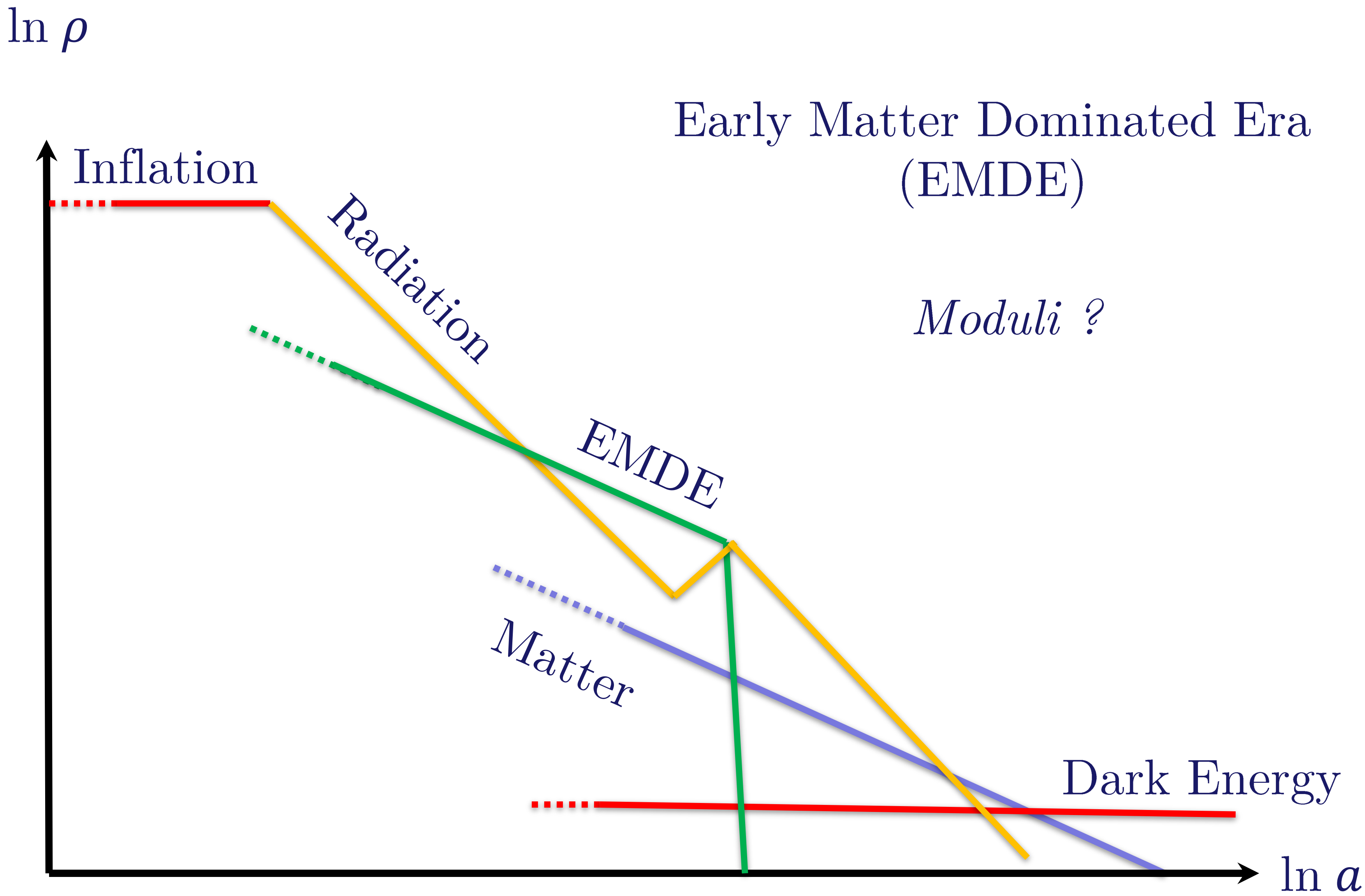
The ΛCDM Ideology



- Nature of the reheating mechanism?
  [See Simon Clery's talk]
- Dark Matter production?
  [See Mathias Pierre's talk]
- Nature/Dynamics of Dark Energy?
- Early periods of non-standard cosmology?
  [See Yann Gouttenoire's talk]

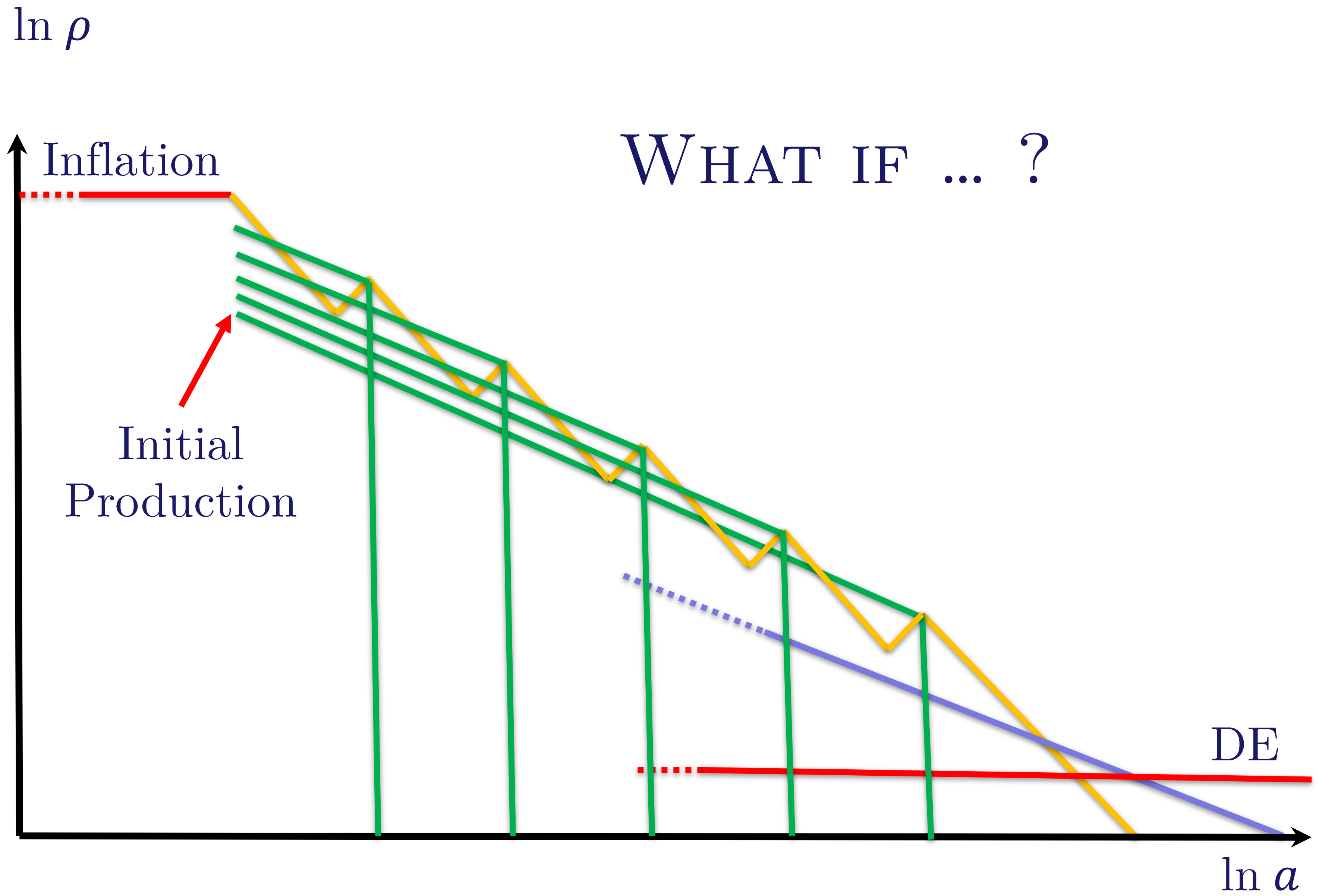
Nature may be
Complex ...

Going Beyond ΛCDM

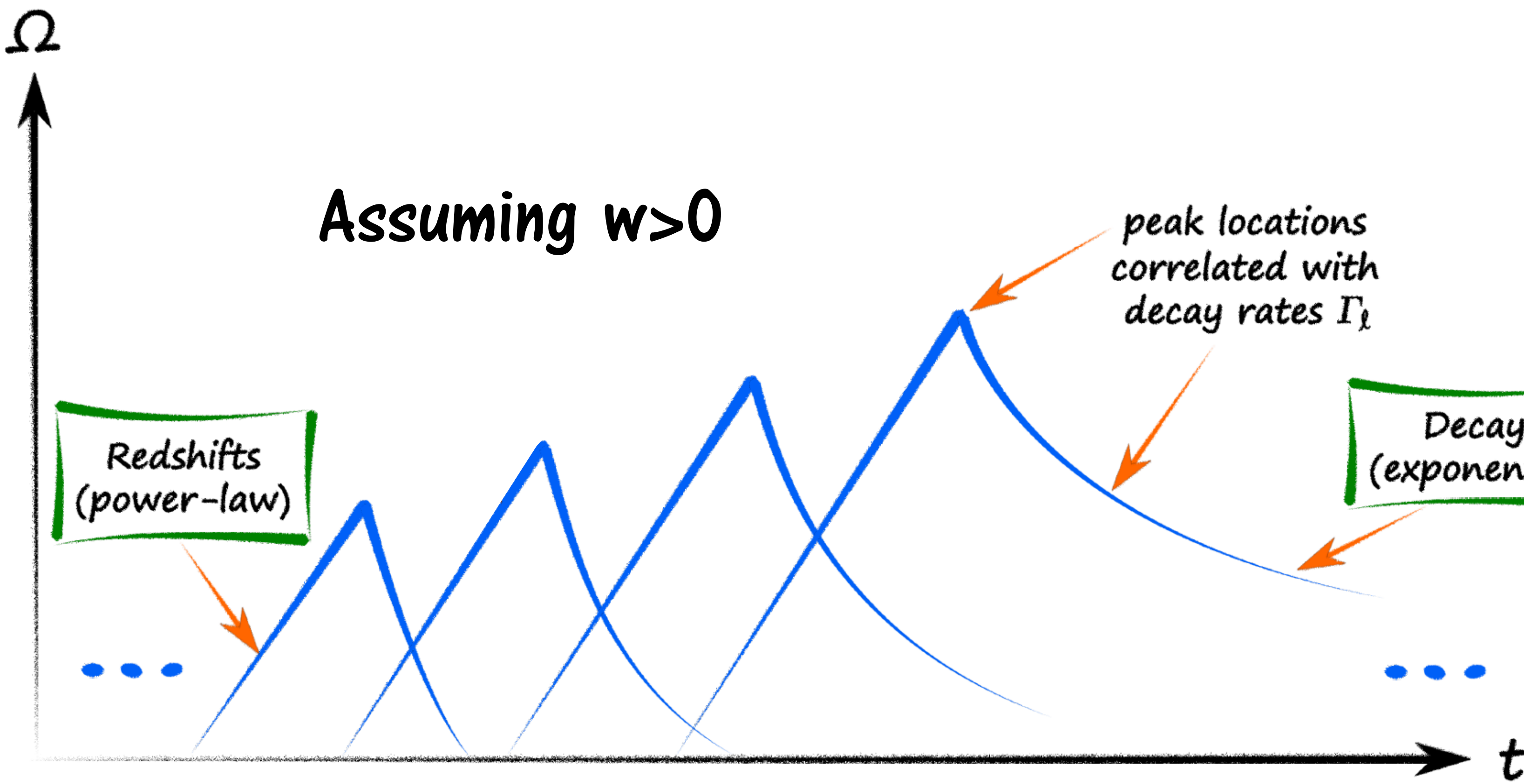


UV theories may be (VERY) non minimal...

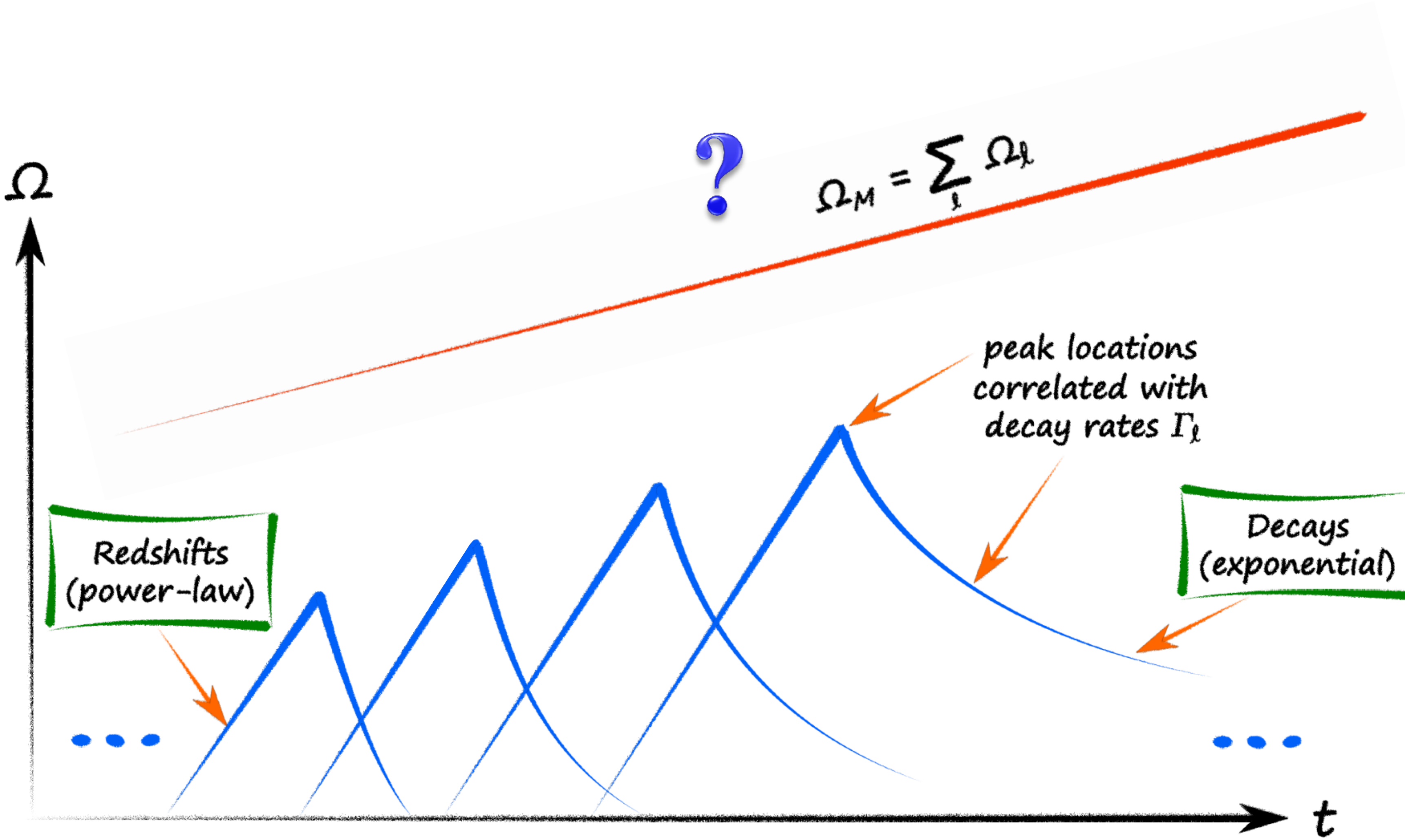
Going (MUCH) Beyond ΛCDM



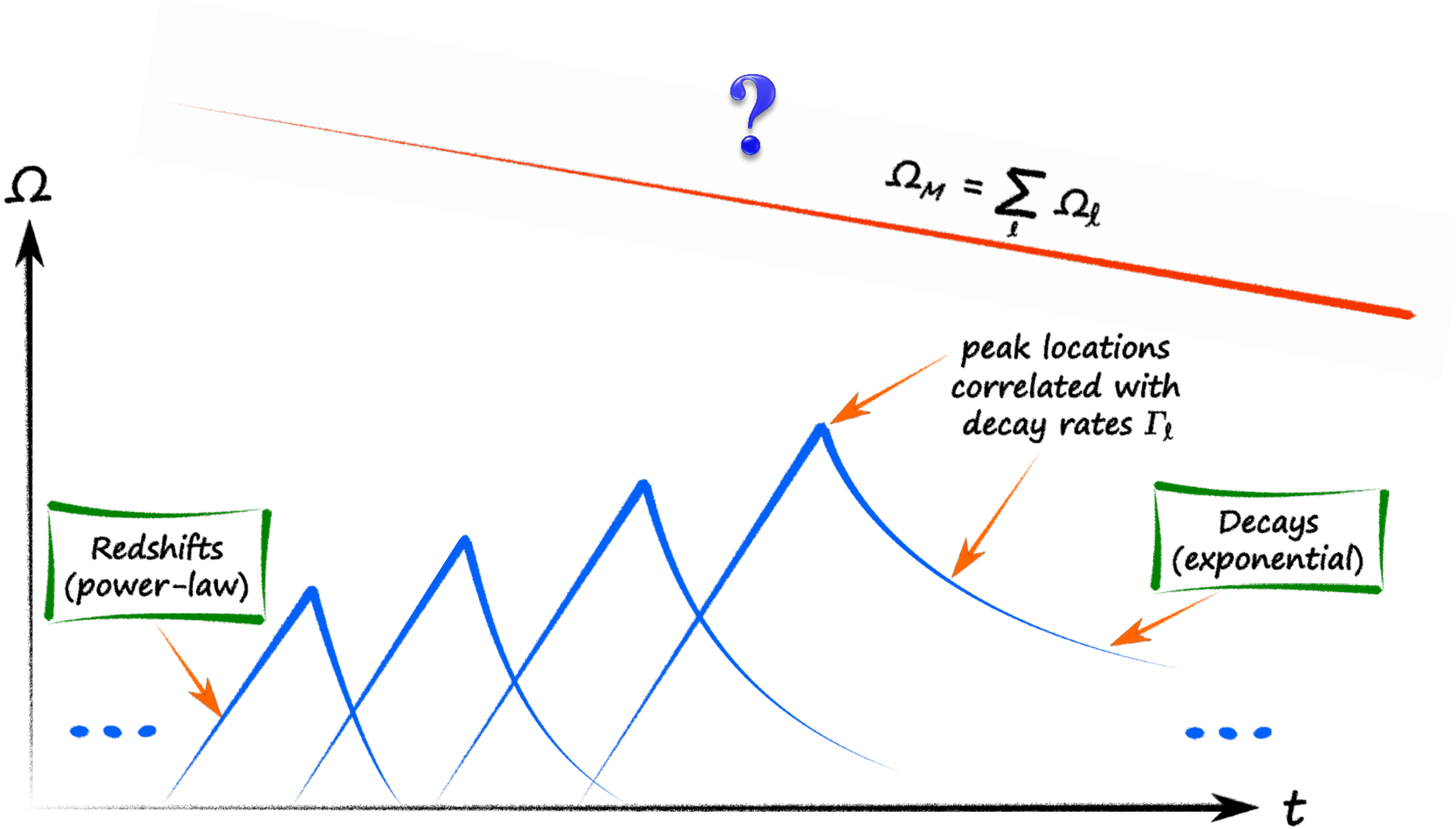
Many states decaying



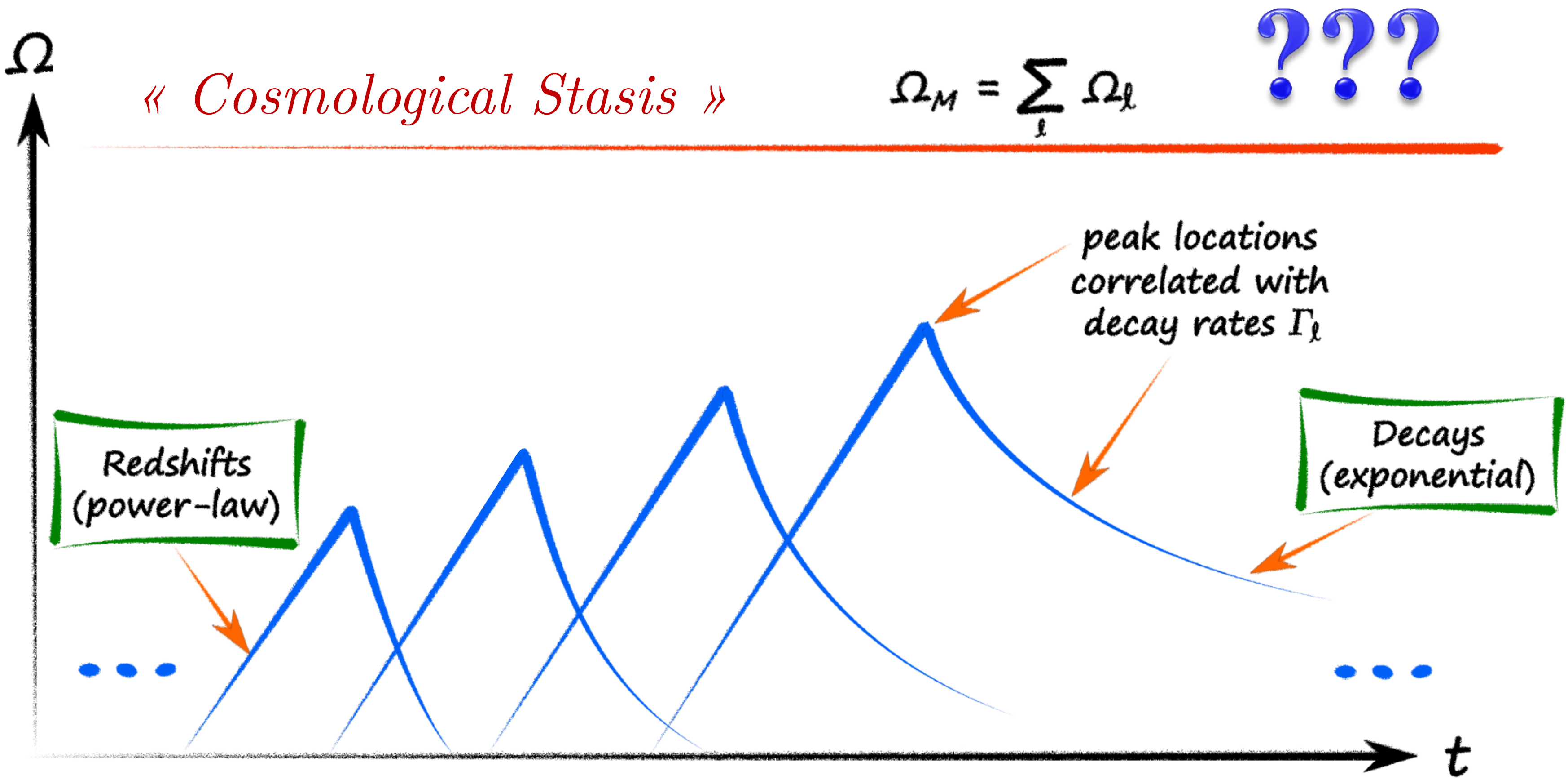
Many states decaying



Many states decaying

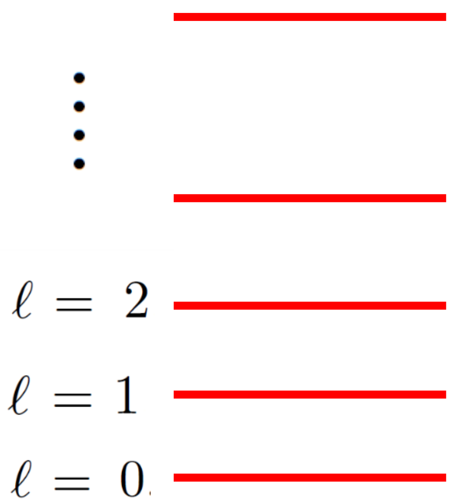


Many states decaying



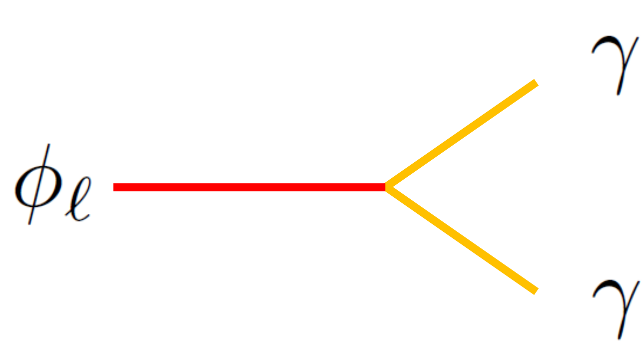
Conditions for Stasis

Mass Spectrum



$$\Omega_i \equiv \frac{8\pi G}{3H^2}\rho_i$$

Decay Processes



$$\Omega_M \equiv \sum_\ell \Omega_\ell$$

$$\Omega_M + \Omega_\gamma = 1$$

Conditions for Stasis

$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell\rho_\ell$$
$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell\rho_\ell$$

Boltzmann Equations

+ Friedmann Equations

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell\Omega_\ell + H\left(\Omega_M - \Omega_M^2\right)$$

Conditions for Stasis

$$\frac{d\Omega_M}{dt} = -\sum_{\ell} \Gamma_\ell \Omega_\ell + H\left(\Omega_M - \Omega_M^2\right)$$

« Cosmological Stasis »

$$\boxed{\sum_{\ell} \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2)\ .}$$

Conditions for Stasis

During Stasis, $\Omega_M = \overline{\Omega}_M$

$$H(t) = \left(\frac{2}{4-\overline{\Omega}_M}\right)\frac{1}{t}$$

$$\Omega_\ell(t) \;=\; \Omega_\ell^* \left(\frac{t}{t_*}\right)^{2-6/(4-\overline{\Omega}_M)} e^{-\Gamma_\ell (t-t_*)}$$

« Cosmological Stasis »

$$\sum_\ell \Omega_\ell(t) \;=\; \overline{\Omega}_M$$

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) \;=\; \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

A Model of Stasis

Mass Spectrum	$m_\ell = m_0 + (\Delta m)\ell^\delta$
Decay Widths	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$
Initial Abundances	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

Free Parameters

$$\{\alpha, \gamma, \delta, m_0, \Delta m, \Gamma_0, \Omega_0^{(0)}, t^{(0)}\}$$

Production time of the states ϕ_ℓ

« *Cosmological Stasis* »

$$\sum_\ell \Omega_\ell(t) = \overline{\Omega}_M$$

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

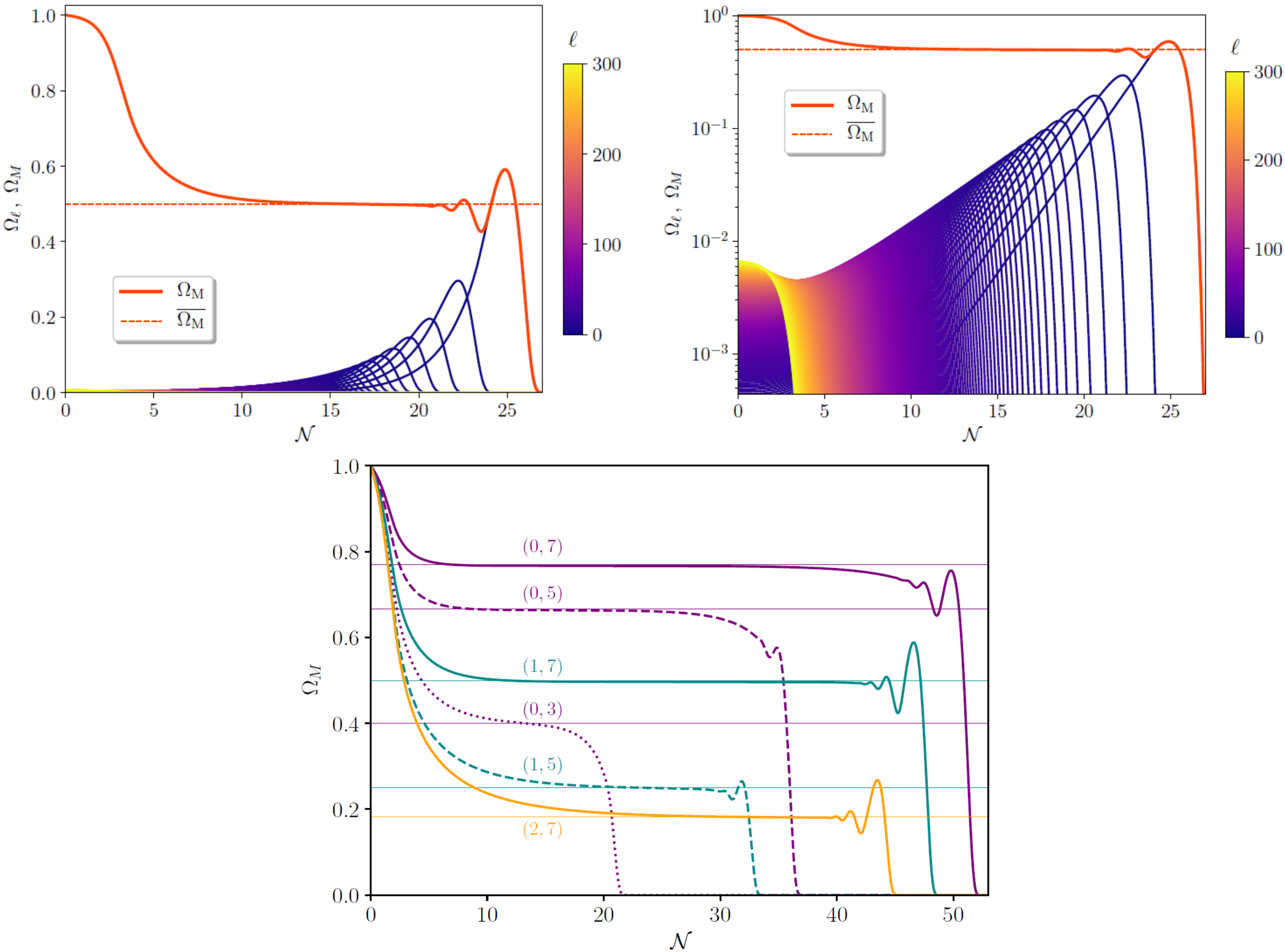
$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

↓

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{\alpha + 1/\delta}{\gamma}\,\frac{1}{t-t^{(0)}}$$

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}\,.$$

Let's try...



$\Omega_\ell\,,\ \Omega_M$
Ω_M
$\overline{\Omega_M}$
ℓ
300
200
100
0
$\mathcal{N}$
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
10^0
10^{-1}
10^{-2}
10^{-3}
Ω_M
(0, 7)
(0, 5)
(1, 7)
(0, 3)
(1, 5)
(2, 7)
30
40
50

Stasis as a Global Attractor

Friedmann Equation

$$\frac{dH}{dt} = -\tfrac{1}{2}H^2\,(4-\Omega_M)$$

$$\frac{1}{H} - \frac{1}{H^{(0)}} = (t-t^{(0)})\left[\frac{4-\langle\Omega_M\rangle}{2}\right]$$

$$\langle\Omega_M\rangle \equiv \frac{1}{t-t^{(0)}}\int_{t^{(0)}}^{t} dt'\,\Omega_M(t')\ .$$

$$\frac{d\Omega_M}{dt} = \frac{\Omega_M}{t-t^{(0)}}\left[\frac{2(1-\Omega_M)}{4-\langle\Omega_M\rangle} - \left(\frac{\alpha+1/\delta}{\gamma}\right)\right]$$

$$\frac{d\langle\Omega_M\rangle}{dt} = \frac{1}{t-t^{(0)}}\left[\Omega_M - \langle\Omega_M\rangle\right]$$

Equilibrium: $\Omega_M = \langle\Omega_M\rangle = \boxed{\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta-(1+\alpha\delta)}\ .}$

Stasis as a Global Attractor

$$\begin{cases} \dfrac{d\Omega_M}{dt} &= \dfrac{1}{t-t^{(0)}}\, f(\Omega_M, \langle\Omega_M\rangle) \\ \dfrac{d\langle\Omega_M\rangle}{dt} &= \dfrac{1}{t-t^{(0)}}\, g(\Omega_M, \langle\Omega_M\rangle)\ , \end{cases}$$

where

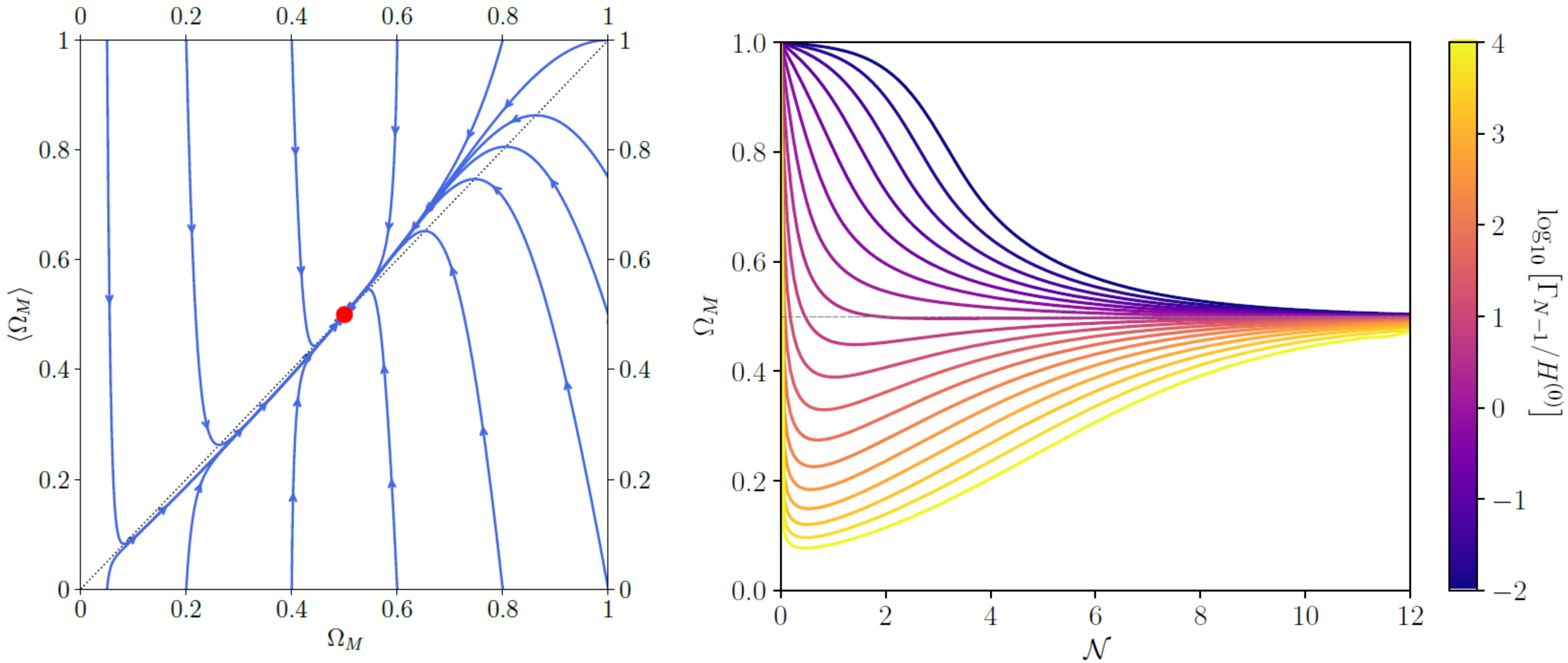
$$f(\Omega_M, \langle\Omega_M\rangle) \equiv \Omega_M \left[\frac{2(1-\Omega_M)}{4-\langle\Omega_M\rangle} - \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\right]$$

$$g(\Omega_M, \langle\Omega_M\rangle) \equiv \Omega_M - \langle\Omega_M\rangle\ .$$

$$\widehat{J} = \begin{pmatrix} \partial_{\Omega_M} f & \partial_{\langle\Omega_M\rangle} f \\ \partial_{\Omega_M} g & \partial_{\langle\Omega_M\rangle} g \end{pmatrix} \qquad \lambda_\pm = \frac{-(4+\overline{\Omega}_M) \pm \sqrt{\overline{\Omega}_M^2 - 16\overline{\Omega}_M + 16}}{2(4-\overline{\Omega}_M)}$$

$$\lambda_\pm < 0 \quad \text{for all} \quad 0 \leq \overline{\Omega}_M \leq 1$$

Stasis as a Global Attractor



The attractor is GLOBAL!!!

Implication for Cosmology

Stasis:

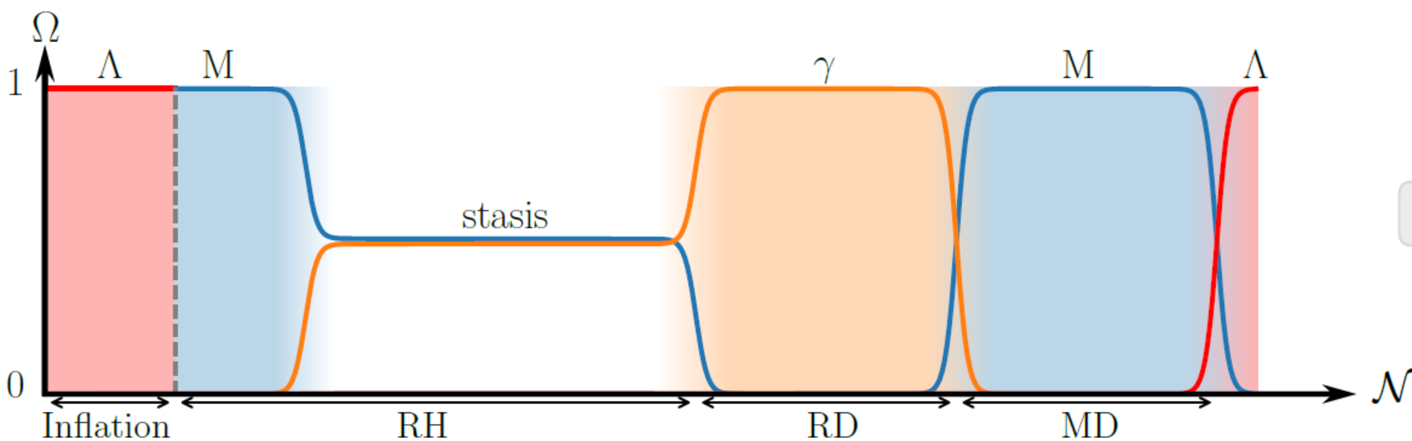
Matter Domination (MD) → Radiation Domination (RD)

Let's splice it in the cosmological timeline!

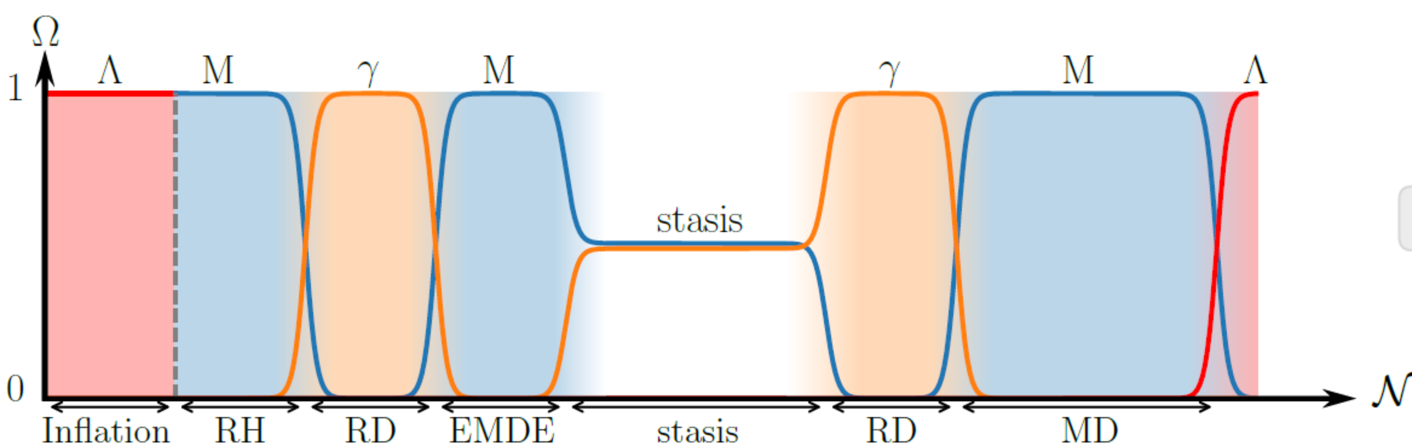
Implication for Cosmology

Stasis:

Matter Domination (MD) → Radiation Domination (RD)



Stasis during Reheating

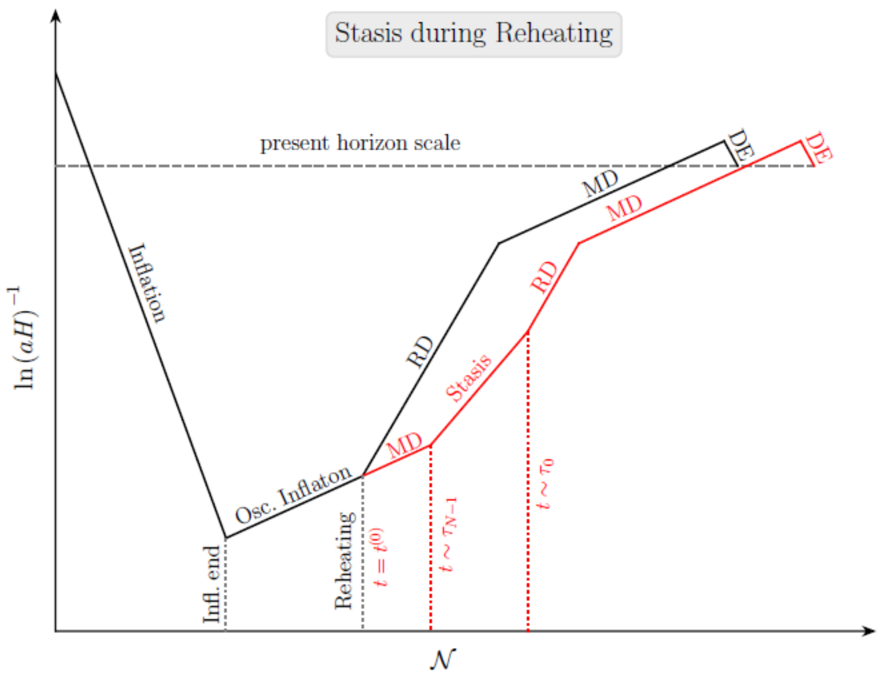


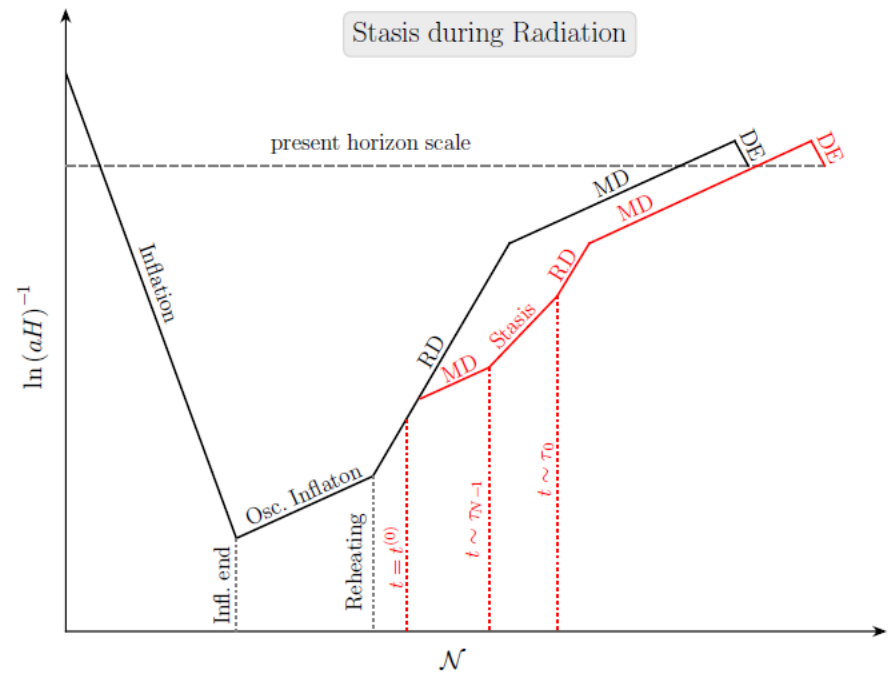
Stasis during Radiation

Implication for Cosmology

Stasis:

Matter Domination (MD) → Radiation Domination (RD)





Implication for Cosmology

- Stasis **modifies the cosmological timeline**
- It **increases the number of *e*-folds** since horizon exit
- It introduces an **era of non-standard cosmology** different from an EMDE

$\longrightarrow$ Dark Matter Production

$\longrightarrow$ Axion Cosmology

$\longrightarrow$ Baryo/Leptogenesis

$\longrightarrow$ Growth of Primordial Perturbations

Conclusion

- Decaying towers of dark states can lead to (very) long periods of stasis;

- The stasis regime is **insensitive to initial conditions**, it is a global attractor;

- Numerous implications: reheating mechanism, constraints on inflation, thermal particle production in the early universe, etc.

- Many possible extensions: production of massive states instead of photons, PBH evaporation, interaction with dark energy, etc.

Much more to come ...

Back Up

Conditions for Stasis

Assume that stasis is established at time t

$$\Omega_\ell(t) \;=\; \Omega_\ell^{(0)}\, h(t^{(0)}, t)\, e^{-\Gamma_\ell (t - t^{(0)})}$$

Non-trivial redshift

$$\sum_\ell \Omega_\ell(t) \;=\; \Omega_0^{(0)}\, h(t^{(0)}, t) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^\alpha e^{-\Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma (t - t^{(0)})}$$

Continuous Limit

$$= \;\frac{\Omega_0^{(0)}}{\delta(\Delta m)^{1/\delta}}\, h(t^{(0)}, t) \int_0^\infty dm\, m^{1/\delta - 1} \left(\frac{m}{m_0}\right)^\alpha e^{-\Gamma_0 \left(\frac{m}{m_0}\right)^\gamma (t - t^{(0)})}$$

$$= \;\frac{\Omega_0^{(0)}}{\gamma\delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} \Gamma\left(\frac{\alpha + 1/\delta}{\gamma}\right) h(t^{(0)}, t) \left[\Gamma_0 (t - t^{(0)})\right]^{-(\alpha + 1/\delta)/\gamma}$$

A Model of Stasis

Mass Spectrum	$m_\ell = m_0 + (\Delta m)\ell^\delta$
Decay Widths	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$
Initial Abundances	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

KK spectrum scalar field compactified on a circle of radius R

[Dienes & Thomas, Phys.Rev.D 85, 083523 / 85, 083524 / 86, 055013]

$mR \ll 1$ or $mR \gg 1$ $\longrightarrow$ $\{m_0, \Delta m, \delta\} = \{m, 1/R, 1\}$ or $\{m_0, \Delta m, \delta\} = \{m, 1/(2mR^2), 2\}$

Bound states of some strongly coupled theory $\longrightarrow$ $\delta = 1/2$ [Dienes, Huang, Su, and Thomas, PRD 95, 043526 (2017)]

A Model of Stasis

Mass Spectrum	$m_\ell = m_0 + (\Delta m)\ell^\delta$
Decay Widths	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$
Initial Abundances	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

Decay Scaling

$$\mathcal{O}_\ell \sim c_n \phi_\ell \mathcal{F}/\Lambda^{d-4}$$

Depends on the microscopic theory $\longrightarrow$

$$\gamma = 2d - 7$$

$$\gamma = \{3, 5, 7\}$$

A Model of Stasis

Mass Spectrum	$m_\ell = m_0 + (\Delta m)\ell^\delta$
Decay Widths	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$
Initial Abundances	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

Abundances

Depends on the production mechanism...

Universal Inflaton Decay $\longrightarrow$ $\alpha = 1$

Stasis with an Extra Component

Ω_X in addition to Ω_M and Ω_γ

$$p_X = w_X \rho_X$$

Stasis requires $d\Omega_X/dt = 0$

$$\begin{cases} \overline{\Omega}_M &= (1-3w_X)(1-\overline{\Omega}_X) \\ \overline{\Omega}_\gamma &= 3w_X(1-\overline{\Omega}_X) \, . \end{cases}$$

$$\boxed{w_X = \frac{\overline{\Omega}_\gamma}{3(\overline{\Omega}_M + \overline{\Omega}_\gamma)} \, .}$$

Line of Attractors...