Exploring the Dualities of Massive Gauge Theories
A_μ vs. $B_{\mu\nu}$

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based on [arXiv:2109.05030]

- 1955. **THE PRINCIPLE OF CONTINUITY**

  by E. Schrödinger and L. Bass

$$S = \int d^4x \left[-\frac{1}{2} tr(F_{\mu\nu}F^{\mu\nu}) + m^2 tr(A_\mu A^\mu) \right]$$

where

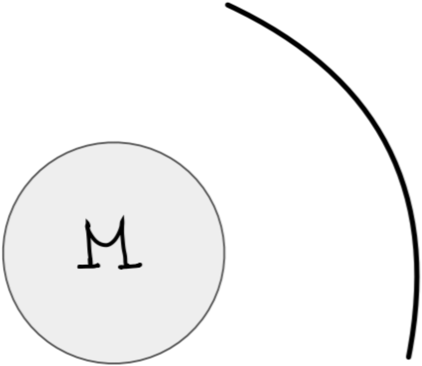
$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$$

- Standard perturbation theory:
  1. not power-counting renormalizable
  2. not unitary

- 1939. Fierz-Pauli massive gravity

- 1970. vDVZ discontinuity

  by H. van Dam, M. J. G. Veltman, V. I. Zakharov, Y. Iwasaki



$$r_V = \left(\frac{GM}{m_G^4}\right)^{1/5}$$

- 1971. Vainshtein, Kriplovich

"...it appears highly probable that outside perturbation theory, a continous zero-mass limit exists and the theory is renormalizable."

- arXiv:2111.00017

The Consequence of Massless Limit on Dualities

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu - J_\mu A^\mu \right)$$

where

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$$

$$S = \int d^4x \left(\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} J_{\mu\nu} B^{\mu\nu} \right)$$

where

$$B_{\mu\nu} = -B_{\nu\mu}$$

and

$$H_{\mu\nu\rho} = B_{\nu\rho,\mu} + B_{\rho\mu,\nu} + B_{\mu\nu,\rho}$$

- Until now: The theories are dual (have the same physics)
- Why?
  - Same number of degrees of freedom
  - Dualization procedure (see eg. (Smailagic *et al.*, 2001))

The goal

- To convince you that these theories do not have the same physics

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- How?

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- How?
- Massless limit of interacting theories

- Proca theory:

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \frac{g^2}{4} \left(A_\mu A^\mu \right)^2 \right]$$

- Kalb-Ramond theory

$$S = \int d^4x \left[\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} + \frac{g^2}{16} \left(B_{\mu\nu} B^{\mu\nu} \right)^2 \right]$$

- Assume $g^2 \ll 1, \; k^2 \sim \frac{1}{L^2} \gg m^2$

1. Express the theory only in terms of the propagating degrees of freedom

2. Find the strong coupling scale

3. Estimate the theory beyond the strong coupling scale

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \frac{g^2}{4} \left(A_\mu A^\mu \right)^2 \right]$$

- Decomposition:

$$A_i = A_i^T + \chi_{,i}, \qquad A_{i,i}^T = 0$$

$$S = \frac{1}{2} \int d^4x \left[A_0 \left(-\Delta + m^2 \right) A_0 + 2 A_0 \Delta \dot{\chi} - \left(\dot{\chi} \Delta \dot{\chi} - m^2 \chi \Delta \chi \right) \right.$$
$$\left. + \dot{A}_i^T \dot{A}_i^T - A_{i,j}^T A_{i,j}^T - m^2 A_i^T A_i^T + \frac{g^2}{2} \left(A_0^2 - A_i A_i \right)^2 \right]$$

- A_0 is not propagating

$$\mathcal{L}_0 = -\frac{1}{2}\chi(\Box + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi - \frac{1}{2}A_i^T(\Box + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4}\left(\chi_{,\mu}\chi^{,\mu}\right)^2 - g^2\chi_{,\mu}\chi^{,\mu}\chi_{,i}A_i^T + \mathcal{O}\left(\frac{g^2\chi^2\left(A^T\right)^2}{L^2}\right)$$

- Canonically normalised variable:

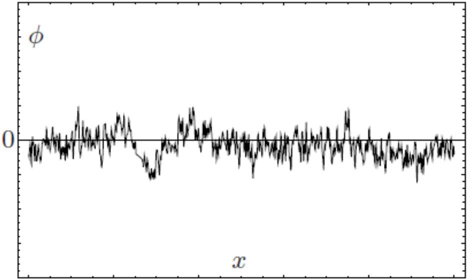
$$\chi_n = m\sqrt{\frac{-\Delta}{-\Delta + m^2}}\chi$$

$$\mathcal{L}_0 = -\frac{1}{2}\chi_n(\Box + m^2)\chi_n - \frac{1}{2}A_i^T(\Box + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4m^4}\left(\chi_{n,\mu}\chi_n^{,\mu}\right)^2 - \frac{g^2}{m^3}\chi_{n,\mu}\chi_n^{,\mu}\chi_{n,i}A_i^T$$

- How to compare the terms?
- **The minimal level of quantum fluctuations**

$$S = \frac{1}{2} \int d^4x \left(\phi_{,\mu} \phi^{,\mu} - m^2 \phi^2 \right)$$



Source: V. Mukhanov, S. Winitzki, *Introduction to Quantum Effects in Gravity*, Cambridge University Press (2007)

- Vacuum fluctuations:

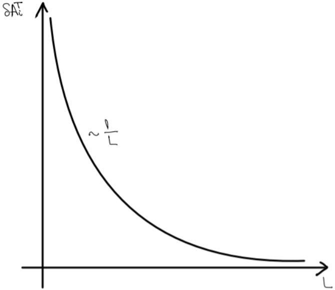
$$\delta\phi_L^2 = \langle 0|\, \phi(\vec{x}, t)\, \phi(\vec{y}, t)\, |0\rangle = \int_0^\infty dk \frac{k^2}{\omega_k} \frac{\sin(kL)}{kL}, \qquad L = |\vec{x} - \vec{y}|$$

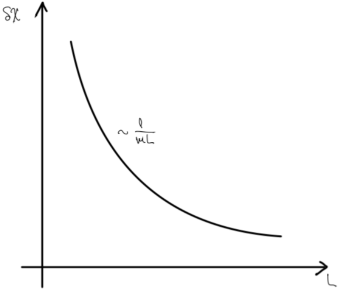
$$\delta\phi_L \sim \left. \frac{k^{3/2}}{\sqrt{\omega_k}} \right|_{k \sim L^{-1}}$$

- For $k^2 \sim \frac{1}{L^2} \gg m^2$ $\qquad \delta\phi_L \sim \frac{1}{L}$

- The minimal level of quantum fluctuations:

$$\delta A_L^T \sim \frac{1}{L} \qquad \delta\chi_{nL} \sim \frac{1}{L} \qquad \delta\chi_L \sim \frac{1}{mL}$$





- The most dominant terms for χ

$$\mathcal{L} \supset -\frac{1}{2}\chi_n(\Box + m^2)\chi_n + \frac{g^2}{4m^4}\left(\chi_{n,\mu}\chi_n^{,\mu}\right)^2$$

- Longitudinal modes enter the strong coupling regime at:

$$L_{str} \sim \frac{\sqrt{g}}{m}$$

- The most dominant terms for χ

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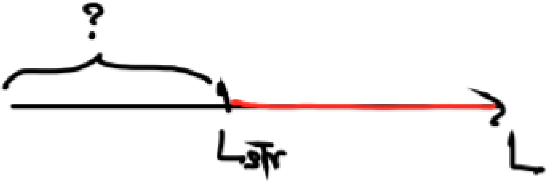


- Most dominant terms for the longitudinal modes

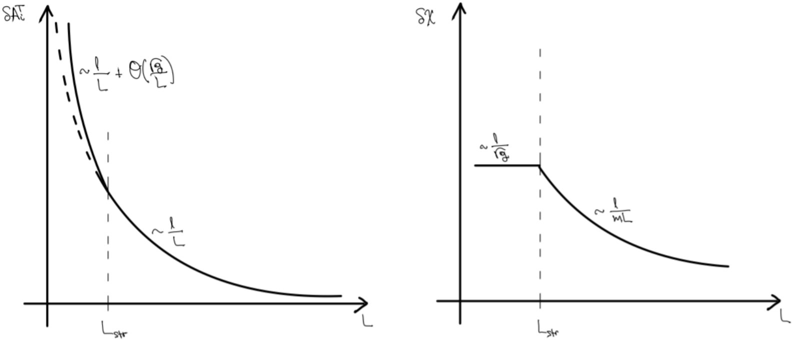
$$\mathcal{L}_{\chi int} = \frac{g^2}{4} (\chi_{,\mu}\chi^{,\mu})^2 \qquad \rightarrow \qquad \delta_L \chi \sim \frac{1}{\sqrt{g}}$$

- For transverse modes the most dominant terms are:

$$\mathcal{L}_{A^T} \sim -\frac{1}{2} A_i^T (\Box + m^2) A_i^T - g^2 \chi_{,\mu}\chi^{,\mu}\chi_{,i} A_i^T$$

- Corrections to the transverse modes:

$$A_i^{T(1)} \sim \frac{\sqrt{g}}{L}$$



$$S = \int d^4x \left[\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} + \frac{g^2}{16} \left(B_{\mu\nu} B^{\mu\nu} \right)^2 \right]$$

- Decomposition:

$$B_{0i} = C_i^T + \mu_{,i}, \qquad C_{i,i}^T = 0$$

$$B_{ij} = \varepsilon_{ijk} B_k, \qquad B_i = B_i^T + \phi_{,i}, \qquad B_{i,i}^T = 0$$

- C_i^T and μ are not propagating

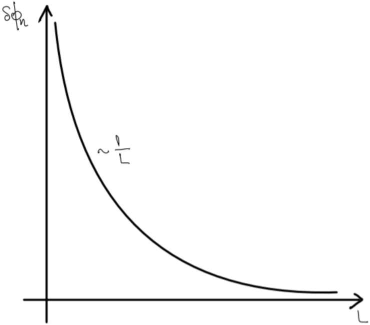
$$\mathcal{L}_0 = -\frac{1}{2} B_i^T (\Box + m^2) \frac{m^2}{-\Delta + m^2} B_i^T - \frac{1}{2} \phi (\Box + m^2)(-\Delta \phi)$$

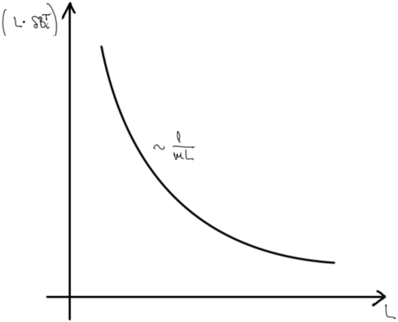
$$\mathcal{L}_{int} \sim g^2 \left(B^T\right)^4 + g^2 \left(B^T\right)^3 \frac{\phi}{L} + \mathcal{O}\left(g^2 \left(B^T\right)^2 \frac{\phi^2}{L^2}\right)$$

- Canonically normalised variables:

$$B_{ni}^T = \sqrt{\frac{m^2}{-\Delta + m^2}} B_i^T \qquad \text{and} \qquad \phi_n = \sqrt{-\Delta} \phi$$

$$\delta B^T_{nL} \sim \frac{1}{L} \qquad \delta\phi_{nL} \sim \frac{1}{L} \qquad B^T_L \sim \frac{1}{mL^2}$$



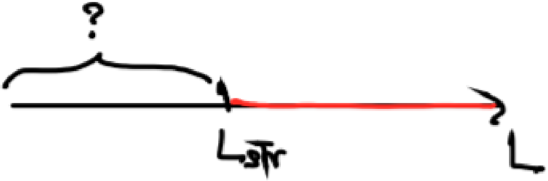


- The most dominant terms for B_i^T

$$\mathcal{L} \supset -\frac{1}{2} B_i^T (\Box + m^2) \frac{m^2}{-\Delta + m^2} B_i^T + g^2 \left(B^T\right)^4$$

- Transverse modes enter the strong coupling regime at:

$$L_{str} \sim \frac{\sqrt{g}}{m}$$



- Most dominant term for the transverse modes

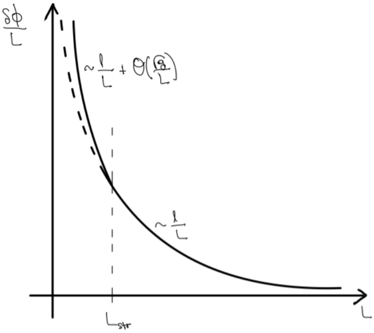
$$\mathcal{L}_{Bint} \sim g^2 \left(B^T\right)^4$$

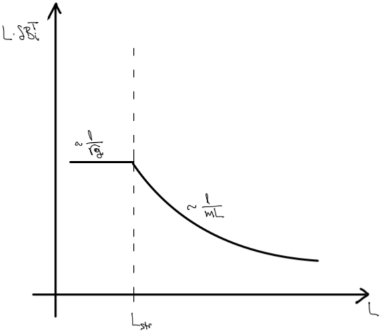
- The minimal level of quantum fluctuations for the original field

$$\delta_L B^T \sim \frac{1}{\sqrt{g}L}$$

- Corrections to the longitudinal mode

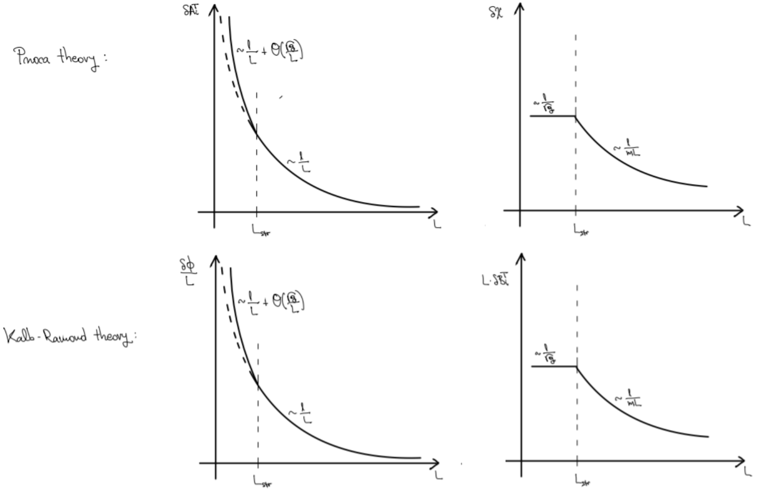
$$g^2 \left(B^T\right)^3 \frac{\phi}{L} \sim \frac{\sqrt{g}}{L^4}$$


δφ
L
~ 1
L
L_str
L


L · δB_i^T
L_str
L

INTERACTING THEORIES:

1. Same Vainshtein scale: $L_{str} \sim \frac{\sqrt{g}}{m}$
2. $L > L_{str}$ divergences in mass are caused by B^T for Kalb-Ramond and χ for Proca theory
3. $L < L_{str}$ In Kalb-Ramond theory the transverse modes are strongly coupled. In Proca theory the longitudinal modes are strongly coupled


Proca theory:
δA_i^T
$\sim \frac{1}{L} + \mathcal{O}(\frac{\sqrt{g}}{L})$
$\sim \frac{1}{L}$
L_{str}
L
$\delta\chi$
$\sim \frac{1}{\sqrt{g}}$
$\sim \frac{1}{mL}$
L_{str}
L
Kalb-Ramond theory:
$\frac{\delta\phi}{L}$
$\sim \frac{1}{L} + \mathcal{O}(\frac{\sqrt{g}}{L})$
$\sim \frac{1}{L}$
L_{str}
L
$L\cdot\delta B_i^T$
$\sim \frac{1}{\sqrt{g}}$
$\sim \frac{1}{mL}$
L_{str}
L

Thank You!