# PLANCK 2021

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## I. Antoniadis (LPTHE) **The on-shell way to BSM**  S. Lavignac (IPHT)

**Alex Pomarol, IFAE & UAB (Barcelona) and CERN**



- **Some motivations for on-shell amplitude methods**
- **EFT (EFfective Theories) from amplitudes, instead of Lagrangians**
- **Renormalization of EFT using on-shell methods:**

*Loops from tree-level amplitudes*

☞ **Simple**, elegant, and **efficient**



- **Some motivations for on-shell amplitude methods**
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*Loops from tree-level amplitudes*

☞ **Simple**, elegant, and **efficient**

**Makes explicit a lot of information that one cannot see from the Feynman approach!**



# **I. Some motivation**



*à la* Feynman !



1 2 3 4





1) Define your fields (hμν)

*à la* Feynman !





1 2 3 4

1) Define your fields (hμν)

*à la* Feynman !

2) Get the Lagrangian (GR)











#### $\dot{a}$  *la* Feynman !  $\overline{a}$  Launman is an arbitrary multiplicity. The complexity of each individual interaction term is a contraction term in the complexity of each interaction term is a contraction term in the complexity of the complexity per striking than the three-gravitons in  $\mathbf{C}$ interaction. In the standard de Donder gauge, *ˆ‹h‹ <sup>µ</sup>* = <sup>1</sup> <sup>2</sup> *ˆµh‹*  $I_a$   $E_a$ evnm: l 6 with polynomial  $\mathbf{r}$ à la Feynman! (2)

 $\Gamma$ 

 $\sum_{c}^{a} 3$ 

*c*

3

 $\mathcal{F}$  is a few  $\mathcal{F}$  gauge three- and four-point vertices in a  $\mathcal{F}$ 

*µ*

م<br>محل

1

 $\stackrel{\shortparallel}{\nu}$ 

1

*a µ*

2

 $\frac{e^{v}}{a}$ 

*a µ*

1

 $\nu$   $\epsilon$   $\beta$   $\sigma$ 

*b*

 $\begin{array}{cc} \nu \rightarrow & \sigma \rightarrow & \sigma \end{array}$ 

*b*

2

 $\epsilon$  $\bigotimes \limits^{\textstyle \bigotimes \limits_{3\big(P_1\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\nu \lambda} \eta_{\sigma \tau} \big) \ - \ \frac{1}{2} P_6(p_{1\nu} p_{1\lambda} \eta_{\mu \rho} \eta_{\sigma \tau}) \ + \ \frac{1}{2} P_3(p_{1}\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\rho \lambda} \eta_{\sigma \tau} \big)$  $\mathcal{O}(\mathcal{O})$   $\mathcal{O}_{\mathcal{O}}(1 \cdot p_2 \eta_{\mu \rho} \eta_{\nu \sigma} \eta_{\lambda \tau}) + 2 P_3(p_{1\nu} p_{1\tau} \eta_{\mu \rho} \eta_{\lambda \sigma}) - P_3(p_{1\lambda} p_{2\mu} \eta_{\rho \nu} \eta_{\sigma \tau})$  $\Gamma_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda} + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{\rho\sigma})$  $+ 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\right]$ *,* (1.6)  $\sqrt{2}$ I' (p -p'~" ~"~"")j, (26)

 $+ \frac{1}{4} Pa_3(p\cdot p'\eta'''\eta'''\eta''') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{2} Pa_3(p'\cdot p'\eta'''\eta'''\eta''') - \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta$  $-\frac{1}{2}P_{12}(p \cdot p' \eta^{r\sigma} \eta^{r\rho} \eta^{k\sigma} \eta^{r\rho}) - \frac{1}{2}P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma} \eta^{k\sigma}) + \frac{1}{2}P_{12}(p^e p^e \eta^{r\gamma} \eta^{w\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{k\sigma} \eta^{r\sigma}) - P_{13}(p^e p' \eta^{k\sigma} \eta^{r\sigma} \eta^{k\sigma}) - P_{14}(p^e p' \eta^{$  $\sum_{\substack{\text{sym}\left[-\frac{1}{3}P_{6}(p\cdot p'\eta^{\omega}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\iota}\right)-\frac{1}{3}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\gamma}\eta^{\rho\lambda}\eta^{\iota\iota})-\frac{1}{4}P_{6}(p^{\sigma}p'\eta^{\tau}\eta^{\rho\lambda}\eta^{\iota\iota})+\frac{1}{3}P_{6}(p\cdot p'\eta^{\mu\sigma}\eta^{\tau\gamma}\eta^{\rho\lambda}\eta^{\iota\iota})}{+^{\frac{1}{4}P_{s}(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\$  $\text{Sym}[-\tfrac{1}{8}P_6(\rho\cdot \rho'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})-\tfrac{1}{8}P_{12}(\rho^{\sigma}\rho^{\tau}\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\nu\lambda})-\tfrac{1}{4}P_6(\rho^{\sigma}\rho'\prime^{\mu}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})+\tfrac{1}{8}P_6(\rho\cdot \rho'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})]$  $+\frac{1}{4}P_6(p\cdot p' \eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{4}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{2}P_6(p^{\sigma}p'\mu\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\frac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$  $+{1\over 4}P_{24}(\rho\cdot \rho'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\mu\rho}\eta^{\tau\kappa}\eta^{\iota\kappa})+{1\over 2}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$  $-\tfrac12 P_{12} (p\cdot p'\eta^{\sigma} \eta^{\tau\rho} \eta^{\lambda\mu} \eta^{\iota\epsilon}) -\tfrac12 P_{12} (p^\sigma p'\mu\eta^{\tau\rho} \eta^{\lambda\kappa} \eta^{\iota\epsilon}) +\tfrac12 P_{12} (p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\nu} \eta^{\iota\epsilon}) -\tfrac12 P_{24} (p\cdot p'\eta^{\mu\nu} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\iota\sigma})$  $\mathcal{H}_{p_{\mathfrak{a}}(p+p' \eta^{*}\rho} \eta^{\lambda\sigma} \eta^{\tau\iota}\eta^{\kappa\mu}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\mu\nu} \eta^{\tau\iota}\eta^{\kappa\lambda}) - \frac{1}{2} P_{12}(p\cdot p' \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\sigma\iota}\eta^{\tau\kappa}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\tau\lambda} \eta^{\mu\iota}\eta^{\nu\kappa})$ *V abc*  $\begin{array}{ccccc} & -P_6(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s r})-P_{24}(p^s p'^s \eta^{s s} \eta^{s s})-P_{12}(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s s})+2P_6(p\cdot p' \eta^{s s} \eta^{s s} \eta^{s s}) \end{array}$ 

$$
A(H_1^- H_2^- H_3^+) = \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 32 \rangle^2}
$$

$$
A(H_1^- H_2^- H_3^+ H_4^+) \quad = \quad \frac{\langle 12 \rangle^4 [34]^4}{stu}
$$



Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip-1) Define your fields (h<sub>μν</sub>)

2) Get the Lagrangian (GR)

3) Get, the Feynman rules

 $\mathcal{L}$ 

 $\overline{p}$ Feynman vertex



#### $\dot{a}$  *la* Feynman !  $\overline{a}$  Launman is an arbitrary multiplicity. The complexity of each individual interaction term is a contraction term in the complexity of each interaction term is a contraction term in the complexity of the complexity per striking than the three-gravitons in  $\mathbf{C}$ interaction. In the standard de Donder gauge, *ˆ‹h‹ <sup>µ</sup>* = <sup>1</sup> <sup>2</sup> *ˆµh‹*  $I_a$   $E_a$ evnm: l 6 with polynomial  $\mathbf{r}$ à la Feynman! (2)

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2

 $\frac{e^{v}}{a}$ 

*a µ*

1

 $\nu$   $\epsilon$   $\beta$   $\sigma$ 

*b*

 $\begin{array}{cc} \nu \rightarrow & \sigma \rightarrow & \sigma \end{array}$ 

*b*

2

 $\epsilon$  $\bigotimes \limits^{\textstyle \bigotimes \limits_{3\big(P_1\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\nu \lambda} \eta_{\sigma \tau} \big) \ - \ \frac{1}{2} P_6(p_{1\nu} p_{1\lambda} \eta_{\mu \rho} \eta_{\sigma \tau}) \ + \ \frac{1}{2} P_3(p_{1}\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\rho \lambda} \eta_{\sigma \tau} \big)$  $\mathcal{O}(\mathcal{O})$   $\mathcal{O}_{\mathcal{O}}$   $\left( \frac{1}{2} \cdot p_2 \eta_{\mu \rho} \eta_{\nu \sigma} \eta_{\lambda \tau} \right) + 2 P_3 (p_{1\nu} p_{1\tau} \eta_{\mu \rho} \eta_{\lambda \sigma}) - P_3 (p_{1\lambda} p_{2\mu} \eta_{\rho \nu} \eta_{\sigma \tau})$  $\Gamma_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda} + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{\rho\sigma})$  $+ 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\right]$  $\sqrt{2}$ 

*,* (1.6)

I' (p -p'~" ~"~"")j, (26)

 $+ \frac{1}{4} Pa_3(p\cdot p'\eta'''\eta'''\eta''') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{2} Pa_3(p'\cdot p'\eta'''\eta'''\eta''') - \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta$  $-\frac{1}{2}P_{12}(p \cdot p' \eta^{r\sigma} \eta^{r\rho} \eta^{k\sigma} \eta^{r\rho}) - \frac{1}{2}P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma} \eta^{k\sigma}) + \frac{1}{2}P_{12}(p^e p^e \eta^{r\gamma} \eta^{w\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{k\sigma} \eta^{r\sigma}) - P_{13}(p^e p' \eta^{k\sigma} \eta^{r\sigma} \eta^{k\sigma}) - P_{14}(p^e p' \eta^{$  $\sum_{\substack{\text{sym}\left[-\frac{1}{3}P_{6}(p\cdot p'\eta^{\omega}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\iota}\right)-\frac{1}{3}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\gamma}\eta^{\rho\lambda}\eta^{\iota\iota})-\frac{1}{4}P_{6}(p^{\sigma}p'\eta^{\tau}\eta^{\rho\lambda}\eta^{\iota\iota})+\frac{1}{3}P_{6}(p\cdot p'\eta^{\mu\sigma}\eta^{\tau\gamma}\eta^{\rho\lambda}\eta^{\iota\iota})}{+^{\frac{1}{4}P_{s}(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\$  $\text{Sym}[-\tfrac{1}{8}P_6(\rho\cdot \rho'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})-\tfrac{1}{8}P_{12}(\rho^{\sigma}\rho^{\tau}\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\nu\lambda})-\tfrac{1}{4}P_6(\rho^{\sigma}\rho'\prime^{\mu}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})+\tfrac{1}{8}P_6(\rho\cdot \rho'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\nu\lambda})]$  $+\frac{1}{4}P_6(p\cdot p' \eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{4}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{2}P_6(p^{\sigma}p'\mu\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\frac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$  $+{1\over 4}P_{24}(\rho\cdot \rho'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\mu\rho}\eta^{\tau\kappa}\eta^{\iota\kappa})+{1\over 2}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$  $-\tfrac12 P_{12} (p\cdot p'\eta^{\sigma} \eta^{\tau\rho} \eta^{\lambda\mu} \eta^{\iota\epsilon}) -\tfrac12 P_{12} (p^\sigma p'\mu\eta^{\tau\rho} \eta^{\lambda\kappa} \eta^{\iota\epsilon}) +\tfrac12 P_{12} (p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\nu} \eta^{\iota\epsilon}) -\tfrac12 P_{24} (p\cdot p'\eta^{\mu\nu} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\iota\sigma})$  $\mathcal{H}_{p_{\mathfrak{a}}(p+p' \eta^{*}\rho} \eta^{\lambda\sigma} \eta^{\tau\iota}\eta^{\kappa\mu}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\mu\nu} \eta^{\tau\iota}\eta^{\kappa\lambda}) - \frac{1}{2} P_{12}(p\cdot p' \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\sigma\iota}\eta^{\tau\kappa}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\tau\lambda} \eta^{\mu\iota}\eta^{\nu\kappa})$ *V abc*  $\begin{array}{ccccc} & -P_6(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s r})-P_{24}(p^s p'^s \eta^{s s} \eta^{s s})-P_{12}(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s s})+2P_6(p\cdot p' \eta^{s s} \eta^{s s} \eta^{s s}) \end{array}$ 

$$
A(H_1^- H_2^- H_3^+) = \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 32 \rangle^2}
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$$



Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip- $\mathcal{L}$ 3) Get, the Feynman rules 4 ) ( *)* 4) Obtain the amplitude 2) Get the Lagrangian (GR) 1) Define your fields (h<sub>μν</sub>)



#### $\dot{a}$  *la* Feynman !  $\overline{a}$  Launman is an arbitrary multiplicity. The complexity of each individual interaction term is a contraction term in the complexity of each interaction term is a contraction term in the complexity of the complexity per striking than the three-gravitons in  $\mathbf{C}$ interaction. In the standard de Donder gauge, *ˆ‹h‹ <sup>µ</sup>* = <sup>1</sup> <sup>2</sup> *ˆµh‹*  $I_a$   $E_a$ evnm: l 6 with polynomial  $\mathbf{r}$ à la Feynman! (2)

 $\Gamma$  $\sim$ 

 $\sum_{c}^{a} 3$ 

*c*

3

 $\mathcal{F}$  is a few  $\mathcal{F}$  gauge three- and four-point vertices in a  $\mathcal{F}$ 

*µ*

 $\mathcal{F}$  and interactions in the interactions in a Feynman interaction interactions in a Feynman interactions in a Feynman in

م<br>محل

1

 $\stackrel{\shortparallel}{\nu}$ 

1

*a µ*

2

 $\frac{e^{v}}{a}$ 

*a µ*

1

 $\nu$   $\epsilon$   $\beta$   $\sigma$ 

*b*

 $\begin{array}{cc} \nu \rightarrow & \sigma \rightarrow & \sigma \end{array}$ 

*b*

2

 $\epsilon$  $\bigotimes \limits^{\textstyle \bigotimes \limits_{3\big(P_1\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\nu \lambda} \eta_{\sigma \tau} \big) \ - \ \frac{1}{2} P_6(p_{1\nu} p_{1\lambda} \eta_{\mu \rho} \eta_{\sigma \tau}) \ + \ \frac{1}{2} P_3(p_{1}\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\rho \lambda} \eta_{\sigma \tau} \big)$  $\mathcal{O}(\mathcal{O})$   $\mathcal{O}_{\mathcal{O}}$   $\left( \frac{1}{2} \cdot p_2 \eta_{\mu \rho} \eta_{\nu \sigma} \eta_{\lambda \tau} \right) + 2 P_3 (p_{1\nu} p_{1\tau} \eta_{\mu \rho} \eta_{\lambda \sigma}) - P_3 (p_{1\lambda} p_{2\mu} \eta_{\rho \nu} \eta_{\sigma \tau})$  $\Gamma_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda} + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{\rho\sigma})$  $+ 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\right]$  $\sqrt{2}$ 

*,* (1.6)

 The choice of terms is not completely unique since momentum conservation may be used to replace a given terms. We give here when we believe (but have not proved} to be expressions containing the smallest number of terms.

 $st$ 

 $\mathcal{L}(\mathbf{e})$ 

I' (p -p'~" ~"~"")j, (26)

 $\begin{align} \eta^{n\star}\eta^{*s} + \lambda \eta^{*s} + \lambda \eta^{*s} + \end{align}$  $-\frac{1}{2}P_{12}(p \cdot p' \eta^{r\sigma} \eta^{r\rho} \eta^{k\sigma} \eta^{r\rho}) - \frac{1}{2}P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma} \eta^{k\sigma}) + \frac{1}{2}P_{12}(p^e p^e \eta^{r\gamma} \eta^{w\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{k\sigma} \eta^{r\sigma}) - P_{13}(p^e p' \eta^{k\sigma} \eta^{r\sigma} \eta^{k\sigma}) - P_{14}(p^e p' \eta^{$  implies a symmetrization in each pair of graviton Lorentz indices *µ* ¡ *fl*, *‹* ¡ *⁄* and *‡* ¡ *·* , Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip-symL —l& (p p'~""0"~'"~'")—l~ (p'p'~""~'"~'")—:I' (p'p'"~"—'I'"~'")+lI' (p p'~"'~"'~'"~'") where we set *Ÿ* = 2, *p<sup>i</sup>* are the momenta of the three gravitons, *÷µ‹* is the flat metric, "Sym"  $+\frac{1}{4}P_6(p\cdot p' \eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{4}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{2}P_6(p^{\sigma}p'\mu\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\frac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$  $+{1\over 4}P_{24}(\rho\cdot p' \eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma} \rho^{\tau}\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+{1\over 4}P_{12}(\rho^{\rho} \rho^{\prime\lambda}\eta^{\mu\sigma}\eta^{\sigma\tau}\eta^{\nu\kappa})+{1\over 2}P_{24}(\rho^{\sigma} \rho^{\prime}\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$ symmetry of the vertex. In total, the vertex has of the order of 100 terms. This generally  $-\tfrac12 P_{12} (p\cdot p'\eta^{\sigma} \eta^{\tau\rho} \eta^{\lambda\mu} \eta^{\iota\epsilon}) -\tfrac12 P_{12} (p^\sigma p'\mu\eta^{\tau\rho} \eta^{\lambda\kappa} \eta^{\iota\epsilon}) +\tfrac12 P_{12} (p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\nu} \eta^{\iota\epsilon}) -\tfrac12 P_{24} (p\cdot p'\eta^{\mu\nu} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\iota\sigma})$  $\mathcal{H}_{p_{\mathfrak{a}}(p+p' \eta^{*}\rho} \eta^{\lambda\sigma} \eta^{\tau\iota}\eta^{\kappa\mu}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\mu\nu} \eta^{\tau\iota}\eta^{\kappa\lambda}) - \frac{1}{2} P_{12}(p\cdot p' \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\sigma\iota}\eta^{\tau\kappa}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\tau\lambda} \eta^{\mu\iota}\eta^{\nu\kappa})$  $\begin{array}{ccccc} & -P_6(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s r})-P_{24}(p^s p'^s \eta^{s s} \eta^{s t} \eta^{s s})-P_{12}(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s s})+2P_6(p\cdot p' \eta^{s s} \eta^{s s} \eta^{s s}) \end{array}$ 

 $A(H_1, H_2, H_3)$  is the three-graviton vertex in matrix  $A(H_1, H_2, H_3)$  is the conflict with  $\frac{1}{2}$  $\left( \begin{array}{cc} 1 & 2 & 3 \end{array} \right)$  $f \leftarrow f$ The reason why the three-graviton vertex is so complicated is that it is gauge-dependent.<sup>1</sup>  $\sqrt{19}\sqrt{4}$  $t(T - H - H^+H^+)$  direct perturbative gravity calculations of  $T$  $\mathcal{L}$ in an  $\mathcal{L}$   $\mathcal{S}$ polarization vectors of  $\mathbf{A} \times \mathbf{r} = \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r}$  $A(H_1 H_2 H_3 H_4)$  = ordered partial tree amplitudes are the coecients of basis elements once the amplitude's  $A(H_1^- H_2^- H_3^+$  $A(H_1^-H_2^-H_3^+H_4^+$  $st$  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **A**( $H_1^ H_2^ H_3^+$ ) =  $\frac{\langle 12 \rangle^6}{\langle 13 \rangle^2}$  $\langle 13 \rangle^2$ +(p.p" p'.p )4'+p'P'4—& 'j (2 g)  $(19)$  $A(H_1^- H_2^- H_3^+ H_4^+) = \frac{\langle 12 \rangle^4}{ct}$  $A(H_1^{\rm -} H_2^{\rm -} H_3^{\rm +})$  $\frac{1}{2}$   $A(H<sup>-</sup> H<sup>-</sup> H<sup>+</sup> H<sup>+</sup>)$  $\ldots$   $\ldots$ 

> the three-vertex with on-shell conditions imposed on external legs, by demanding that the <sup>1</sup>While somewhat less complicated than the three-graviton vertex, the three-gluon vertex is also gauge-

> $\mu$  and appear to be a set appear to the corresponding to the correspondi  $v_{\text{c}} = -1.$  These considerations suggests that gravity is much more complicated that gravity is much more complicated that gravity is much more complicated that  $(12)^{\circ}$





Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip- $F_{A(H_1 H_2 H_3 H_4)} = -\frac{1}{st}$  in a  $\mathcal{A}(1 \cdot 2 \cdot 3 \cdot 4 \cdot ) = 0$  $\mathcal{L}$ 3) Get, the Feynman rules 4 ) ( J 4) Obtain the amplitude  $\mathcal{A}(1^+2^+3^+4^+) = 0$ 2) Get the Lagrangian (GR) 1) Define your fields (h<sub>μν</sub>)



#### $\dot{a}$  *la* Feynman !  $\overline{a}$  Launman is an arbitrary multiplicity. The complexity of each individual interaction term is a contraction term in the complexity of each interaction term is a contraction term in the complexity of the complexity per striking than the three-gravitons in  $\mathbf{C}$ interaction. In the standard de Donder gauge, *ˆ‹h‹ <sup>µ</sup>* = <sup>1</sup> <sup>2</sup> *ˆµh‹*  $I_a$   $E_a$ evnm: l 6 with polynomial  $\mathbf{r}$ à la Feynman! (2)

 $\Gamma$  $\sim$  $A(H_1^- H_2^- H_3^+$ 

 $A(H_1^- H_2^- H_3^+)$  =

 $\sum_{c}^{a} 3$ 

*c*

3

 $\mathcal{F}$  is a few  $\mathcal{F}$  gauge three- and four-point vertices in a  $\mathcal{F}$ 

*µ*

 $\mathcal{F}$  and interactions in the interactions in a Feynman interaction interactions in a Feynman interactions in a Feynman in

 $A(H_1^{\rm -} H_2^{\rm -} H_3^{\rm +})$  $\frac{1}{2}$   $\frac{1}{2}$ 

polarization vectors of  $\mathbf{A} \times \mathbf{r} = \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r} + \mathbf{r} \mathbf{r}$  $A(H_1 H_2 H_3 H_4)$  = ordered partial tree amplitudes are the coecients of basis elements once the amplitude's

color factors are expressed in the trace color basis, and the coupling *g* is set to unity. They are  $A(H<sup>-</sup> H<sup>-</sup> H<sup>+</sup> H<sup>+</sup>)$  $\ldots$   $\ldots$ 

م<br>محل

1

 $\stackrel{\shortparallel}{\nu}$ 

1

*a µ*

2

 $\frac{e^{v}}{a}$ 

*a µ*

1

 $\nu$   $\epsilon$   $\beta$   $\sigma$ 

*b*

 $\begin{array}{cc} \nu \rightarrow & \sigma \rightarrow & \sigma \end{array}$ 

*b*

2

 $\epsilon$  $\bigotimes \limits^{\textstyle \bigotimes \limits_{3\big(P_1\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\nu \lambda} \eta_{\sigma \tau} \big) \ - \ \frac{1}{2} P_6(p_{1\nu} p_{1\lambda} \eta_{\mu \rho} \eta_{\sigma \tau}) \ + \ \frac{1}{2} P_3(p_{1}\ \cdot \ p_2 \eta_{\mu \nu} \eta_{\rho \lambda} \eta_{\sigma \tau} \big)$  $\mathcal{O}(\mathcal{O})$   $\mathcal{O}_{\mathcal{O}}$   $\left( \frac{1}{2} \cdot p_2 \eta_{\mu \rho} \eta_{\nu \sigma} \eta_{\lambda \tau} \right) + 2 P_3 (p_{1\nu} p_{1\tau} \eta_{\mu \rho} \eta_{\lambda \sigma}) - P_3 (p_{1\lambda} p_{2\mu} \eta_{\rho \nu} \eta_{\sigma \tau})$  $\Gamma_{1\sigma} p_{2\tau} \eta_{\mu\nu} \eta_{\rho\lambda} + P_6(p_{1\sigma} p_{1\tau} \eta_{\mu\nu} \eta_{\rho\lambda}) + 2P_6(p_{1\nu} p_{\rho\sigma})$  $+ 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\right]$  $\sqrt{2}$ 

 $+ \frac{1}{4} Pa_3(p\cdot p'\eta'''\eta'''\eta''') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{2} Pa_3(p'\cdot p'\eta'''\eta'''\eta''') - \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta'''\eta'') + \frac{1}{4} Pa_3(p'\cdot p'\eta'''\eta$  $-\frac{1}{2}P_{12}(p \cdot p' \eta^{r\sigma} \eta^{r\rho} \eta^{k\sigma} \eta^{r\rho}) - \frac{1}{2}P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma} \eta^{k\sigma}) + \frac{1}{2}P_{12}(p^e p^e \eta^{r\gamma} \eta^{w\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{r\sigma} \eta^{k\sigma}) - P_{12}(p^e p' \eta^{k\sigma} \eta^{r\sigma}) - P_{13}(p^e p' \eta^{k\sigma} \eta^{r\sigma} \eta^{k\sigma}) - P_{14}(p^e p' \eta^{$  implies a symmetrization in each pair of graviton Lorentz indices *µ* ¡ *fl*, *‹* ¡ *⁄* and *‡* ¡ *·* , Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip-symL —l& (p p'~""0"~'"~'")—l~ (p'p'~""~'"~'")—:I' (p'p'"~"—'I'"~'")+lI' (p p'~"'~"'~'"~'") where we set *Ÿ* = 2, *p<sup>i</sup>* are the momenta of the three gravitons, *÷µ‹* is the flat metric, "Sym"  $+\frac{1}{4}P_6(p\cdot p' \eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{4}P_{12}(p^{\sigma}p^{\tau}\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\frac{1}{2}P_6(p^{\sigma}p'\mu\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\frac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$  $+{1\over 4}P_{24}(\rho\cdot \rho'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+{1\over 4}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\mu\rho}\eta^{\tau\kappa}\eta^{\iota\kappa})+{1\over 2}P_{24}(\rho^{\sigma}\rho'\rho'\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$  $-\tfrac12 P_{12} (p\cdot p'\eta^{\sigma} \eta^{\tau\rho} \eta^{\lambda\mu} \eta^{\iota\epsilon}) -\tfrac12 P_{12} (p^\sigma p'\mu\eta^{\tau\rho} \eta^{\lambda\kappa} \eta^{\iota\epsilon}) +\tfrac12 P_{12} (p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\nu} \eta^{\iota\epsilon}) -\tfrac12 P_{24} (p\cdot p'\eta^{\mu\nu} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\iota\sigma})$  $\mathcal{H}_{p_{\mathfrak{a}}(p+p' \eta^{*}\rho} \eta^{\lambda\sigma} \eta^{\tau\iota}\eta^{\kappa\mu}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\mu\nu} \eta^{\tau\iota}\eta^{\kappa\lambda}) - \frac{1}{2} P_{12}(p\cdot p' \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\sigma\iota}\eta^{\tau\kappa}) - P_{12}(p^{\sigma} p^{\rho} \eta^{\tau\lambda} \eta^{\mu\iota}\eta^{\nu\kappa})$ *V abc*  $\begin{array}{ccccc} & -P_6(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s r})-P_{24}(p^s p'^s \eta^{s s} \eta^{s s})-P_{12}(p^s p'^s \eta^{s s} \eta^{s s} \eta^{s s})+2P_6(p\cdot p' \eta^{s s} \eta^{s s} \eta^{s s}) \end{array}$ 

 $\mu$ <sup>o</sup>

the three-vertex with on-shell conditions imposed on external legs, by demanding that the <sup>1</sup>While somewhat less complicated than the three-graviton vertex, the three-gluon vertex is also gauge-





Figure 1: Gauge theories have three- and four-point vertices in a Feynman diagrammatic descrip- $\mathcal{L}$ 3) Get, the Feynman rules 4 ) ( J 4) Obtain the amplitude 2) Get the Lagrangian (GR) 1) Define your fields (h<sub>μν</sub>)

 $v_{\text{c}} = -1.$  These considerations suggests that gravity is much more complicated that gravity is much more complicated that gravity is much more complicated that  $(12)^{\circ}$  $A(H_1, H_2, H_3)$  is the three-graviton vertex in the three-graviton vertex in the three-gravitons of conflict with  $H_1$  $\frac{1}{2}$  in Eq. (1.4). The left and right sets visible into left and right sets visible in Eq. (1.4). The set of  $\frac{1}{2}$ first term in Eq. (1.6), for example, contains a factor *÷µfl* which explicitly contracts a left The reason why the three-graviton vertex is so complicated is that it is gauge-dependent.<sup>1</sup>  $(19)4[31]4$  $t(T-T-T+H+T)$  and  $t(T-T+H+T+T)$  $\lim_{\epsilon \to 0} \frac{1}{1} \lim_{\epsilon \to 0} \frac{1}{1} \lim_{\epsilon \to 0} \frac{1}{1} \lim_{\epsilon \to 0} \frac{1}{1}$  $s\alpha$  $A(H_1^- H_2^- H_3^+ H_4^+) \quad = \quad \frac{\langle 12 \rangle^4 [34]^4}{stu}$  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  $\mathcal{L}$ +(p.p" p'.p )4'+p'P'4—& 'j (2 g)  $A(H_1^- H_2^- H_3^+ H_4^+)$  =  $\frac{\langle 12 \rangle^4 [34]^4}{stu}$  $\binom{+}{3}$  =  $\frac{\langle 12 \rangle^6}{\langle 13 \rangle^2/3^6}$  $\langle 13\rangle^2 \langle 32\rangle^2$ 

*,* (1.6)

 The choice of terms is not completely unique since momentum conservation may be used to replace a given terms. We give here when we believe (but have not proved} to be expressions containing the smallest number of terms.

I' (p -p'~" ~"~"")j, (26)

1

*stu*

 $F_A(H_1 H_2 H_3 H_4) = \frac{1}{\pi}$  in  $\mathcal{A}(1 \quad 2 \quad 3 \quad 4 \quad ) \quad \mathcal{X} =$  $A(1-2+3-4+)\propto$ 







• Under Little group 
$$
\left\{ \begin{aligned} |p\rangle_{\alpha} &\to e^{-i\theta/2}|p\rangle_{\alpha} &\text{Helicity - 1/2} \\ |p]_{\dot{\alpha}} &\to e^{i\theta/2}|p]_{\dot{\alpha}} &\text{Helicity + 1/2} \end{aligned} \right.
$$

*pµ*  $\int$   $\alpha$  $p$  $\mu_{\alpha} = \mu_{\alpha} \mu_{\beta}$ • Lorentz invariance:  $\langle pq \rangle \equiv \langle p|^\alpha |q\rangle_\alpha$  ;  $[pq] \equiv [p|_\alpha |q]^\dot{\alpha}$  $\dot{\alpha}$ 



only by *g*'s that, up to change of basis, are only non-vanishing for *a*<sup>1</sup> = *a*<sup>2</sup> = *a*3, i.e. which **dim analysis & single-pole structure**

# **II. EFT (EFfective Theories) from on-shell amplitudes**



#### specified later, that the this sector preserves lepton and baryon number. By integrating out this sector preserves lepton and baryon number. By integrating out this sector preserves lepton and baryon number. By integratin **section and performing and performing and the SM**  $\alpha$  **over**  $\alpha$  or  $\alpha$ **Ordinary EFT approach**

obtain an early control of local operators: which is a strong of local

$$
\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2} \mathcal{L} \left( \frac{D_\mu}{\Lambda} , \frac{g_H H}{\Lambda} , \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}} , \frac{g F_{\mu\nu}}{\Lambda^2} \right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots
$$

#### specified later, that the this sector preserves lepton and baryon number. By integrating out this sector preserves lepton and baryon number. By integrating out this sector preserves lepton and baryon number. By integratin **section and performing and performing and the SM**  $\alpha$  **over**  $\alpha$  or  $\alpha$ **Ordinary EFT approach**

obtain an early control of local operators: which is a strong of local

✓*D<sup>µ</sup>*



# **An important gain in simplicity:**

*the power of being on-shell!*



Ghosts, Golstones,... only physical states  $(p^2=0)$ (p2≠0) <sup>䡿</sup> **definite helicity**  $(h = \mp)$ 



#### by the SM gauge boson interactions. For the SM gauge boson in the SM gauge boson in the SM gauge boson in the a<br>Final state and the adjoint the adjoint to the adjoint the adjoint to the adjoint to the adjoint to the adjoin Expansion: ⟨ij⟩/Λ<sup>2</sup> , [ij]/Λ2

# **SM "Building-blocks":**



…

#### *E*<sup>2</sup>*/*⇤<sup>2</sup> (up to complex conjugation): to the structure constants. **At O(E2/Λ2):**only arise for non-abelian gauge bosons, in which case the full amplitude is proportional amplitude is proportional

For a generic theory of (*i*) vector bosons *V<sup>±</sup>* with helicity *h* = *±*1, (*ii*) Weyl fermions m = number of external state that is a new have the following building building building building building build<br>The helicity of the amplitudes and order the following building building building building building building  $n = number of external states$  $\mathbf{A} \mathbf{t} \mathbf{O}(\mathbf{E}^2/\Lambda^2)$ :  $\vert \mathbf{h} \vert = \mathbf{h}$ elicity of the amplitude possibility. It is important to notice that Eq. (2) is antisymmetric under *i* \$ *j*, and can

$$
\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle
$$
   
h=-3

that has *h* = 3. It is quite straightforward to see that this is the only amplitude at

$$
\begin{cases}\n n=3 \\
h=-3\n\end{cases}
$$

 $\int h=-2$ 

**}** 

**}** 

n=4

 $h = -2$ 

n=4

 $h=0$ 

$$
\mathcal{A}_{F^{2}\phi^{2}}(1_{V_{-}}, 2_{V_{-}}, 3_{\phi}, 4_{\phi}) = \frac{C_{F^{2}\phi^{2}}}{\Lambda^{2}} \langle 12 \rangle^{2},
$$
\n
$$
\mathcal{A}_{F\psi^{2}\phi}(1_{V_{-}}, 2_{\psi}, 3_{\psi}, 4_{\phi}) = \frac{C_{F\psi^{2}\phi}}{\Lambda^{2}} \langle 12 \rangle \langle 13 \rangle,
$$
\n
$$
\mathcal{A}_{\psi^{4}}(1_{\psi}, 2_{\psi}, 3_{\psi}, 4_{\psi}) = (C_{\psi^{4}} \langle 12 \rangle \langle 34 \rangle + C_{\psi^{4}}' \langle 13 \rangle \langle 24 \rangle) \frac{1}{\Lambda^{2}}
$$

h*ij*i[*ji*]=2*p<sup>i</sup> ·p<sup>j</sup>* =0(*i, j* = 1*,* 2*,* 3), that forces the vanishing of either all [*ij*], in which case

$$
\mathcal{A}_{\Box\phi^{4}}(1_{\phi}, 2_{\phi}, 3_{\phi}, 4_{\phi}) = (C_{\Box\phi^{4}}\langle 12\rangle[12] + C'_{\Box\phi^{4}}\langle 13\rangle[13])\frac{1}{\Lambda^{2}}
$$
\n
$$
\mathcal{A}_{\psi\bar{\psi}\phi^{2}}(1_{\psi}, 2_{\bar{\psi}}, 3_{\phi}, 4_{\phi}) = \frac{C_{\psi\bar{\psi}\phi^{2}}}{\Lambda^{2}}\langle 13\rangle[23],
$$
\n
$$
\mathcal{A}_{\psi^{2}\bar{\psi}^{2}}(1_{\psi}, 2_{\psi}, 3_{\bar{\psi}}, 4_{\bar{\psi}}) = \frac{C_{\psi^{2}\bar{\psi}^{2}}}{\Lambda^{2}}\langle 12\rangle[34].
$$

#### **At O(E<sup>2</sup>/Λ<sup>2</sup>):** only arise for non-abelian gauge bosons, in which case the full amplitude is proportional amplitude is proportional

#### For a generic theory of (*i*) vector bosons *V<sup>±</sup>* with helicity *h* = *±*1, (*ii*) Weyl fermions m = number of external state that is a new have the following building building building building building build<br>The helicity of the amplitudes and order the following building building building building building building  $n = number of external states$  $\mathbf{A} \mathbf{t} \mathbf{O}(\mathbf{E}^2/\Lambda^2)$ :  $\vert \mathbf{h} \vert = \mathbf{h}$ elicity of the amplitude possibility. It is important to notice that Eq. (2) is antisymmetric under *i* \$ *j*, and can

$$
\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle
$$
   
h=-3

h*ij*i[*ji*]=2*p<sup>i</sup> ·p<sup>j</sup>* =0(*i, j* = 1*,* 2*,* 3), that forces the vanishing of either all [*ij*], in which case

$$
\begin{cases}\n n=3 \\
h=-3\n\end{cases}
$$

$$
A_{F^{2}\phi^{2}}(1_{V_{-}}, 2_{V_{-}}, 3_{\phi}, 4_{\phi}) = \frac{C_{F^{2}\phi^{2}}}{\Lambda^{2}} \langle 12 \rangle^{2},
$$
\n
$$
A_{F\psi^{2}\phi}(1_{V_{-}}, 2_{\psi}, 3_{\psi}, 4_{\phi}) = \frac{C_{F\psi^{2}\phi}}{\Lambda^{2}} \langle 12 \rangle \langle 13 \rangle,
$$
\n
$$
A_{\psi^{4}}(1_{\psi}, 2_{\psi}, 3_{\psi}, 4_{\psi}) = (C_{\psi^{4}} \langle 12 \rangle \langle 34 \rangle + C_{\psi^{4}}' \langle 13 \rangle \langle 24 \rangle) \frac{1}{\Lambda^{2}}
$$
\n
$$
A_{\Box\phi^{4}}(1_{\phi}, 2_{\phi}, 3_{\phi}, 4_{\phi}) = (C_{\Box\phi^{4}} \langle 12 \rangle [12] + C_{\Box\phi^{4}}' \langle 13 \rangle [13]) \frac{1}{\Lambda^{2}}
$$
\n
$$
A_{\psi\bar{\psi}\phi^{2}}(1_{\psi}, 2_{\bar{\psi}}, 3_{\phi}, 4_{\phi}) = \frac{C_{\psi\bar{\psi}\phi^{2}}}{\Lambda^{2}} \langle 13 \rangle [23],
$$

$$
\mathcal{A}_{\Box\phi^{4}}(1_{\phi}, 2_{\phi}, 3_{\phi}, 4_{\phi}) = (C_{\Box\phi^{4}}\langle 12\rangle[12] + C_{\Box\phi^{4}}^{\prime}\langle 13\rangle[13])\overline{\Lambda^{2}}
$$
\n
$$
\mathcal{A}_{\psi\bar{\psi}\phi^{2}}(1_{\psi}, 2_{\bar{\psi}}, 3_{\phi}, 4_{\phi}) = \frac{C_{\psi\bar{\psi}\phi^{2}}}{\Lambda^{2}}\langle 13\rangle[23],
$$
\n
$$
\mathcal{A}_{\psi^{2}\bar{\psi}^{2}}(1_{\psi}, 2_{\psi}, 3_{\bar{\psi}}, 4_{\bar{\psi}}) = \frac{C_{\psi^{2}\bar{\psi}^{2}}}{\Lambda^{2}}\langle 12\rangle[34].
$$

$$
\mathcal{A}_{\psi^2\phi^3}(1_{\psi}, 2_{\psi}, 3_{\phi}, 4_{\phi}, 5_{\phi}) = \frac{C_{\psi^2\phi^3}}{\Lambda^2} \langle 12 \rangle \qquad \qquad \text{n=5}
$$

$$
\mathcal{A}_{\phi^{6}}(1_{\phi}, 2_{\phi}, 3_{\phi}, 4_{\phi}, 5_{\phi}, 6_{\phi}) = \frac{C_{\phi^{6}}}{\Lambda^{2}}
$$
\nn=6\nh=0

# **III. EFT renormalization via amplitude methods**



# **Of great importance:**

In particular, the **RG-running** of the (Wilson) coefficients of the amplitudes (the anomalous dimensions) needed for making contact with low-energy experiments

*e.g. for muon g-2, they must "run" down to E~m*μ







$$
d_{4}I_{4} + \sum_{\mathcal{A}_{j}} \frac{d_{4}I_{4}}{\sigma_{4}^{2}I_{2}} + \sum_{\mathcal{A}_{j}} \frac{d_{4}I_{3}}{\sigma_{4}^{2}I_{2}} + \sum_{\mathcal{A}_{j}} \frac{d_{4}I_{4}}{\sigma_{4}^{2}I_{2}} + \text{rational} \frac{d_{4}I_{4}}{\sigma_{4}I_{2}} + \text{rational} \frac{d_{4}I_{4}}{\sigma_{4}I_{2}} + \text{rational} \frac{d_{4}I_{4}}{\sigma_{4
$$

#### $\frac{1}{2}$  gration  $\frac{2}{3}$  sum over internal states phase-space integration  $\alpha$  sum over internal states phase-space integration & sum over internal states

*<sup>d</sup>*3*I*<sup>3</sup> <sup>+</sup>



1505.01844 (also by susy techniques:1412.7151) (up to self-renormalization). This is equivalent to calculate the anomalous dimension of the coecient **C**<sub>*F*</sub>  $\frac{1}{2}$  2, defined in Eq. (4),  $\frac{1}{2}$   $\frac{1}{2}$ 

#### *u* **No 4-fermion (ψ** $\overline{\psi}^{\mu}$ **ψ)<sup>2</sup> corrections to dipoles Ho 4-fermion (ψγ<sup>μ</sup>ψ)<sup>2</sup> corrections to dipoles**



 $F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi\,H$ 



1505.01844 (also by susy techniques:1412.7151) (up to self-renormalization). This is equivalent to calculate the anomalous dimension of the coecient **C**<sub>*F*</sub>  $\frac{1}{2}$  2, defined in Eq. (4),  $\frac{1}{2}$   $\frac{1}{2}$ 

#### *u* **No 4-fermion (ψ** $\overline{\psi}^{\mu}$ **ψ)<sup>2</sup> corrections to dipoles Ho 4-fermion (ψγ<sup>μ</sup>ψ)<sup>2</sup> corrections to dipoles**



 $\frac{1}{2}$   $F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi\,H$ 

that the *JJ*-operators *<sup>O</sup>*<sup>4</sup>*<sup>f</sup>* and *<sup>O</sup><sup>f</sup>* do not renormalize the loop-operators. For $\alpha A_{\text{H}}$  =  $\alpha A_{\text{H}}$  (*D*  $\alpha A_{\text{H}}$  (1)  $\alpha A_{\text{H}}$  (1)  $\alpha A_{\text{H}}$  (*A*<sup>*i*</sup> 2<sup>*i*</sup> 2<sub>iiii</sub> 4<sub>ii</sub>)  $d\mathcal{H}W = \frac{1}{4\pi^3}\int d\mathbf{L}d\mathbf{L}$  is  $\mathcal{H}_{luge}(\mathbf{1}_e, \mathbf{2}_l, \mathbf{0}_e, \mathbf{4}_{\bar{l}}) \times \mathcal{H}_{\text{SM}}(\mathbf{4}_e, \mathbf{0}_l, \mathbf{0}_e)$  $\gamma {\cal A}_{WHle} = - \frac{1}{4 \pi}$  $4\pi^3$ z<br>Zanada<br>Zanada  $d\text{LIPS} \mathcal{A}_{luge}(1_{e}, 2_{l}, 3'_{\overline{e}}, 4'_{\overline{l}}) \times \mathcal{A}_{\text{SM}}(4'_{e}, 3'_{l}, 3_{W_{-}^a}, 4_{H^{\dagger}})$ 



1505.01844 (also by susy techniques:1412.7151) (up to self-renormalization). This is equivalent to calculate the anomalous dimension of the coecient **C**<sub>*F*</sub>  $\frac{1}{2}$  2, defined in Eq. (4),  $\frac{1}{2}$   $\frac{1}{2}$ 

#### *u* **No 4-fermion (ψ** $\overline{\psi}^{\mu}$ **ψ)<sup>2</sup> corrections to dipoles Ho 4-fermion (ψγ<sup>μ</sup>ψ)<sup>2</sup> corrections to dipoles**



#### **No p2H4 corrections to Hγγ** *u u <sup>A</sup><sup>µ</sup>*  $F_{\alpha\beta}F^{\alpha\beta}h^2$ *<sup>µ</sup> A*  $(H^\dagger D_\mu H)^2$ e.g. **-1 -1**

Figure 1: *<sup>A</sup> potential contribution from <sup>O</sup><sup>q</sup> to <sup>O</sup><sup>D</sup>.*

●



#### **No p2H4 corrections to Hγγ** *u u <sup>A</sup><sup>µ</sup>*  $F_{\alpha\beta}F^{\alpha\beta}h^2$ *<sup>µ</sup> A*  $(H^\dagger D_\mu H)^2$ e.g. **-1 -1**

Figure 1: *<sup>A</sup> potential contribution from <sup>O</sup><sup>q</sup> to <sup>O</sup><sup>D</sup>.*

●

#### **No p2H4 corrections to Hγγ** ● *u u <sup>A</sup><sup>µ</sup>*  $F_{\alpha\beta}F^{\alpha\beta}h^2$ *<sup>µ</sup> A*  $(H^\dagger D_\mu H)^2$ e.g. **-1 -1**

# Figure 1: *<sup>A</sup> potential contribution from <sup>O</sup><sup>q</sup> to <sup>O</sup><sup>D</sup>. Potential**Absent* **in the SM**  $h_{total} = -2$

# But the on-shell methods also tell us about the non-zero result

**Contributions to dipoles from Feynman** approach:



## very **different** contributions

# But the on-shell methods also tell us about the non-zero result

 $\sqrt{u}$   $\sqrt{u}$   $\sqrt{u}$   $\sqrt{u}$   $\sqrt{u}$ From on-shell approach:  $\gamma \mathcal{A}_{i} \sim \sum \mathcal{A}_{j} \mathcal{A}_{SM}$ 





# But the on-shell methods also tell us about the non-zero result

From on-shell approach:  $\gamma A_i \sim \sum A_j A_{\text{SM}}$  $\gamma A_{\mathbf{i}} \sim \sum A_{\mathbf{j}} A_{\mathbf{S}\mathbf{M}}$ 



 $\mathcal{A}_{\rm SM}(1_{\bar{w}}, 2_{\bar{w}}, 3_{V_{-}}, 4_{H^+})$ 

from the same SM amplitude! ⇤ from the same SM amplitude!



from the same SM amplitude! ⇤ from the same SM amplitude!

# But there is more to say by **angular-momentum decomposition (partial-waves)**







it is in the called to recall the correct of the c<br>in the correct of th<br> anguian - ... **m** ☞ **angular-momentum selection rules:**

see also arXiv:2001.044 see also arXiv:2001.04481

 write four-fermion $\blacktriangleright$   $\blacktriangleright$   $\blacktriangleright$   $\blacktriangleleft$   $\mathsf{cannot}$  contribute to dipoles *i* must be completed a loop of the complete of the complete  $\mathbf{r}$ write four-fermion<br>La latina mente de contribute de operators, such as (*q*<sup>†</sup>*q*†*p* $\alpha$ **<sup>***n***</sup>** Auphous with  $j$ <sup>2</sup> cannot contribute to urpores **Amplitudes with J≠1cannot contribute to dipoles**

#### **Anomalous Dimensions as a product of partial-waves**

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

 $\gamma_i \sim a^J_{\rm SM} a^J_{\rm BSM}$ • 1/Λ<sup>2</sup> amplitude



2005.06983 2005.12917 2112.12131 *DH* ! *<sup>F</sup>µ*⌫ ¯*<sup>H</sup>* which is the only one







J. Elias-Miro, C. Fernandez, M. Gümüs, AP 2112.12131

 $(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{e}_{R}\gamma^{\mu}\mu_{R})$  affects  $\mu \rightarrow e\gamma$  at the two-loop level: **䡿 Z→μe**  $\forall x \rightarrow \mu e$  of contributions to be the dominant ones because they are proportional to be the dominant ones because they are proportional to be the dominant ones because they are proportional to be the set of contribution o



where on the l.h.s. the gauge boson must be attached in all possible ways to the Higgs (dashed product of tree-level amplitudes



# **Difficult** in general, **but** simplifies a lot for BSM calculations, where new physics scale **M >> Eexp**

**New insights** from the **amplitude** method!







N. Arkani-Hamed, K. Harigaya 2106.01373

**Finite terms to g-2**

# **No** contribution *O*(1/M2) to **dipoles from** a **heavy singlet + doublet fermion**:



**even under S↔L** 

**Finite terms to g-2**

# **No** contribution *O*(1/M2) to **dipoles from** a **heavy singlet + doublet fermion**:



**Finite terms to g-2**

# **No** contribution *O*(1/M2) to **dipoles from** a **heavy singlet + doublet fermion**:

![](_page_48_Figure_2.jpeg)

![](_page_49_Picture_0.jpeg)

L. Delle Rosse, B. von Harling, AP in 2201.10572

Following the same argument, more zeros can be found:

• **Scalar + heavy doublet + charged fermion**:

![](_page_49_Figure_4.jpeg)

*relevant 2-cuts.* • **Beyond g-2: Zeros in h**

![](_page_49_Figure_6.jpeg)

![](_page_50_Picture_0.jpeg)

![](_page_50_Picture_1.jpeg)

- Allows to construct **BSM without Lagrangians**
- Calculation of loop effects:

# **Simpler with easy recycling**

- ☞ many "emergent" **selection rules**
- ☞ many relations **between anomalous dimensions**

**where Feynman approach is quite obscure**

*A lot to do! Stay Tuned!*