Self-tuning of the cosmological constant in brane-worlds with Cuscuton bulk field

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based on 2203.16322 with S. Mukohyama (YITP)

Introduction

- Cosmological constant problem can be stated several *(related)* ways *(see numerous talks of Planck 2022)*
- Vacuum energy contributions due to SM loops are huge compared to observed CC . How can it be so well protected ? *Fine tuning of bare CC.*
- *Other way.* Why is there a huge hierarchy between EW scale and CC scale ?
- One idea: observed CC not related to vacuum energy of 4d theory, which curves **extra-dimensions**

  (motivated from UV considerations such as string theory)

Outline of the talk

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2. Brane-world scenario
3. Self-tuning of the cosmological constant
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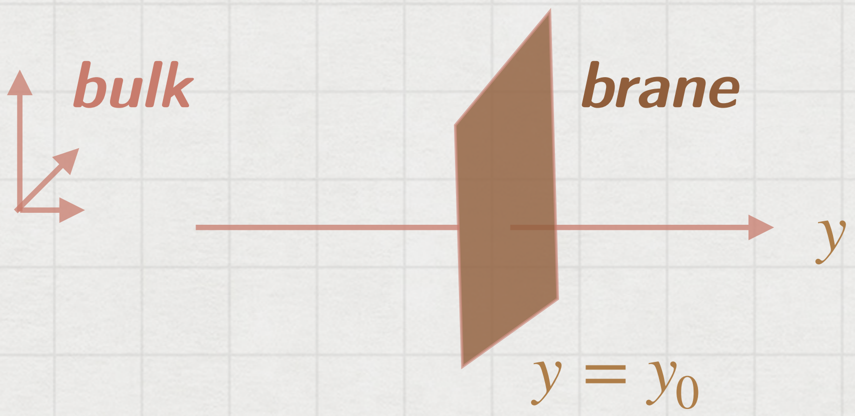
Brane-world setup

- Setup with one extra dimension: $5d = 4d + 1d$
- Observed Universe (4d) living on a **3-brane**, a thin surface layer spanning 3 spatial directions



- Total space-time (full space-time) **bulk.** The 3-brane has a (fixed) position in the bulk. *we take $y = y_0 = 0$ in what follows*
- Some **localized** fields 'live' on the brane

  → surface energy-momentum tensor (delta function) $S_{\mu\nu}$
- Other fields live in the bulk: **gravity** + ... coupled to brane through **brane tension**

Warped metric

- We consider a factorized **warped** metric

μ, ν *4d indices*
M, N *5d indices*

$$ds^2 = g_{MN}dx^M dx^N = a^2(y) g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- $a^2(y)$ warping factor, we assume *maximally symmetric* brane i.e. *4d* dS, AdS or Minkovski metric $g_{\mu\nu}$. *+ we impose $\mathbb{Z}_2$ sym $y \leftrightarrow -y$*

- Action decomposed as

$$S = \int d^5x \sqrt{-g}\left(\frac{1}{2\kappa_5^2}R + \mathscr{L}_{bk}\right) + \int d^5x \sqrt{-g_4}\,\mathscr{L}_{bn}\,\delta(y)$$

- Bulk fluid: $T_{MN} = -\dfrac{2}{\sqrt{g}}\dfrac{\delta(\sqrt{-g}\,\mathscr{L}_{bk})}{\delta g_{MN}} = -p_5 g_{MN} + (\rho_5 + p_5)u_M u_N$

- Brane (surface) energy tensor: $S_{\mu\nu} = -\dfrac{2}{\sqrt{g}}\dfrac{\delta(\sqrt{-g}\,\mathscr{L}_{bn})}{\delta g_{\mu\nu}}$

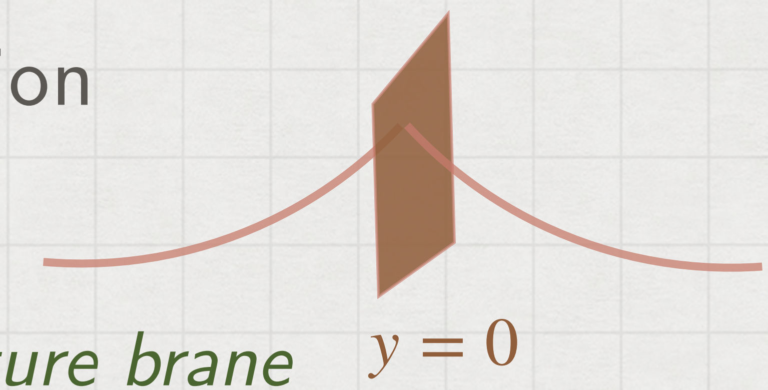
Background equations and junction conditions

- Einstein tensor warped geo. $G_{y\mu} = 0,\ G_{yy} = 6\frac{a'^2}{a^2},\ G_{\mu\nu} = 3(a'^2 + a''a)g_{\mu\nu}$
- Lead to background 'Friedmann eqs' $\quad ' \equiv \partial_y$

$$H^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{\kappa_5^2}{6}\rho_5 \qquad\qquad H' + H^2 = \frac{a''}{a} = -\frac{\kappa_5^2}{6}(2p_5 + \rho_5)$$

- At the brane, a continuous, but possibly ∃ **discontinuity** in derivatives a', a'', supported by the brane tension

Israel junction conditions (IJC)



$$[K_{\mu\nu}] = -\kappa_5^2(S_{\mu\nu} - \frac{1}{3}g_{\mu\nu}S)$$

$K_{\mu\nu}$ *: extrinsic curvature brane*

$[f] = f(0^+) - f(0^-)$ *: discontinuity at the brane*

- *In flat brane case:* $g_{\mu\nu} = \eta_{\mu\nu}$, *leads to* $K_{\mu\nu} = a'a\eta_{\mu\nu}$

$$S_{\mu\nu} = g_{\mu\nu}T\delta(y) = a^2(0)\eta_{\mu\nu}T\delta(y)$$

IJC: $\frac{[a']}{a(0)} = -\frac{\kappa_5^2}{3}T$

Randall-Sundrum scenarios

- They considered brane in AdS *Randall, Sundrum '99*

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\Big(R \ - \Lambda \Big) + \int d^5x\sqrt{-g_4}\mathscr{L}_{bn}\delta(y) \qquad \mathscr{L}_{bk} = -\Lambda,$$

- This gives: $\boxed{\rho_5 = -\Lambda \qquad p_5 = \Lambda}$ $\quad (T_{MN} = -p_5 g_{MN} + (\rho_5 + p_5)u_M u_N)$

- Friedman eq. gives warp factor $a(y) \propto e^{-\sqrt{-\Lambda}/\kappa_5 \, y}$

- **IJC** relates Λ to V_0 (brane scalar pot.)→ unique scale k

$$\Lambda \sim -k^2/\kappa_5^2, \qquad V_0 \sim k/\kappa_5^2$$

- 4d gravity ? 4d Planck mass $M_P^2 = 8\pi/\kappa$ with

$$\frac{\kappa_5^2}{\kappa} = \int a^2(y)dy \sim 1/k \qquad \textit{finite}$$

$\in \mathscr{L}_{bn}$

- 4d physical (renormalized) parameters **suppressed** $m = m_0 e^{-k}$

  'natural' hierarchy

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Bulk scalar field

- In RS scenario, Λ has to be tuned to balance brane vacuum energy V_0 *CC problem not 'solved' (not even addressed)*
- New mechanism: introduce **bulk scalar** $\phi(y)$

Arkani-Hamed, Dimopoulos, Kaloper, Dvali '00
Kachru, Schultw, Silverstein '00

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}R - \frac{3}{2}\partial_M\phi\partial^M\phi - V_{bk}(\phi)\right] + \int d^5x\sqrt{-g_4}V_{bn}(\phi)\delta(y)$$

$$\partial_M\phi = \partial_y\phi \equiv \phi' \qquad\qquad V_{bn}(\phi) = V_0 f(\phi)$$

$$\rightarrow \quad \rho_5 = \frac{1}{2}\partial_M\phi\partial^M\phi + V(\phi), \qquad p_5 = \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)$$

- **IJC** *(flat case)*: $\dfrac{[a']}{a(0)} = -\dfrac{\kappa_5^2}{3}T = \dfrac{\kappa_5^2}{3}V_{bn}(\phi_0)$
- New junction condition for brane scalar: $[\phi'] \propto \kappa_5^2\dfrac{\partial V_{bn}}{\partial\phi}(\phi_0)$

Self-tuning of the CC

- Original example: particular coupling

$$V_{bk} = 0, \qquad V_{bn}(\phi) = V_0 f(\phi) = V_0 e^{\gamma\phi}$$

- **IJC** for curved brane with 4d Poincaré inv. :

(h brane curvature) $$\frac{[a']^2}{a(0)^2} - \frac{h^2}{a(0)^2} = \frac{\kappa_5^2}{3} V_{bn}^2(\phi_0)$$

- Rewrite the brane curvature as:

$$\frac{h^2}{a(0)^2} = \frac{[a']^2}{a(0)^2} - \frac{\kappa_5^2}{6} V_{bn}^2(\phi_0) \sim \frac{1}{24} V_{bn,\,\phi}^2(\phi_0) - \frac{\kappa_5^4}{6} V_{bn}^2(\phi_0)$$

$$[a']/a \leftrightarrow [\rho_5] \leftrightarrow [\phi'] \leftrightarrow V_{bn,\,\phi}$$

$$\sim V_0^2 \left(\frac{\gamma^2}{24} - \frac{\kappa_5^4}{6} \right) e^{2\gamma\phi_0} = 0$$ *for* $\gamma = 2\kappa_5$

- *For any V_0, flat brane solution enforced,* ***vanishing 4d CC***
brane vacuum energy converted into a current through IJC

Singularities

- Previous model: *warping factor and bulk scalar*

$$a^2(y) = \left(1 - \frac{2V_0 e^{\gamma\phi_0}}{3}|y|\right)^{1/4} \qquad \phi = \phi_0 - \ln\left(1 - \frac{2V_0 e^{2\gamma\phi_0}}{3}|y|\right)$$

→ *singularities at* $|y| = \frac{3}{2V_0} e^{-\gamma\phi_0}$

- Original paper: current ***flows*** from brane to singularities.

- Singularities generic to self-tuning *(other mechanisms, e.g including $V_{bk}(\phi)$)* How to resolve singularities ?
  - additional branes at singularities *Forste, Lalak, Lavignac, Nilles '00*
  - non-linear bulk fluid: $\rho_5 = \gamma p_5^{\lambda}$ *Antoniadis, Cotsakis, Klaoudatou '21*

- Both solutions seems to kill the self-tuning mechanism *(e.g. additional brane tensions have to be tuned to V_0)*

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k-essence actions and reconstruction

- More evolved kinetic term for bulk scalar $\phi(y)$? Additional freedom ?
  Generic k-essence scalar field

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}R + P(X,\phi)\right] \qquad X = -\frac{1}{2}g^{AB}\partial_A\phi\partial_B\phi = -\frac{1}{2}\phi'^2$$

- Analog to bulk fluid with $p_5 = -P(X,\phi),\quad \rho_5 = P(X) - 2XP_X(X,\phi)$

$$u^M = \partial^M\phi\Big/\sqrt{-2X} = \phi' e_y$$

- Can reconstruct any *(regular enough)* warping factor: *(e.g.* $V_{bk} = 0$*)*
  - ➡ From $a(y)$, $a'(y)$, $a''(y)$ deduce H, H'
  - ➡ Friedmann eqs. $p_5(y) = -3/\kappa_5^2(H' + 2H^2)(y)$ invert and write
    $\rho_5(y) = 3/\kappa_5^2 H'(p_5) - p_5 = F(-p_5)$
  - ➡ Solve ODE: $2XP_X + F(P) - P = 0$ and find $P(X,\phi)$

- Can also find reconstruct any fluid e.o.s.

Junction conditions and self-tuning

- k-essence scalar junction conditions expressed in terms of $P(X)$ only:

assume again $\mathbb{Z}_2$ sym, $[f]_0 = 2f(0^+)$

$$-8XP_X^2|_{0^+} = V_{bn,\phi}^2(\phi_0) = V_0^2 f_\phi(\phi_0)^2$$

$$(P - 2XP_X)|_{0^+} \pm \frac{6h^2}{\kappa_5^2 a^2(y_0)} = \frac{\kappa_5^2}{6} V_{bn}^2(\phi_0) = \frac{\kappa_5^2}{6} V_0^2 f(\phi_0)^2$$

- **Self-tuning** if expression of $P(X)$ enforces $h = 0$ through junction cdts

- For original example: $V_{bn}(\phi) = V_0 e^{\gamma\phi}$, $\qquad f(\phi) = e^{\gamma\phi}$

$$\frac{4\kappa_5^2}{3\gamma^2} XP_X^2 - 2XP_X + P = 0 \qquad \Longrightarrow \qquad h = 0$$

ODE equivalent to: $\frac{du}{dx}x + u = \pm\sqrt{1-u} + 1, \qquad P = ux, \qquad x \equiv 3\gamma^2/4\kappa_5^2\ X$

Solutions: 1) $P = x = \frac{3\gamma^2}{4\kappa_5^2} X$ $\quad$ 2) $P = \pm 2\sqrt{c(\phi)x} - c(\phi)$ $\quad$ 3) ... ?

- For generic $f(\phi)$? write $P - 2XP_X \equiv g(-8XP_X^2)$, then self-tuning iff

$\forall V_0 \quad 0 = g(V_0 f_\phi^2(\phi_0)) - \frac{\kappa_5^2}{6} V_0 f(\phi_0)^2 = g(V_0 f_\phi^2(\phi_0)) - V_0 g(f_\phi^2(\phi_0))$ $\quad \rightarrow$ original example

Cuscuton bulk scalar (1)

- For original example: $V_{bn}(\phi) = V_0 e^{\gamma\phi}$, $f(\phi) = e^{\gamma\phi}$

  Solutions: 1) $P = x = \frac{3\gamma^2}{4\kappa_5^2} X$ 2) $P = \pm 2\sqrt{cx} - c$

- Solution 2) $P = -2\sqrt{c(\phi)x} - c(\phi)$ 'Cuscuton' *Afshordi, Chung, Geshnizjani '06*

→ *'pressure' & 'density':* $p_5 = \frac{\sqrt{3}\gamma}{\kappa_5}\sqrt{c(\phi)X} + c(\phi)$, $\rho_5 = -c(\phi)$

→ *eom system: Friedman eq.* $H^2 = -\frac{\kappa_5^2}{6} c(\phi)$ *scalar:* $4HP_X\phi' - c_\phi(\phi) = 0$

- ***Inconsistency*** *for constant* $c(\phi) = c$, $c_\phi = 0$ *(bulk CC)*

∃ *solution for* $c_\phi(\phi) \neq 0$, *if* $c_\phi(\phi) = -2\gamma c(\phi)$ *i.e* $\boxed{c(\phi) = -c_0 e^{2\gamma\phi}}$ $c_0 > 0$.

→ $$S = \int d^5x\sqrt{-g}\left\{\frac{R}{2\kappa_5^2} - \frac{\sqrt{3}\gamma}{\kappa_5} e^{\gamma\phi}\sqrt{-c_0 X} + c_0 e^{2\gamma\phi}\right\} - \int d^5x\sqrt{-g_4}\, V_0 e^{\gamma\phi}\delta(y)\,.$$

Cuscuton bulk scalar (2)

- Action $S = \int d^5x\sqrt{-g}\left\{\frac{R}{2\kappa_5^2} - \frac{\sqrt{3}\gamma}{\kappa_5}e^{\gamma\phi}\sqrt{-c_0 X} + c_0 e^{2\gamma\phi}\right\} - \int d^5x\sqrt{-g_4}\; V_0 e^{\gamma\phi}\delta(y)$

  Redefine $\phi \to \tilde{\phi} = \frac{\sqrt{c_0}}{\gamma}e^{\gamma\phi}$ to get 'standard' Cuscuton kinetic term

$$S = \int d^5x\sqrt{-g}\left\{\frac{R}{2\kappa_5^2} - \frac{\sqrt{3}\gamma}{\kappa_5}\sqrt{-\tilde{X}} + \gamma^2\tilde{\phi}^2\right\} - \int d^5x\sqrt{-g_4}\; \frac{\gamma V_0}{\sqrt{c_0}}\tilde{\phi}\,\delta(y)$$

- Scalar field and warp factor profiles. Solve Friedman eqs.

$$\phi = \frac{1}{\gamma}\left\{\ln|H| + \frac{1}{2}\ln\left(\frac{6}{c_0\kappa_5^2}\right)\right\}$$

→ *any given warp factor $a(y)$ (or yet $H(y)$) can be supported by $\phi(y)$.*

In particular those with finite $\frac{1}{\kappa} = \frac{M_P^2}{8\pi} = \frac{1}{\kappa_5^2}\int dy\, a(y)^2$

→ *Profile expected to be determined from IC and evolution of brane fields*

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Summary & Outlook

- ***Brane-world scenarios:*** ✓ observed Universe on a 4d brane living in a higher dimensional bulk (at least 1 extra dim)
  - ✓ SM localized on the brane, gravity+bulk fields in the bulk

- ***Can realize:*** ✓ gravity localized on the brane (finite 4d M_P), hierarchy between scales due to warping
  - ✓ Self-tuning of 4d CC to flat brane but.... singularities
  - ✓ k-essence bulk scalar a way to construct generic warp profile

- ***Cuscuton bulk scalar:*** ✓ self-tuning of 4d CC
  - ✓ background solution seems stable
  - ✓ warp profile determined from IC conditions and brane cosmology
    - → *Our solution: final stage of cosmic evolution, once brane fields diluted by expansion.*
    - → *Before dilution, brane fields determine warp factor profile through coupling to bulk scalar. TO DO: cosmology*

Thank you

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