

# PLANCK 2022

Paris, May 30 - June 3, 2022

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## The on-shell way to BSM

**Alex Pomarol, IFAE & UAB (Barcelona)  
and CERN**

# Outline

- **Some motivations for on-shell amplitude methods**
- **EFT (Effective Theories) from amplitudes, instead of Lagrangians**
- **Renormalization of EFT using on-shell methods:**
  - Loops from tree-level amplitudes***
  - ☞ **Simple, elegant, and efficient**

# Outline

- **Some motivations for on-shell amplitude methods**
- **EFT (Effective Theories) from amplitudes, instead of Lagrangians**
- **Renormalization of EFT using on-shell methods:**

***Loops from tree-level amplitudes***

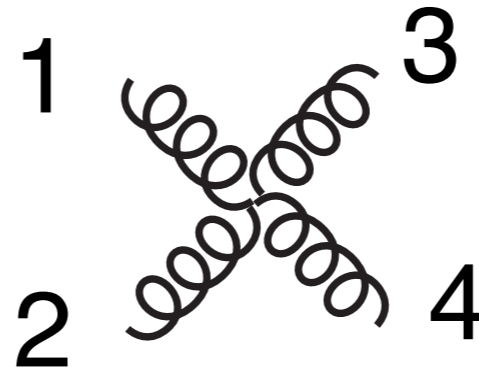
☞ **Simple, elegant, and efficient**

*Makes explicit a lot of information  
that one cannot see from the Feynman approach!*



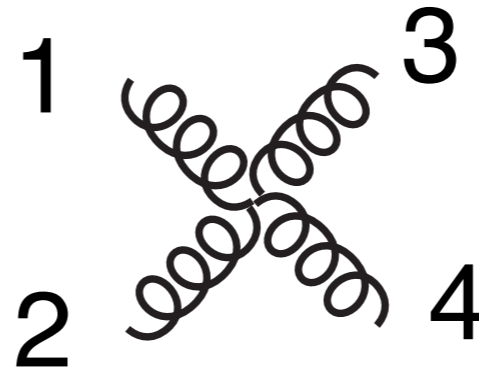
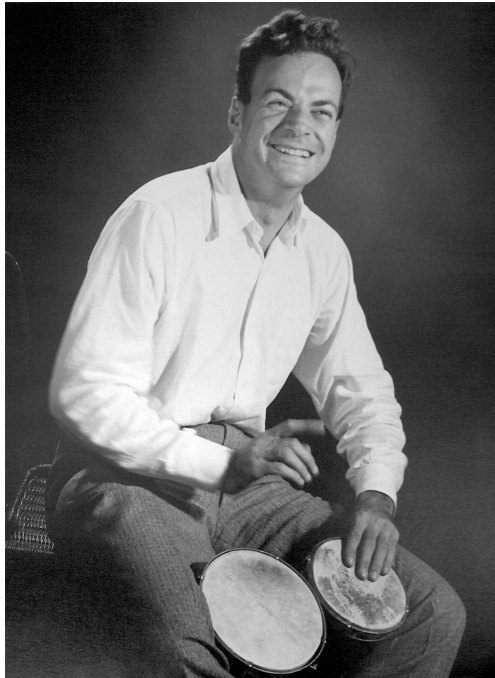
# **I. Some motivation**

graviton + graviton  $\rightarrow$  graviton + graviton



*à la Feynman !*

graviton + graviton  $\rightarrow$  graviton + graviton



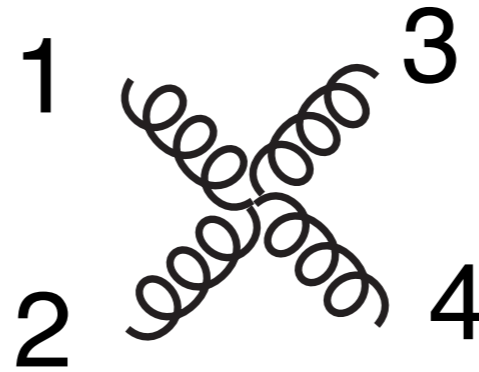
I) Define your fields ( $h_{\mu\nu}$ )

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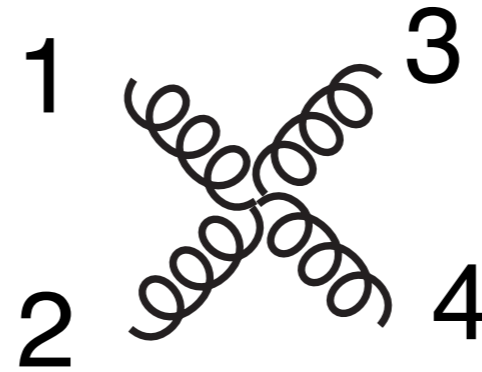
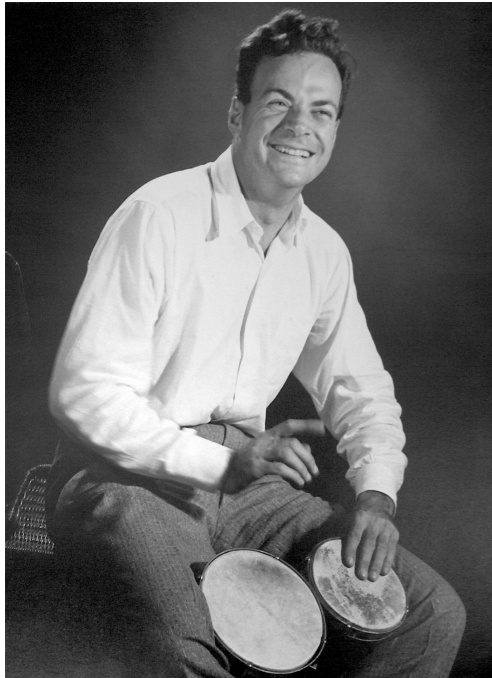


*à la* Feynman !



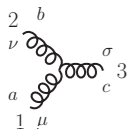
- 1) Define your fields ( $h_{\mu\nu}$ )
- 2) Get the Lagrangian (GR)

# graviton + graviton $\rightarrow$ graviton + graviton



*à la* Feynman !

- 1) Define your fields ( $h_{\mu\nu}$ )
- 2) Get the Lagrangian (GR)
- 3) Get the Feynman rules



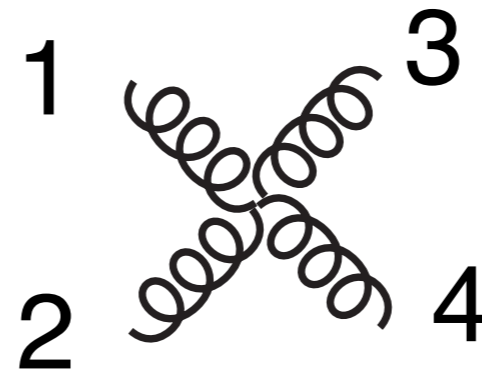
$$i\text{Sym} \left[ -\frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\lambda} \eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu} p_{1\lambda} \eta_{\mu\rho} \eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\nu} \eta_{\rho\lambda} \eta_{\sigma\tau}) \right. \\ \left. + P_6(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\sigma} \eta_{\lambda\tau}) + 2P_3(p_{1\nu} p_{1\tau} \eta_{\mu\rho} \eta_{\lambda\sigma}) - P_3(p_{1\lambda} p_{2\mu} \eta_{\rho\nu} \eta_{\sigma\tau}) \right. \\ \left. + P_3(p_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{2\tau} \eta_{\lambda\mu} \eta_{\rho\sigma}) \right. \\ \left. + 2P_3(p_{1\nu} p_{2\mu} \eta_{\lambda\sigma} \eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2 \eta_{\rho\nu} \eta_{\lambda\sigma} \eta_{\tau\mu}) \right],$$



$$\text{Sym} \left[ -\frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. + \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) + \frac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. + P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. - P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + 2P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right]$$

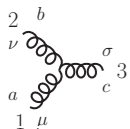


# graviton + graviton $\rightarrow$ graviton + graviton



*à la* Feynman !

- 1) Define your fields ( $h_{\mu\nu}$ )
- 2) Get the Lagrangian (GR)
- 3) Get the Feynman rules
- 4) Obtain the amplitude

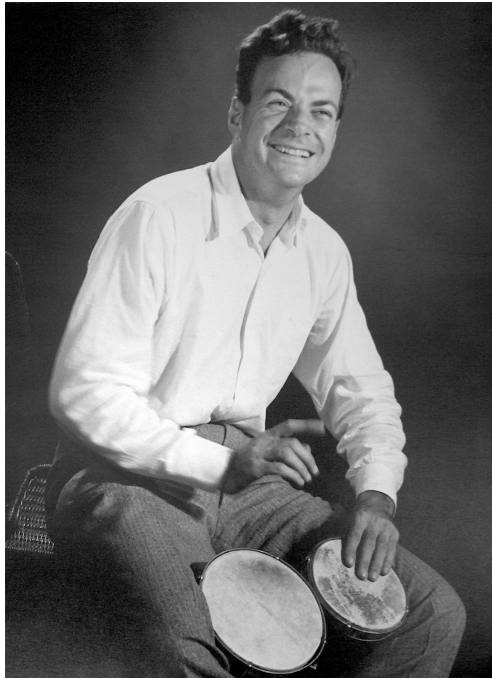


$$i\text{Sym} \left[ -\frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\lambda} \eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu} p_{1\lambda} \eta_{\mu\rho} \eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\nu} \eta_{\rho\lambda} \eta_{\sigma\tau}) \right. \\ \left. + P_6(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\sigma} \eta_{\lambda\tau}) + 2P_3(p_{1\nu} p_{1\tau} \eta_{\mu\rho} \eta_{\lambda\sigma}) - P_3(p_{1\lambda} p_{2\mu} \eta_{\rho\nu} \eta_{\sigma\tau}) \right. \\ \left. + P_3(p_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{2\tau} \eta_{\lambda\mu} \eta_{\rho\sigma}) \right. \\ \left. + 2P_3(p_{1\nu} p_{2\mu} \eta_{\lambda\sigma} \eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2 \eta_{\rho\nu} \eta_{\lambda\sigma} \eta_{\tau\mu}) \right],$$

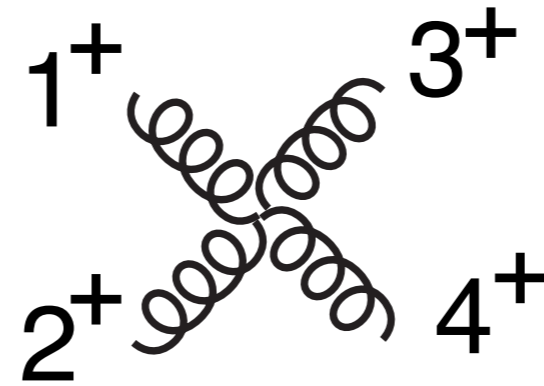


$$\text{Sym} \left[ -\frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - \frac{1}{2}P_{12}(p^\sigma p'^\tau \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\rho\delta}) - \frac{1}{2}P_6(p^\sigma p'^\mu \eta^{\nu\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) \right. \\ \left. + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_{12}(p^\sigma p'^\tau \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_6(p^\sigma p'^\mu \eta^{\nu\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) \right. \\ \left. + \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_{24}(p^\sigma p'^\tau \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_{12}(p^\sigma p'^\lambda \eta^{\mu\nu} \eta^{\tau\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\tau\delta}) \right. \\ \left. - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - \frac{1}{2}P_{12}(p^\sigma p'^\mu \eta^{\nu\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) + \frac{1}{2}P_{12}(p^\sigma p'^\tau \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\rho\delta}) - \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) \right. \\ \left. - P_{12}(p^\sigma p'^\tau \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\rho\delta}) - P_{12}(p^\sigma p'^\lambda \eta^{\mu\nu} \eta^{\tau\kappa} \eta^{\rho\delta}) - P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - P_{12}(p^\sigma p'^\lambda \eta^{\mu\nu} \eta^{\tau\kappa} \eta^{\rho\delta}) \right. \\ \left. + P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\tau\delta}) - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\tau\delta}) \right. \\ \left. - P_6(p^\sigma p'^\lambda \eta^{\mu\nu} \eta^{\tau\kappa} \eta^{\rho\delta}) - P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\kappa} \eta^{\tau\delta}) - P_{12}(p^\sigma p'^\mu \eta^{\nu\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) + 2P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\lambda\kappa} \eta^{\rho\delta}) \right]$$

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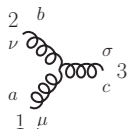


*à la* Feynman !



$$\pm \text{ (hand pointing right) } h = \pm 2$$

- 1) Define your fields ( $h_{\mu\nu}$ )
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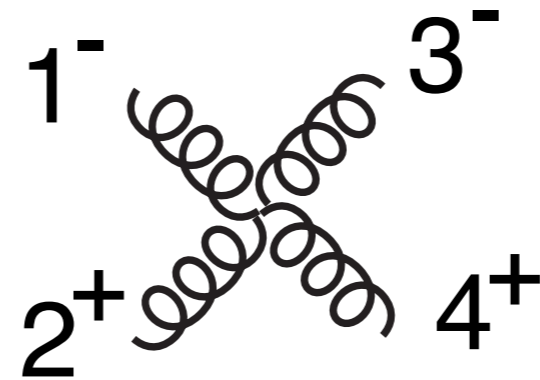
$$i\text{Sym} \left[ -\frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\lambda} \eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu} p_{1\lambda} \eta_{\mu\rho} \eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\nu} \eta_{\rho\lambda} \eta_{\sigma\tau}) \right. \\ \left. + P_6(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\sigma} \eta_{\lambda\tau}) + 2P_3(p_{1\nu} p_{1\tau} \eta_{\mu\rho} \eta_{\lambda\sigma}) - P_3(p_{1\lambda} p_{2\mu} \eta_{\rho\nu} \eta_{\sigma\tau}) \right. \\ \left. + P_3(p_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{2\tau} \eta_{\lambda\mu} \eta_{\rho\sigma}) \right. \\ \left. + 2P_3(p_{1\nu} p_{2\mu} \eta_{\lambda\sigma} \eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2 \eta_{\rho\nu} \eta_{\lambda\sigma} \eta_{\tau\mu}) \right],$$



$$\text{Sym} \left[ -\frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. + \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. + \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) + \frac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + \frac{1}{2}P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right. \\ \left. - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. + P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - \frac{1}{2}P_{12}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) \right. \\ \left. - P_6(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{24}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) - P_{12}(p^\sigma p'^\rho \eta^{\mu\nu} \eta^{\lambda\eta}) + 2P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\lambda\eta}) \right]$$

$$\mathcal{A}(1^+ 2^+ 3^+ 4^+) = 0$$

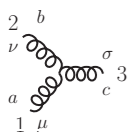
# graviton + graviton $\rightarrow$ graviton + graviton



$$\pm \rightarrow h = \pm 2$$

*à la* Feynman !

- 1) Define your fields ( $h_{\mu\nu}$ )
- 2) Get the Lagrangian (GR)
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$$i\text{Sym} \left[ -\frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\lambda} \eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu} p_{1\lambda} \eta_{\mu\rho} \eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1 \cdot p_2 \eta_{\mu\nu} \eta_{\rho\lambda} \eta_{\sigma\tau}) + P_6(p_1 \cdot p_2 \eta_{\mu\rho} \eta_{\nu\sigma} \eta_{\lambda\tau}) + 2P_3(p_{1\nu} p_{1\tau} \eta_{\mu\rho} \eta_{\lambda\sigma}) - P_3(p_{1\lambda} p_{2\mu} \eta_{\rho\nu} \eta_{\sigma\tau}) + P_3(p_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{2\tau} \eta_{\lambda\mu} \eta_{\rho\sigma}) + 2P_3(p_{1\nu} p_{2\mu} \eta_{\lambda\sigma} \eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2 \eta_{\rho\nu} \eta_{\lambda\sigma} \eta_{\tau\mu}) \right],$$

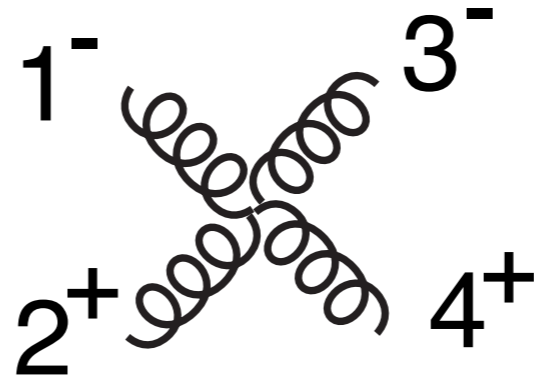


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$$\mathcal{A}(1^- 2^+ 3^- 4^+) \propto \frac{1}{stu}$$

graviton + graviton  $\rightarrow$  graviton + graviton

## On-shell amplitude methods

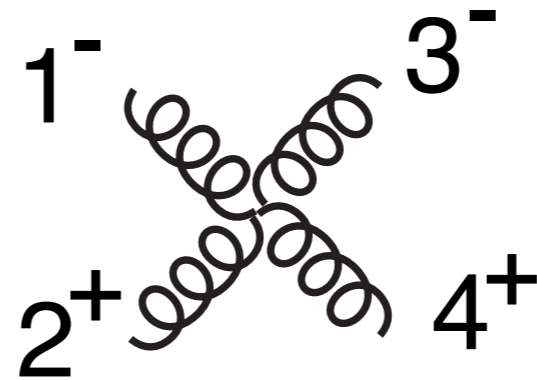


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Demanding: • Lorentz invariance  
• Unitarity & locality:

(single poles & factorization)

# Spinor-helicity formalism

Lorentz



SL(2,C)

$p^\mu$



$$p_{\alpha\dot{\alpha}} = |p\rangle_\alpha [p]_{\dot{\alpha}}$$

$$(1/2, 1/2) = (1/2, 0) \otimes (0, 1/2)$$

“angle” spinor

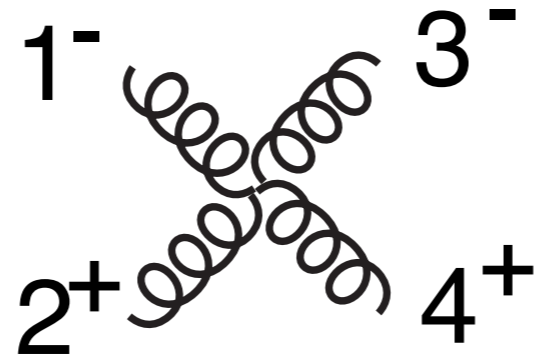
“squared” spinor

• Under Little group

$$\left\{ \begin{array}{ll} |p\rangle_\alpha \rightarrow e^{-i\theta/2} |p\rangle_\alpha & \text{Helicity } -1/2 \\ [p]_{\dot{\alpha}} \rightarrow e^{i\theta/2} [p]_{\dot{\alpha}} & \text{Helicity } +1/2 \end{array} \right.$$

• Lorentz invariance:  $\langle pq \rangle \equiv \langle p |^\alpha |q \rangle_\alpha$  ;  $[pq] \equiv [p |_{\dot{\alpha}} |q]_{\dot{\alpha}}$

# 4-graviton amplitude:

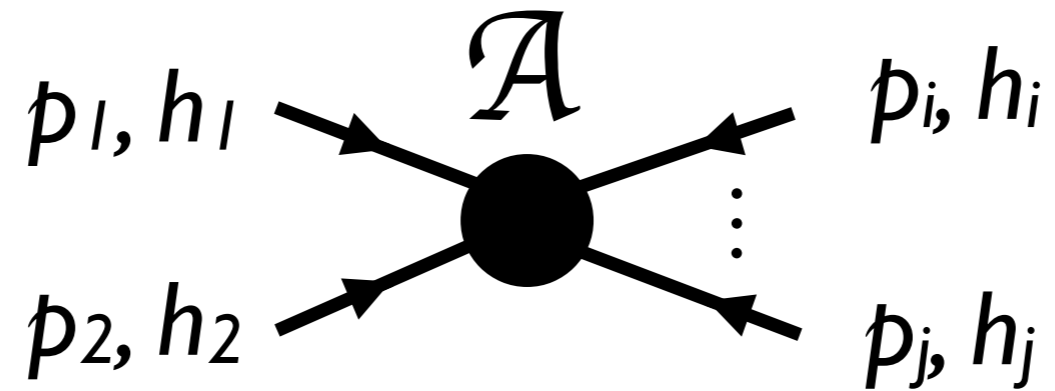


helicities (Little Group)

$$\mathcal{A}(1^- 2^+ 3^- 4^+) = \frac{\langle 13 \rangle^4 [24]^4}{stu} \times G_N$$

dim analysis & single-pole structure

# II. EFT (Effective Theories) from on-shell amplitudes





# Ordinary EFT approach

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2} \mathcal{L} \left( \frac{D_\mu}{\Lambda}, \frac{g_H H}{\Lambda}, \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}, \frac{g F_{\mu\nu}}{\Lambda^2} \right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

**SM**



# Ordinary EFT approach

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2} \mathcal{L} \left( \frac{D_\mu}{\Lambda}, \frac{g_H H}{\Lambda}, \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}, \frac{g F_{\mu\nu}}{\Lambda^2} \right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

**SM**

One must eliminate redundancies

Many missed in original papers (Buchmuller, Wyler,...)!

$$\begin{aligned} \mathcal{O}_H &= \frac{1}{2} (\partial^\mu |H|^2)^2 \\ \mathcal{O}_T &= \frac{1}{2} \left( H^\dagger \overleftrightarrow{D}_\mu H \right)^2 \\ \mathcal{O}_6 &= \lambda |H|^6 \end{aligned}$$

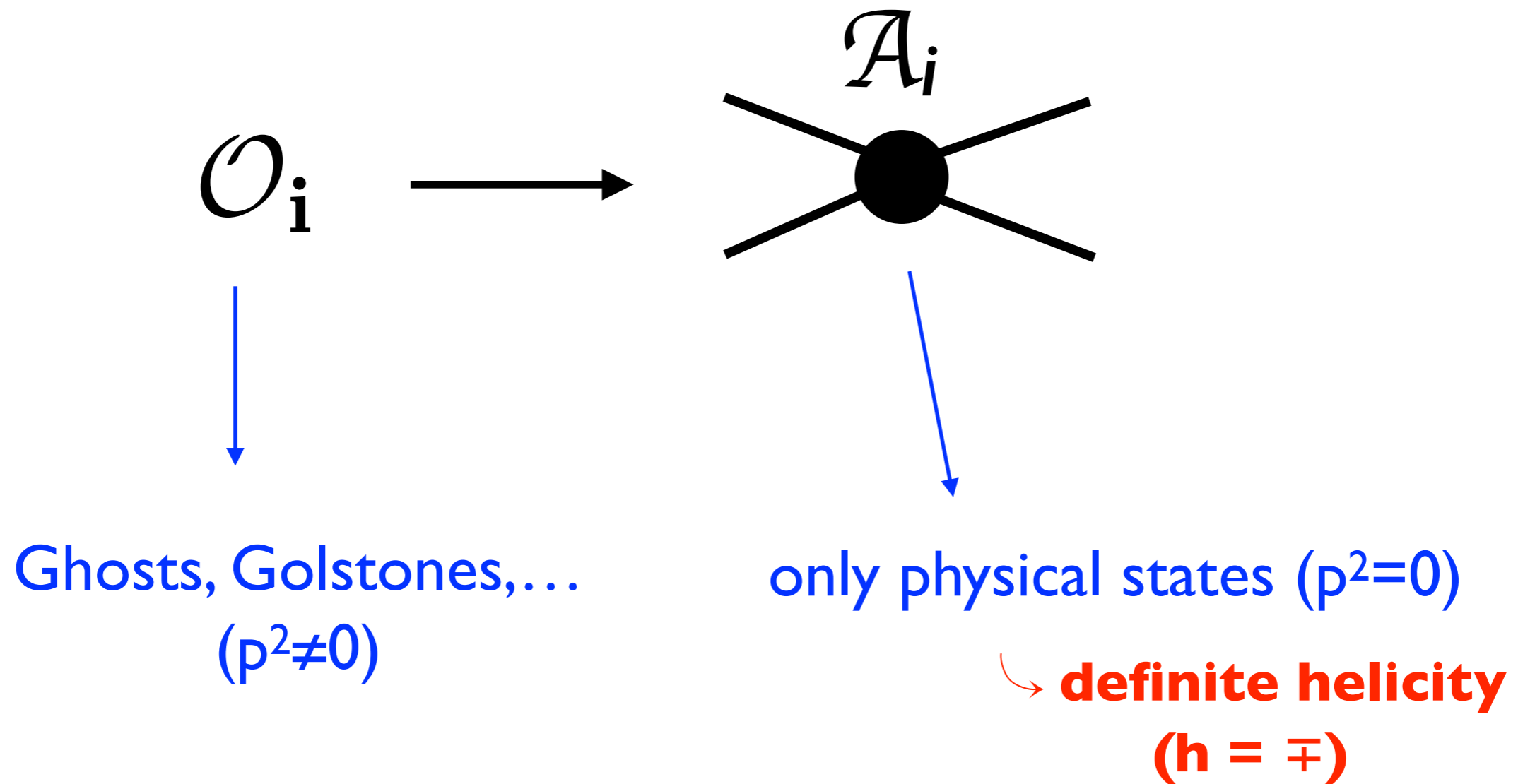
$$\begin{aligned} \mathcal{O}_W &= \frac{ig}{2} \left( H^\dagger \sigma^a \overleftrightarrow{D}^\mu H \right) D^\nu W_{\mu\nu}^a \\ \mathcal{O}_B &= \frac{ig'}{2} \left( H^\dagger \overleftrightarrow{D}^\mu H \right) \partial^\nu B_{\mu\nu} \\ \mathcal{O}_{2W} &= -\frac{1}{2} (D^\mu W_{\mu\nu}^a)^2 \\ \mathcal{O}_{2B} &= -\frac{1}{2} (\partial^\mu B_{\mu\nu})^2 \\ \mathcal{O}_{2G} &= -\frac{1}{2} (D^\mu G_{\mu\nu}^A)^2 \end{aligned}$$

$$\begin{aligned} \mathcal{O}_{BB} &= g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \\ \mathcal{O}_{GG} &= g_s^2 |H|^2 G_{\mu\nu}^A G^{A\mu\nu} \\ \mathcal{O}_{HW} &= ig (D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a \\ \mathcal{O}_{HB} &= ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\ \mathcal{O}_{3W} &= \frac{1}{3!} g \epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu} \\ \mathcal{O}_{3G} &= \frac{1}{3!} g_s f_{ABC} G_\mu^{A\nu} G_{\nu\rho}^B G^{C\rho\mu} \end{aligned}$$

$\mathcal{O}_{y_u} = y_u  H ^2 \bar{Q}_L \tilde{H} u_R$ $\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{u}_R \gamma^\mu u_R)$ $\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{Q}_L \gamma^\mu Q_L)$ $\mathcal{O}_L^{(3)q} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H) (\bar{Q}_L \gamma^\mu \sigma^a Q_L)$ $\mathcal{O}_{LR}^u = (\bar{Q}_L \gamma^\mu Q_L) (\bar{u}_R \gamma^\mu u_R)$ $\mathcal{O}_{LR}^{(8)u} = (\bar{Q}_L \gamma^\mu T^A Q_L) (\bar{u}_R \gamma^\mu T^A u_R)$ $\mathcal{O}_{RR}^u = (\bar{u}_R \gamma^\mu u_R) (\bar{u}_R \gamma^\mu u_R)$ $\mathcal{O}_{LL}^q = (\bar{Q}_L \gamma^\mu Q_L) (\bar{Q}_L \gamma^\mu Q_L)$ $\mathcal{O}_{LL}^{(3)q} = (\bar{Q}_L \gamma^\mu T^A Q_L) (\bar{Q}_L \gamma^\mu T^A Q_L)$ $\mathcal{O}_{LL}^{ql} = (\bar{Q}_L \gamma^\mu Q_L) (\bar{L}_L \gamma^\mu L_L)$ $\mathcal{O}_{LL}^{(3)ql} = (\bar{Q}_L \gamma^\mu \sigma^a Q_L) (\bar{L}_L \gamma^\mu \sigma^a L_L)$ $\mathcal{O}_{LR}^{qe} = (\bar{Q}_L \gamma^\mu Q_L) (\bar{e}_R \gamma^\mu e_R)$ $\mathcal{O}_{LR}^{lu} = (\bar{L}_L \gamma^\mu L_L) (\bar{u}_R \gamma^\mu u_R)$ $\mathcal{O}_{RR}^{ud} = (\bar{u}_R \gamma^\mu u_R) (\bar{d}_R \gamma^\mu d_R)$ $\mathcal{O}_{RR}^{(8)ud} = (\bar{u}_R \gamma^\mu T^A u_R) (\bar{d}_R \gamma^\mu T^A d_R)$ $\mathcal{O}_{RR}^{ue} = (\bar{u}_R \gamma^\mu u_R) (\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{y_d} = y_d  H ^2 \bar{Q}_L H d_R$ $\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{d}_R \gamma^\mu d_R)$ $\mathcal{O}_{LR}^d = (\bar{Q}_L \gamma^\mu Q_L) (\bar{d}_R \gamma^\mu d_R)$ $\mathcal{O}_{LR}^{(8)d} = (\bar{Q}_L \gamma^\mu T^A Q_L) (\bar{d}_R \gamma^\mu T^A d_R)$ $\mathcal{O}_{RR}^d = (\bar{d}_R \gamma^\mu d_R) (\bar{d}_R \gamma^\mu d_R)$ $\mathcal{O}_{LR}^{ld} = (\bar{L}_L \gamma^\mu L_L) (\bar{d}_R \gamma^\mu d_R)$ $\mathcal{O}_{RR}^{de} = (\bar{d}_R \gamma^\mu d_R) (\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{y_e} = y_e  H ^2 \bar{L}_L H e_R$ $\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{e}_R \gamma^\mu e_R)$ $\mathcal{O}_L^l = (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{L}_L \gamma^\mu L_L)$ $\mathcal{O}_L^{(3)l} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H) (\bar{L}_L \gamma^\mu \sigma^a L_L)$ $\mathcal{O}_{LR}^e = (\bar{L}_L \gamma^\mu L_L) (\bar{e}_R \gamma^\mu e_R)$ $\mathcal{O}_{RR}^e = (\bar{e}_R \gamma^\mu e_R) (\bar{e}_R \gamma^\mu e_R)$ $\mathcal{O}_{LL}^l = (\bar{L}_L \gamma^\mu L_L) (\bar{L}_L \gamma^\mu L_L)$
$\mathcal{O}_R^{ud} = y_u y_d (iH^\dagger \overleftrightarrow{D}_\mu H) (\bar{u}_R \gamma^\mu d_R)$ $\mathcal{O}_{y_u y_d} = y_u y_d (\bar{Q}_L^r u_R) \epsilon_{rs} (\bar{Q}_L^s d_R)$ $\mathcal{O}_{y_u y_d}^{(8)} = y_u y_d (\bar{Q}_L^r T^A u_R) \epsilon_{rs} (\bar{Q}_L^s T^A d_R)$ $\mathcal{O}_{y_u y_e} = y_u y_e (\bar{Q}_L^r u_R) \epsilon_{rs} (\bar{L}_L^s e_R)$ $\mathcal{O}'_{y_u y_e} = y_u y_e (\bar{Q}_L^r e_R) \epsilon_{rs} (\bar{L}_L^s u_R)$ $\mathcal{O}_{y_e y_d} = y_e y_d (\bar{L}_L e_R) (\bar{d}_R Q_L)$		
$\mathcal{O}_{DB}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \tilde{H} g' B_{\mu\nu}$ $\mathcal{O}_{DW}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \tilde{H} g W_{\mu\nu}^a$ $\mathcal{O}_{DG}^u = y_u \bar{Q}_L \sigma^{\mu\nu} T^A u_R \tilde{H} g_s G_{\mu\nu}^A$	$\mathcal{O}_{DB}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R H g' B_{\mu\nu}$ $\mathcal{O}_{DW}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R \sigma^a H g W_{\mu\nu}^a$ $\mathcal{O}_{DG}^d = y_d \bar{Q}_L \sigma^{\mu\nu} T^A d_R H g_s G_{\mu\nu}^A$	$\mathcal{O}_{DB}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R H g' B_{\mu\nu}$ $\mathcal{O}_{DW}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R \sigma^a H g W_{\mu\nu}^a$

# An important gain in simplicity:

*the power of being on-shell!*



# The SM as an EFT = *EF*fective *T*heory

Expansion:  $\langle ij \rangle / \Lambda^2$ ,  $[ij] / \Lambda^2$

## SM “Building-blocks”:

$$\begin{array}{l} 1 \psi \\ 2 \bar{\psi} \end{array} \text{---} \bullet \text{---} 3 \nu_{-} = \frac{\langle 13 \rangle^2}{\langle 12 \rangle}$$

$$\begin{array}{l} 1 \psi \\ 2 \psi \end{array} \text{---} \bullet \text{---} 3 H = \langle 12 \rangle$$

...

**At  $\mathcal{O}(E^2/\Lambda^2)$ :**

**$n$  = number of external states  
 $h$  = helicity of the amplitude**

$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle \quad \left. \vphantom{\frac{C_{F^3}}{\Lambda^2}} \right\} \begin{array}{l} n=3 \\ h=-3 \end{array}$$

$$\begin{aligned} \mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_\phi, 4_\phi) &= \frac{C_{F^2\phi^2}}{\Lambda^2} \langle 12 \rangle^2, \\ \mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_\psi, 3_\psi, 4_\phi) &= \frac{C_{F\psi^2\phi}}{\Lambda^2} \langle 12 \rangle \langle 13 \rangle, \\ \mathcal{A}_{\psi^4}(1_\psi, 2_\psi, 3_\psi, 4_\psi) &= (C_{\psi^4} \langle 12 \rangle \langle 34 \rangle + C'_{\psi^4} \langle 13 \rangle \langle 24 \rangle) \frac{1}{\Lambda^2} \end{aligned} \quad \left. \vphantom{\frac{C_{F\psi^2\phi}}{\Lambda^2}} \right\} \begin{array}{l} n=4 \\ h=-2 \end{array}$$

$$\begin{aligned} \mathcal{A}_{\square\phi^4}(1_\phi, 2_\phi, 3_\phi, 4_\phi) &= (C_{\square\phi^4} \langle 12 \rangle [12] + C'_{\square\phi^4} \langle 13 \rangle [13]) \frac{1}{\Lambda^2} \\ \mathcal{A}_{\psi\bar{\psi}\phi^2}(1_\psi, 2_{\bar{\psi}}, 3_\phi, 4_\phi) &= \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2} \langle 13 \rangle [23], \\ \mathcal{A}_{\psi^2\bar{\psi}^2}(1_\psi, 2_\psi, 3_{\bar{\psi}}, 4_{\bar{\psi}}) &= \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2} \langle 12 \rangle [34]. \end{aligned} \quad \left. \vphantom{\frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2}} \right\} \begin{array}{l} n=4 \\ h=0 \end{array}$$

**At  $\mathcal{O}(E^2/\Lambda^2)$ :**

**$n$  = number of external states  
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$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle \quad \left. \vphantom{\frac{C_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \right\} \begin{array}{l} n=3 \\ h=-3 \end{array}$$

$$\mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_\phi, 4_\phi) = \frac{C_{F^2\phi^2}}{\Lambda^2} \langle 12 \rangle^2,$$

$$\mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_\psi, 3_\psi, 4_\phi) = \frac{C_{F\psi^2\phi}}{\Lambda^2} \langle 12 \rangle \langle 13 \rangle, \quad \longleftrightarrow \quad F^{\mu\nu} \psi \sigma_{\mu\nu} \psi H$$

$$\mathcal{A}_{\psi^4}(1_\psi, 2_\psi, 3_\psi, 4_\psi) = (C_{\psi^4} \langle 12 \rangle \langle 34 \rangle + C'_{\psi^4} \langle 13 \rangle \langle 24 \rangle) \frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\square\phi^4}(1_\phi, 2_\phi, 3_\phi, 4_\phi) = (C_{\square\phi^4} \langle 12 \rangle [12] + C'_{\square\phi^4} \langle 13 \rangle [13]) \frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\psi\bar{\psi}\phi^2}(1_\psi, 2_{\bar{\psi}}, 3_\phi, 4_\phi) = \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2} \langle 13 \rangle [23],$$

$$\mathcal{A}_{\psi^2\bar{\psi}^2}(1_\psi, 2_\psi, 3_{\bar{\psi}}, 4_{\bar{\psi}}) = \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2} \langle 12 \rangle [34].$$

**$n=4$   
 $h=0$**

$$\mathcal{A}_{\psi^2\phi^3}(1_\psi, 2_\psi, 3_\phi, 4_\phi, 5_\phi) = \frac{C_{\psi^2\phi^3}}{\Lambda^2} \langle 12 \rangle \quad \begin{array}{l} n=5 \\ h=-1 \end{array}$$

$$\mathcal{A}_{\phi^6}(1_\phi, 2_\phi, 3_\phi, 4_\phi, 5_\phi, 6_\phi) = \frac{C_{\phi^6}}{\Lambda^2} \quad \begin{array}{l} n=6 \\ h=0 \end{array}$$

# **III. EFT renormalization via amplitude methods**



# One-loop mixing

$$A_i^{1\text{-loop}} = \text{Diagram 1} + \text{Diagram 2}$$

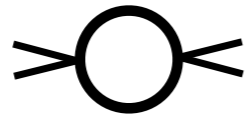
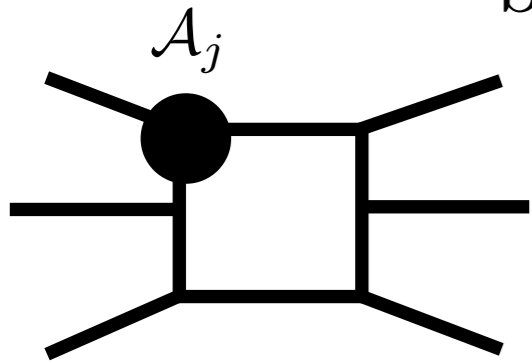
## Of great importance:

In particular, the **RG-running** of the (Wilson) coefficients of the amplitudes (the anomalous dimensions) needed for making contact with low-energy experiments

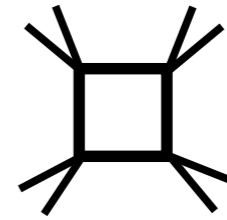
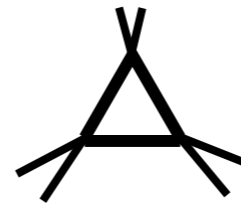
*e.g. for muon  $g-2$ , they must “run” down to  $E \sim m_\mu$*

# After one-loop reduction to Passarino-Veltman integrals

$$A_i = \sum_{\text{bubble}} c_2 I_2 + \sum_{\text{triangle}} c_3 I_3 + \sum_{\text{box}} c_4 I_4 + \text{rational}$$

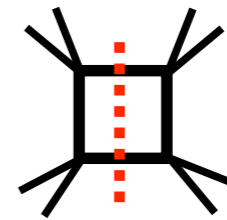
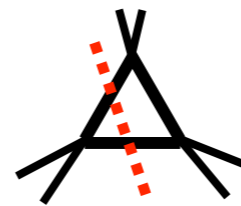
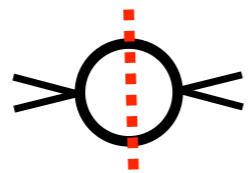
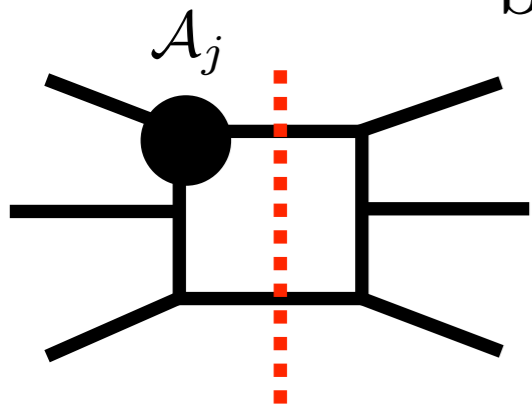


divergent   $c_2 = \text{anomalous dimensions}$



# After one-loop reduction to Passarino-Veltman integrals

$$A_i = \sum_{\text{bubble}} c_2 I_2 + \sum_{\text{triangle}} c_3 I_3 + \sum_{\text{box}} c_4 I_4 + \text{rational}$$

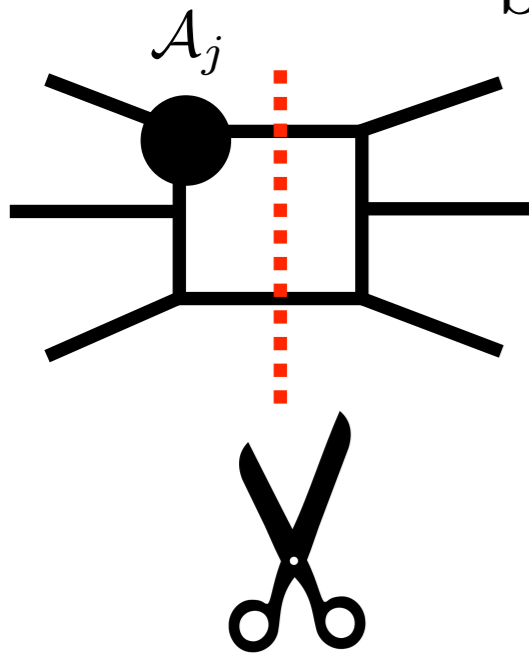


**double cut**

(internal particles on-shell)

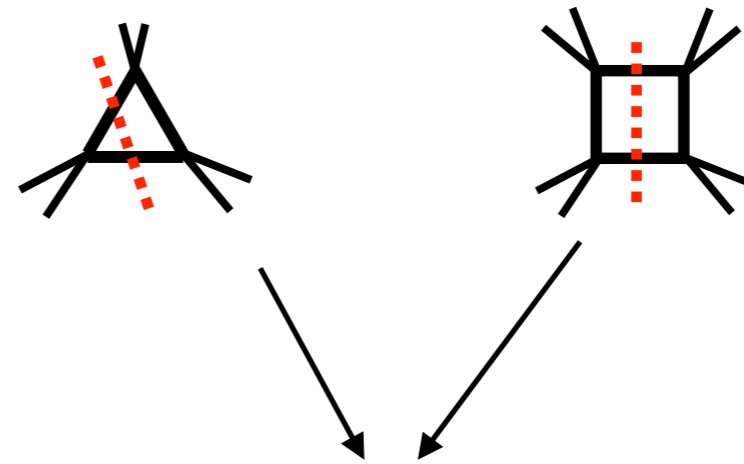
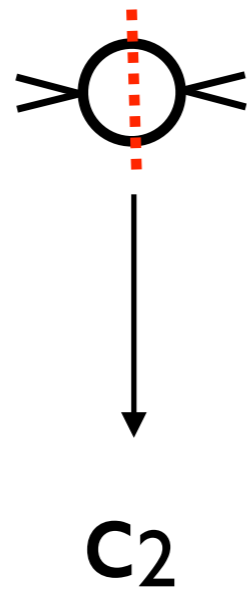
# After one-loop reduction to Passarino-Veltman integrals

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**double cut**

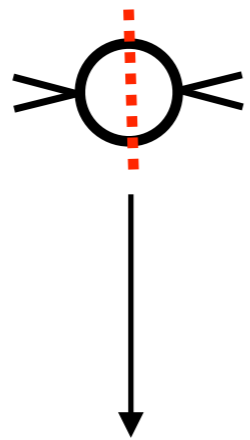
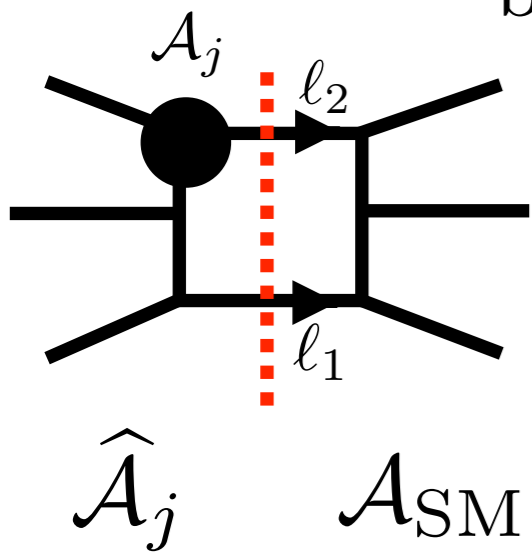
(internal particles on-shell)



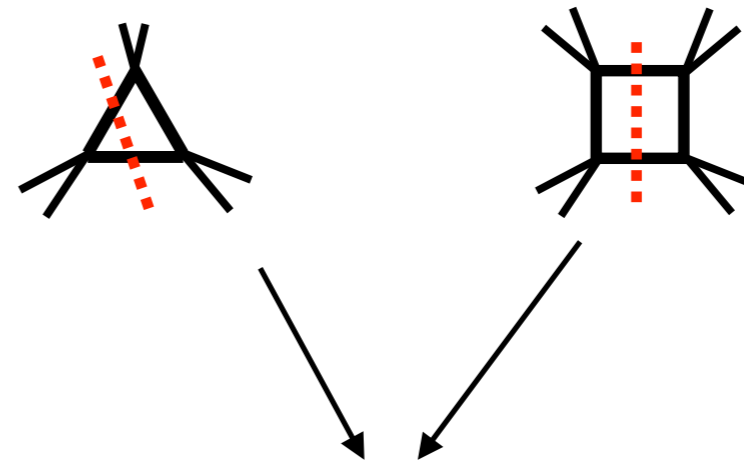
**Zero contribution**  
(after extracting IR-div)

# After one-loop reduction to Passarino-Veltman integrals

$$A_i = \sum_{\text{bubble}} c_2 I_2 + \sum_{\text{triangle}} c_3 I_3 + \sum_{\text{box}} c_4 I_4 + \text{rational}$$



$c_2$



Zero contribution  
(after extracting IR-div)

P. Baratella, C. Fernandez, AP 2005.07129

$$\gamma_{ij} \mathcal{A}_i(1, 2, \dots, n) = -\frac{1}{4\pi^3} \frac{C_i}{C_j} \int d\text{LIPS} \sum_{\substack{\text{ext. legs} \\ \text{distrib.}}} \sum_{l_1, l_2} \hat{A}_j(\dots, -l_1, -l_2) \times \mathcal{A}_{SM}(l_2, l_1, \dots)$$

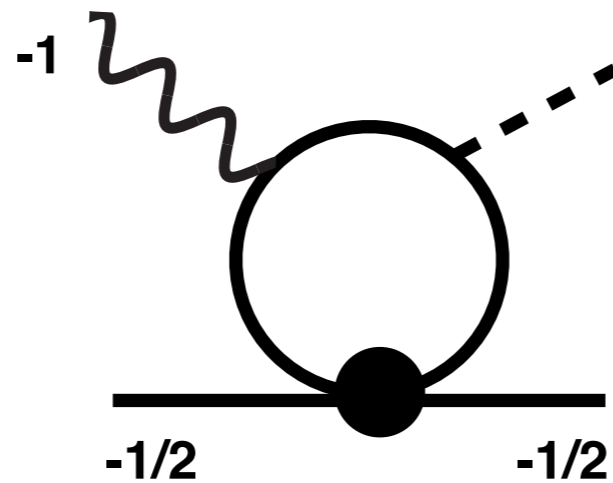
phase-space integration & sum over internal states

# “Emergent” selection rules

1505.01844 (also by susy techniques:1412.7151)

## No 4-fermion $(\psi\bar{\gamma}^\mu\psi)^2$ corrections to dipoles

$$\mathcal{A}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger})$$



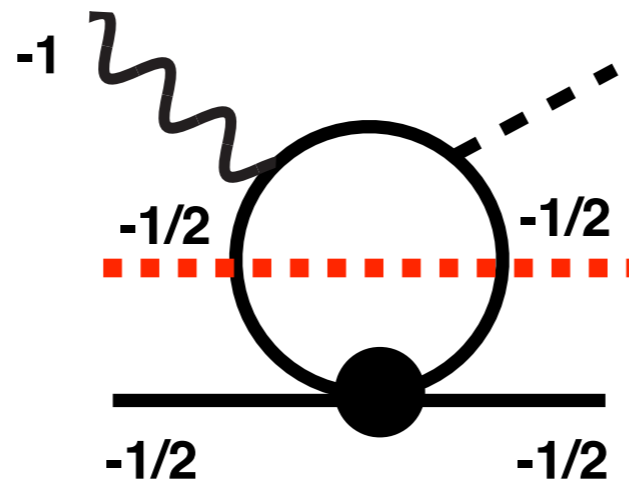
$$F^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi H$$

# “Emergent” selection rules

1505.01844 (also by susy techniques:1412.7151)

## No 4-fermion $(\psi\bar{\psi})^2$ corrections to dipoles

$$\mathcal{A}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger})$$



$$F^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi H$$

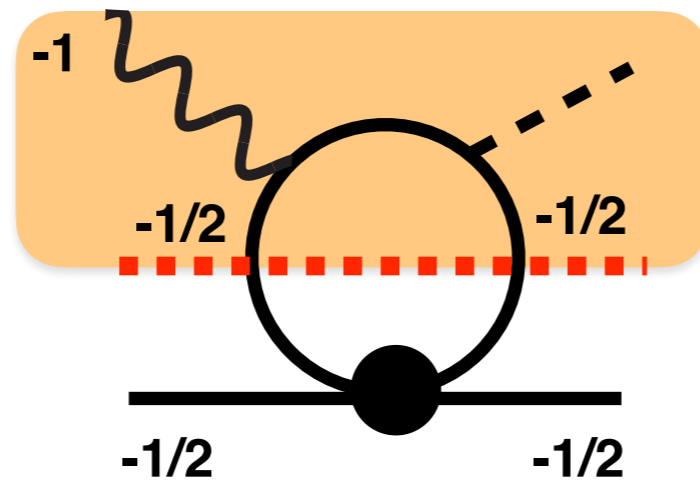
$$\gamma \mathcal{A}_{WHle} = -\frac{1}{4\pi^3} \int d\text{LIPS} \mathcal{A}_{luqe}(1_e, 2_l, 3'_{\bar{e}}, 4'_l) \times \mathcal{A}_{\text{SM}}(4'_e, 3'_l, 3_{W_-^a}, 4_{H^\dagger})$$

# “Emergent” selection rules

I505.01844 (also by susy techniques: I412.7151)

## No 4-fermion $(\psi\bar{\psi})^2$ corrections to dipoles

$$\mathcal{A}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger})$$



$$F^{\mu\nu} \bar{\psi} \sigma_{\mu\nu} \psi H$$

$$\mathbf{h}_{\text{total}} = -2$$

**Absent in the SM**

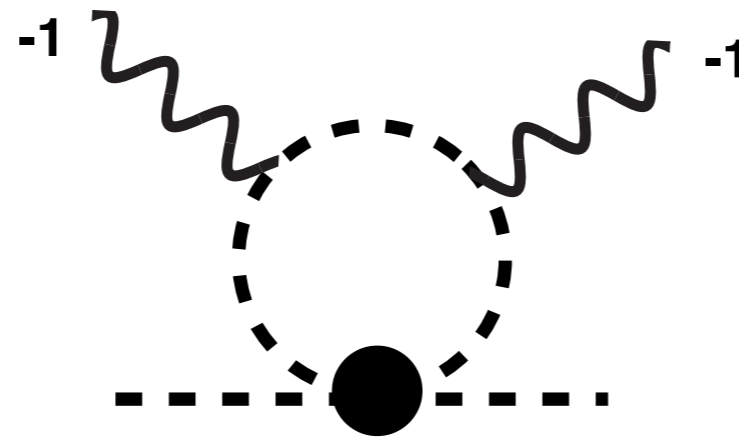
$$\gamma \mathcal{A}_{WHle} = -\frac{1}{4\pi^3} \int d\text{LIPS} \mathcal{A}_{luqe}(1_e, 2_l, 3'_{\bar{e}}, 4'_l) \times \mathcal{A}_{\text{SM}}(4'_e, 3'_l, 3_{W_-^a}, 4_{H^\dagger})$$



# No $p^2 H^4$ corrections to $H\gamma\gamma$

e.g. ↙

$$(H^\dagger D_\mu H)^2$$

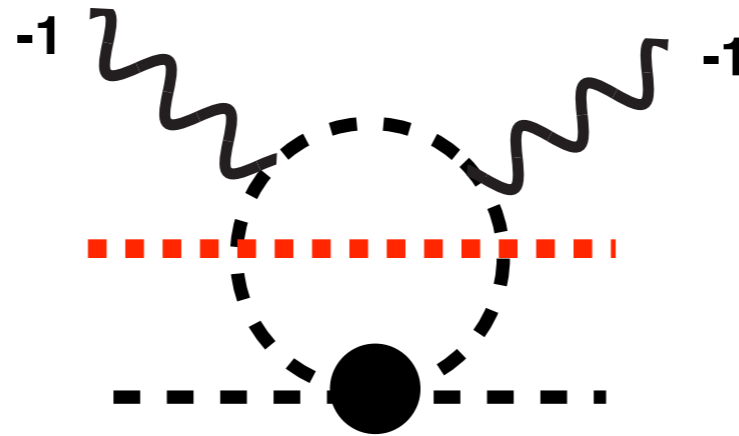


$$F_{\alpha\beta} F^{\alpha\beta} h^2$$

# No $p^2 H^4$ corrections to $H\gamma\gamma$

e.g. ↙

$$(H^\dagger D_\mu H)^2$$

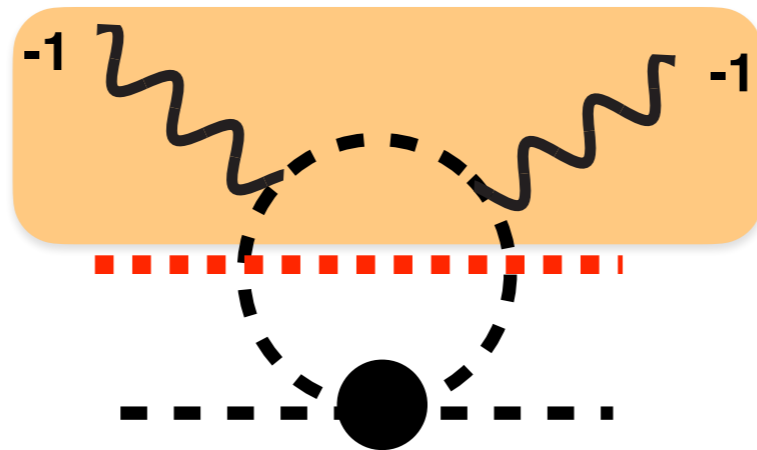


$$F_{\alpha\beta} F^{\alpha\beta} h^2$$

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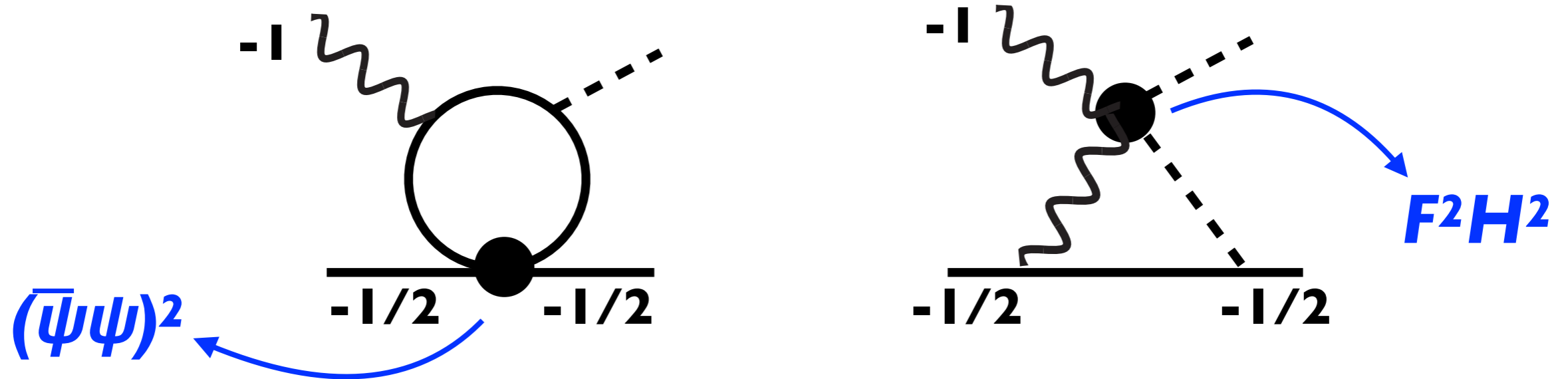


$$F_{\alpha\beta} F^{\alpha\beta} h^2$$

$h_{\text{total}} = -2$   
**Absent in the SM**

But the **on-shell methods** also tell us about the non-zero result

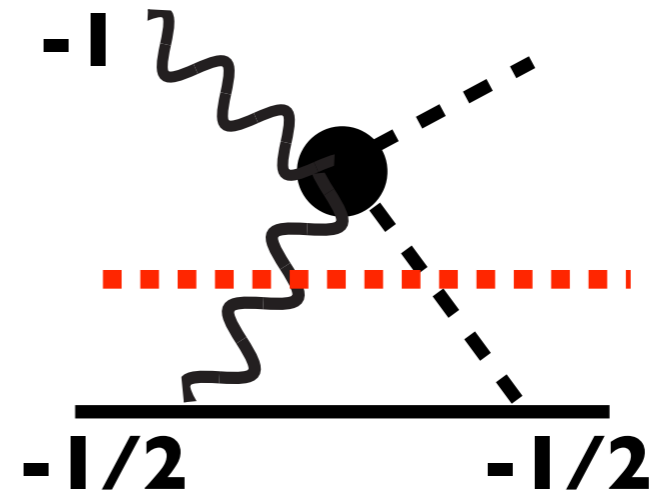
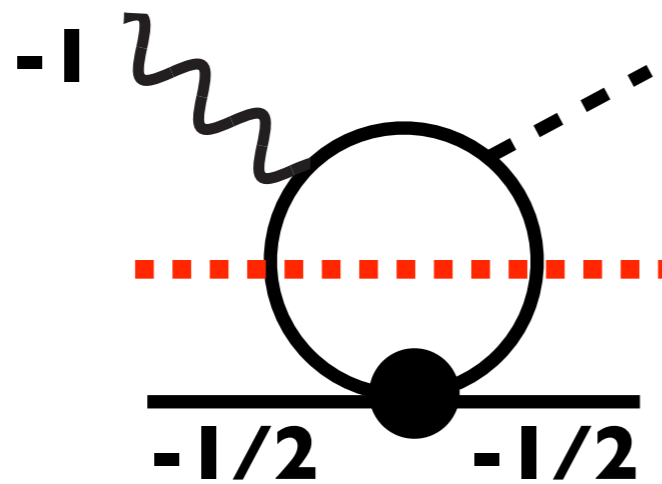
Contributions to **dipoles** from **Feynman** approach:



very **different** contributions

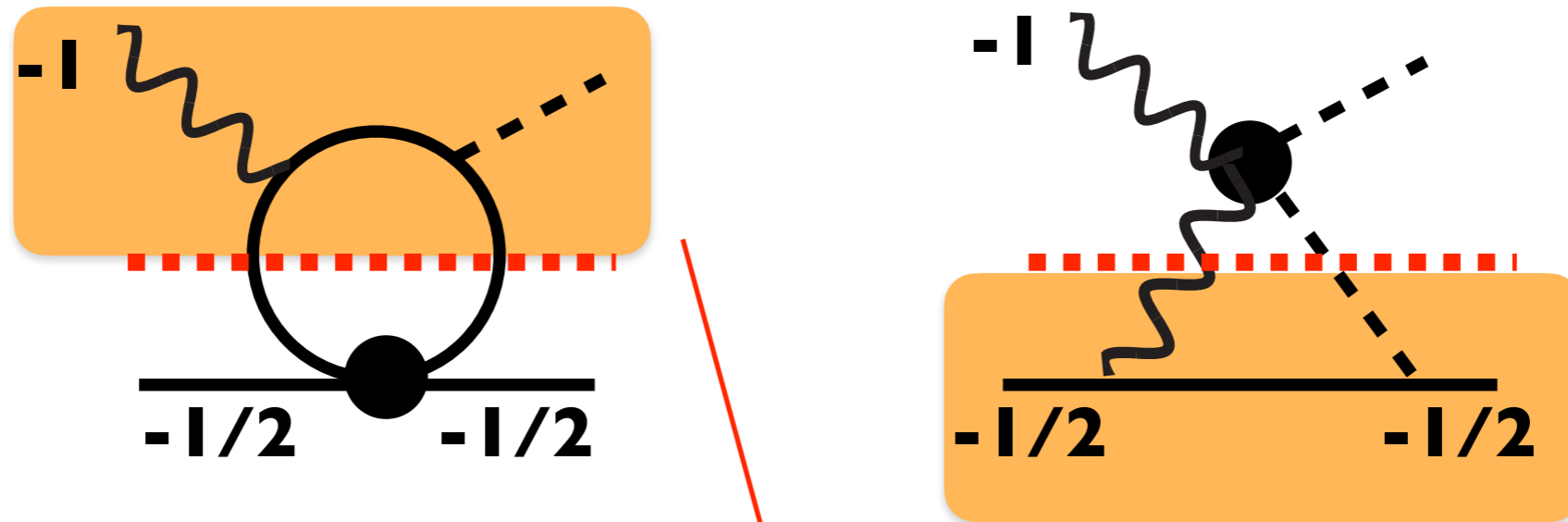
But the **on-shell methods** also tell us about the non-zero result

From **on-shell** approach:  $\gamma A_i \sim \sum_j A_j A_{SM}$



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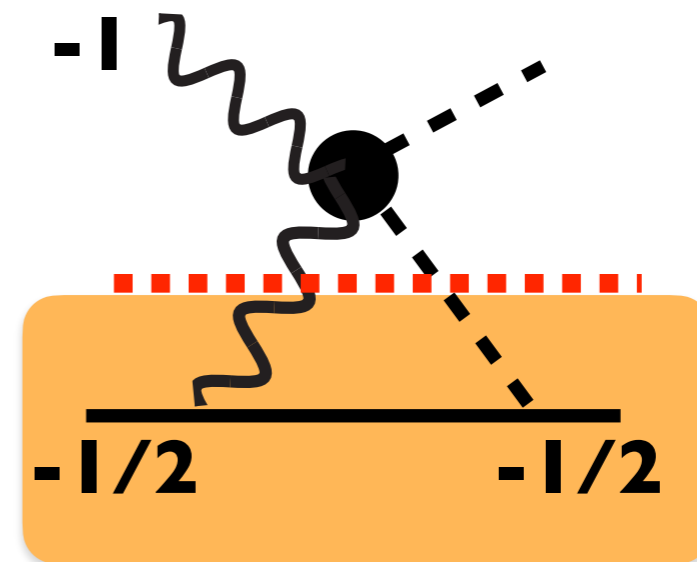
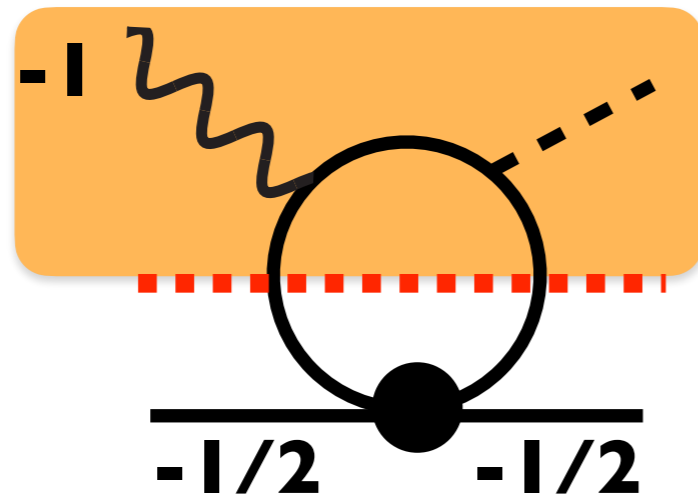


$$A_{SM}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^+})$$

from the same SM amplitude!

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From **on-shell** approach:  $\gamma A_i \sim \sum_j A_j A_{SM}$



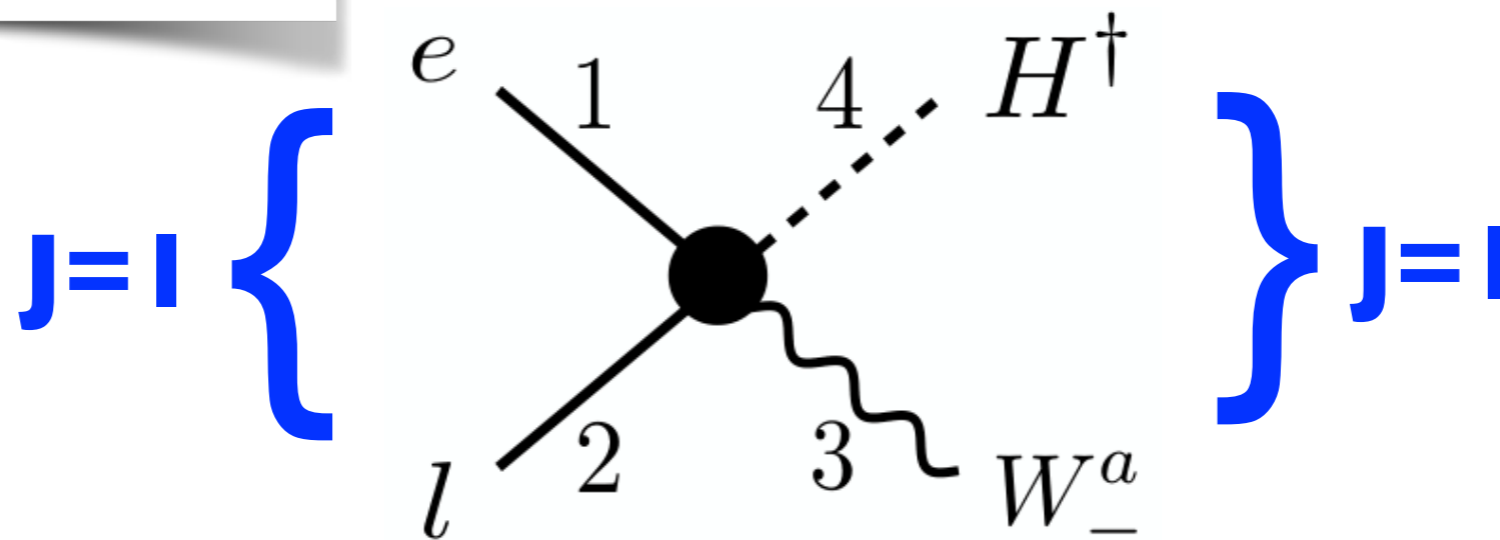
**No calculation wasted in the on-shell method**

$$A_{SM}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^+})$$

from the same SM amplitude!

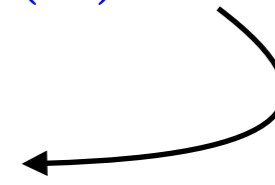
# But there is more to say by angular-momentum decomposition (partial-waves)

Example of dipoles:



$$\mathcal{A}(1_e, 2_l, 3_{W_-}, 4_{H^+}) = 3e^{-i\phi} d_{01}^{J=1}(\theta) a^{J=1}$$

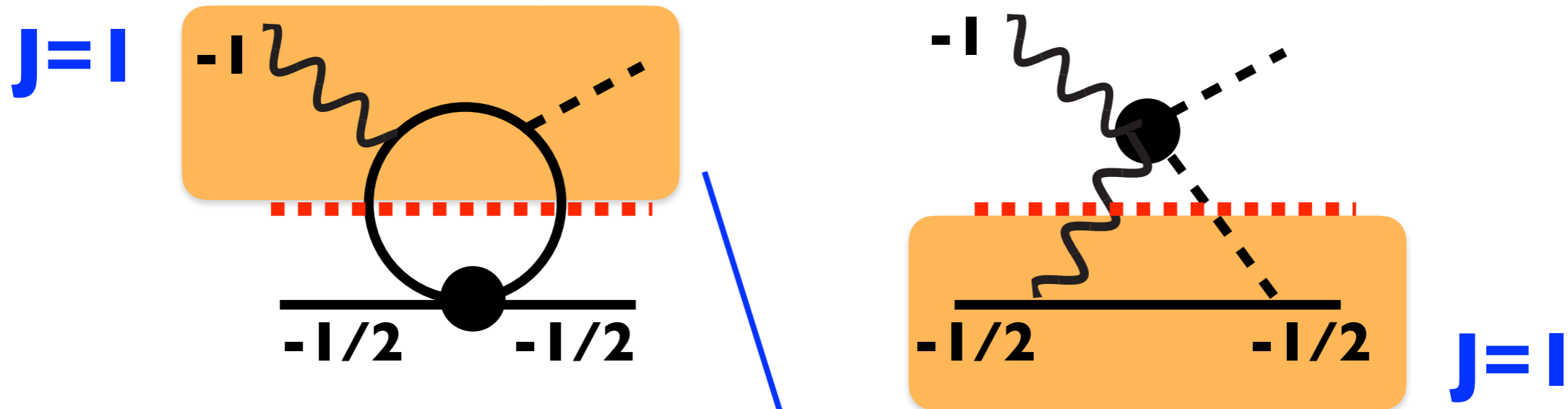
**only one partial-wave!**





# But there is more to say by angular-momentum decomposition (partial-waves)

Example of dipoles:



Not needed the full  
SM amplitude, only:

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

 **angular-momentum selection rules:**

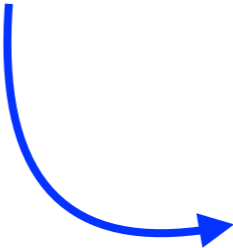
see also arXiv:2001.04481

**Amplitudes with  $J \neq 1$  cannot contribute to dipoles**

# Anomalous Dimensions as a product of partial-waves

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

$$\gamma_i \sim a_{\text{SM}}^J a_{\text{BSM}}^J$$

  $1/\Lambda^2$  amplitude

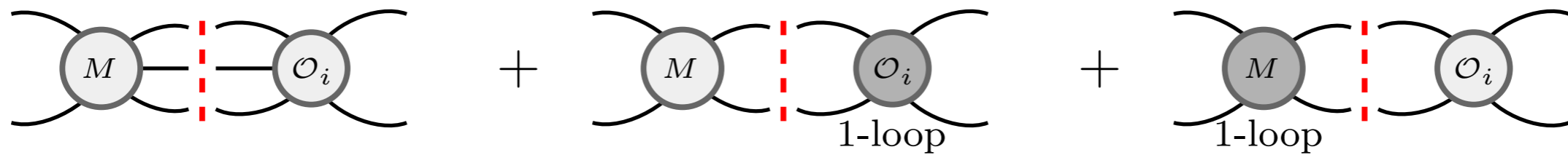
# Beyond one-loop

2005.06983

2005.12917

2112.12131

Two-loop:

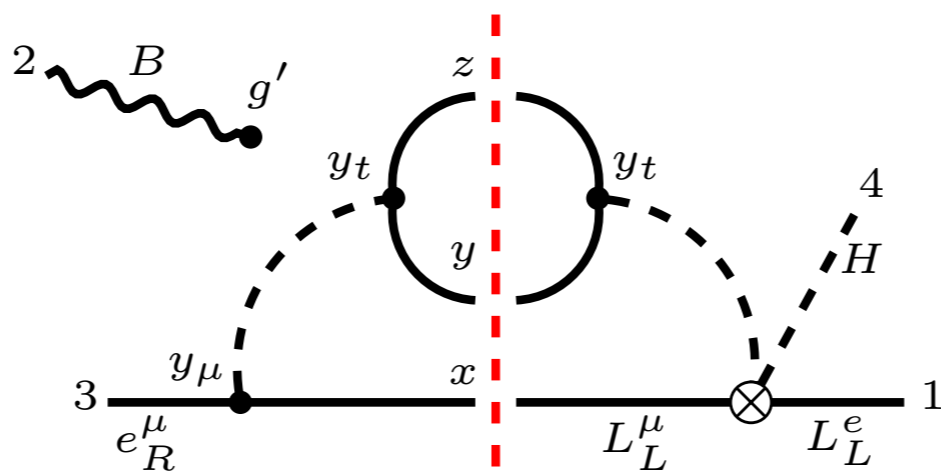


# Two-loops for $\mu \rightarrow e\gamma$

J. Elias-Miro, C. Fernandez, M. Gümüs, AP 2112.12131

$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu \mu_R)$  affects  $\mu \rightarrow e\gamma$  at the two-loop level:

↪ **Z** →  $\mu e$



product of tree-level amplitudes

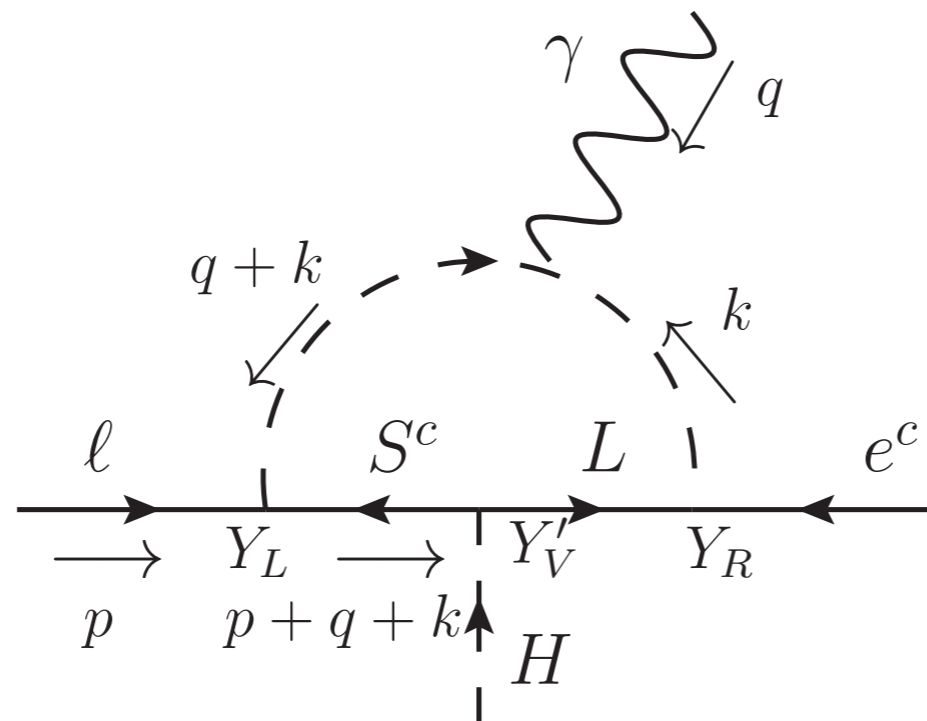
# Finite terms?

**Difficult** in general, **but** simplifies a lot for BSM calculations, where new physics scale  $\mathbf{M} \gg \mathbf{E}_{\text{exp}}$

**New insights** from the **amplitude** method!

# Finite terms to g-2

**No contribution  $O(1/M^2)$  to dipoles from a heavy singlet + doublet fermion:**



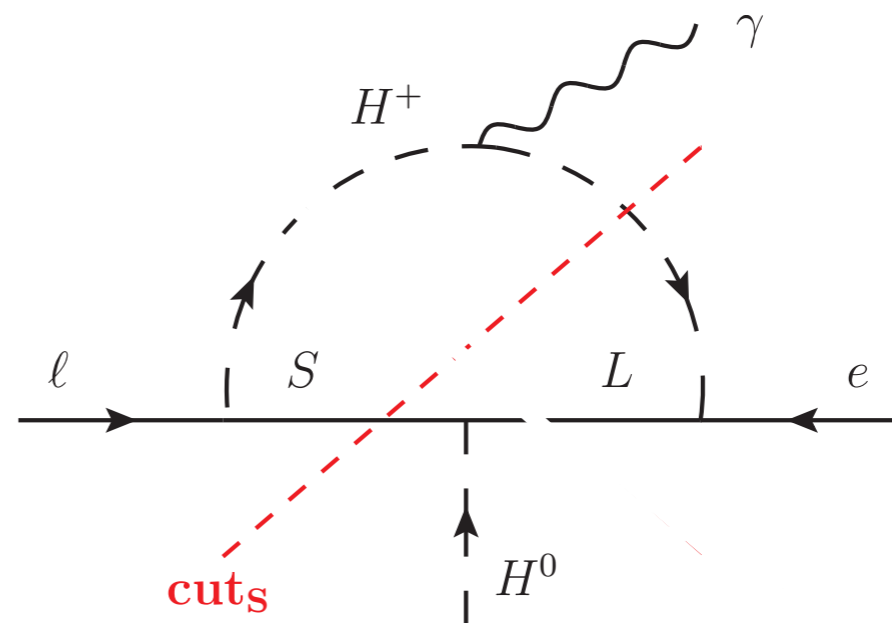
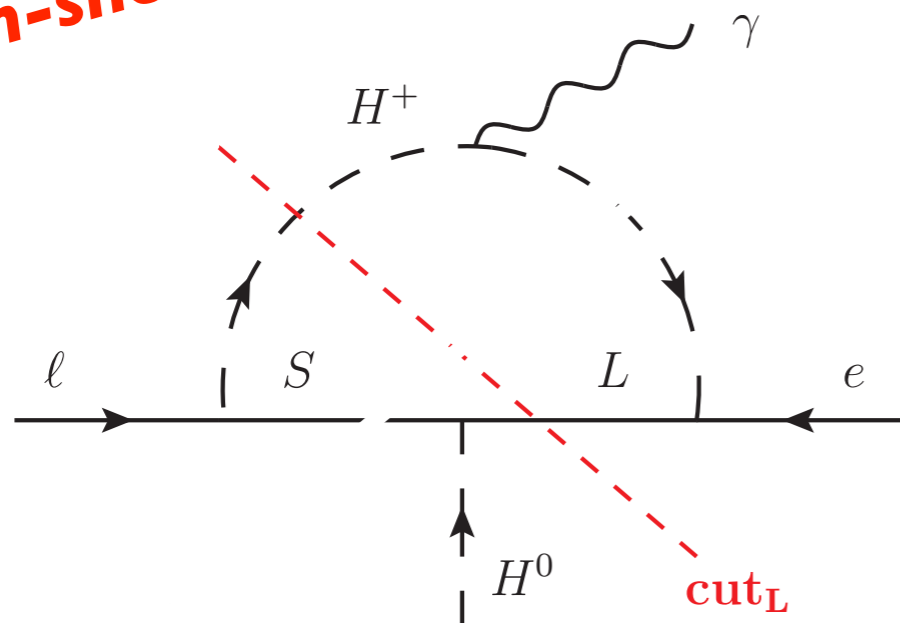
$$\sim O(1/M^4)$$

# Finite terms to $g-2$

**No contribution  $O(1/M^2)$  to **dipoles**  
from a **heavy singlet + doublet fermion**:**

from on-shell methods:

L. Delle Rosse, B. von Harling, AP in 2201.10572



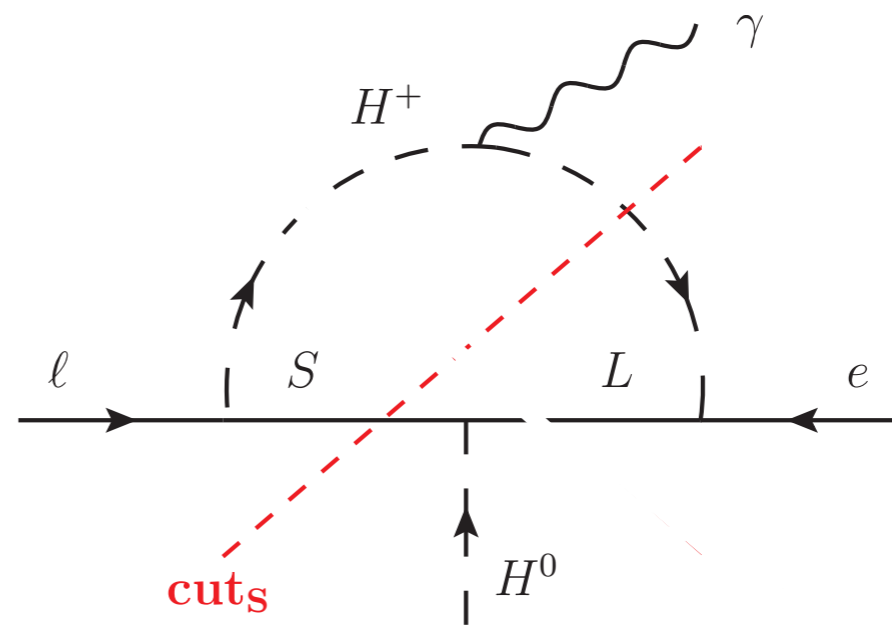
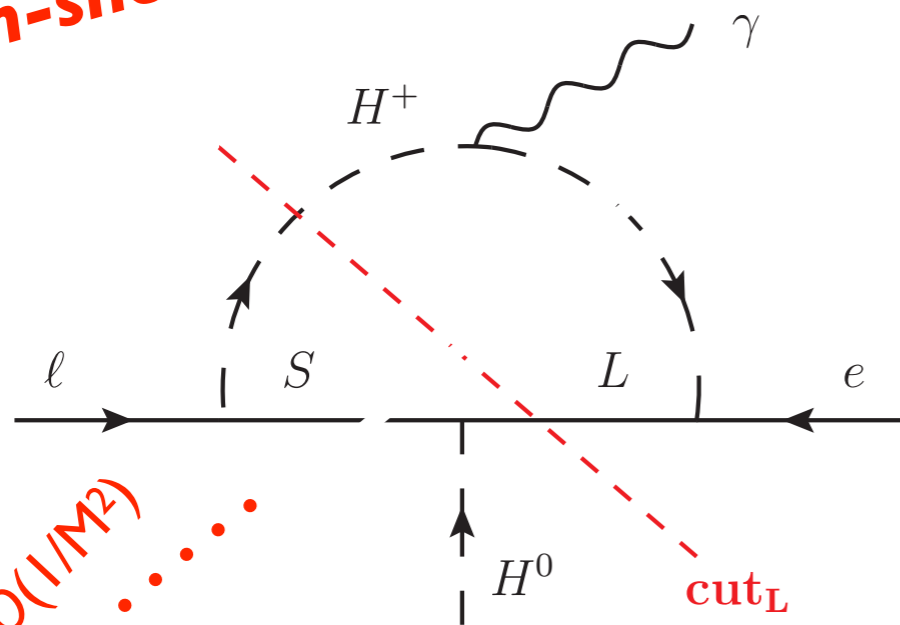
**even under  $S \leftrightarrow L$**

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from on-shell methods:



amplitude at  $O(1/M^2)$

$$\frac{1}{M_L^2 - M_S^2}$$

**even under  $S \leftrightarrow L$**

**odd under  $S \leftrightarrow L$**

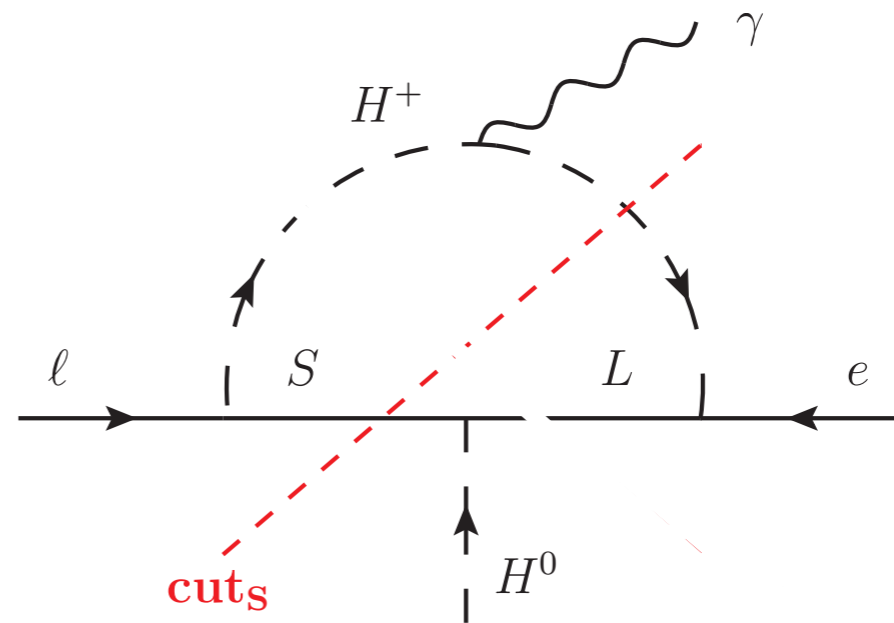
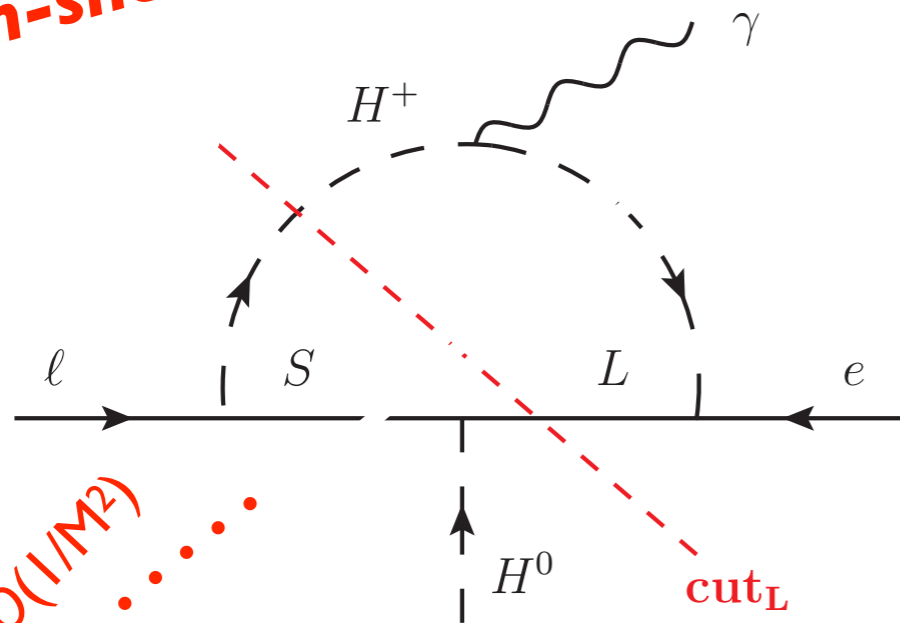


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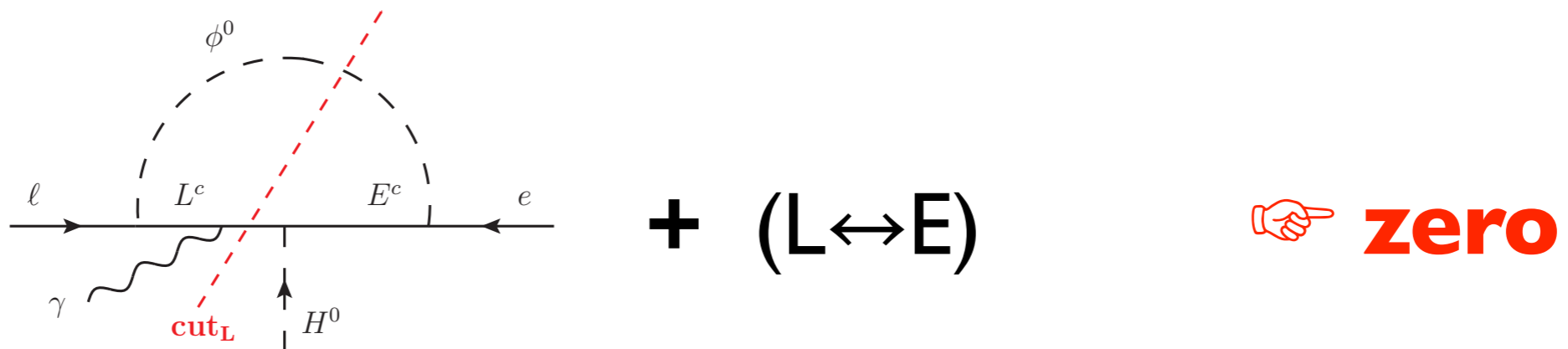
zero

# Finite terms to g-2

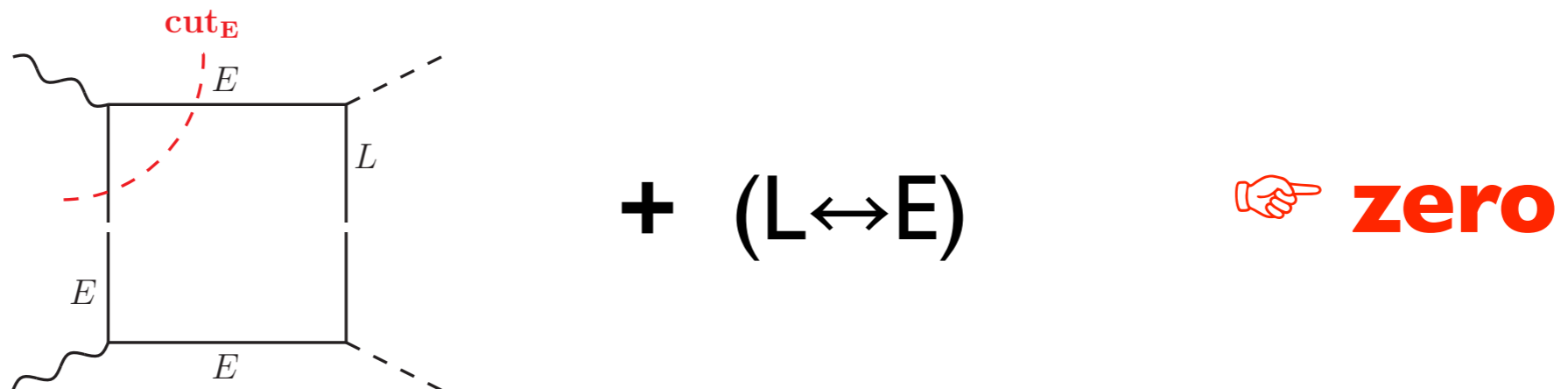
L. Delle Rosse, B. von Harling, AP in 2201.10572

Following the same argument, more zeros can be found:

- **Scalar + heavy doublet + charged fermion:**

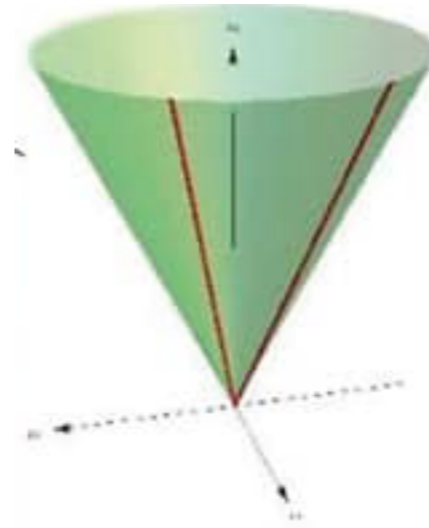


- **Beyond g-2: Zeros in  $h\gamma\gamma$**



# Conclusions

## ***Get on-shell!***



- Allows to construct **BSM without Lagrangians**
- Calculation of loop effects:

**Simpler with easy recycling**

☞ many “emergent” **selection rules**

☞ many **relations between anomalous dimensions**

**where Feynman approach is quite obscure**

*A lot to do! Stay Tuned!*