

Testing General Relativity with Cosmological Large Scale Structure Observations

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Center for Astroparticle Physics
GENEVA

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- 1 Introduction
- 2 Very large scale galaxy surveys
- 3 The angular power spectrum and the correlation function of galaxy number density fluctuations
 - The transversal power spectrum
- 4 Measuring the lensing potential
- 5 Measuring the growth rate of perturbations
- 6 Conclusions

Einstein's theory of gravity has been tested in many ways and passed all the tests with flying colors:

- Light deflection
- Perihel advance of mercury & many other binary systems
- Shapiro time delay
- ...
- Gravitational waves

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All these observations test GR on solar system size scales. Furthermore, they essentially test vacuum solutions of Einstein's equations,

$$R_{\mu\nu} = 0.$$

Can we also test these equations with matter,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi GT_{\mu\nu} \quad ?$$

The Friedmann-Lemaître solution of cosmology is a non-vacuum solution of Einstein's equation:

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \quad z + 1 = a_0/a(t)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \left(\rho + \frac{\Lambda}{8\pi G} \right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3P - \frac{\Lambda}{4\pi G} \right)$$

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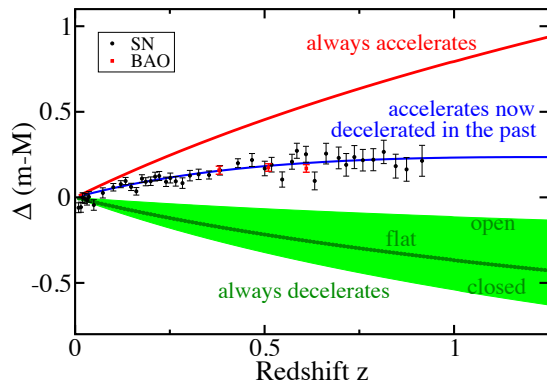
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Have we 'tested' these equations with cosmological observations?

What have we truly measured:

$$F(z) = \frac{L}{4\pi d_L(z)^2}$$

$$d_L(z) = (1+z)\chi_K \left(\int_0^z \frac{dz'}{H(z')} \right), \quad \chi_K(r) = \frac{\sin(\sqrt{K}r)}{\sqrt{K}}$$



$$m - M \propto \log d_L$$

$$\Delta(m - M) \propto \log(d_L/d_L^{\text{Milne}})$$

$$\Omega_\Lambda \simeq 0.7$$

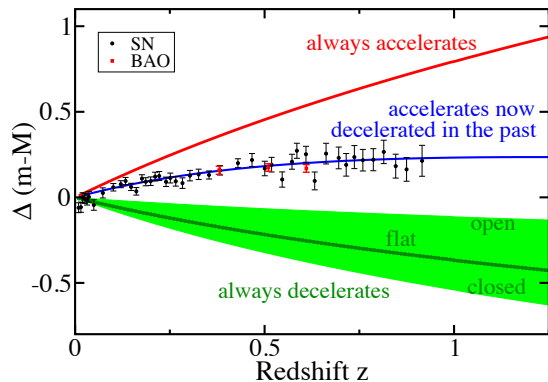
$$\Omega_m \simeq 0.3$$

$$\Omega_X = \rho_X/\rho_c$$

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Compilation by Huterer & Shafer '17.

Binned from 870 SNe Ia (black) and 3 BAO points (from BOSS DR12, red).



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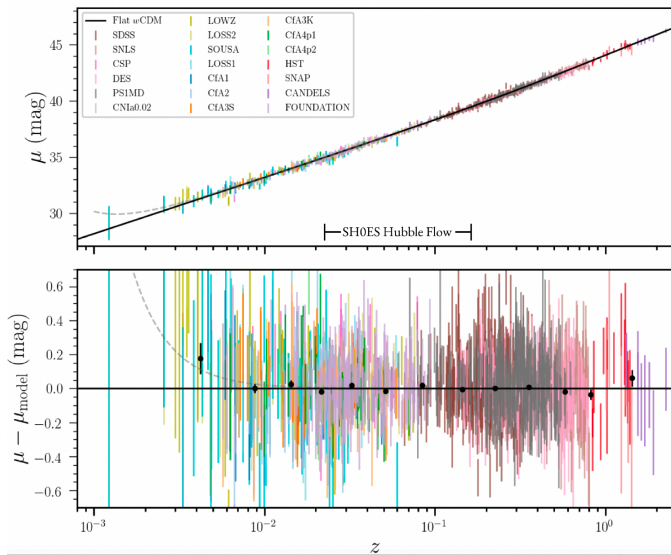
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NO !

We have 'postulated' the existence of dark matter and dark energy to fit this data.

Pantheon+ (1550 Type Ia supernovae)



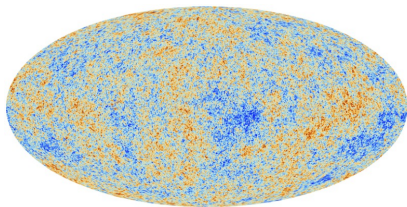
Compilation 1701 light curves from 18 different samples, Brout et al. 2022.

In this talk I shall show that with the help of **clustering observations**, i.e. using the fact that the Universe is not perfectly homogeneous and isotropic, we can actually test Einstein's equations to some extent. . .

We shall do this using the statistics of the galaxy distribution. In this talk I shall only consider the 2-point function and its power spectrum, but also higher statistics are very relevant especially in the non-linear regime, and they are very sensitive to the theory of gravity.

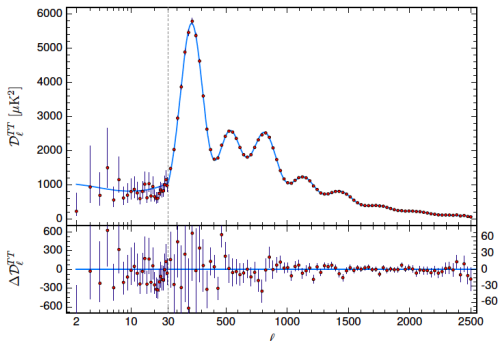
The CMB

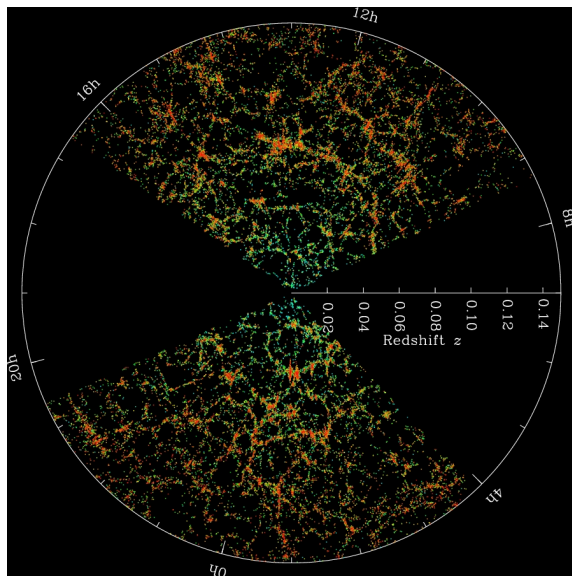
CMB sky as seen by Planck



$$T(\mathbf{n}) = \sum a_{\ell m} Y_{\ell m}(\mathbf{n})$$
$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell\ell'} \delta_{mm'} C_{\ell}$$
$$D_{\ell} = \ell(\ell + 1) C_{\ell} / (2\pi)$$

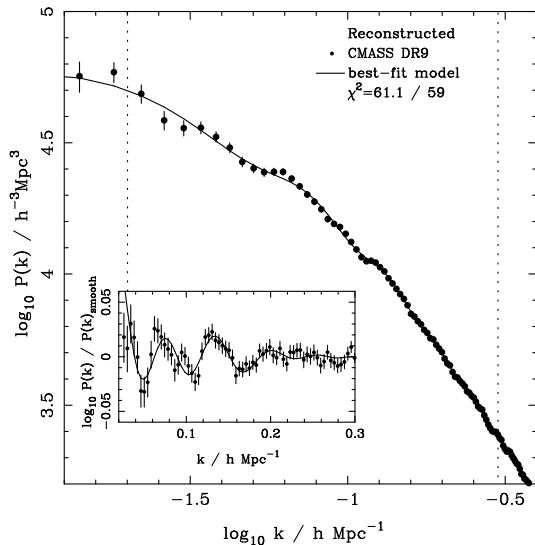
The Planck Collaboration:
Planck results 2018
[1807.06209]





M. Blanton and the Sloan Digital Sky Survey Team.

Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)



from [Anderson et al. '12](#)

SDSS-III (BOSS)
power spectrum.

Galaxy surveys \simeq
matter density fluctuations,
biasing and redshift space
distortions.

But...

- We have to take fully into account that all observations are made on our **past lightcone** which is itself perturbed.
We see density fluctuations which are further away from us, further in the past.
We cannot observe 3 spatial dimensions but **2 spatial and 1 lightlike**, more precisely we measure **2 angles and a redshift**.

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- For small galaxy catalogs, these effects are not very important, but when we go out to **$z \sim 1$ or more**, they become relevant. Already for SDSS BOSS which goes out to $z \simeq 0.7$ (BOSS) or DES which goes to $z \simeq 0.8$.

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- But of course much more for **future surveys like DESI, Euclid, LSST, SKA and WFIRST**.

Cosmological distances

In a Friedmann Universe the (comoving) radial distance is

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}}$$

In cosmology we infer distances by measuring redshifts and calculating them, via this relation. **The result depends on the cosmological model.**

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Depending on the observational situation we measure directly $r(z)$ or

$$d_A(z) = \frac{1}{(1+z)} \chi_K(r(z)) \quad \text{the angular diameter distance}$$

$$d_L(z) = (1+z) \chi_K(r(z)) \quad \text{the luminosity distance.}$$

At small redshift all distances are $d(z) = z/H_0 + \mathcal{O}(z^2)$, for $z \ll 1$, $[d] = h^{-1}\text{Mpc}$.
At larger redshifts, the distance depends strongly on $\Omega_K, \Omega_\Lambda, \dots$.

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- Whenever we convert a measured **redshift and angle into a length scale**, we make assumptions about the **underlying cosmology**.

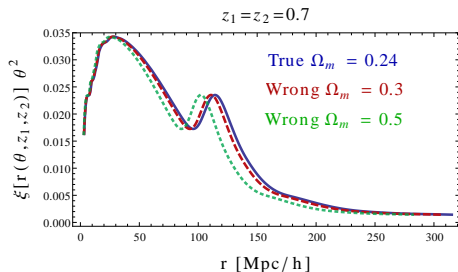
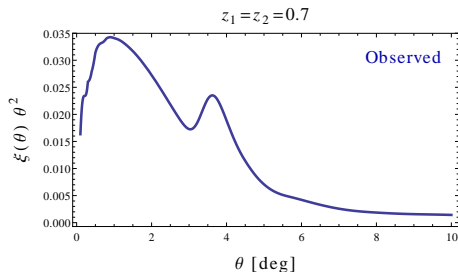
Very large scale galaxy surveys

If we convert the **measured** correlation function $\xi(\theta, z_1, z_2)$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales.

$$r(z_1, z_2, \theta) \stackrel{(K=0)}{=} \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta}.$$

$$r_i = r(z_i) = \int_0^{z_i} \frac{dz}{H(z)}$$

(Figure by F. Montanari)



We now consider fluctuations in the matter distribution and in the geometry first to linear order. (See [J. Yoo et al. 2009](#), [J. Yoo 2010](#); [C. Bonvin & RD 2011](#); [Challinor & Lewis, 2011](#))

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For each galaxy in a catalog we measure

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We can count the galaxies inside a redshift bin and small solid angle, $N(\mathbf{n}, z)$ and measure the fluctuation of this count:

$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

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$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle, \quad \mathbf{n} \cdot \mathbf{n}' = \cos \theta.$$

This quantity is directly measurable.

The total galaxy density fluctuation per redshift bin, per solid angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations from scalar perturbations to 1st order as function of the observed redshift z and direction \mathbf{n}

$$\begin{aligned}\Delta(\mathbf{n}, z) &= bD - 3\mathcal{H}V - (2 - 5s)\Phi + \Psi + \frac{1}{\mathcal{H}} \left[\dot{\Phi} + \partial_r(\mathbf{V} \cdot \mathbf{n}) \right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2 - 5s}{r(z)\mathcal{H}} + 5s \right) \left(\Psi + \mathbf{V} \cdot \mathbf{n} + \int_0^{r(z)} dr (\dot{\Phi} + \dot{\Psi}) \right) \\ &\quad - \frac{2 - 5s}{2} \int_0^{r(z)} dr \left[\frac{r(z) - r}{r(z)r} \Delta_{\Omega}(\Phi + \Psi) - 2(\Phi + \Psi) \right].\end{aligned}$$

(Bonvin & RD '11, Challinor & Lewis '11)

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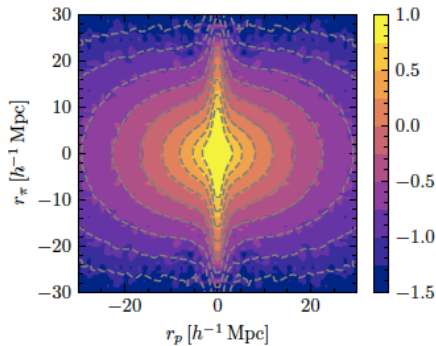
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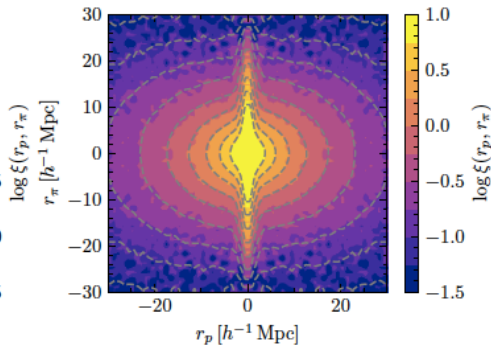
(C. Bonvin & RD '11, Challinor & Lewis '11)

Redshift space distortions in the BOSS survey

(from [Lange et al. \[2101.12261\]](#))



$0.18 < z < 0.3$



$0.3 < z < 0.42$

The angular power spectrum of galaxy density fluctuations

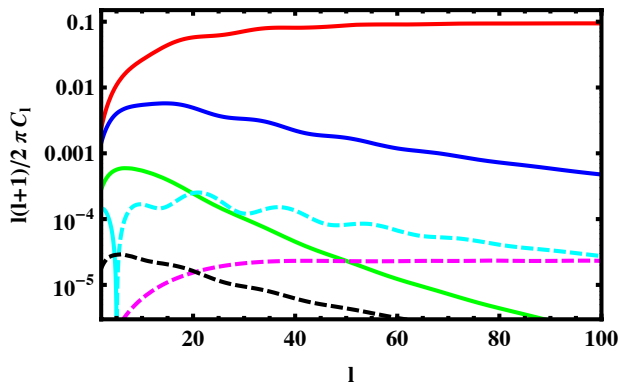
For fixed z , we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \quad C_{\ell}(z, z') = \langle a_{\ell m}(z) a_{\ell m}^*(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}(z, z') P_{\ell}(\cos \theta)$$
$$\cos \theta = \mathbf{n} \cdot \mathbf{n}'$$

The transversal power spectrum

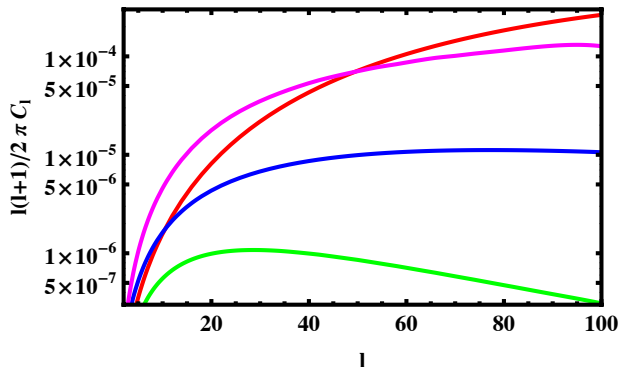
Contributions to the transverse power spectrum at redshift $z = 0.1$, $\Delta z = 0.01$
(from [Bonvin & RD '11](#))



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{Doppler}$ (cyan), $C_\ell^{lensing}$ (magenta), C_ℓ^{grav} (black).

The transversal power spectrum

Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$
(from [Bonvin & RD '11](#))



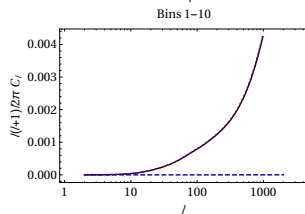
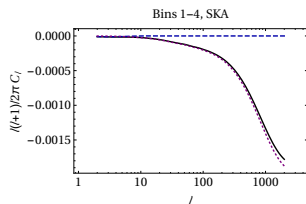
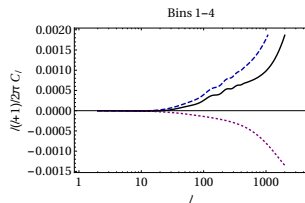
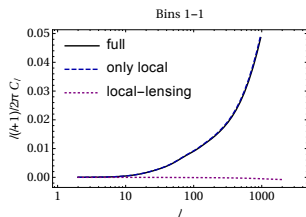
C_l^{DD} (red), C_l^{zz} (green), $2C_l^{Dz}$ (blue), C_l^{lensing} (magenta).

Measuring the lensing potential with Euclid

Well separated redshift bins measure mainly the lensing-density correlation:

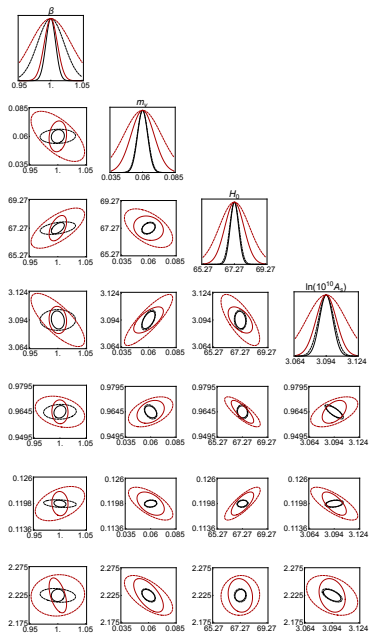
$$\langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle \simeq \langle \Delta^L(\mathbf{n}, z) \delta(\mathbf{n}', z') \rangle \quad z > z'$$

$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$



(Montanari & RD)
[1506.01369]

Testing GR with the lensing potential



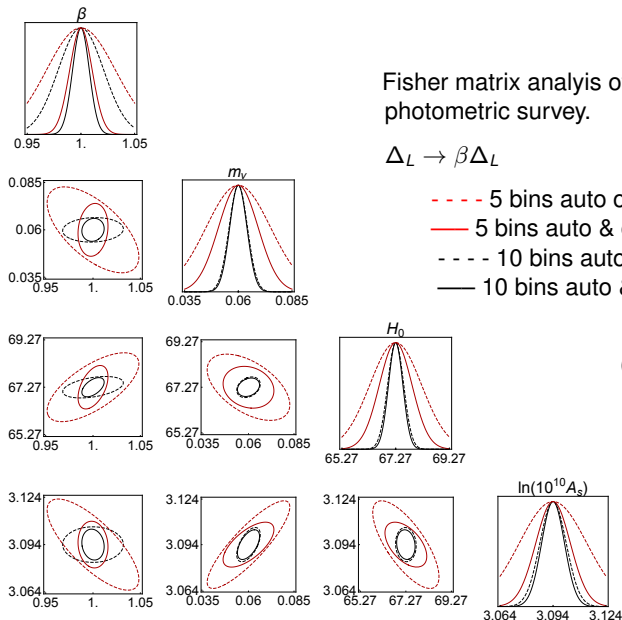
Fisher matrix analysis of an Euclid-like photometric survey.

$$\Delta_L \rightarrow \beta \Delta_L$$

- 5 bins auto only
- 5 bins auto & cross
- 10 bins auto only
- 10 bins auto & cross

(Montanari & RD 2015)

Testing GR with the lensing potential



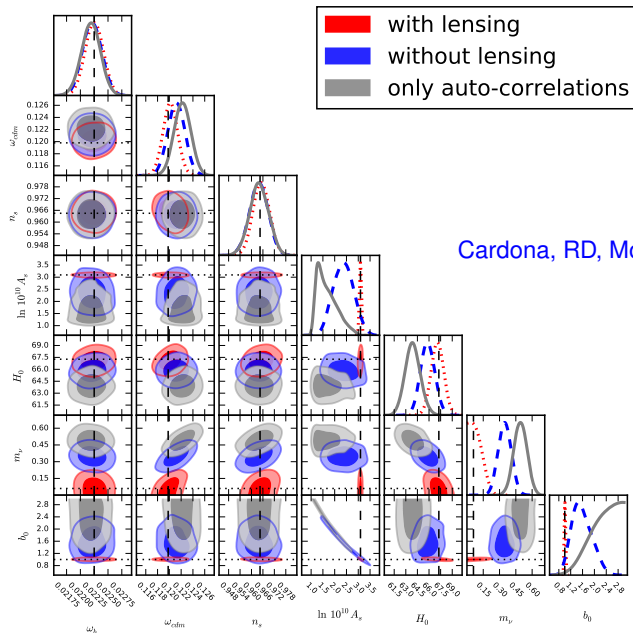
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Neglecting the lensing potential biases cosmological parameters



An estimator for the lensing potential

$$\Delta(\ell, z) = \tilde{\Delta}(\ell, z) + g(\ell, z)\phi(\ell, z) + \int \frac{d^2\ell'}{2\pi} K(\ell', \ell, z)\tilde{\Delta}(\ell', z)\phi(\ell - \ell', z) + \mathcal{O}(\phi^2)$$

$$\langle \Delta(\mathbf{L}) \rangle_\phi = g(L, z)\phi(\mathbf{L}),$$

$$\langle \Delta(\ell)\Delta(\mathbf{L} - \ell) \rangle_\phi = \frac{1}{2\pi} f(\ell, \mathbf{L} - \ell)\phi(\mathbf{L}) \quad (\text{for } \mathbf{L} \neq 0).$$

$$\hat{\phi}(\mathbf{L}, z) = A(L, z)N(L, z) \int \frac{d^2\ell}{2\pi} \Delta(\ell, z)\Delta(\mathbf{L} - \ell, z)F(\ell, \mathbf{L} - \ell, z) \\ + (1 - A(L, z)) \frac{\Delta(\mathbf{L}, z)}{g(L, z)}$$

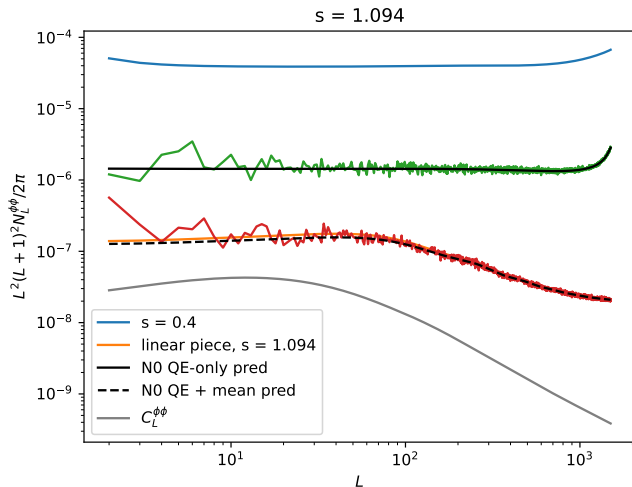
with

$$F(\ell_1, \ell_2, z) = \frac{f(\ell_1, \ell_2, z)}{2C_{\ell_1}(z)C_{\ell_2}(z)},$$

$$N(L, z) = \left[\int \frac{d^2\ell}{(2\pi)^2} f(\ell, \mathbf{L} - \ell, z)F(\ell, \mathbf{L} - \ell, z) \right]^{-1}$$

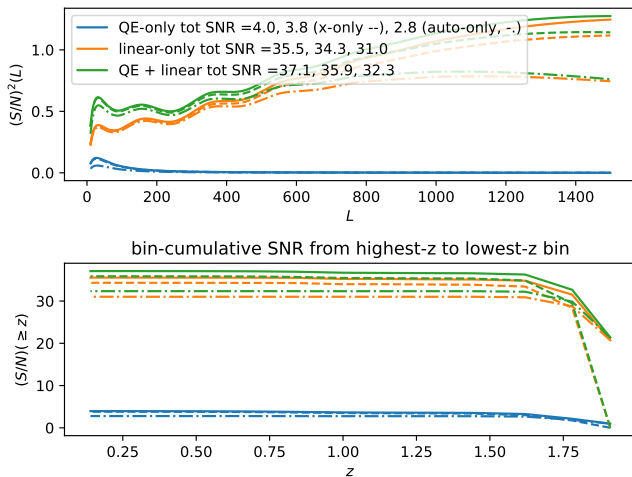
$$A(L, z) = \frac{C_L(z)}{g(L, z)^2 N(L, z) + C_L(z)}.$$

An estimator for the lensing potential



(from Nistane, Jalilvand, Carron, RD & Kunz, arXiv:2201.04129)

An estimator for the lensing potential

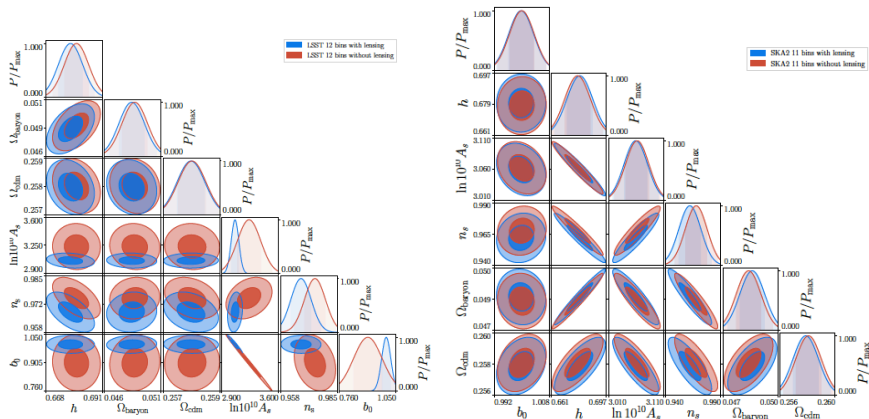


(from Nistane, Jalilvand, Carron, RD & Kunz, arXiv:2201.04129)

Measuring the growth rate of perturbations

- The **growth rate of perturbations** is very sensitive to the theory of gravity.
- A cosmological constant is the only form of dark energy which exhibits absolutely no clustering.
- **Redshift space distortions** are most sensitive to the growth rate. hence to measure it we need good redshift resolution → a **spectroscopic survey**.
- Even though '**lensing convergence**' is not very relevant for standard cosmological parameter estimation with spectroscopic surveys, it does **significantly affect the growth rate**.

Standard parameter estimation from Vera Rubin Observatory (LSST) and SKA2 galaxy number counts



(Lepori, Jelic-Cizmek, Bonvin, RD 2020)

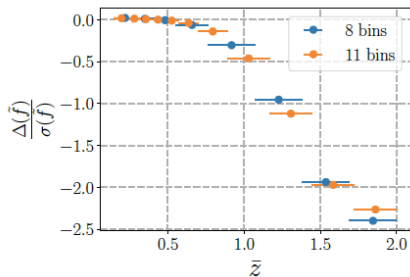
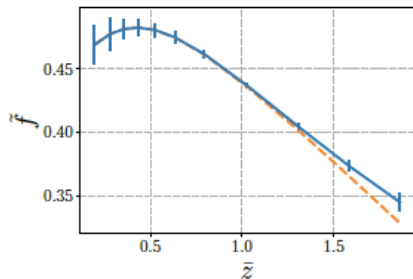
Errobars on std parameters from LSST will be similar to those from SKA2
 h_0 , n_s and Ω_{cdm} will even be better determined with LSST than with SKA2 !

Growth rate estimation from SKA2 galaxy number counts

The growth rate is best estimated with RSD. However, in the k-power spectrum lensing is not easily included.

Including lensing, SKA2 will be able to determine it at the few % level (2 - 3% in a Fisher analysis).

$$\tilde{f}(z) = f(z)\sigma_8(z) \text{ (neglecting lensing / including lensing in the analysis)}$$



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Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$ or $B(k_1, k_2, k_3) \dots$. These are easier to measure (less noisy) but:
 - they require an fiducial **input cosmology** converting redshift and angles to length scales,

$$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z') \cos \theta}.$$

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- Upcoming large & precise 3d galaxy catalogs like **Euclid, DESI, SKA, LSST** etc. will be able to determine directly the measured 3d correlation functions and spectra, $\xi(\theta, z, z')$ and $C_\ell(z, z')$ and $b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3) \dots$ from the data.

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- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (**density**) but also to the velocity via (**redshift space distortions**) and to the perturbations of spacetime geometry (**lensing**).

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- We can therefore in principle determine both, the components of the energy momentum tensor and the geometry which allows us to test Einstein's equations.
 - We can define an estimator for the lensing potential and in principle generate tomographic maps of the lensing potential from galaxy number counts.
 - To test GR e.g. with the growth rate of perturbations it is important to include lensing even in the analysis of spectroscopic surveys.
 - The spectra $C_\ell(z, z')$ and $b_{\ell_1, \ell_2, \ell_2}(z_1, z_2, z_3)$ depend sensitively and in several different ways on the theory of gravity (growth factor, relation between Ψ and Φ), on the matter and baryon densities, and on the velocity. Their measurements provide a new route to estimate cosmological parameters and, especially, to test General Relativity on cosmological scales.
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