The Tadpole Conjecture in the Asymptotic Limits of Moduli Space

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Based on [arXiv:2204.05331]
with M. Graña, T. Grimm, D. van de Heisteeg, E. Plauschinn

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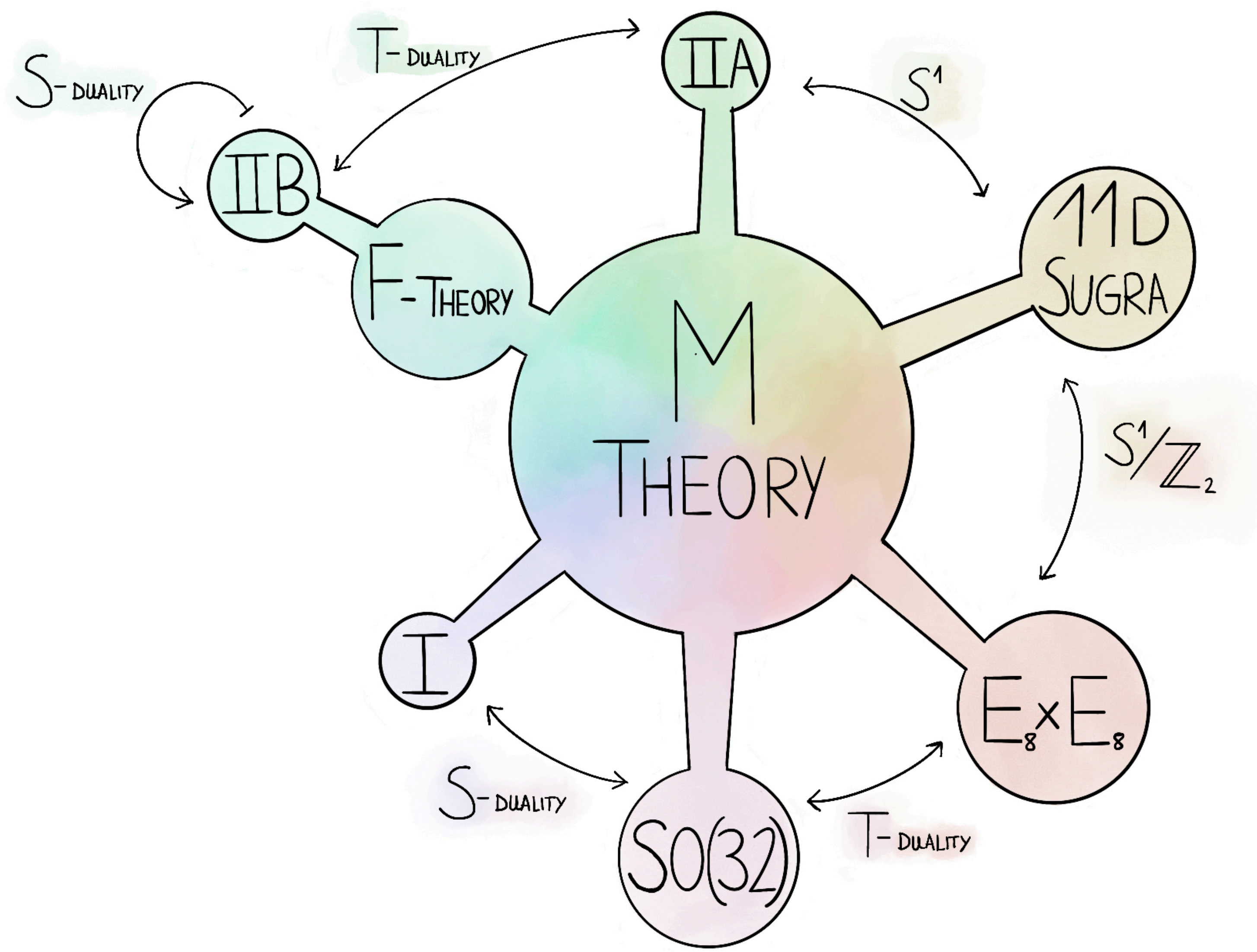
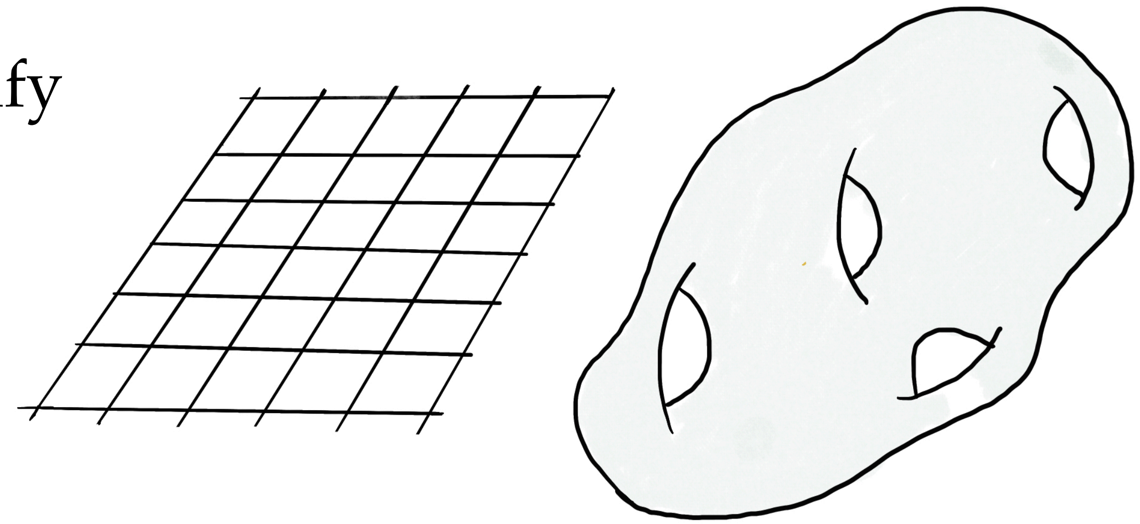
June 2, 2022

The Landscape of Supersymmetric String Theories



Compactifications and moduli stabilization

- To go to four dimensions we need to compactify



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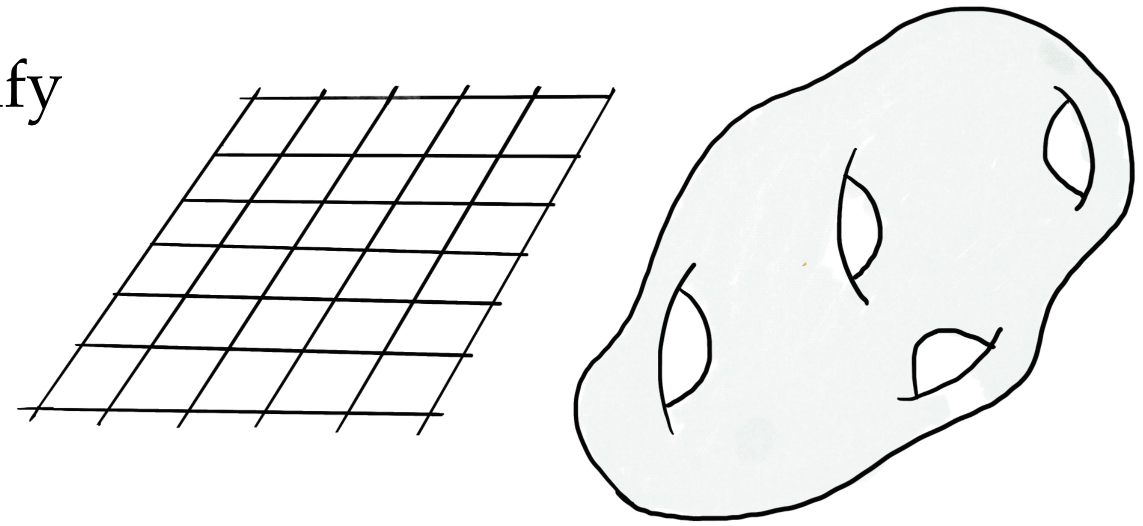

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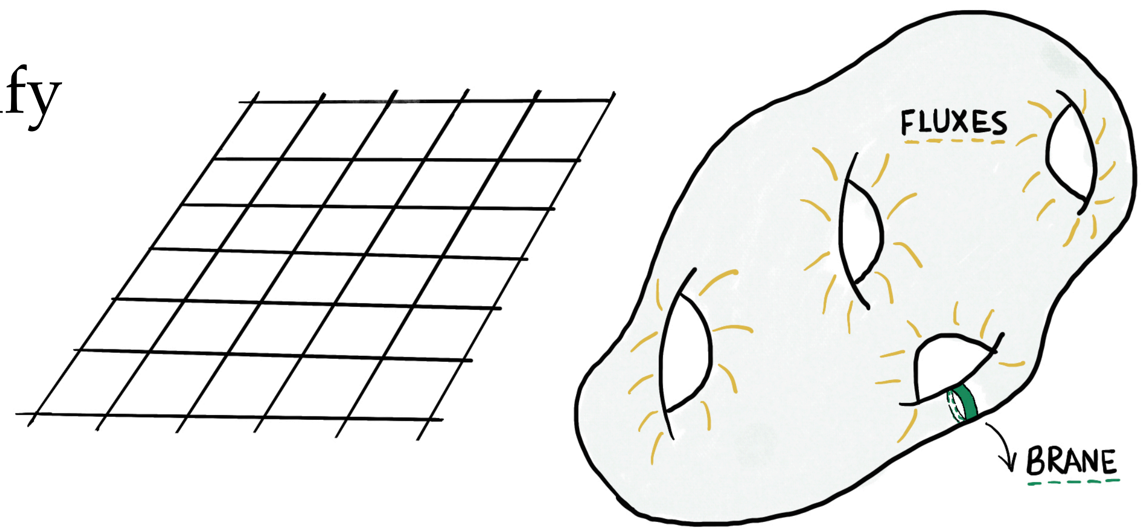

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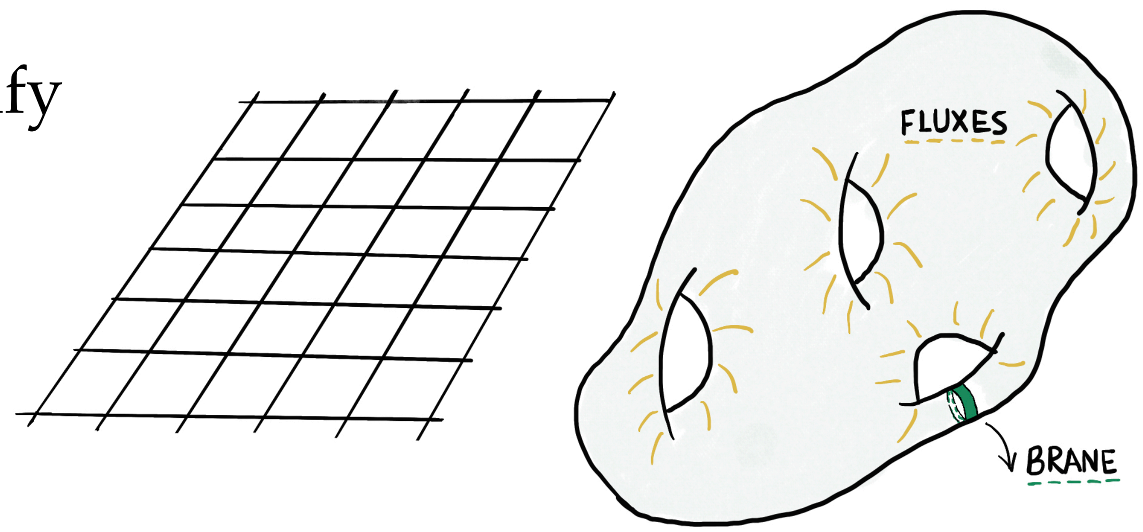

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- Bianchi identity / EoM integrated over compact space $\longrightarrow$ Tadpole cancellation condition

$$dF_p = \text{sources} \quad \longrightarrow \quad 0 = \int_{Y_n} \text{sources}$$

F-theory Compactifications

- Consider F-theroy on a Calabi-Yau fourfold with fluxes

Review: [Denef '08]
Effective action:
[Grimm '10]
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- Hodge decomposition and Hodge star: $H^4_{\text{prim}} = H^{4,0} \oplus H^{3,1} \oplus H^{2,2}_{\text{prim}} \oplus H^{1,3} \oplus H^{0,4}$

$$\star\, v^{p,q} = i^{p-q}\, v^{p,q}$$

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Tadpole Conjecture: The flux contribution to the tadpole needed to stabilize a large number of moduli grows as

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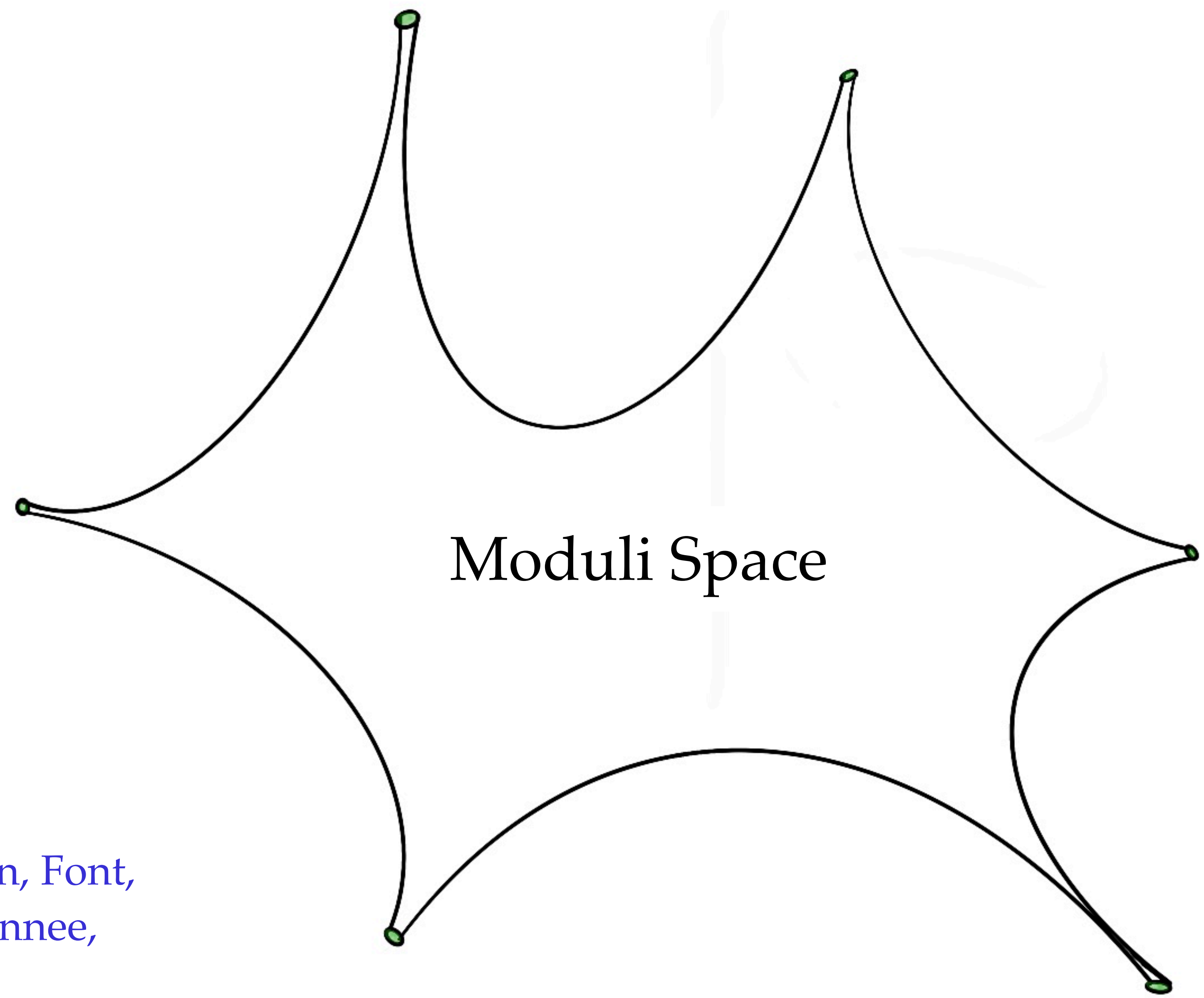
GOAL: Prove this in the strict asymptotic region of moduli space

Asymptotic Hodge Theory
-Asymptotic limits-

[Griffiths, Deligne, Schmid, Cattani, Kaplan…]



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  $t^i = \phi^i + is^i$
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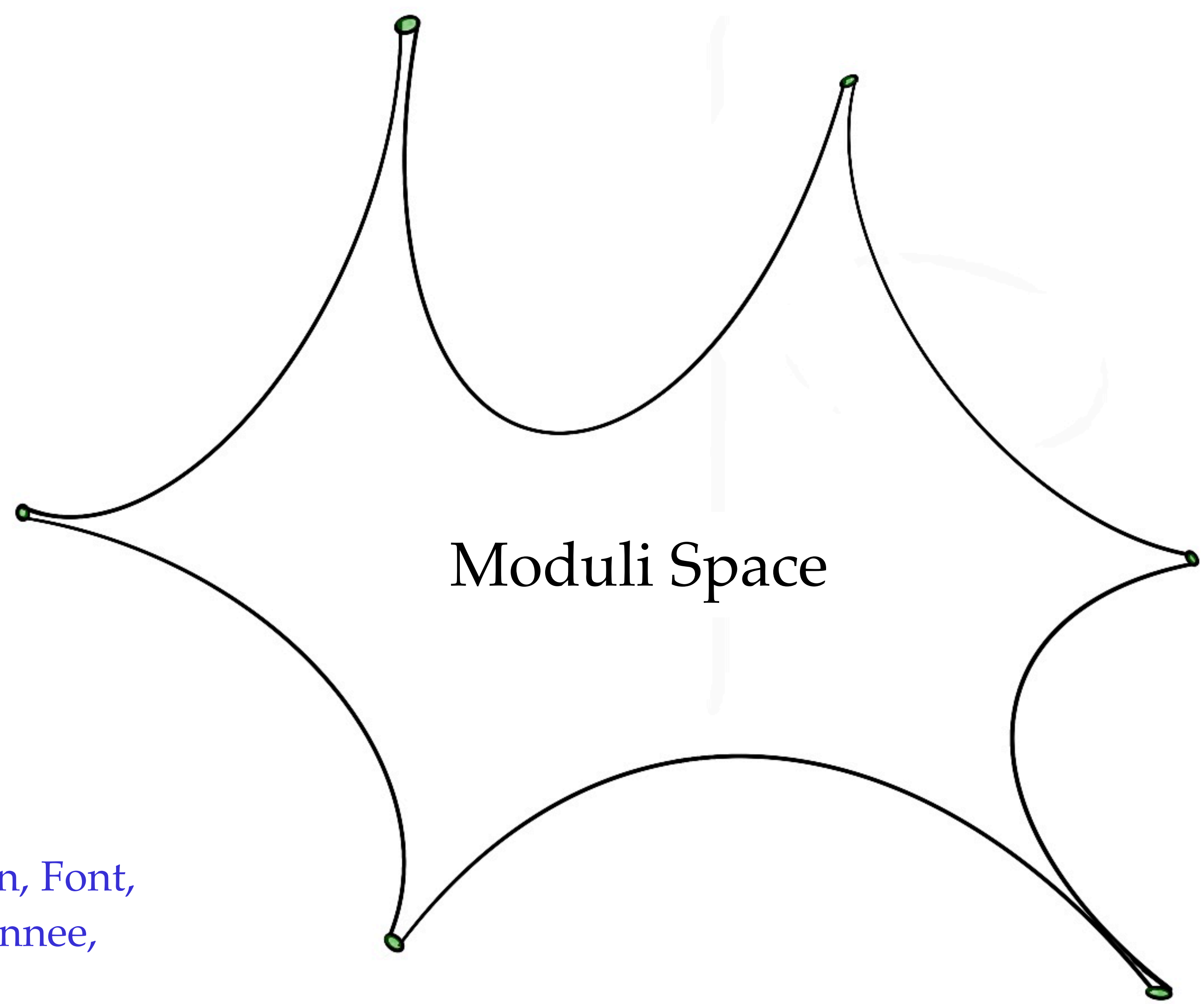

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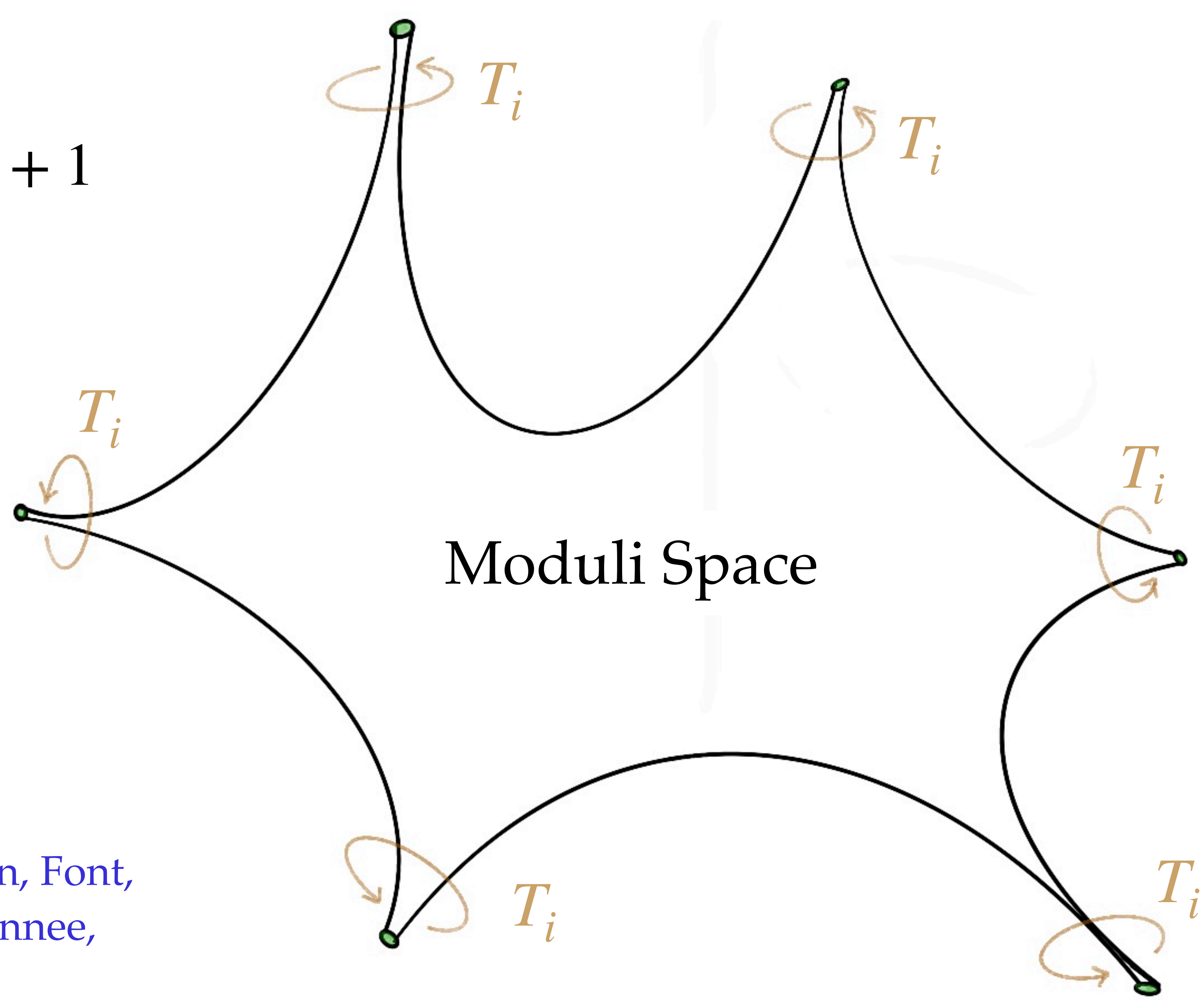
$T_i = e^{N_i}$



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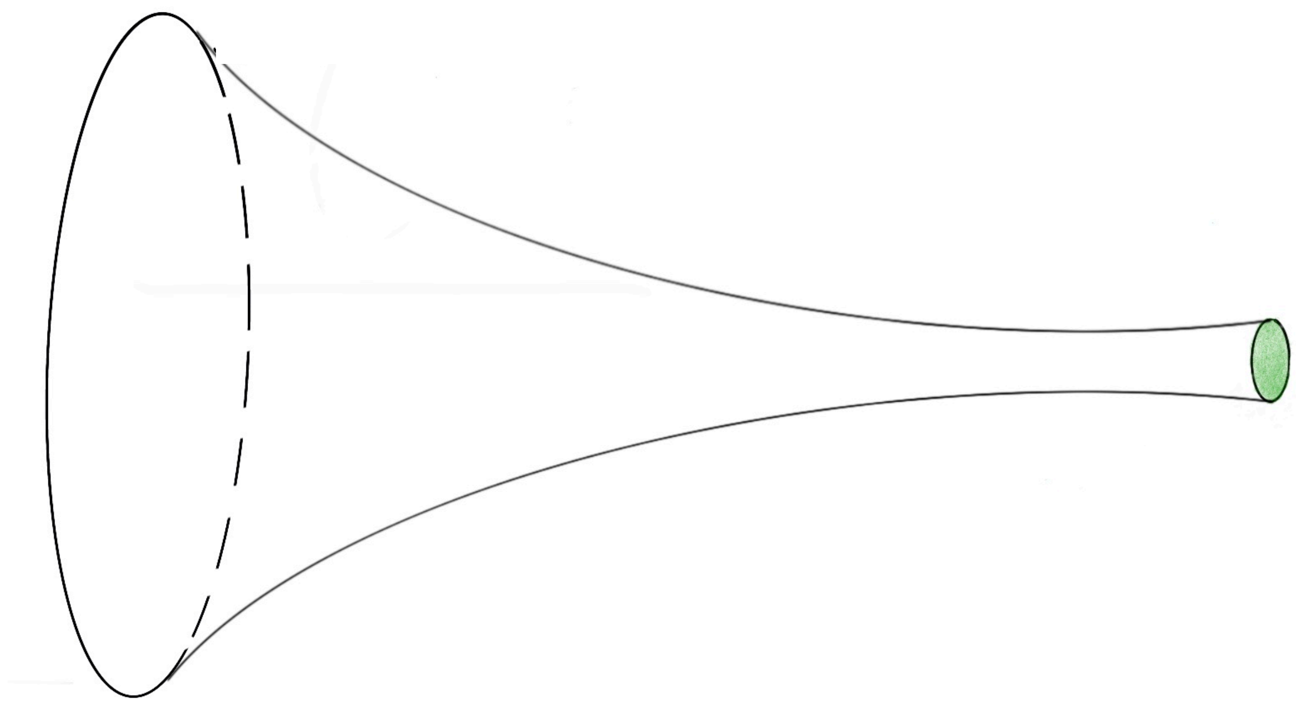

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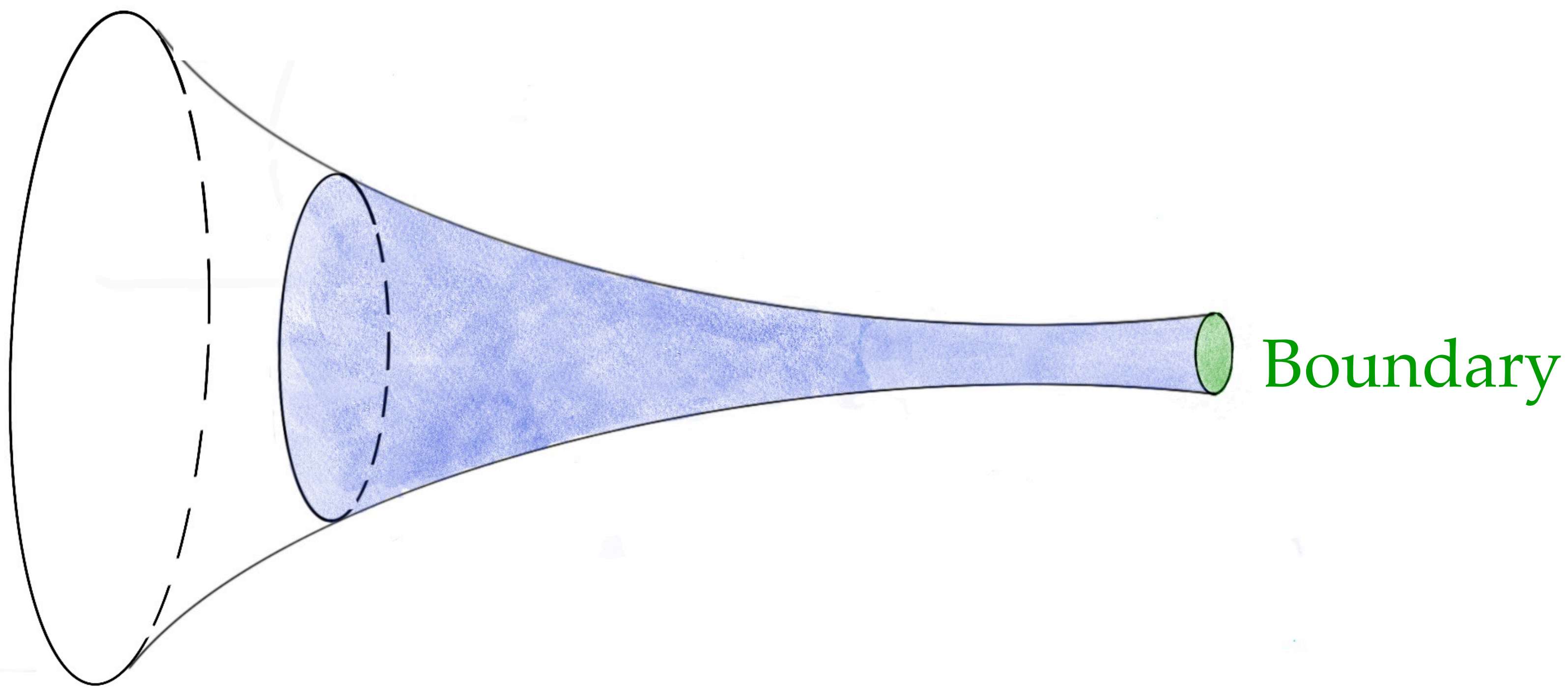

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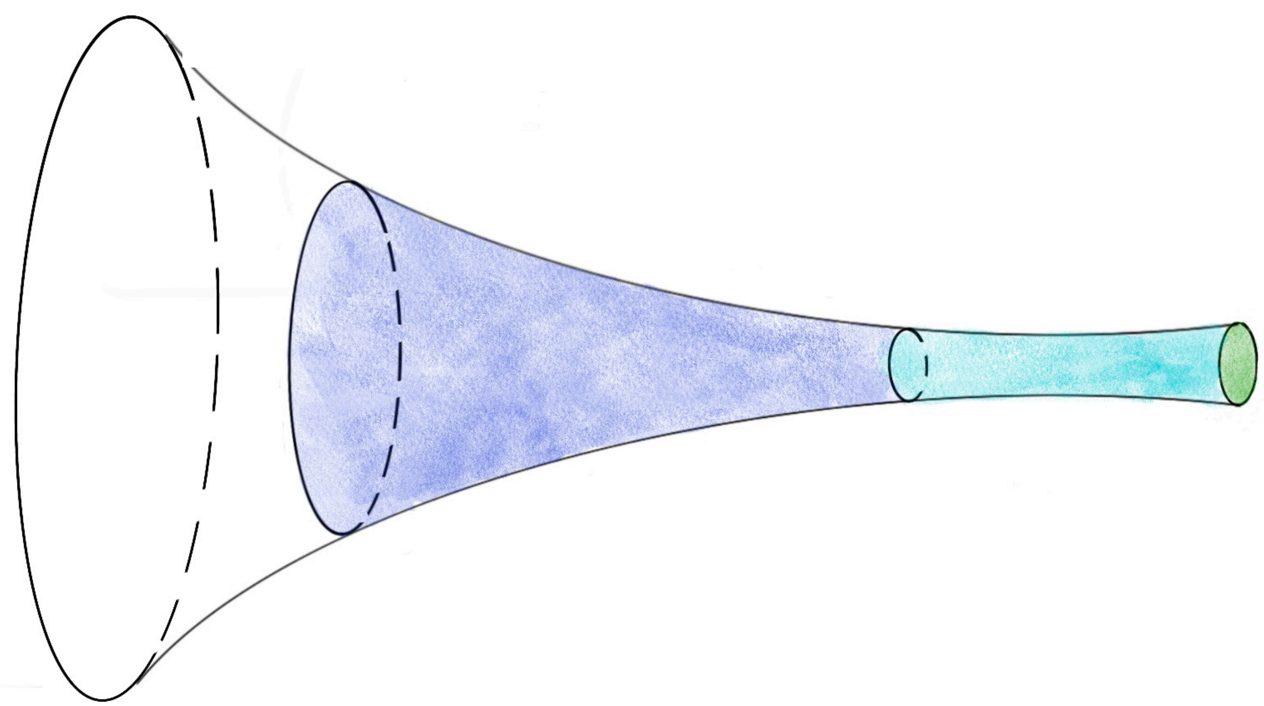

- **Strict** Asymptotic Region ⟶ Introduce an **ordering** (drop polynomial corrections)

$$\frac{s^1}{s^2} > \gamma, \quad \frac{s^2}{s^3} > \gamma, \quad \ldots, \quad \frac{s^{n-1}}{s^n} > \gamma, \quad s^n > \gamma \qquad \gamma \gg 1$$

Asymptotic Hodge Theory
-Boundary structures-

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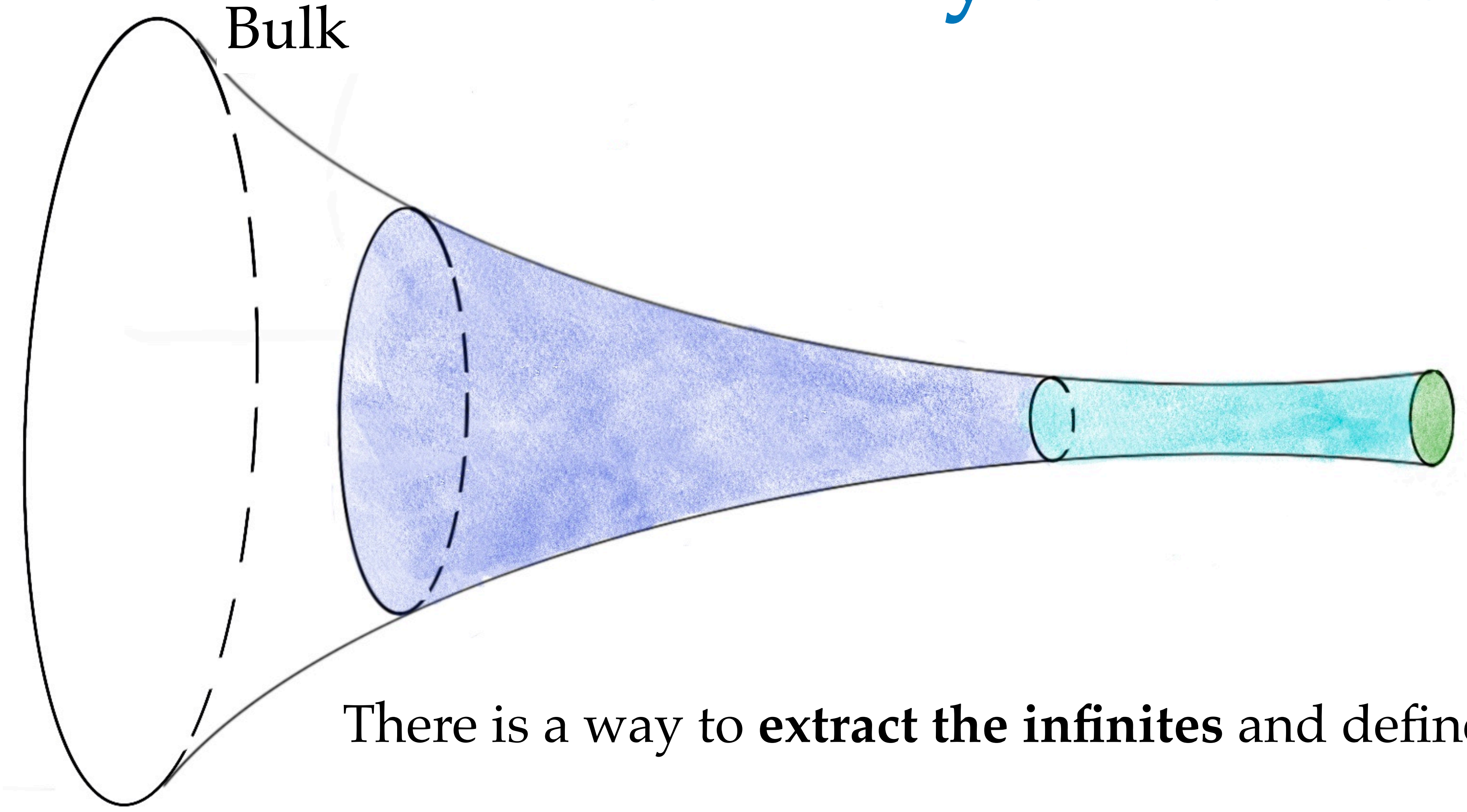

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There is a way to **extract the infinites** and define boundary structures

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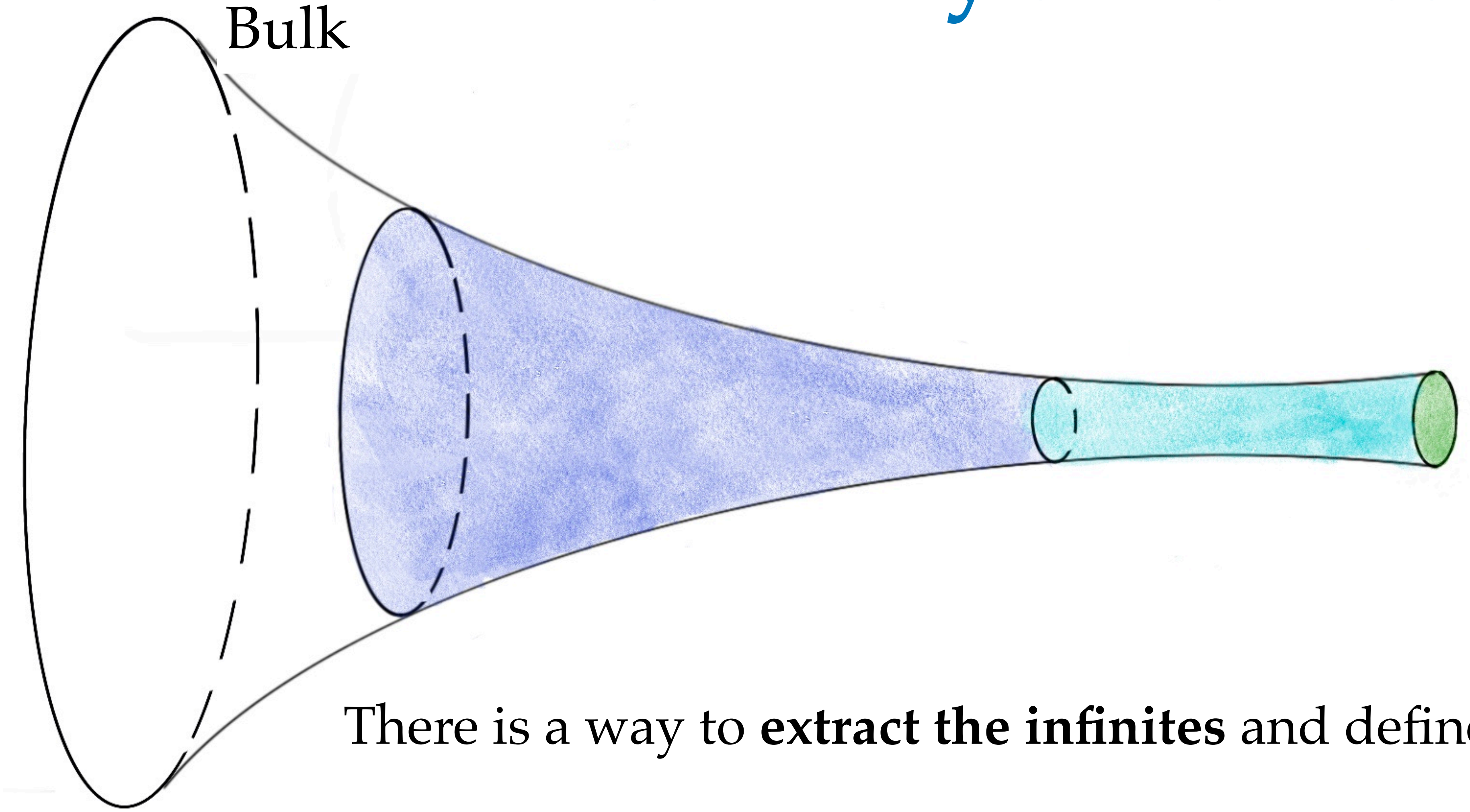

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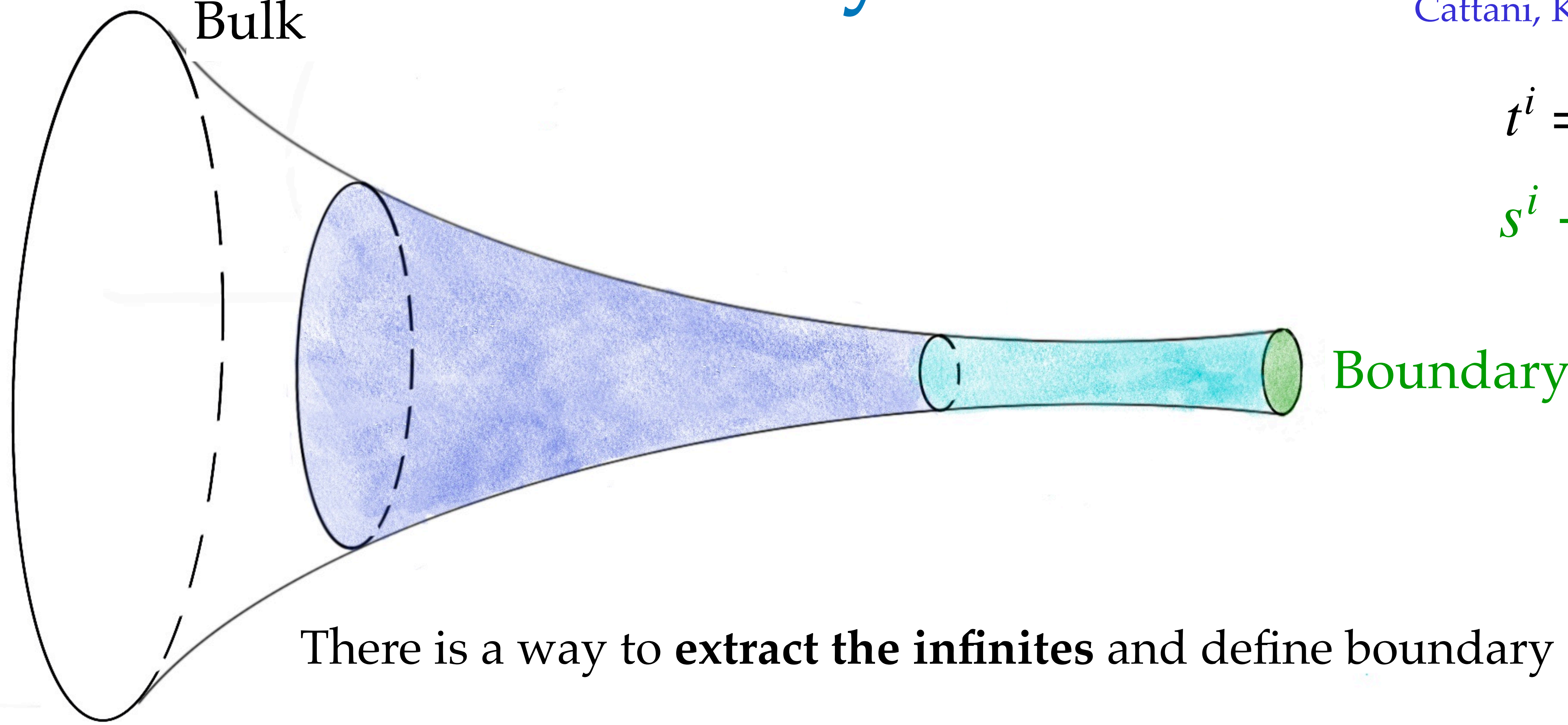

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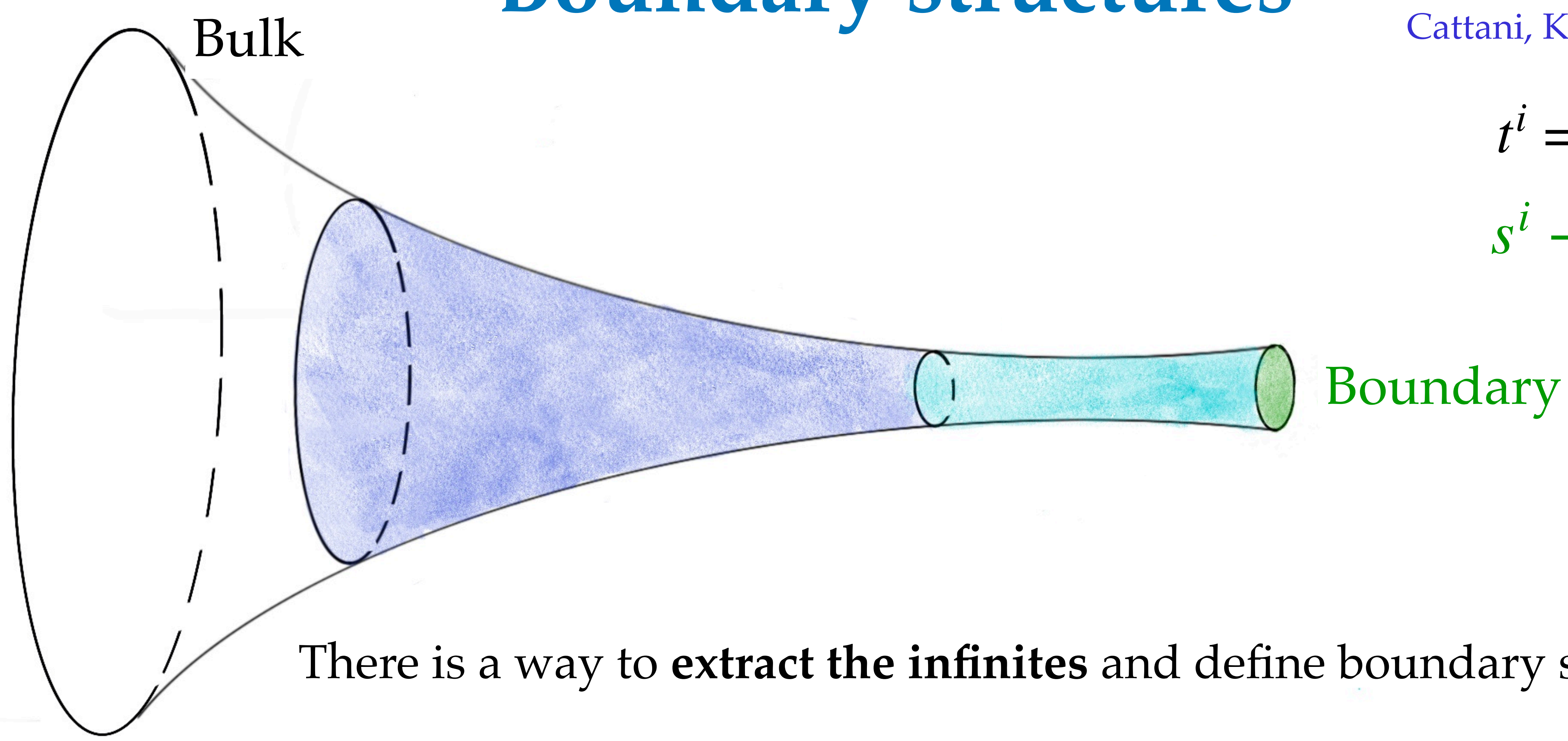

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- Decompose fluxes into sl(2) reps: $H^4_{\mathrm{prim}}\left(Y_4, \mathbb{R}\right) = \bigoplus V_\ell$ $\ell = (\ell_1, \ldots, \ell_n)$

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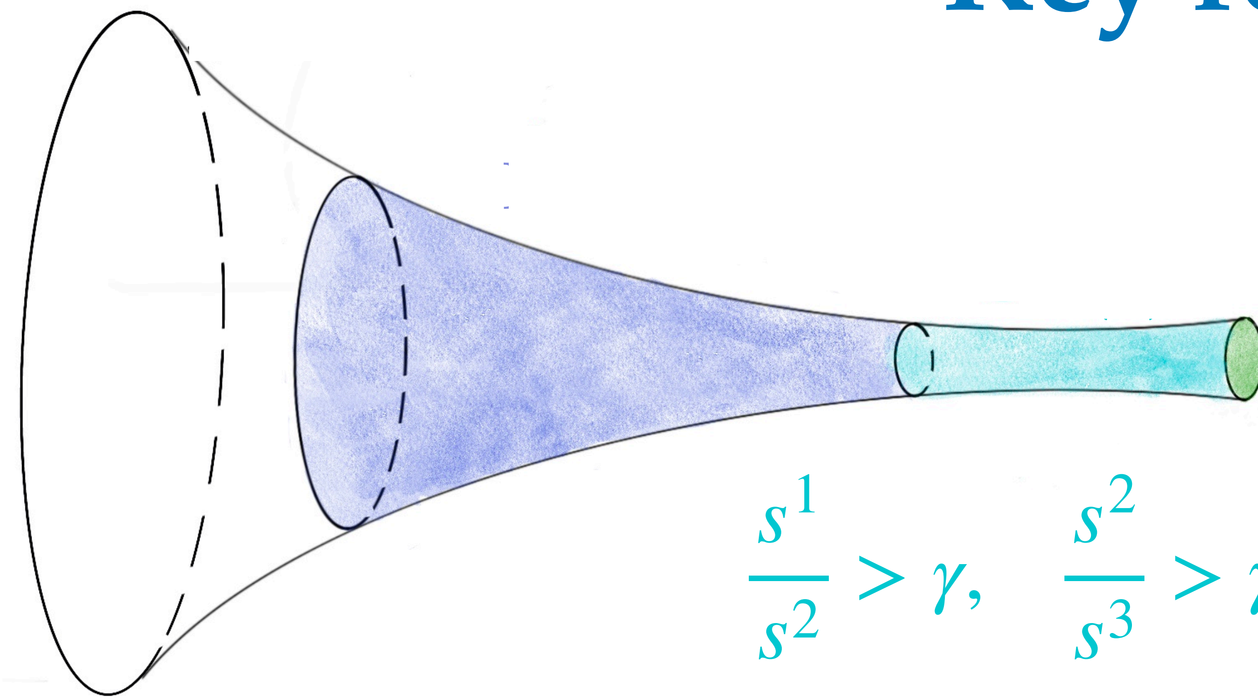

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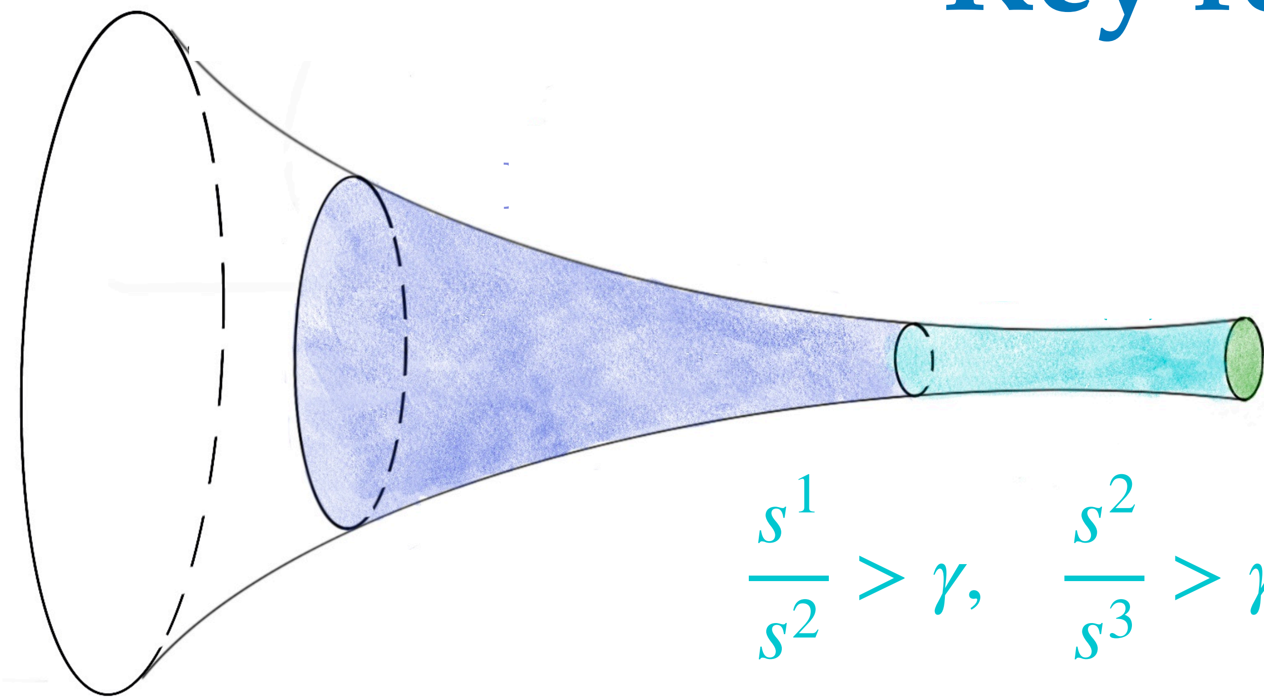

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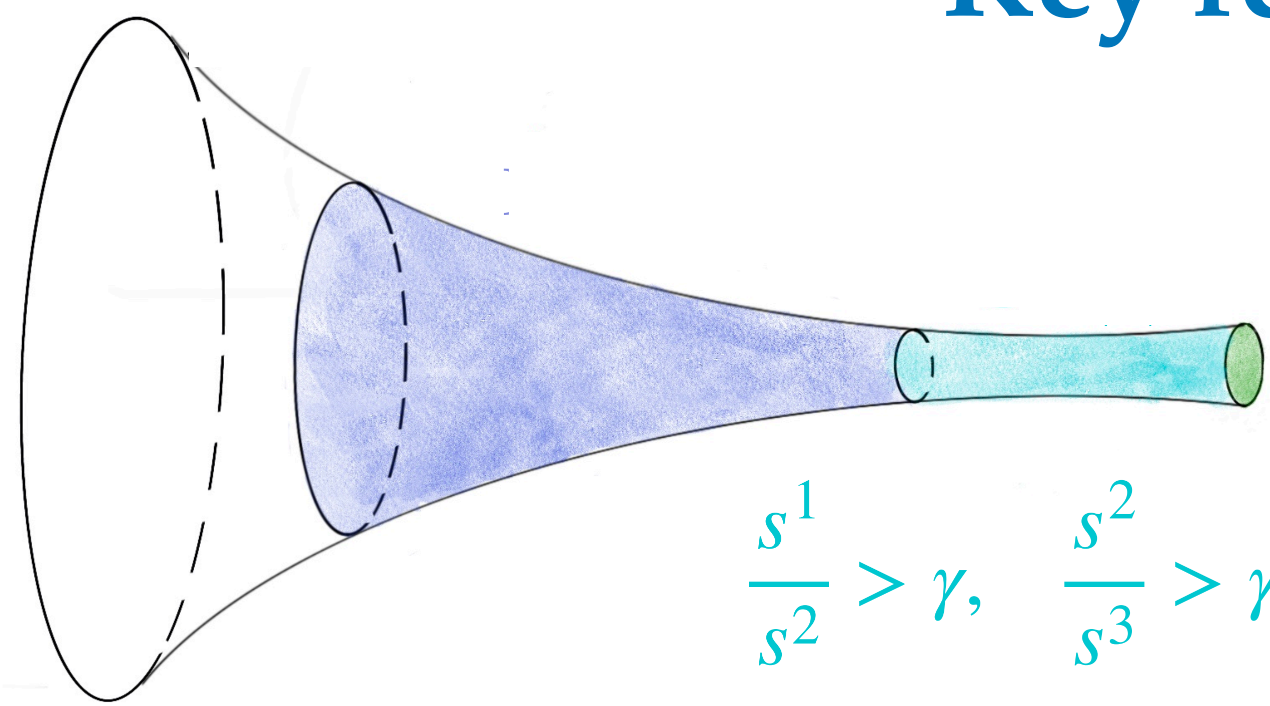

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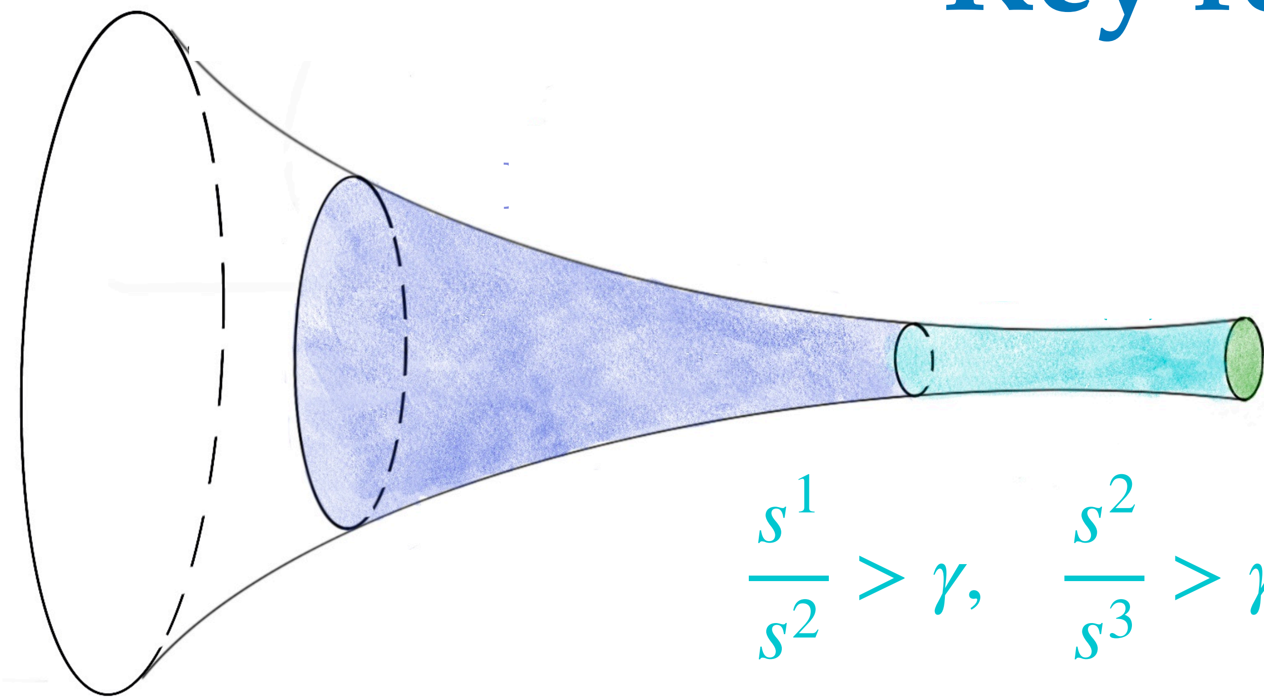

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- Bdry and sl(2) Hodge star: $\star \longrightarrow \star_{\mathrm{sl}(2)}$

$$\begin{cases} \star_\infty : V_\ell \to V_{-\ell} \\ ||v_\ell||^2_{\mathrm{sl}(2)} = \left(\frac{s^1}{s^2}\right)^{\ell_1} \left(\frac{s^2}{s^3}\right)^{\ell_2} \cdots \left(\frac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} (s^n)^{\ell_n} \; ||v_\ell||^2_\infty \end{cases}$$

Moduli stabilization: Generalities

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- Combining many sl(2) reps only imposes compatibility constraints between the fluxes, but never lowers the tadpole for a fixed modulus.
- Tadpole-wise, the most economic thing is to turn on only one sl(2) for each modulus.

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q labels sl(2)-representations

$\ell > 0$ labels positive weights

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At least linear scaling with (a large number of) stabilized moduli

Moduli stabilization and Tadpole -Key results-

- For **large number of stabilized moduli**, most of them (i.e. at least $(n-4)$ of them) have to be **fixed with fluxes from these representations**

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- Key **qualitative difference** between **small and large number of moduli**


شكراً جزيلاً
ngiyabonga
рахмат
danke
謝謝
Баярлалаа
spas
mersi
kia ora
barka
welalin
tack
teşekkür ederim
mahalo
tapadh leat
спасибо
faafetai lava
vinaka
спасибі
blagodaram
dank je
misaotra
paldies
grazzi
matondo
kiitos
dankie
nanni
nandri
dhanyavad
thank you
gracias
хвала
asante
manana
obrigada
tenki
bedankt
bayarlalaa
gràçie
hvala
mauruuru
köszönöm
dankon
aciū
akun
djiere dieuf
tau
mochchakkeram
murakoze
enkosi
dziękuję
дякую
mamnun
chokrane
chnorakaloutioun
gratias ago
gràcies
sulpáy
go raibh maith agat
sobodi
dēkuji
sagolun
sukriya
kop khun krap
taiku
arigatô
takk
dakujem
trugarez
obrigado
mèsi
didi madloba
najis tuke
ありがとう
tanemirt
grazie
rahmet
terima kasih
rahmat
kam sah hamnida
ευχαριστώ
diolch
dhanyavadagalu
shukriya
mercê
мерси
তোমাকে ধন্যবাদ
감사합니다
xièxie
merci

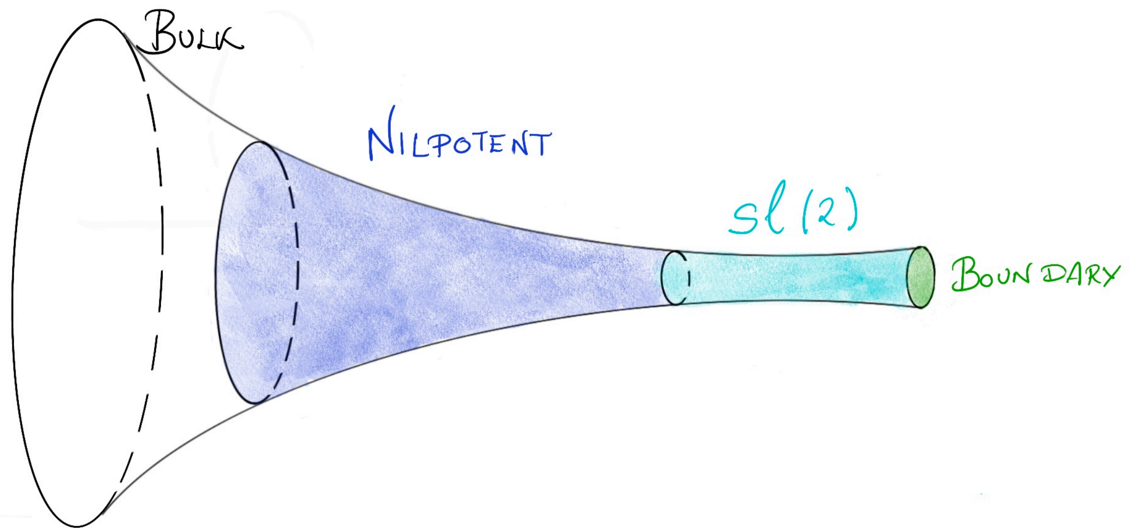
Backup slides

Charge quantization and extensions

- **Charge quantization** in the sl(2)-basis (over $\mathbb{Q}$ instead of over $\mathbb{Z}$) ⟶

Exclude scaling of $||G_\ell||^2_\infty \sim \frac{1}{n\,\gamma^{\sum_i \ell_i}}$ ⟶ Found numerical evidence

$$Q \geq \sum_q \gamma^{\sum \ell'_i} ||G_{q,\ell'}||^2_\infty$$

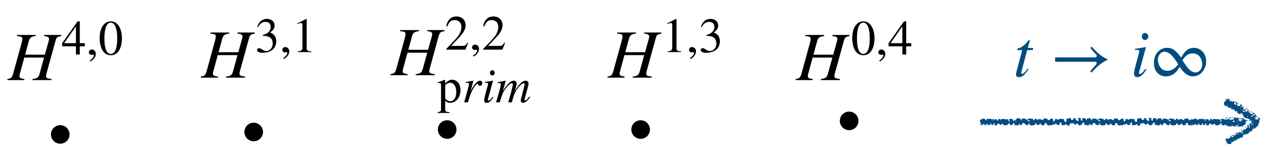


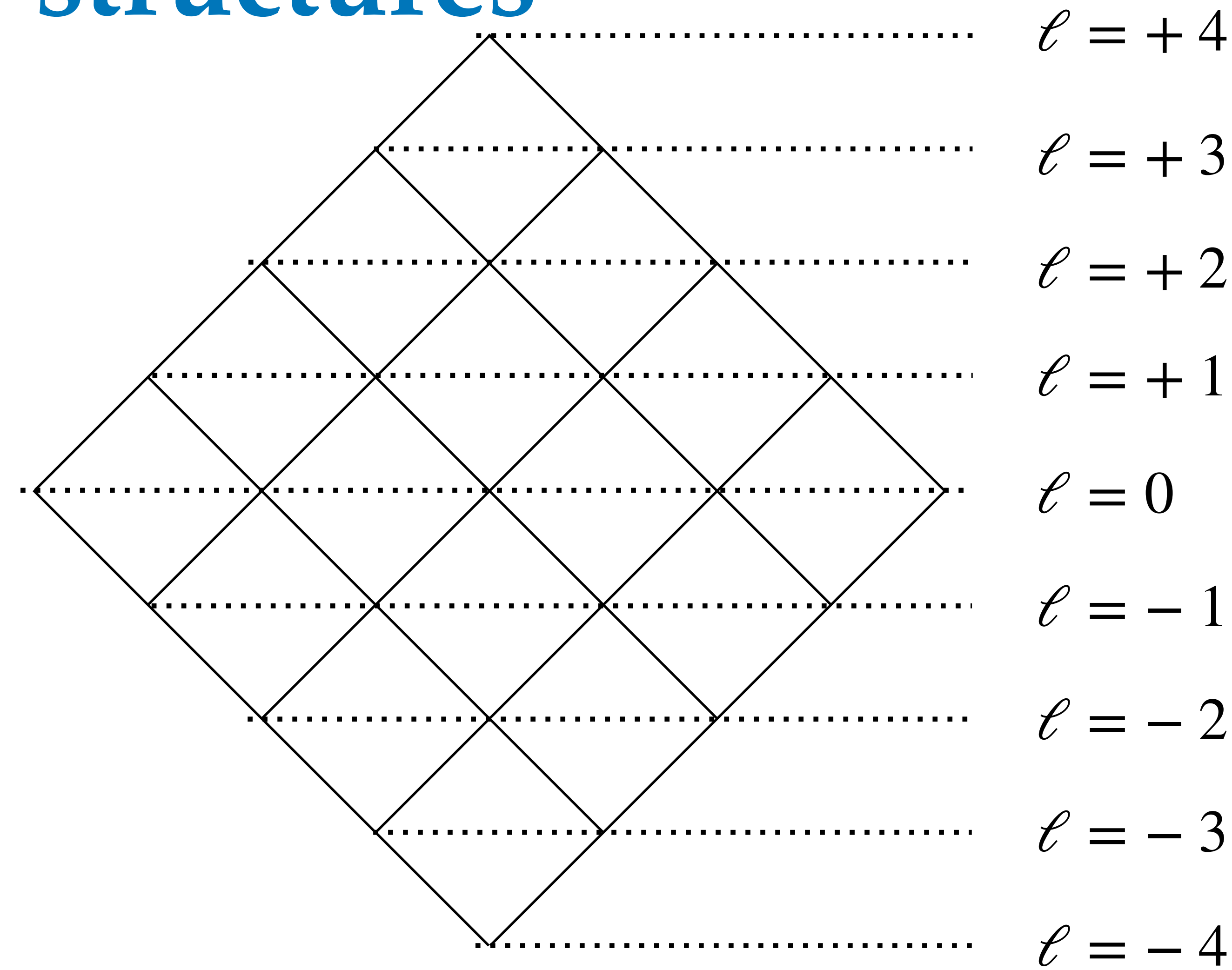
- Extend to the **interior of moduli space**

1) Include polynomial corrections (~~strict~~ asymptotic limit) ⟶ "Linear scenario"
[Grimm '20]
[Marchesano, Prieto, Wiesner '21]
[Palti, Tasinato, Ward '08]
See also: [Plauschinn '22] [Lüst '22]

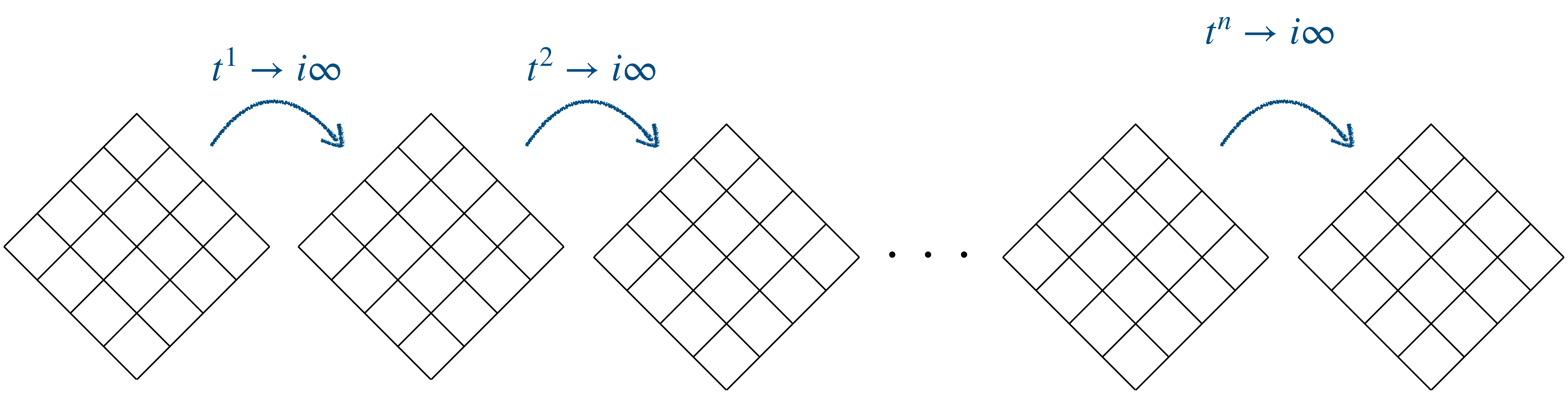
3) Interior of moduli space ⟶ Hodge loci of $G_4^{2,2}$ fluxes is algebraic
[Bakker, Grimm, Schnell, Tsimerman '21] [Cattani, Deligne, Kaplan '95]

Asymptotic Hodge Theory
-Boundary structures-



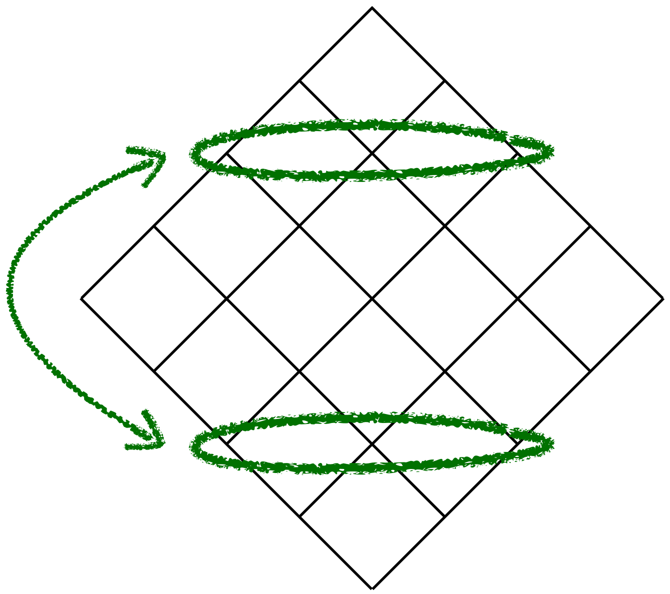


$$H^4_{\text{prim}}(Y_4, \mathbb{R}) = \bigoplus V_\ell$$



Asymptotic Hodge Theory
-Hodge star close to the boundary-

- The boundary Hodge decomposition naturally includes a boundary Hodge star operator:



$$\star_\infty : V_\ell \;\rightarrow\; V_{-\ell}$$

$$\langle v_\ell, v_{\ell'} \rangle = 0 \qquad \text{for} \;\; \ell \neq -\ell'$$

$$\langle v_\ell, \star_\infty v_{\ell'} \rangle = 0 \qquad \text{for} \;\; \ell \neq \ell'$$

- Allows to express the Hodge star in the strict asymptotic limit: $\star \longrightarrow \star_{\mathrm{sl}(2)}$

Vanishing axions: $\star_{\mathrm{sl}(2)}\, v_\ell = \left(\frac{s^1}{s^2}\right)^{\ell_1} \left(\frac{s^2}{s^3}\right)^{\ell_2} \cdots \left(\frac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} \left(s^n\right)^{\ell_n} \star_\infty v_l$

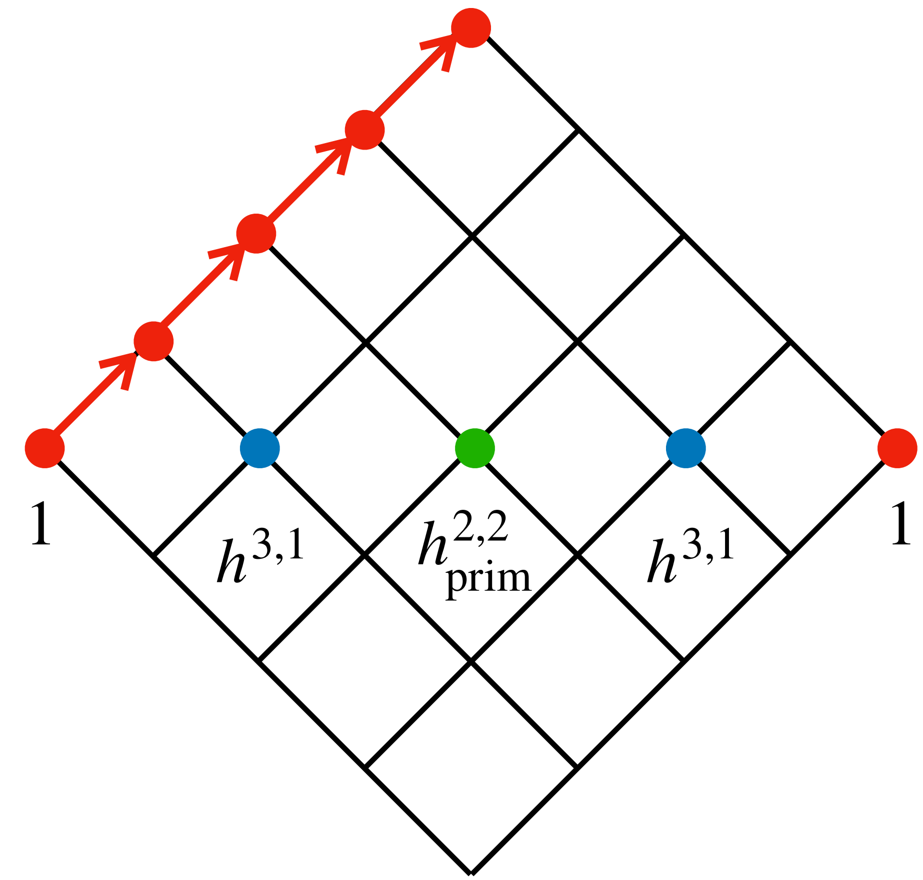
Non-vanishing axions ⟶ Mixing with lower subspaces via $e^{\phi^i N_i^-}\; v_l$

Asymptotic Hodge Theory
-Highest weight spaces-

- Elements in $H^4_{\text{prim}}\left(Y_4,\mathbb{R}\right)$ arrange into irreps of the n commuting sl(2) $\longrightarrow$ Generated by applying N_i^- to **highest weight states**

- What are the possible highest weight states?

Only **ONE**, corresponding to the **(4,0)-form**, can move along the exterior line of the diagram



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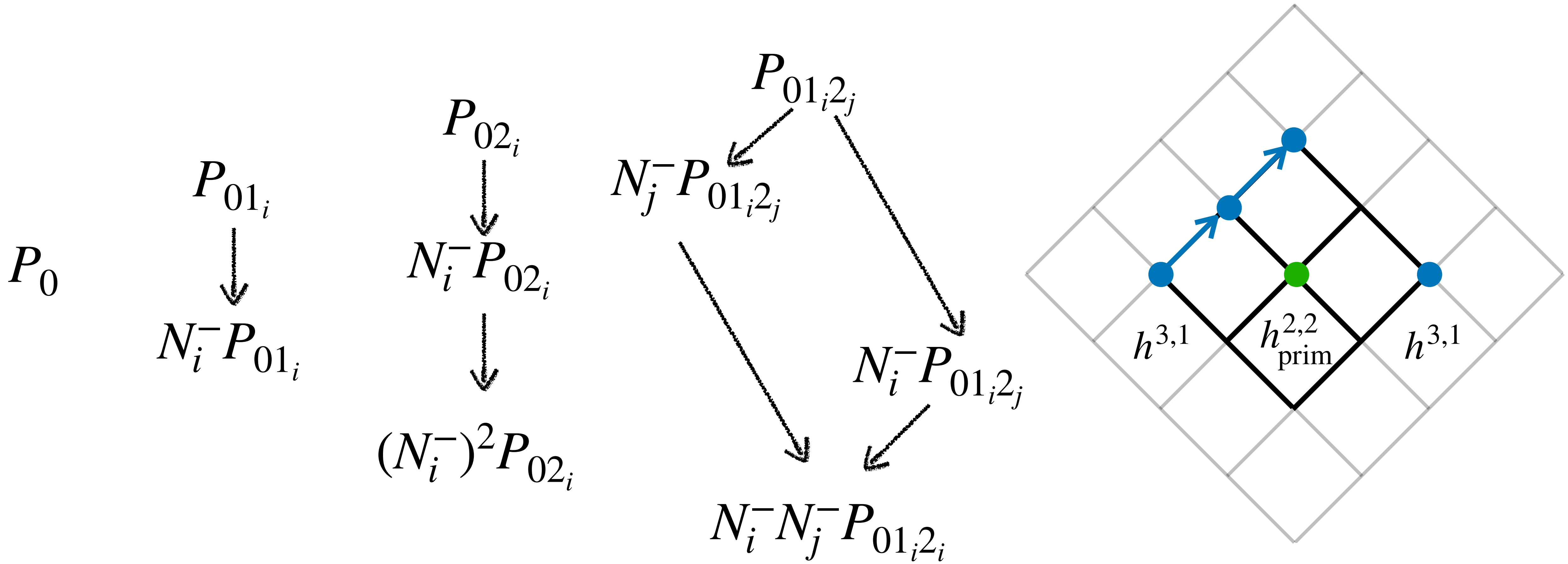


~ $h^{3,1}$ copies of K3 surfaces

Moduli stabilization at work

$$\hat{G}_{-\ell} = \left(\frac{s^1}{s^2}\right)^{\ell_1} \cdots \left(\frac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} \left(s^n\right)^{\ell_n} \star_\infty \hat{G}_{+\ell}$$

$$\hat{G}_4 = e^{\phi^i N_i^-} G_4$$

- Example: Flux from the sl(2) rep generated by P_{02_i}

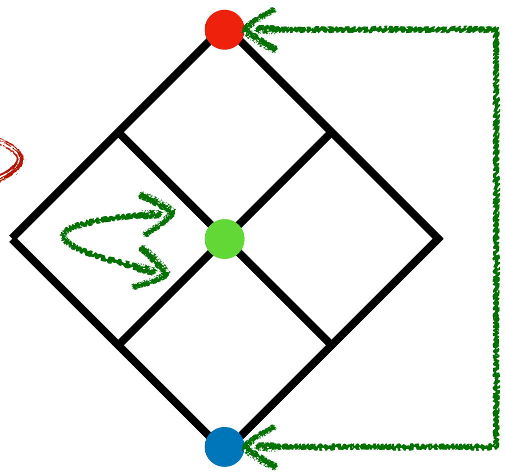
$$G_4 = G_{02}\, v_{02} + G_0\, v_0 + G_{0-2} v_{0-2} \longrightarrow Q = \frac{1}{2}\langle G_4, G_4\rangle = G_{02}G_{0-2} - \frac{1}{2}G_0^2$$

$$\hat{G}_4 = G_{02}\, v_{02} + \left(G_0 - \phi\, G_{02}\right) v_0 + \left(G_{0-2} - \phi\, G_0 + \frac{1}{2}\left(\phi\right)^2 G_{02_i}\right) v_{0-2}$$

- Self-duality conditions:

$$\frac{G_{02}}{2}(s)^2 = G_{0-2} - \phi G_0 + \frac{1}{2}\left(\phi\right)^2 G_{02} \longrightarrow s = \frac{\sqrt{2G_{0-2}G_{02} - G_0^2}}{G_{02}}$$

$$\left(G_0 - \phi G_{02}\right) = -\left(G_0 - \phi G_{02}\right) \longrightarrow \phi = \frac{G_0}{G_{02}}$$



sl(2) Hodge star

- Extend boundary Hodge star to the interior: $\star_\infty \longrightarrow \star_{\mathrm{sl}(2)}$

$$\star_{\mathrm{sl}(2)} = e^{+\phi^i N_i^-}\left[e^{-\frac{1}{2}\log\left(s^i\right)N_i^0}\star_\infty\, e^{+\frac{1}{2}\log\left(s^i\right)N_i^0}\right]e^{-\phi^i N_i^-}$$

$$H^4_{\mathrm{prim}}\left(Y_4,\mathbb{C}\right) = H^{4,0}_{\mathrm{sl}(2)} \oplus H^{3,1}_{\mathrm{sl}(2)} \oplus H^{2,2}_{\mathrm{sl}(2)} \oplus H^{1,3}_{\mathrm{sl}(2)} \oplus H^{0,4}_{\mathrm{sl}(2)}$$

$$H^{p,q}_{\mathrm{sl}(2)} = e^{\phi^i N_i^-} e^{-\frac{1}{2}\log\left(s^i\right)N_i^0} H^{p,q}_\infty$$