SCUOLA NORMALE SUPERIORE

Closing the window on WIMP Dark Matter

Salvatore Bottaro

Based on: 2107.09688 and 2205.04486

with D.Buttazzo, M.Costa, R.Franceschini, P.Panci, D.Redigolo, L.Vittorio

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    1. Direct Detection (LZ, DARWIN, XenonNT…)
    2. Indirect Detection (CTA, LHAASO)
    3. Muon collider (?)

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- Minimal, predictive theoretical framework:
    1. SM increased with a single EW multiplet
    2. Three parameters (n, Y, M)
    3. M determined by gauge interactions through freeze-out

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- Minimal, predictive theoretical framework:
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    2. Three parameters (n, Y, M)
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- Not fully nor systematically explored

Real WIMPs - Odd n, Y = 0

2107.09688

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$$\mathscr{L}_\mathrm{s} = \frac{1}{2}\left(D_\mu \chi\right)^2 - \frac{1}{2}M_\chi^2\chi^2 - \frac{\lambda_H}{2}\chi^2|H|^2 - \frac{\lambda_\chi}{4}\chi^4\,,$$

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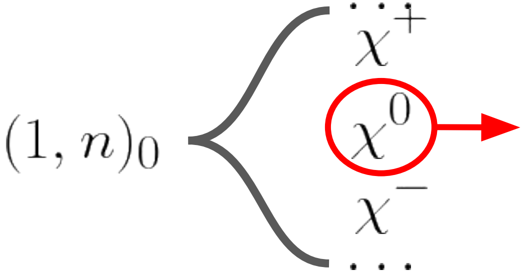

Naturally vanishing coupling to the Z-boson

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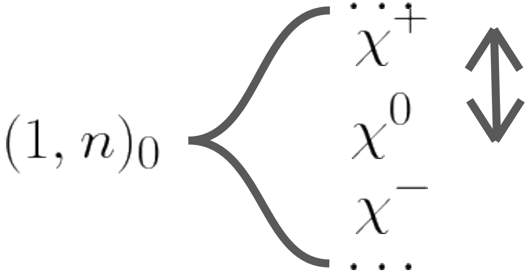
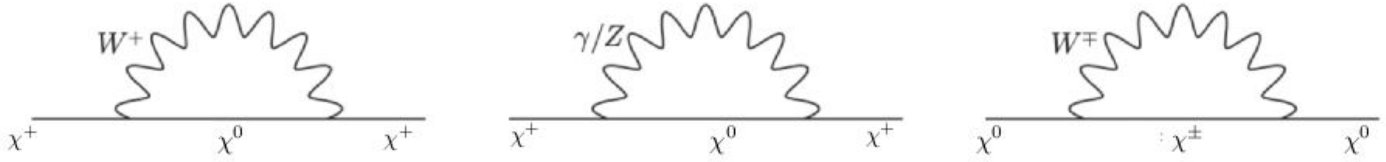


$\Delta M = (167 \pm 4)$ MeV



Cheng ‘98
Feng ‘99
Gherghetta ‘99
Ibe ‘12
McKay ‘18

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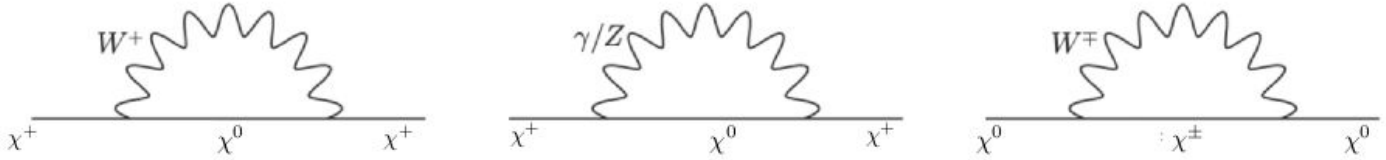
$(1, n)_0$ → $\ldots$, χ^+, χ^0, χ^-, $\ldots$

ΔM = (167 ± 4) MeV

$$(\chi^T T^a \chi)(H^\dagger T^a H) \rightarrow 0$$



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Computing the DM Relic Abundance

Boltzmann equation: $$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\frac{s(x)}{xH(x)}\langle\sigma v\rangle\left(1 - \frac{x}{3g_*(x)}\frac{\mathrm{d}g_*}{\mathrm{d}x}\right)(Y^2(x) - Y_{eq}^2(x))$$

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Std. tree-level cross-section:

$$\langle\sigma v\rangle_0 = \frac{\pi\alpha_2^2(2n^4 + 17n^2 - 19)}{16 g_\chi M_\chi^2}$$

Correct...

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... but inaccurate! Important physics is missing

- Sommerfeld enhancement
- Bound states formation

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Large *non-perturbative, non-relativistic* effects!

Sommerfeld Effect (SE) & Bound States (BS)

SE: Potentials deform the wave function of incoming particles

$$-\frac{\nabla^2\psi}{M_\chi} + V\psi = E\psi \qquad \langle\sigma v\rangle_0 \to \begin{cases} \langle\sigma v\rangle = S_{Som}(x)\langle\sigma v\rangle_0 \\ S_{Som}(x) \propto |\psi(0)|^2 \end{cases}$$

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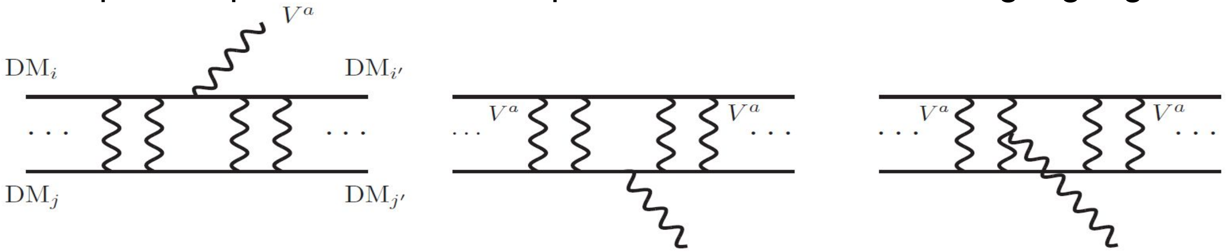

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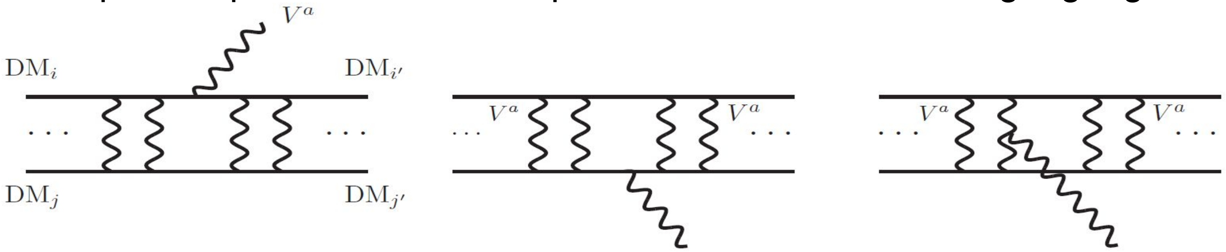


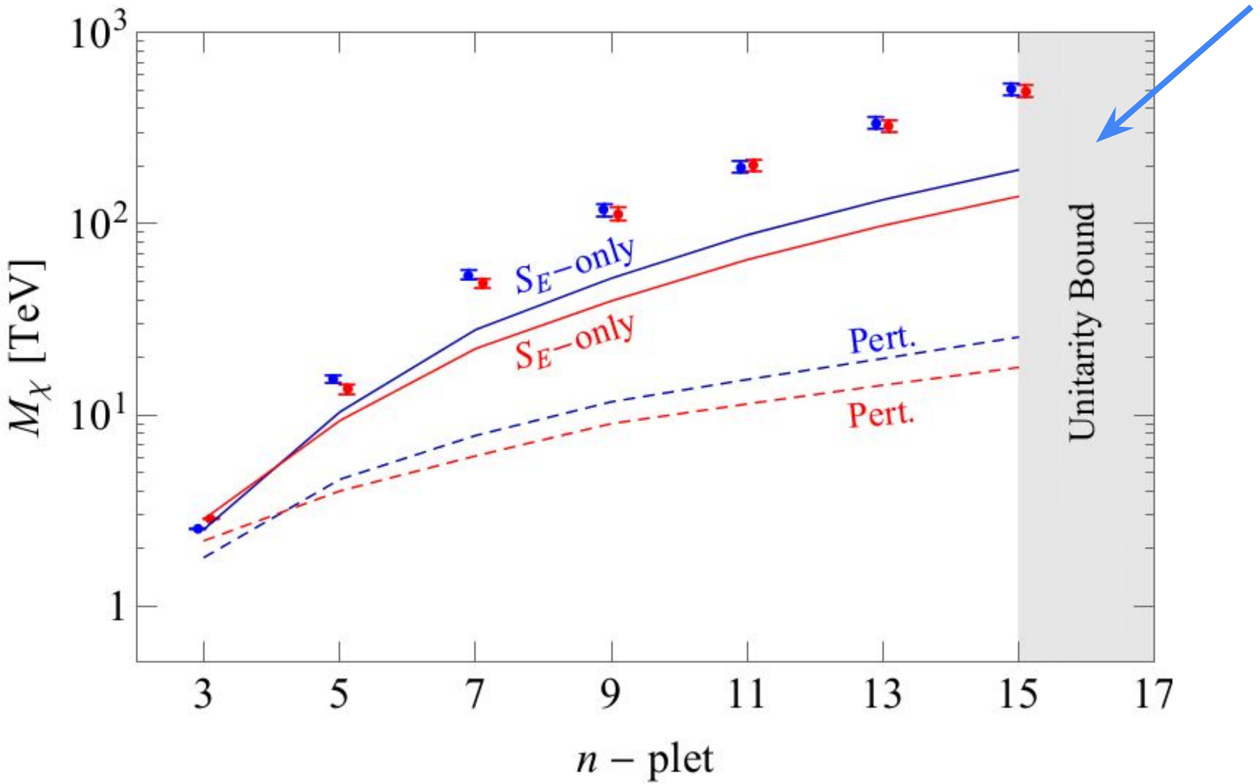
The pair in the bound state later annihilates into SM (annihilation enhancement)

Mitridate '17

$$S(x) = S_{Som}(x) + \left[\frac{\langle\sigma v\rangle_0}{\langle\sigma_I v\rangle} + \frac{g_\chi^2\langle\sigma v\rangle_0 M_\chi^3}{2g_I\Gamma_{ann}}\left(\frac{1}{4\pi x}\right)^{\frac{3}{2}} e^{-xE_{B_I}/M_\chi}\right]^{-1}$$

Real WIMPs - Odd n, Y = 0

$$(\sigma v_{\rm rel})^J \leq \frac{4\pi(2J+1)}{M_{\rm DM}^2 v_{\rm rel}}$$



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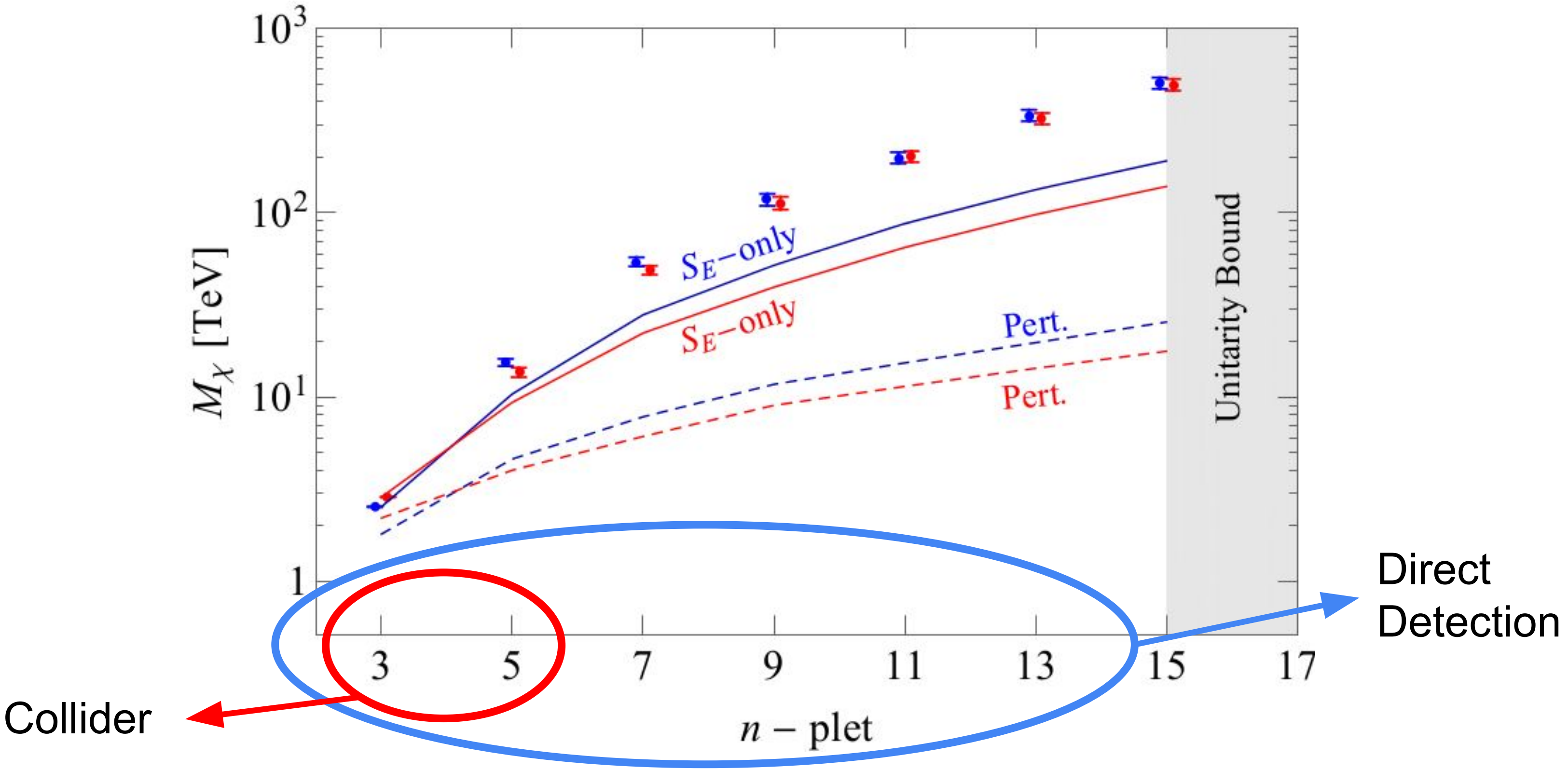
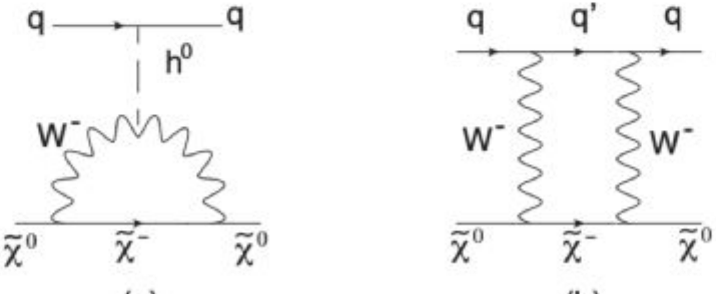


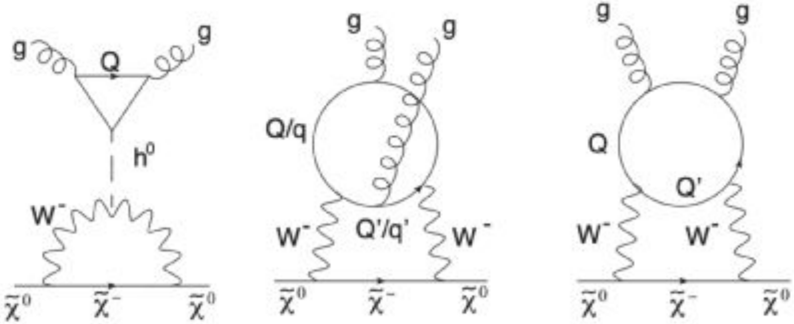
Direct Detection

1-loop



(a) (b)

2-loop



$$\mathscr{L}_{\rm eff}^{\rm SI} = \bar{\chi}\chi\left(f_q m_q \bar{q}q + f_G G_{\mu\nu}G^{\mu\nu}\right) + \frac{g_q}{M_\chi}\bar{\chi} i\partial^\mu\gamma^\nu\chi\mathcal{O}^q_{\mu\nu}$$

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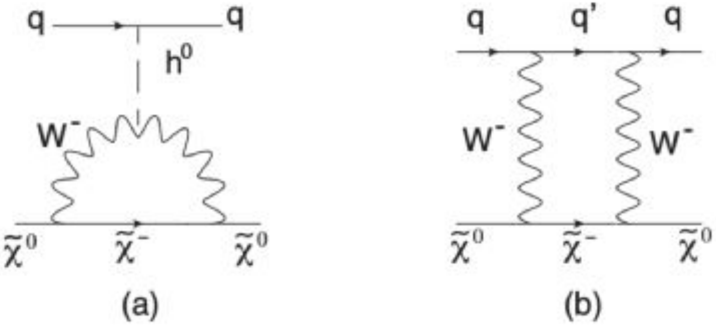
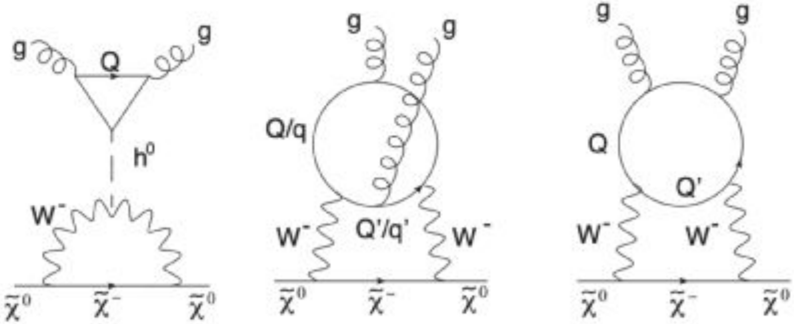


2-loop



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Hisano ‘05, Hisano ‘10

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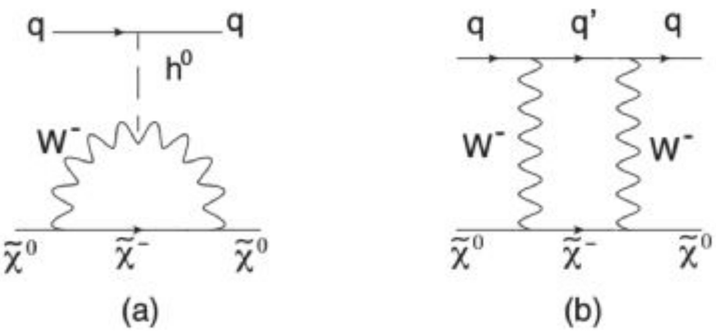

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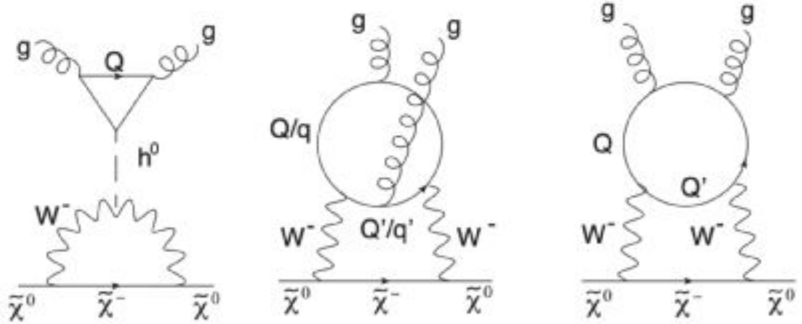

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Flag '20

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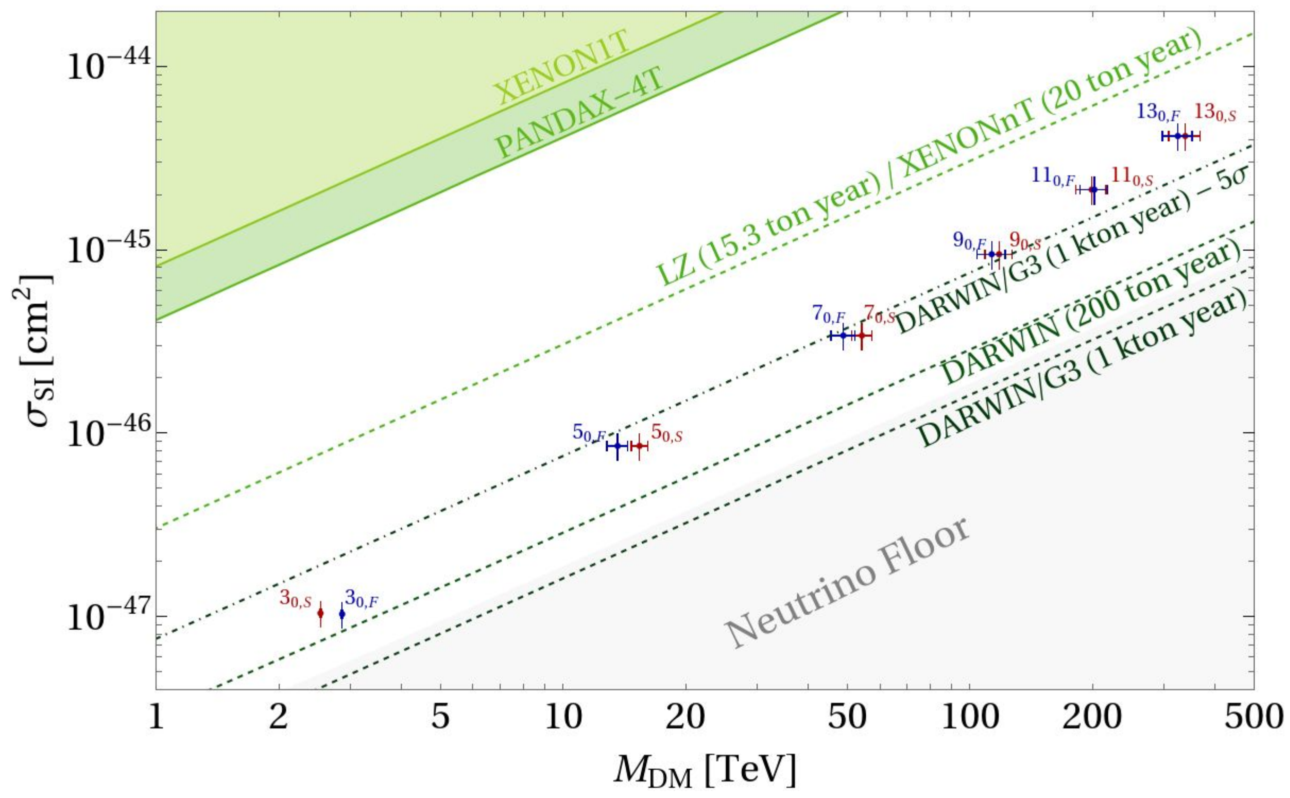

Complex WIMPs - $Y \neq 0$

2205.04486

Complex WIMPs - Y≠0

2205.04486

$$\mathscr{L}_{\mathrm{D}} = \overline{\chi}\left(i\not{D} - M_{\chi}\right)\chi + \frac{y_0}{\Lambda_{\mathrm{UV}}^{4Y-1}}\mathcal{O}_0 + \frac{y_+}{\Lambda_{\mathrm{UV}}}\mathcal{O}_+ + \mathrm{h.c.}$$

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→

$$\chi_{Q=0} \to \chi_0, \chi_{\rm DM}$$

$$\delta m_0 = 4 y_0 c_{nY0}\Lambda_{\rm UV}\left(\frac{v}{\sqrt{2}\Lambda_{\rm UV}}\right)^{4Y}$$

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Dirac   Majorana

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Dirac → $\chi_{Q=0}$ → $\chi_0, \chi_{\rm DM}$ ← Majorana

$$\delta m_0 = 4 y_0 c_{nY0} \Lambda_{\rm UV}\left(\frac{v}{\sqrt{2}\Lambda_{\rm UV}}\right)^{4Y}$$

$$\mathscr{L}_Z = \frac{ieY}{\sin\theta_W\cos\theta_W}\overline{\chi}_0 \not{Z}\chi_{\rm DM} \longrightarrow \frac{1}{2}\mu v_{\rm rel}^2 < \delta m_0\ , \quad \mu = \frac{M_{\rm DM} m_N}{M_{\rm DM} + m_N}$$

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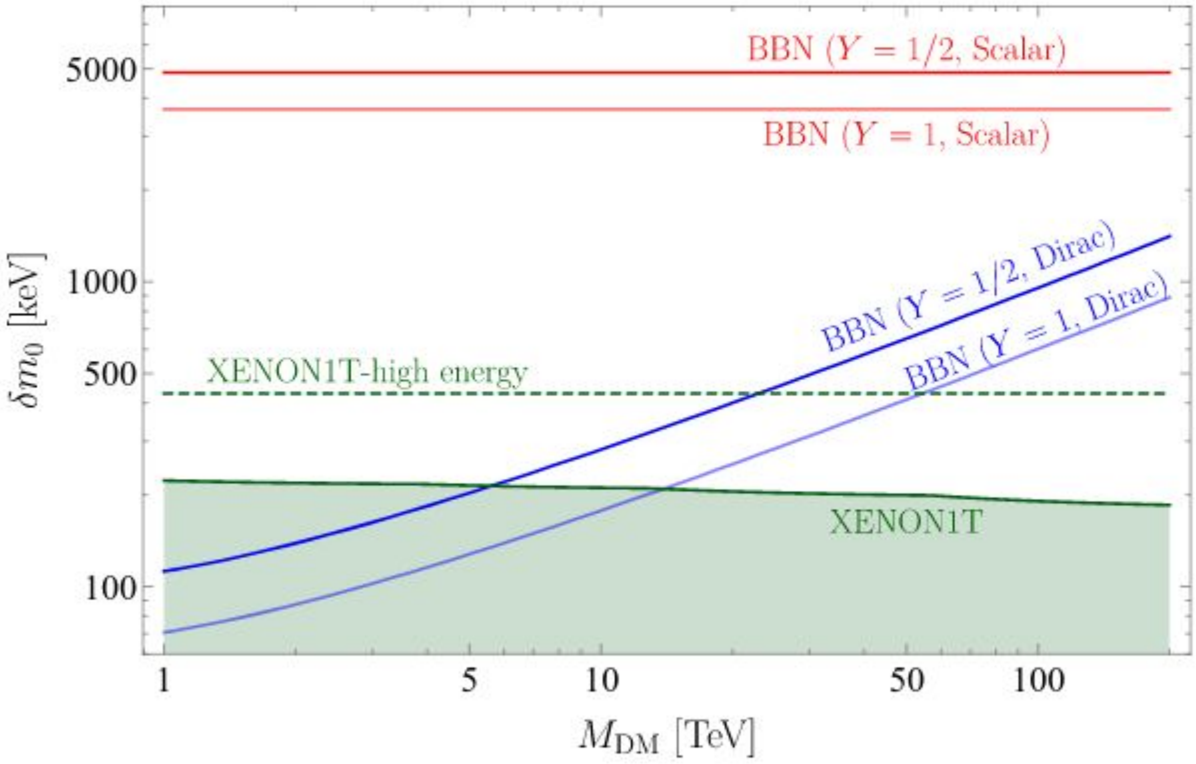
$$\Gamma(\chi_0 \to \chi_{\rm DM} SM) > \tau_{\rm BBN}^{-1}$$



BBN ($Y = 1/2$, Scalar)
BBN ($Y = 1$, Scalar)
BBN ($Y = 1/2$, Dirac)
BBN ($Y = 1$, Dirac)
XENON1T-high energy
XENON1T
5000
1000
500
100
δm_0 [keV]
1
5
10
50
100
$M_{\rm DM}$ [TeV]

Complex WIMPs - Y≠0

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$$\Delta M_{\rm gauge} = 167\ {\rm MeV}\left(Q^2 + \frac{2QY}{\cos\theta_W}\right)$$

→

$\mathcal{O}_+$ necessary to make DM the lightest component of the multiplet unless

$$Y = 0, \quad |Y| = \frac{n-1}{2}$$

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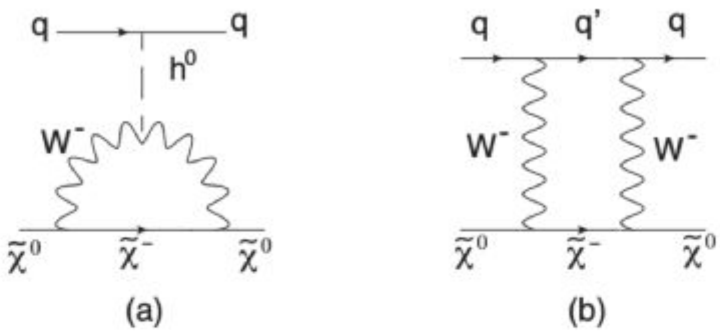
Surviving candidates:

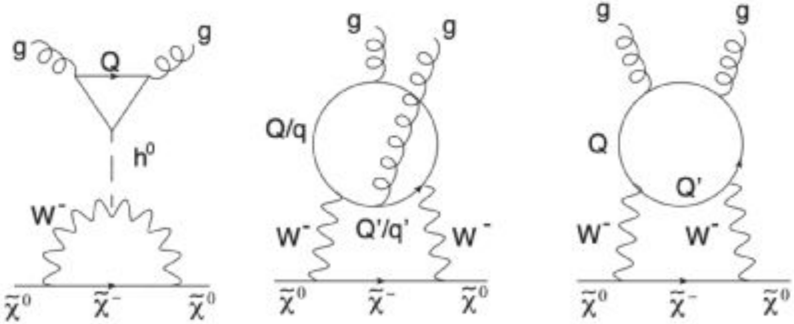
- Y=½, n<13 (perturbative unitarity bound)
- Y=1, n= 3, 5 (perturbativity of mass splitting)
- Y>1 are non-perturbative!

Direct Detection

1-loop



2-loop



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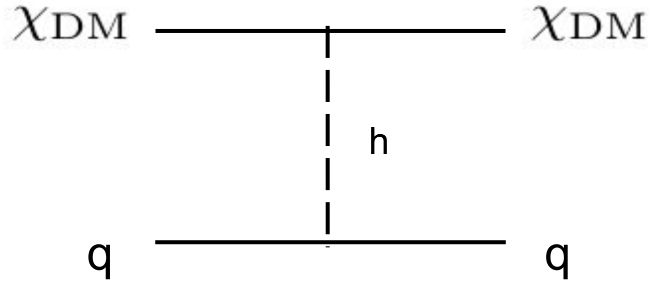
Direct Detection

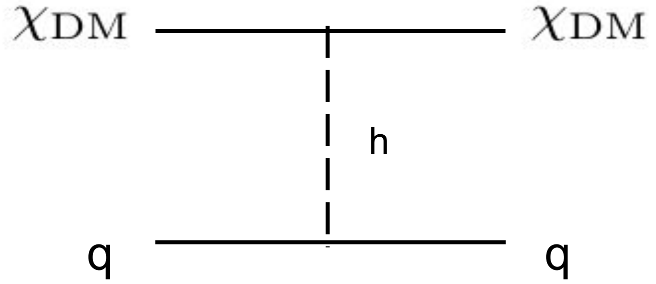
generate tree-level
coupling to the Higgs

$\mathcal{O}_0, \mathcal{O}_+ \longrightarrow$



Direct Detection

$\mathcal{O}_0, \mathcal{O}_+$ → generate tree-level coupling to the Higgs



$$\mathscr{L}_{D,h} = -\frac{\lambda_D v}{2\Lambda_{\rm UV}} \chi_{\rm DM}^2 h$$

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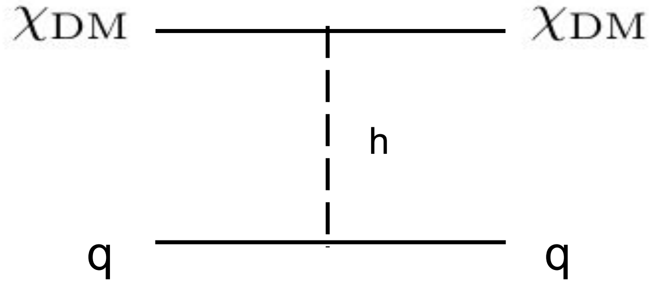

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$$\mathscr{L}_{\rm eff,h}^{\rm SI} = \frac{\lambda_D}{2 m_h^2 \Lambda_{\rm UV}} \chi^2 \left(m_q \bar{q} q - \frac{\alpha_s}{4\pi} G_{\mu\nu}^a G^{a\mu\nu} \right)$$

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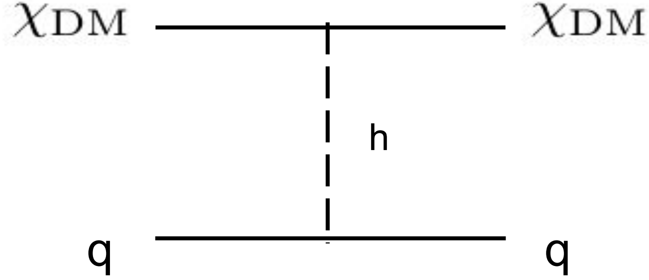
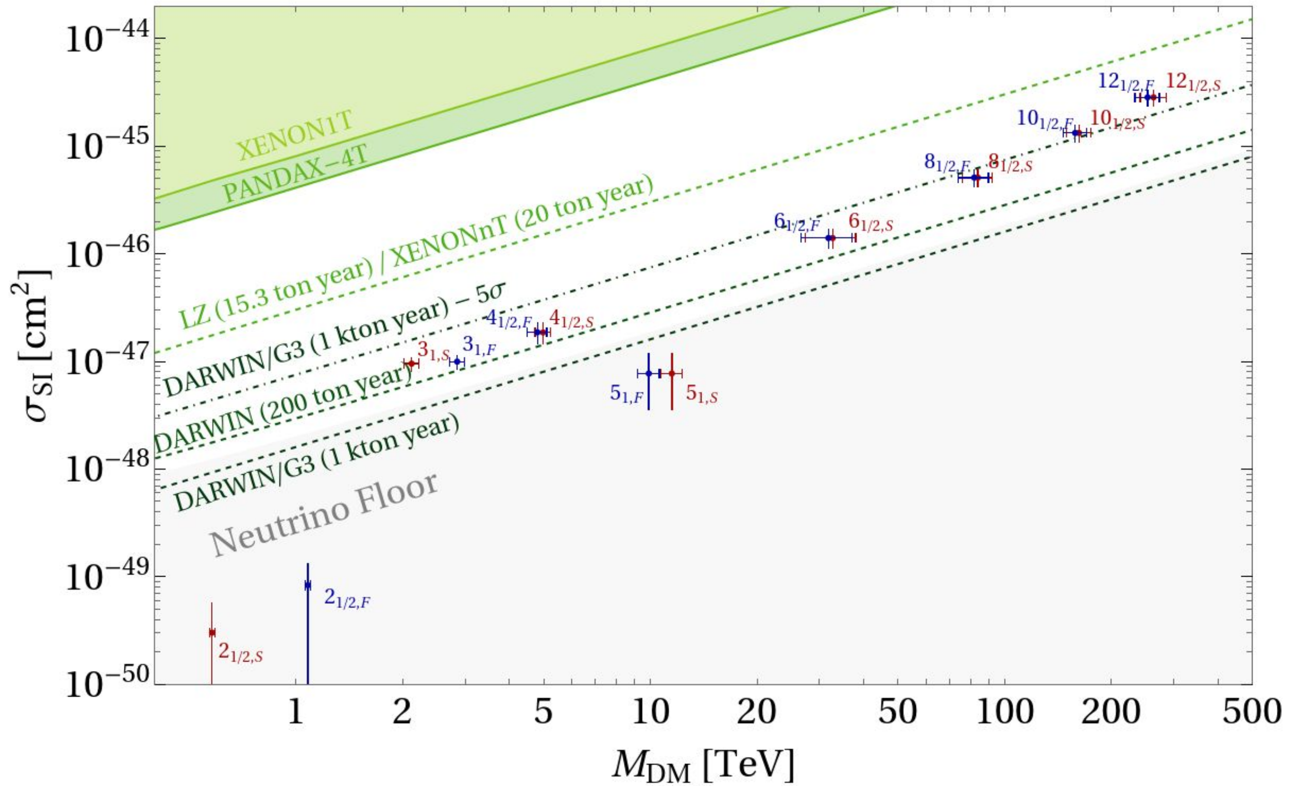


$$\mathscr{L}_{D,h} = -\frac{\lambda_D v}{2\Lambda_{\rm UV}} \chi^2_{\rm DM} h$$

$$\mathscr{L}^{\rm SI}_{\rm eff,h} = \frac{\lambda_D}{2m_h^2 \Lambda_{\rm UV}} \chi^2 \left(m_q \bar{q} q - \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} \right)$$

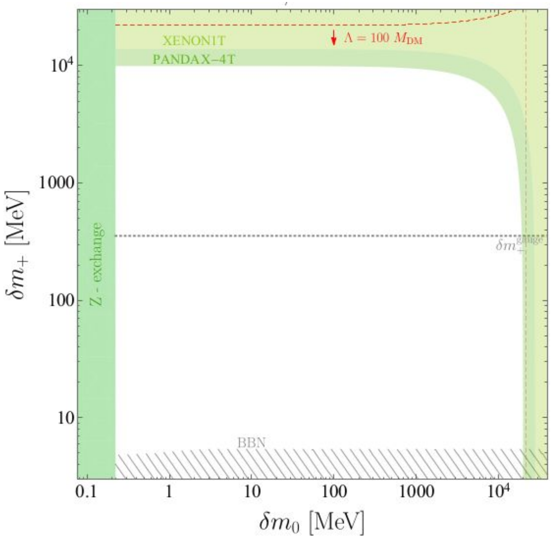
$$f_q = f_q^{\rm EW} + \frac{\lambda_D}{2m_h^2 \Lambda_{\rm UV}}, \; f_G = f_G^{\rm EW} - \frac{\alpha_s}{8\pi} \frac{\lambda_D}{2m_h^2 \Lambda_{\rm UV}}$$

Direct Detection - Minimal Splitting

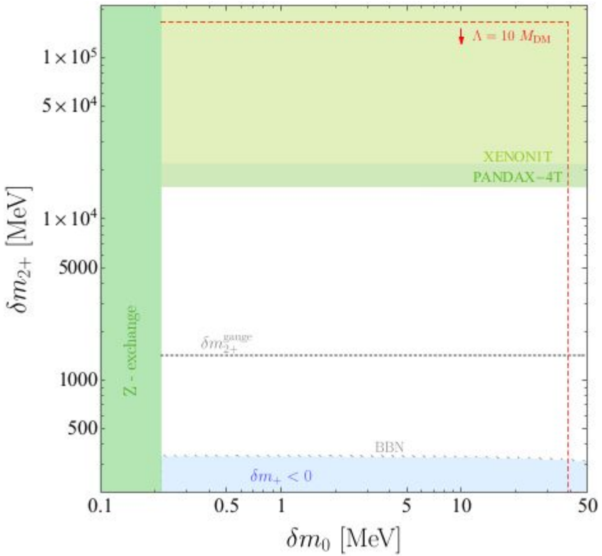


Direct Detection - Non minimal splitting

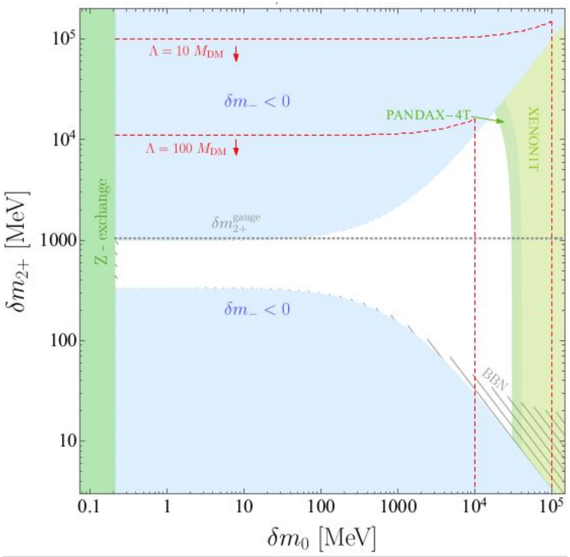
n = 2



n = 3

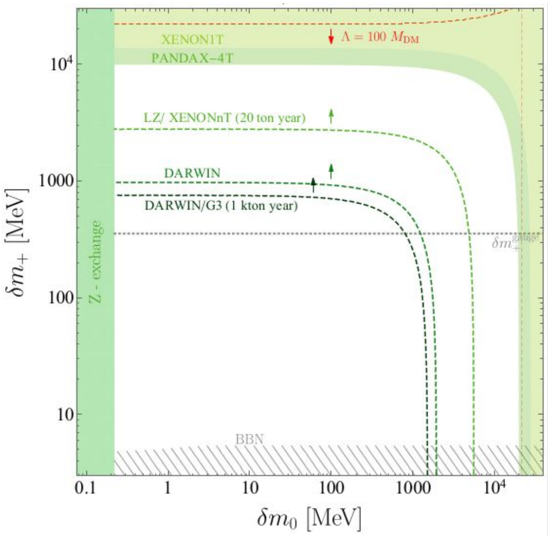


n = 4

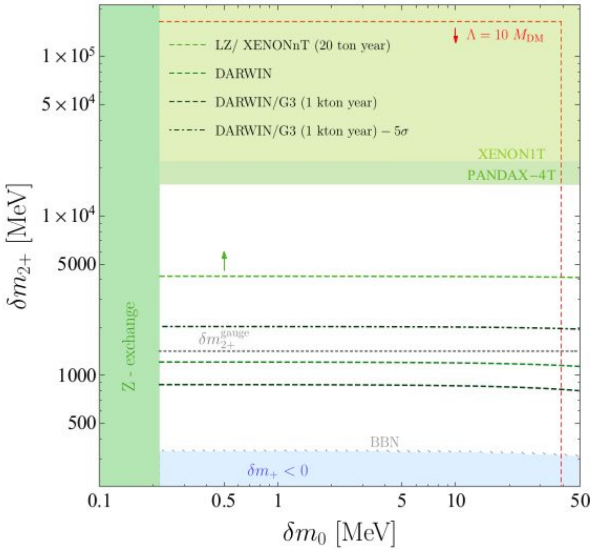


Direct Detection - Non minimal splitting

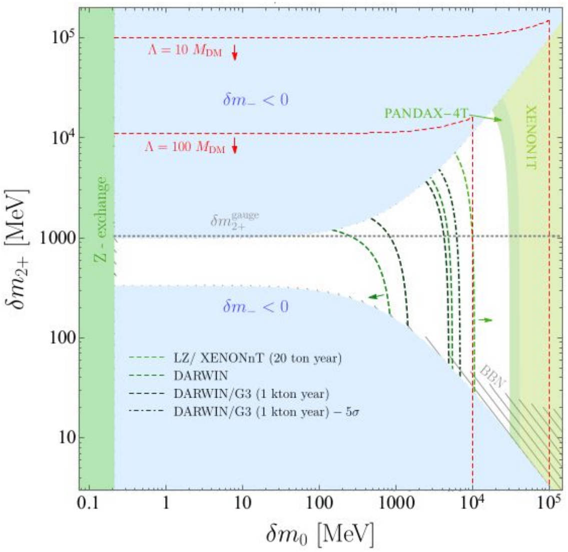
n = 2



n = 3

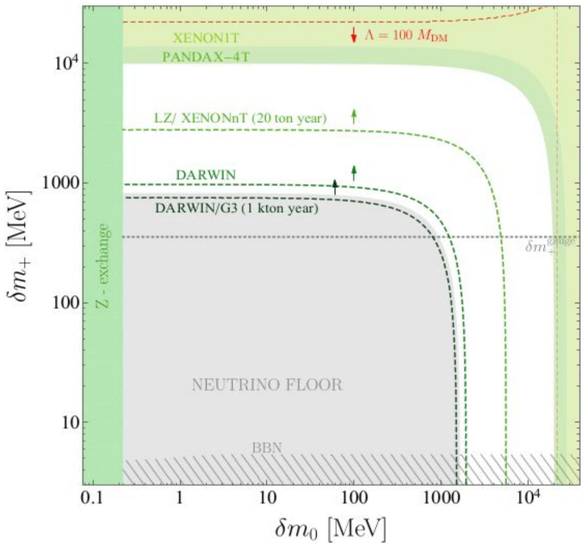


n = 4

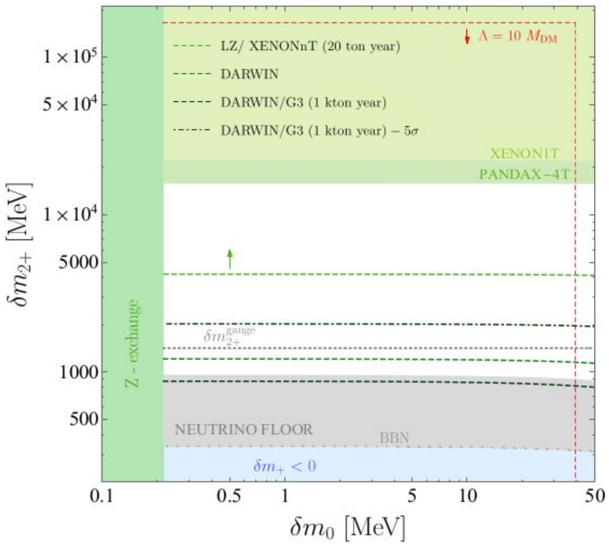


Direct Detection - Non minimal splitting

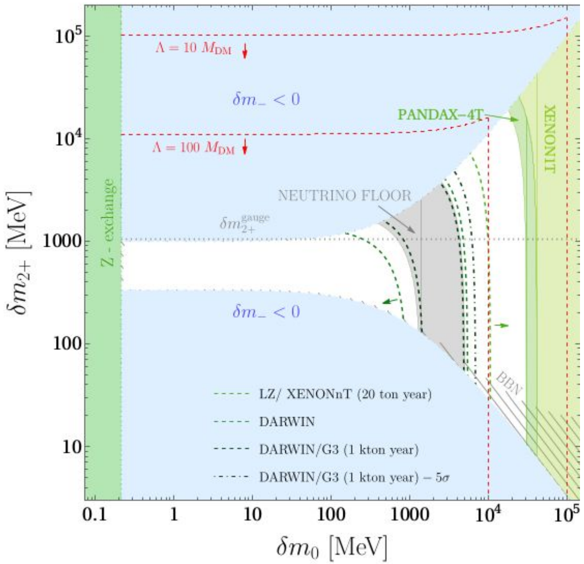
n = 2



n = 3



n = 4



Conclusions

- We computed the thermal mass of all perturbative WIMP candidates
- Real candidates can all be excluded by high exposure (> 200 ton x year) Xenon experiments like DARWIN
- Complex candidates with $Y \neq 0$ and minimal splitting can also be excluded by DARWIN, with the exception of n=2 and 5
- Future DD experiments can close most of the parameter space spanned by mass splittings
- Collider can close the parameter space for light multiplets, while ID for the heavier ones (future work)

Thanks for the attention

Back-up

Real WIMPs

DM spin	EW n-plet	M_χ (TeV)	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$	$\Lambda_{\rm Landau}/M_{\rm DM}$	$\Lambda_{\rm UV}/M_{\rm DM}$
Real scalar	3	2.53 ± 0.01	–	2.4×10^{37}	$4 \times 10^{24}*$
	5	15.4 ± 0.7	0.002	7×10^{36}	3×10^{24}
	7	54.2 ± 3.1	0.022	7.8×10^{16}	2×10^{24}
	9	117.8 ± 15.4	0.088	3×10^{4}	2×10^{24}
	11	199 ± 42	0.25	62	1×10^{24}
	13	338 ± 102	0.6	7.2	2×10^{24}
Majorana fermion	3	2.86 ± 0.01	–	2.4×10^{37}	$2 \times 10^{12}*$
	5	13.6 ± 0.8	0.003	5.5×10^{17}	3×10^{12}
	7	48.8 ± 3.3	0.019	1.2×10^{4}	1×10^{8}
	9	113 ± 15	0.07	41	1×10^{8}
	11	202 ± 43	0.2	6	1×10^{8}
	13	324.6 ± 94	0.5	2.6	1×10^{8}

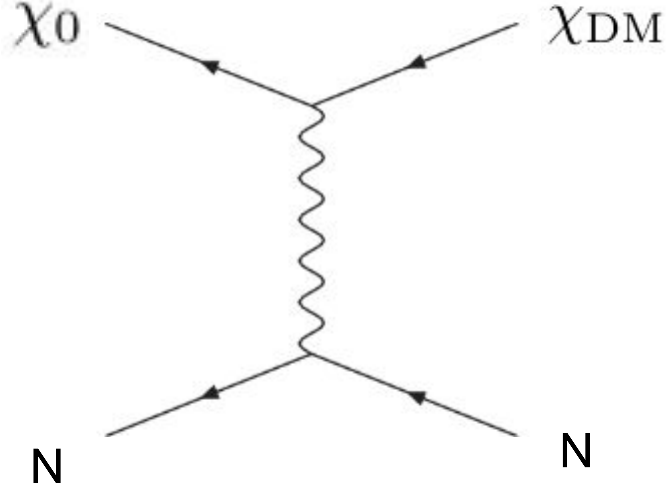
Complex WIMPs Y=0

DM spin	n_ϵ	$M_{\rm DM}$ (TeV)	$\Lambda_{\rm Landau}/M_{\rm DM}$	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$
Complex scalar	3	$1.60 \pm 0.01 - 2.4^*$	$> M_{\rm Pl}$	-
	5	11.3 ± 0.6	$> M_{\rm Pl}$	0.003
	7	47 ± 3	2×10^6	0.02
	9	118 ± 9	110	0.09
	11	217 ± 17	7	0.25
	13	352 ± 30	3	0.6
Dirac fermion	3	$2.0 \pm 0.1 - 2.4^*$	$> M_{\rm Pl}$	-
	5	9.1 ± 0.5	4×10^6	0.002
	7	45 ± 3	80	0.02
	9	115 ± 9	6	0.09
	11	211 ± 16	2.4	0.3
	13	340 ± 27	1.6	0.7

Complex WIMPs Y≠0

DM spin	n_Y	$M_{\rm DM}$ (TeV)	$\Lambda_{\rm Landau}/M_{\rm DM}$	$(\sigma v)^{J=0}_{\rm tot}/(\sigma v)^{J=0}_{\rm max}$	δm_0 [MeV]	$\Lambda^{\rm max}_{\rm UV}/M_{\rm DM}$	δm_{Q_M} [MeV]
Dirac fermion	$2_{1/2}$	1.08 ± 0.02	$> M_{\rm Pl}$	-	0.22 - 2×10^4	10^7	4.8 - 10^4
	3_1	2.85 ± 0.14	$> M_{\rm Pl}$	-	0.22 - 40	60	312 - 1.6×10^4
	$4_{1/2}$	4.8 ± 0.3	$\simeq M_{\rm Pl}$	0.001	0.21 - 3×10^4	5×10^6	20 - 1.9×10^4
	5_1	9.9 ± 0.7	3×10^6	0.003	0.21 - 3	25	$10^3 - 2 \times 10^3$
	$6_{1/2}$	31.8 ± 5.2	2×10^4	0.01	0.5 - 2×10^4	4×10^5	100 - 2×10^4
	$8_{1/2}$	82 ± 8	15	0.05	0.84 - 10^4	10^5	440 - 10^4
	$10_{1/2}$	158 ± 12	3	0.16	1.2 - 8×10^3	6×10^4	1.1×10^3 - 9×10^3
	$12_{1/2}$	253 ± 20	2	0.45	1.6 - 6×10^3	4×10^4	2.3×10^3 - 7×10^3
Complex scalar	$2_{1/2}$	0.58 ± 0.01	$> M_{\rm Pl}$	-	4.9 - 1.4×10^4	-	4.2 - 7×10^3
	3_1	2.1 ± 0.1	$> M_{\rm Pl}$	-	3.7 - 500	120	75 - 1.3×10^4
	$4_{1/2}$	4.98 ± 0.25	$> M_{\rm Pl}$	0.001	4.9 - 3×10^4	-	17 - 2×10^4
	5_1	11.5 ± 0.8	$> M_{\rm Pl}$	0.004	3.7 - 10	20	650 - 3×10^3
	$6_{1/2}$	32.7 ± 5.3	$\simeq 6 \times 10^{13}$	0.01	4.9 - 8×10^4	-	50 - 5×10^4
	$8_{1/2}$	84 ± 8	2×10^4	0.05	4.9 - 6×10^4	-	150 - 6×10^4
	$10_{1/2}$	162 ± 13	20	0.16	4.9 - 4×10^4	-	430 - 4×10^4
	$12_{1/2}$	263 ± 22	4	0.4	4.9 - 3×10^4	-	10^3 - 3×10^4

Inelastic DM



$$v_{\min}(E_R) = \frac{E_R + \delta m_0}{\sqrt{2 m_N E_R}}$$