



Weak Nonlocality, Extra Dimensions & Fuzzy Branes

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Introduction – Hierarchies in Particle Physics

Several hierarchies in particle physics \Rightarrow BSM dynamical explanation?

- Gauge hierarchy (EW vs gravity scales): $\Lambda_{EW} \sim 100 \text{ GeV} \ll \Lambda_P \sim 10^{18} \text{ GeV}$.
- Flavor hierarchy (neutrinos vs electron vs top quark): $M_\nu \ll M_e \ll M_t$.

BSM scales \gg TeV-scale.

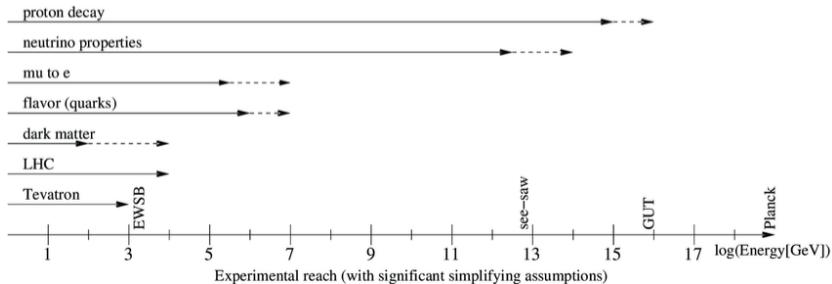
\Rightarrow Stability of Higgs boson mass wrt BSM scales: $\delta M_H^2 \propto \Lambda_{BSM}^2$.

\Rightarrow Models: weak scale SUSY; composite Higgses; extra dims. & branes; higher-deriv.; classicalization; etc.

BUT: No smoking guns of our favorite models at LHC \Rightarrow New (exotic) model building issues?

Talk:

- Motivations for weak nonlocality in particle physics.
- Revisiting famous models with extra dims. of space & branes + weak nonlocality.



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I. Motivations – Local Braneworld Effective Field Theories

Sundrum, arXiv:hep-ph/9805471

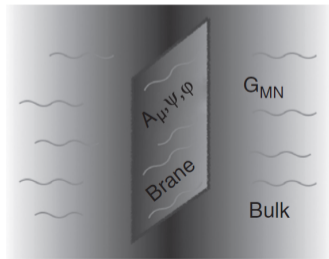
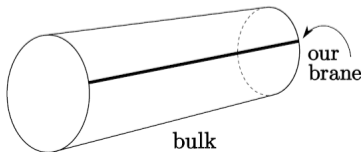
- Local quantum field theory (QFT) with compactified extra dims. of space & δ -like branes.
 - δ -like brane = Hypersurface with localized fields and/or interactions.
 - Bulk = Whole spacetime outside branes.
 - Generates natural hierarchies thanks to geometry.
- Higher-dim. gauge theories (nonrenorm.) \Rightarrow effective field theories (EFT).
- Model with 1 flat extra dim. & N branes:

$$S_{5D} = \int d^4x dy \left[\mathcal{L}_{bulk} + \sum_{i=1}^N \delta(y - y_i) \mathcal{L}_{brane}^{(i)} \right].$$

- Kaluza-Klein (KK) dim. reduction (5D \rightarrow 4D) \Rightarrow 4D KK-modes ϕ_n & bulk wave functions f_n :

$$\Phi(x, y) = \sum_n^{\infty} f_n(y) \phi_n(x).$$

\Rightarrow 1 massless/light 0-mode + KK-tower with mass gap M_{KK} .



I. Motivations – Weak Nonlocality – Basic Notions

Buoninfante, Lambiase, Mazumdar, arXiv:1805.03559 [hep-th]

- **Local QFT**: Operators = Product of fields evaluated at **same spacetime event**.
- **Weakly nonlocal theory**: Interpolates btw **IR local QFT** & **UV nonlocal** theory above $\Lambda = 1/\ell$.
⇒ **Smeared interaction vertices**.
- **Interest**: Better **UV behavior** wrt **local QFT** ⇒ quantum gravity; stability of the EW scale?
- **Usually**: **quantum gravity/spacetime** ⇒ **min. length scale** ⇒ **weak nonlocality!**
- **Causality**: **microcausality** replaced by **macrocausality** ⇒ **cluster decomposition principle** satisfied!
- **Effect**: **Smearing** of **pointlike** sources (∞ -**derivative** operator).
⇒ **EFT**: Pointlike source/vertex **dressed** by higher-dim. operators.
ex: Heat kernel (Gaussian):

$$\begin{aligned} e^{\ell^2 \partial_x^2} \delta(x) &= \sqrt{\frac{1}{4\pi\ell^2}} e^{-\frac{x^2}{4\ell^2}}, \\ &= \delta(x) + \sum_{n=1}^{N-1} \frac{\ell^{2n}}{n!} \delta^{(n)}(x) + \mathcal{O}(\ell^{2N}). \end{aligned}$$

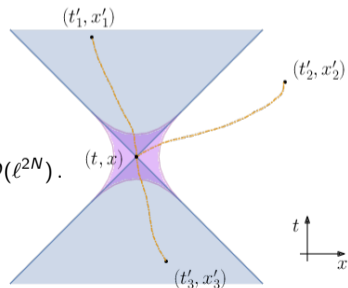
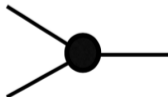
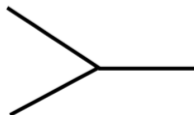


Figure: Interaction vertex & light cone: local vs weakly nonlocal

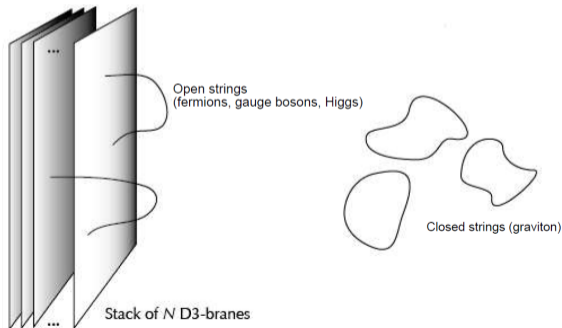
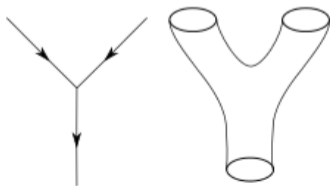
I. Motivations – Weak Nonlocality – Example I: String Theory

Witten, Nucl.Phys.B 268 (1986) 253-294 / Calcagni, Modesto, arXiv:1310.4957 [hep-th] / Calcagni, Modesto, arXiv:1404.2137 [hep-th]

- **String theory** \Rightarrow 1D-extended objects (strings) \Rightarrow nonlocal scale = string scale $M_s = 1/\ell_s$.
- **String field theory (SFT)** = QFT formulation of string theory: ∞ -deriv. operators \Rightarrow weak nonlocality.
- Truncation to 0-level sector (open bosonic SFT) \Rightarrow Tachyon action $(- + \dots +)$:

$$S = \int d^D x \left[\frac{1}{2} \phi e^{-2\ell_s^2 \square} (\square + \mu^2) \phi - \frac{g}{3!} \phi^3 \right] \iff S' = \int d^D x \left[\frac{1}{2} \phi (\square + \mu^2) \phi - \frac{g}{3!} \left(e^{\ell_s^2 \square} \phi \right)^3 \right].$$

- **UV-completion of braneworlds**: 6 extra dims. of space, D-brane stacks, UV-finiteness.



I. Motivations – Weak Nonlocality – Example II: Asymptotically Lee-Wick Field Theories

Pure gauge/gravity: Tomboulis, arXiv:hep-th/9702146 / Modesto, arXiv:1107.2403 [hep-th] / Modesto, Rachwal, arXiv:1407.8036 [hep-th] /

Modesto, Rachwal, arXiv:1503.00261 [hep-th] / Modesto, Piva, Rachwal, arXiv:1506.06227 [hep-th] / Modesto, Rachwal, Int.J.Mod.Phys.D 26 (2017) 11, 1730020

Matter/Higgs: Modesto, arXiv:2103.04936 [gr-qc] / Modesto, arXiv:2103.05536 [hep-th]

- Ghost-free ∞ -deriv. QFT's: weakly nonlocal form factors = transcendental entire functions.
- Asympt. polynomial form factors (pure gauge/gravity theories; nonlocal scale $\Lambda = 1/\ell$).
 - ⇒ Deep-UV \sim Lee-Wick QFT's.
 - ⇒ Superrenorm. & asympt. free / UV-finite & scale inv. (any spacetime dim. D).
 - ⇒ Unitary (no new poles in propagators \sim no ghost-like resonances).
- Prototype of pure gauge Yang-Mills theory ($- + \dots +$):

$$\mathcal{L}_{YM} = -\frac{1}{4g_{YM}} \left[\text{tr} F e^{H(\ell^2 D^2)} F + \mathcal{V}_{YM} \right].$$

- Prototype of pure gravity theory ($- + \dots +$):

$$\mathcal{L}_{gr} = -\frac{2}{\kappa_D^2} \sqrt{|g|} \left[R - \frac{1}{2} R \left(\frac{e^{H(\ell^2 \square)} - 1}{\ell^2 \square} \right) R + R_{\mu\nu} \left(\frac{e^{H(\ell^2 \square)} - 1}{\ell^2 \square} \right) R^{\mu\nu} + \mathcal{V}_{gr} \right].$$

- Recent extension to include matter & Higgs fields.

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II. Applications – Flavor Hierarchy via Split Fermions – Shadow Extra Dimensions

Nortier, arXiv:2112.15592 [hep-th]

Toy model:

- 1 flat extra dim. compactified on the orbifold $S^1/\mathbb{Z}_2 \sim$ interval $[0, \pi\rho]$.
- 1 bulk scalar field $\Phi(x, y)$ + associated smeared field ($\Lambda = 1/\ell$):

$$\tilde{\Phi}(x, y) = e^{\ell^2(\partial_\mu^2 + \partial_y^2)} \Phi(x).$$

- SFT-inspired model: **local kinetic term** + **nonlocal interactions**.
- KK-decomposition & normalization of bulk wave functions:

$$\Phi(x, y) = \sum_n \phi_n(x) f_n(y), \quad M_n = \frac{n}{\rho}, \quad \oint dy f_n(y) f_m(y) = \delta_{nm}.$$

- Smeared KK-fields & bulk wave functions

$$\tilde{\phi}_n(x) = e^{\ell^2 \partial_\mu^2} \phi_n(x), \quad \tilde{f}_n(x) = e^{-\left(\frac{n\ell}{\rho}\right)^2} f_n(y).$$

\Rightarrow **KK-modes** with $M_n \gg \Lambda$ have **suppressed 4D effective couplings** ($\propto f_n$'s overlaps) & **4D propagators**.
 \Rightarrow **Shadow extra dim.** = Suppressed KK-mode effects wrt local theory! \Rightarrow **Way to hide an extra dim.!**

II. Applications – Gauge Hierarchy via Warp Transmutation of Scales – Local Model

Randall, Sundrum, arXiv:hep-ph/9905221 / Gherghetta, Pomarol, arXiv:hep-ph/0003129 / Hosotani, Mabe, arXiv:hep-ph/0503020

Local Randall–Sundrum model (RS1):

- Spacetime = Slice of AdS_5 (− + + + +) of proper length ρ :

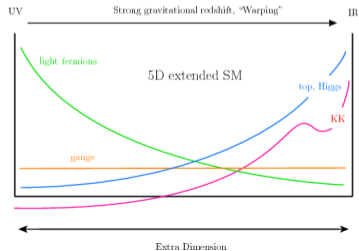
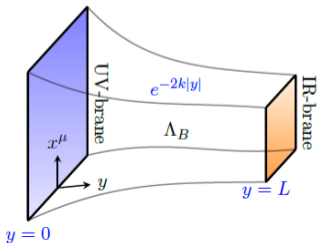
$$ds^2 = g_{MN} dx^M dx^N = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2.$$

- 4D EW Higgs field localized on IR-brane \Rightarrow Redshifted Higgs VEV on IR-brane:

$$v_4 = e^{-k\rho} v_5 \sim 100 \text{ GeV} \Rightarrow \text{Warp transmutation of } \Lambda_{EW}!$$

+ Redshifted nonpert. IR-brane cutoff $\Rightarrow \Lambda_{IR} = e^{-k\rho} \Lambda_{UV} \Rightarrow$ Stability wrt radiative corrections!

- Bulk fermions (quasilocalized 0-modes): 4D Yukawa \propto overlap with Higgs field \Rightarrow Flavor hierarchy!
- LHC bounds $\Rightarrow \Lambda_{IR} \gg M_{KK} = e^{-k\rho} k \geq \mathcal{O}(1) \text{ TeV} \Rightarrow$ Strong little hierarchy problem: .
 \Rightarrow Gauge-Higgs unification: Higgs field = extra components of 5D gauge fields (technically natural).



II. Applications – Gauge Hierarchy via Warp Transmutation of Scales – Weakly Nonlocal Model

4D models: Krasnikov, Theor.Math.Phys. 73 (1987) 1184-1190 / Moffat, Phys.Rev.D 41 (1990) 1177-1184 / Biswas, Okada, arXiv:1407.3331 [hep-ph]

4D bounds: Biswas, Okada, arXiv:1407.3331 [hep-ph] / Su, Li, Nicolaidou, Chen, Wu, Paganis, arXiv:2108.10524 [hep-ph]

RS1 model: Nortier, arXiv:2112.15592 [hep-th]

Stringy warped throats: Klebanov, Strassler, arXiv:hep-th/0007191 / Reece, Wang, arXiv:1003.5669 [hep-ph]

- 4D weakly nonlocal toy models:

- Nonlocal scale Λ stabilizes Higgs-like scalar masses $\Rightarrow \delta M_H^2 \sim \Lambda^2$.
- Nonlocal scale bounds from toy models: $\Lambda = 1/\ell \geq \mathcal{O}(1)$ TeV.

- String-inspired weak nonlocality in RS1 \Rightarrow Higgs field Φ is smeared:

$$\tilde{\Phi}(x) = e^{H(\square/\Lambda_5^2)} \Phi(x), \quad H(\square/\Lambda_5^2) = H\left(\frac{g^{\mu\nu} \partial_\mu \partial_\nu}{\Lambda_5}\right).$$

$\Rightarrow \Lambda_4 = e^{-k\rho} \Lambda_5 \Rightarrow$ Warp transmutation of nonlocal scale!

\Rightarrow Stabilizes brane-localized Higgs boson mass at TeV-scale (alternative to gauge-Higgs unification).

- If $\Lambda_5 \sim M_5$ (string scale) \Rightarrow Redshift of M_5 in warped throat models: Klebanov-Strassler (KS, Superstring IIB).

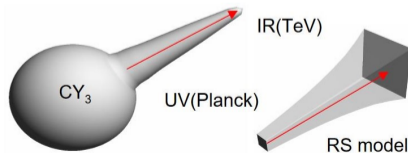
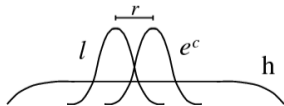
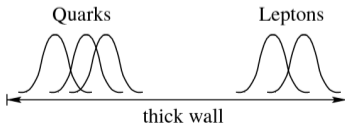
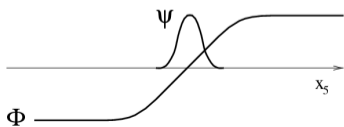


Figure: KS throat versus RS model

Local Arkani-Hamed–Schmaltz (AS) model:

- **Goal:** Generate **natural small interaction couplings**.
⇒ suppressed proton decay, flavor physics, neutrino physics, etc.
- **Ingredients:**
 - 1 flat **extra dim.** of space: **interval** of length $\sim \rho \Rightarrow M_{KK} = 1/\rho$ (KK-scale).
 - **Only bulk fields:** gauge bosons + scalar Φ + fermions Ψ + Higgs boson h .
- **Domain wall** from **bulk scalar Φ** coupled to **bulk fermions Ψ** :
 - **Trapped chiral fermions** (0-modes) inside domain wall.
 - Position of Gaussian wave functions controlled by bulk masses.⇒ **Suppressed 4D fermion operators from suppressed overlaps!**
- **Main constraints:** FCNC's via KK-gauge bosons $\Rightarrow M_{KK} \geq 100$ TeV.



II. Applications – Flavor Hierarchy via Split Fermions – Multiple Fuzzy Branes

Nortier, arXiv:2112.15592 [hep-th]

Goal: \neq UV-origin of split fermions: **domain wall** \rightarrow **multiple fuzzy branes**
 \Rightarrow Same low-energy pheno. & constraints as AS model BUT \neq UV-origin.

Model (5D EFT):

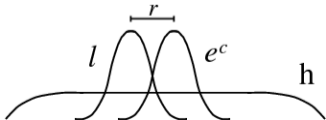
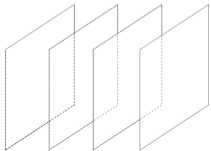
- Yukawa interactions btw 1 **local bulk Higgs field** $H(x, y)$ & **4D weakly nonlocal Weyl fermions** $\psi_L^{(1)}(x)$ & $\psi_R^{(2)}(x)$ localized on 2 \neq **δ -like branes** (nonlocal length scale $\ell \ll \rho$):

$$\mathcal{L} = \mathcal{L}_H - \sum_{i=1}^2 \delta(y - y_i) \psi_{L/R}^{(i)\dagger} (i\cancel{\partial}) \psi_{L/R}^{(i)} + Y \tilde{\Psi}_L^{(1)\dagger} H \tilde{\Psi}_R^{(2)} + \text{H.c.}$$

Smearred fermions:

$$\tilde{\Psi}_{L/R}^{(1/2)}(x, y) = e^{\ell^2 (\partial_\mu^2 + \partial_y^2)} \psi_{L/R}^{(1/2)}(x) \delta(y - y_{1/2}) \Rightarrow y_4 \propto \sqrt{\frac{\ell}{\rho}} e^{\frac{-r^2}{8\ell^2}} \ll 1$$

- **Natural 5D Yukawa couplings** $Y \sim \ell^{3/2}$ + large interbrane distance $r = |y_2 - y_1| \gg \ell$
 \Rightarrow **Suppressed effective 4D Yukawa couplings** $y_4 \ll 1!$
 \Rightarrow **Nonlocal length scale** $\ell \sim$ **domain wall width** in AS model.



Motivations for weak nonlocality:

- Include gravity in UV complete quantum theory of Nature (string theory, asymptotically Lee-Wick QFT's).
- Soft UV-behavior + smearing effect on interaction vertices & pointlike sources.
- Potential path towards asympt. freedom/safety?
- Potential new path to stabilize EW scale?

Applications to braneworlds:

- Shadow extra dim.: KK-excitations ($M_n \gg \Lambda$) have suppressed couplings.
- Fuzzy branes: Suppressed couplings btw fields localized on $2 \neq$ branes.
- Warp transmutation of scales: Nonlocal scale redshifted along a warped extra dim.

⇒ New model building issues for energy frontier!

Outlook:

- Towards a weakly nonlocal SM ⇒ hierarchy problem? quantum gravity?
- Towards a weakly nonlocal RS-like model ⇒ new features of fuzzy branes? UV-complete?
- Study weakly nonlocal pheno. ⇒ collider signatures?
- ...