Università degli Studi di Padova

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# To Break or Not To Break:

## Chiral Symmetry Breaking in QCD-like Theories from Anomaly Matching and Supersymmetry

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work in progress with Luca Ciambriello, Roberto Contino;
hep-th/2202.01239 with Andrea Luzio

I will talk about QCD, the theory we “know” and love, in the following aspects:

- Sketch of a rigorous proof of chiral symmetry breaking from ’t Hooft anomaly matching

- Implications from supersymmetry and anomaly-mediated supersymmetry breaking (AMSB)

# Strong Dynamics at IR

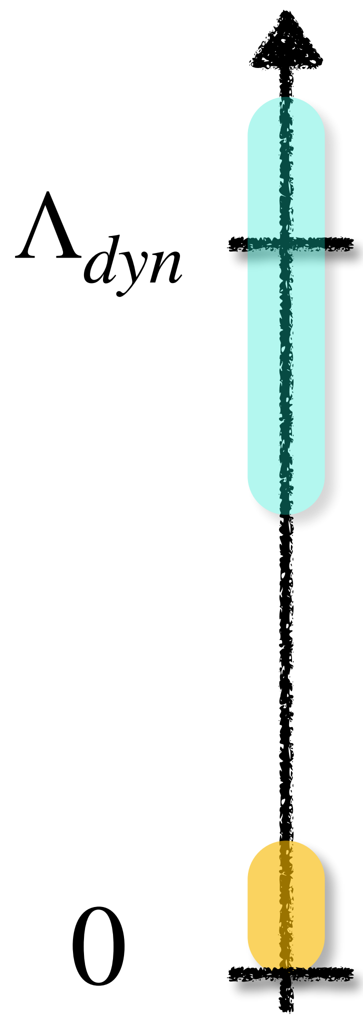
Theories become strongly-coupled at IR

Assuming confinement happens if $N_f$ is below conformal window

Bound states are color singlets and classified under global symmetries, which in QCD-like theories is $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$

Typical spectrum:

# Strong Dynamics at IR




massive composite resonances

(almost) massless states

The existence of these massless states
is implied by 't Hooft anomaly matching
condition for the global symmetries

't Hooft, 1979

**To break or not to break:**

Nambu-Goldstone bosons;

Massless composite <u>chiral</u> fermions.

# ’t Hooft Anomaly Matching

One starts by assuming chiral symmetry is not broken, and checks whether massless fermions can match the anomalies of $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$

$$\sum_{BS} l(BS)\ A(BS) = \sum_{q} l(q)\ A(q)$$

To claim chiral symmetry breaking, we need to consider all the massless composite fermions and prove they do not match the ’t Hooft anomalies

# ’t Hooft Anomaly Matching

One starts by assuming chiral symmetry is <u>not</u> broken, and checks whether massless fermions can match the anomalies of $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$

$$\sum_{BS} l(BS)\, A(BS) = \sum_{q} l(q)\, A(q)$$

To claim chiral symmetry breaking, we need to consider <u>all</u> the massless composite fermions and prove they do <u>not</u> match the ’t Hooft anomalies

However, we do <u>not</u> have good control on the massless composite fermions.
Typically only the simplest baryons are considered in literature

This is the difficulty of the problem, but at the same time motivates our study

# Characterizing Bound States with Tensors

1) Bound states are color singlets: $\boxed{n_q - n_{\bar{q}} = bN_c}$

2) Bound states are charged under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$

$$\boxed{\text{Massless Bound States} \sim \left\{ T^{\{n_L\};\{n_R\}}_{\{\bar{n}_L\};\{\bar{n}_R\};\Delta} \right\}}$$

$$n_q = n_L + n_R + \Delta$$
$$n_{\bar{q}} = \bar{n}_L + \bar{n}_R + \Delta$$

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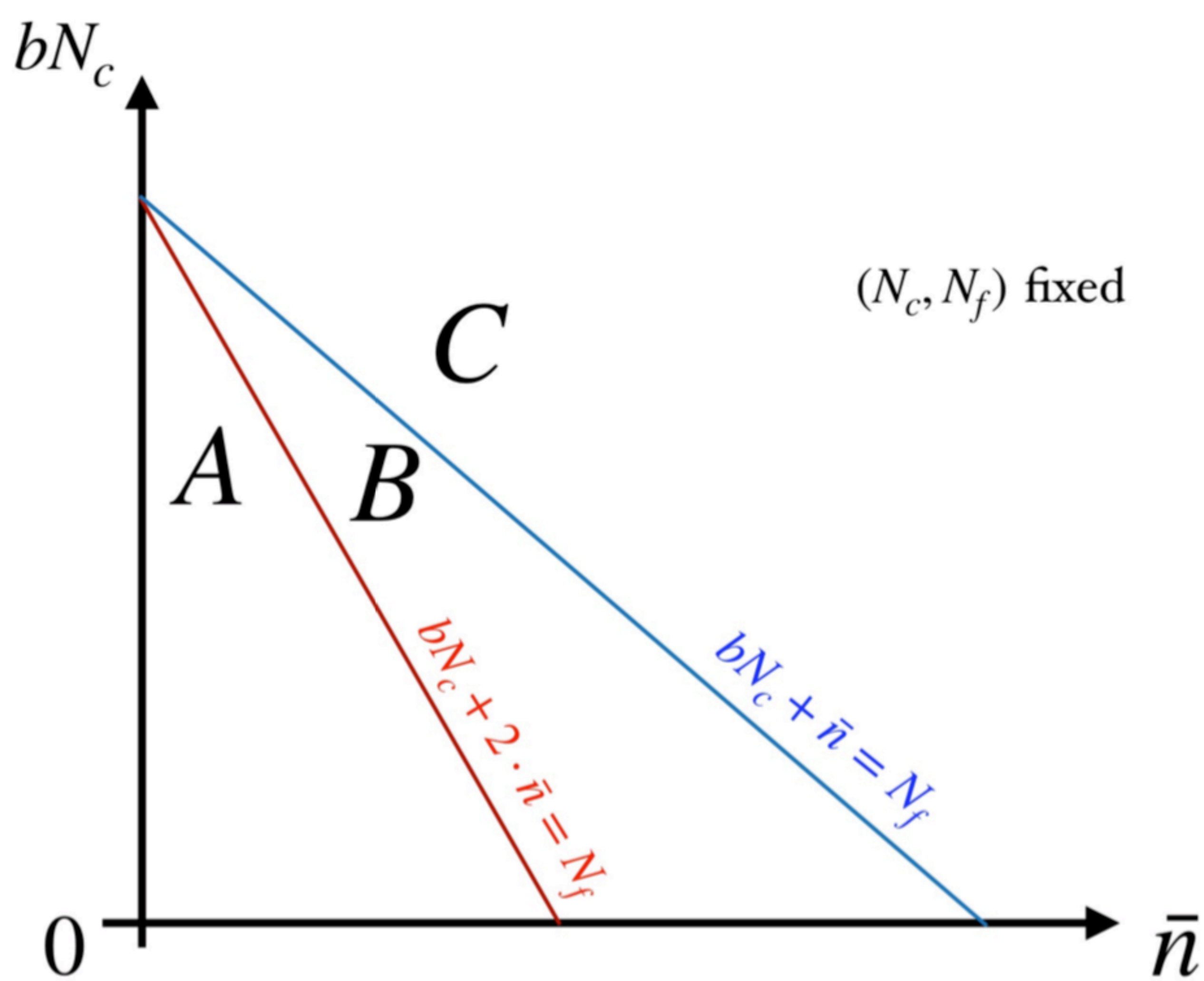
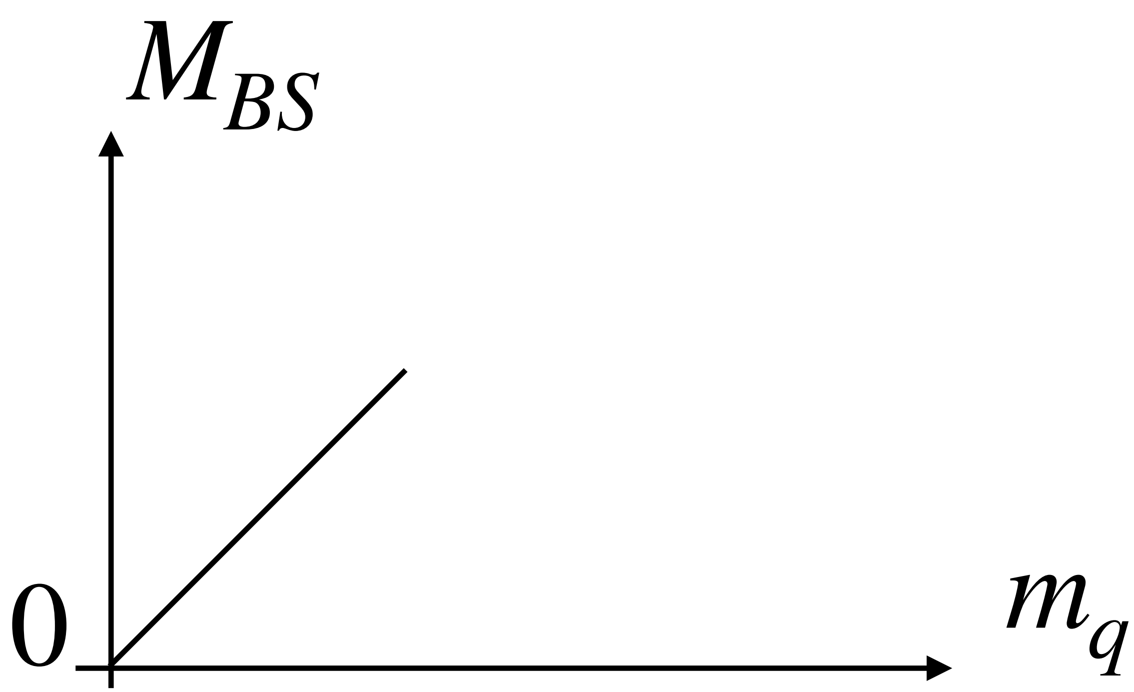
$T^{bN_c+\bar{n}}_{\bar{n}}$ :




We will focus on Class A bound states (no equivalent tensors) in this talk

# Persistent Mass Condition

One can deform the UV theory infinitesimally (namely adding mass terms for quark), and see how the massless composite fermions at IR respond to the deformation




Preskill-Weinberg;
Vafa-Witten (assuming $\theta$ is zero)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(N_f-1)_L \times SU(N_f-1)_R \times U(1)_H \times U(1)_B$$

PMC: $T^{\{n+w\}}_{\{\bar{n}+\bar{w}\};\Delta} \to T^{\{n\}}_{\{\bar{n}\};\Delta}$ at least one of $w, \bar{w}$ or $\Delta$ is not vanishing

# AMC[$N_f$] and PMC[$N_f$] Equations

AMC[$N_f$] for $[SU(N_f)_L]^2 U(1)_B$ and $[SU(N_f)_L]^3$:

$$\sum_{\{n\}} \sum_{\{\bar{n}\}} \sum_{\Delta} \ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f\right) d(\{n_R; \bar{n}_R\}, N_f)\, A_i(\{n_L; \bar{n}_L\}, N_f) = N_c\,.$$

PMC[$N_f$]:

$$\begin{aligned} 0 &= \sum_{T \sim R} \ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f - 1\right) \\ &= \sum_{w,\bar{w},\Delta} k\left(T^{\{n+w\}}_{\{\bar{n}+\bar{w}\};\Delta} \to T^{\{n\}}_{\{\bar{n}\};\Delta}\right) \ell\left(T^{\{n+w\}}_{\{\bar{n}+\bar{w}\};\Delta}, N_f\right)\,. \end{aligned}$$

PMC depend on decomposition of tensors, which in turn only implicitly depend on $N_f$

For the simplest example ($N_c = 3,\ N_f \geq 3,\ b = 1$ baryons), see e.g. in 't Hooft 1979; Weinberg QFT textbook (vol.2, sec.22.5)

# $N_f$ Independence and its Consequence

**$N_f$ independence**: *A finite set of real values $\{\ell\}$ which solves the system of $AMC[N_f] + PMC[N_f]$ for a confining theory with $N_f$ flavors is also a solution of the same equations for any $N'_f \geq N_f$.*

$AMC[N_f]$ :

$$a_h(\{l\})\, N_f^{\,h} + a_{h-1}(\{l\})\, N_f^{\,h-1} + \cdots + a_2(\{l\})\, N_f^{\,2} + a_1(\{l\})\, N_f + a_0(\{l\}) = 0$$

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$AMC[N_f]$ :

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$AMC[N_f+1]$ :

$$a_h(\{l\})\ (N_f+1)^h + a_{h-1}(\{l\})\ (N_f+1)^{h-1} + \cdots + a_2(\{l\})\ (N_f+1)^2 + a_1(\{l\})\ (N_f+1) + a_0(\{l\}) = 0$$

$AMC[N_f+2]$ :

$$a_h(\{l\})\ (N_f+2)^h + a_{h-1}(\{l\})\ (N_f+2)^{h-1} + \cdots + a_2(\{l\})\ (N_f+2)^2 + a_1(\{l\})\ (N_f+2) + a_0(\{l\}) = 0$$

and so on

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$AMC[N_f+2]:$

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and so on

$$a_i(\{l\}) = 0, \quad \text{for} \quad 0 \leq i \leq h$$

$a_0(\{l\}) = 0$ implies chiral symmetry breaking (G. Farrar)

# Proof of $N_f$ Independence

**Lemma 1.** *Let* $\left\{\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f\right)\right\}$ *be a real solution of* $PMC[N_f]$ *such that the multiplicity of any tensor with* $n+\bar{n}=2\bar{n}+bN_c \geq N_f$ *identically vanishes; then* $\left\{\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N'_f\right)\right\}$ *is a solution of* $PMC[N'_f]$ *for any* $N'_f \geq N_f$ *if one defines* $\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N'_f\right) = \ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f\right)$.

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**Lemma 2.** *Let* $\left\{\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f\right)\right\}$ *be a real solution of* $AMC[N_f] + PMC[N_f]$ *such that the multiplicity of any tensor with* $n+\bar{n} = 2\bar{n} + bN_c \geq N_f$ *and* $\Delta > 0$ *identically vanishes; then* $\left\{\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N'_f\right)\right\}$ *is a solution of* $AMC[N'_f] + PMC[N'_f]$ *for any* $N'_f \geq N_f$ *if one defines* $\ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N'_f\right) = \ell\left(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f\right)$.

# Proof of Lemma 2

AMC[$N_f+1$]:

$$N_c = \sum_{\{n\}} \sum_{\{\bar{n}\}} \sum_{\Delta} \ell\Big(T^{\{n\}}_{\{\bar{n}\};\Delta}, N_f+1\Big)\, d(\{n_R;\bar{n}_R\}, N_f+1)\, A_i(\{n_L;\bar{n}_L\}, N_f+1)$$

$$d(\{n_R;\bar{n}_R\}, N_f+1) = \sum_{\{m_R\}} \sum_{\{\bar{m}_R\}} k(\{n_R\}\to\{m_R\})\, k(\{\bar{n}_R\}\to\{\bar{m}_R\}) \times d(\{m_R;\bar{m}_R\}, N_f)\,,$$

$$A_i(\{n_L;\bar{n}_L\}, N_f+1) = \sum_{\{m_L\}} \sum_{\{\bar{m}_L\}} k(\{n_L\}\to\{m_L\})\, k(\{\bar{n}_L\}\to\{\bar{m}_L\}) \times A_i\left(\{m_L;\bar{m}_L\}, N_f\right)\,,$$

$$N_c = \sum_{\{m\}} \sum_{\{\bar{m}\}} d(\{m_R;\bar{m}_R\}, N_f) A_i\left(\{m_L;\bar{m}_L\}, N_f\right) \times \sum_{\{w\}} \sum_{\{\bar{w}\}} \sum_{\Delta} \ell\Big(T^{\{m+w\}}_{\{\bar{m}+\bar{w}\};\Delta}, N_f+1\Big)\, k\Big(T^{\{m+w\}}_{\{\bar{m}+\bar{w}\};\Delta} \to T^{\{m\}}_{\{\bar{m}\};\Delta}\Big)\,,$$

PMC[$N_f+1$] implies that only the term with $w=\bar{w}=\Delta=0$ survive

$$N_c = \sum_{\{m\}} \sum_{\{\bar{m}\}} \ell\Big(T^{\{m\}}_{\{\bar{m}\};\Delta=0}, N_f\Big)\, d(\{m_R;\bar{m}_R\}, N_f) A_i\left(\{m_L;\bar{m}_L\}, N_f\right)$$

AMC[$N_f$] with only BS with $\Delta=0$ have non-vanishing multiplicities

# Proof of $N_f$ Independence

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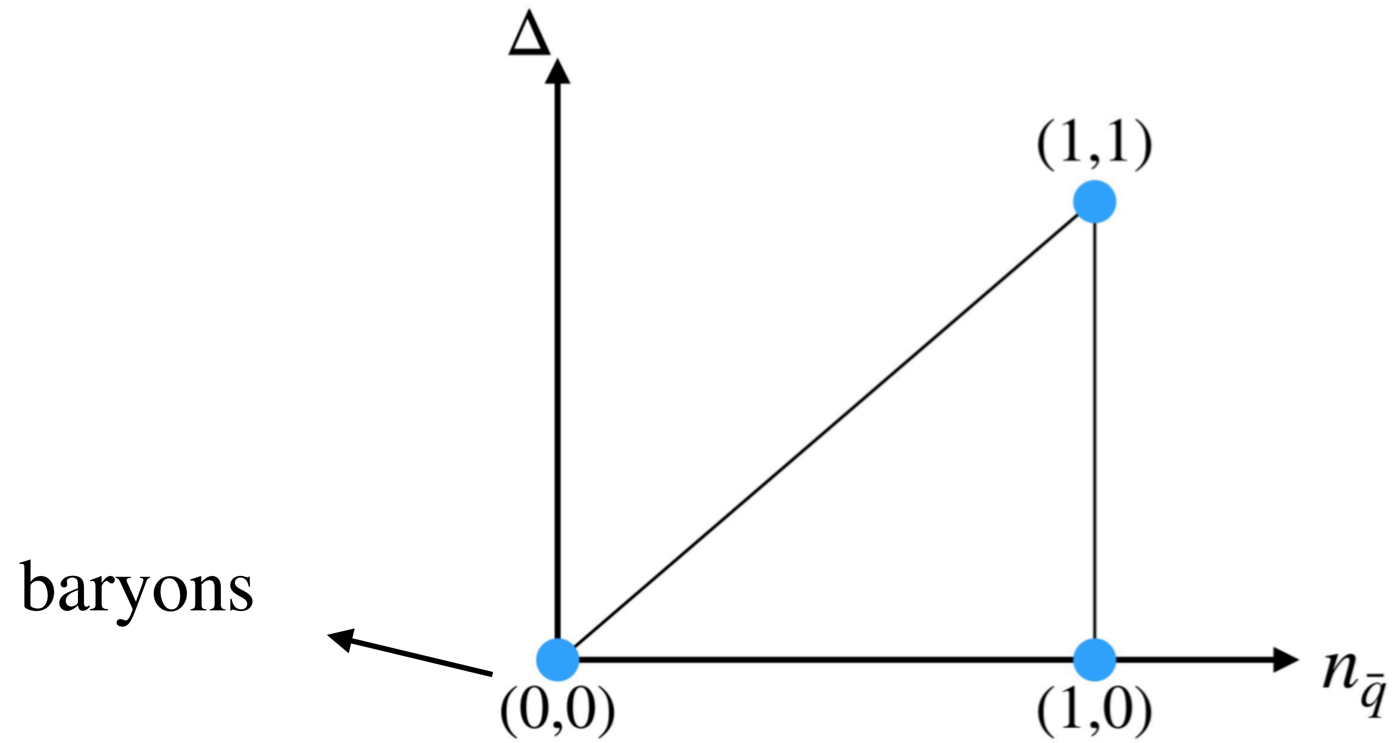
To summarize, Lemma 1 and 2 hold iff

1. Only massless bound states of class A have non-vanishing multiplicities;
2. Massless bound states with $\bar{q}q$ flavor singlets have vanishing multiplicity.

# Exotics ($n_{\bar{q}}$=1) are vectorlike




for fixed $N_c, N_f, b$

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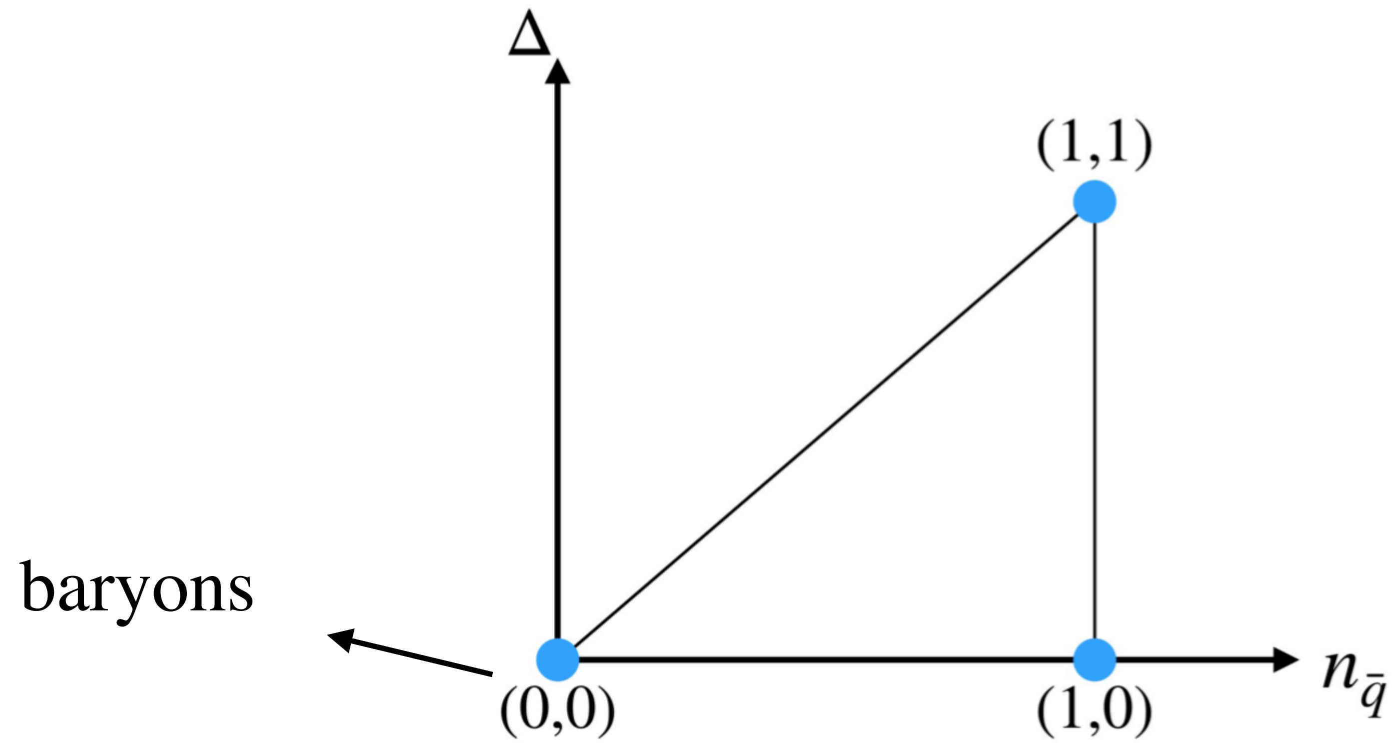


for fixed $N_c, N_f, b$

$$0 = \ell\left(T^{\{n_{max}-i\}}_{\{1\};\Delta=0}, N_f - 1\right) = \sum_{\{n_{max}\}} \ell\left(T^{\{n_{max}\}}_{\{1\};\Delta=0}, N_f\right) k(\{n_{max}\} \to \{n_{max} - i\})$$

$n_{max} = bN_c + 1$

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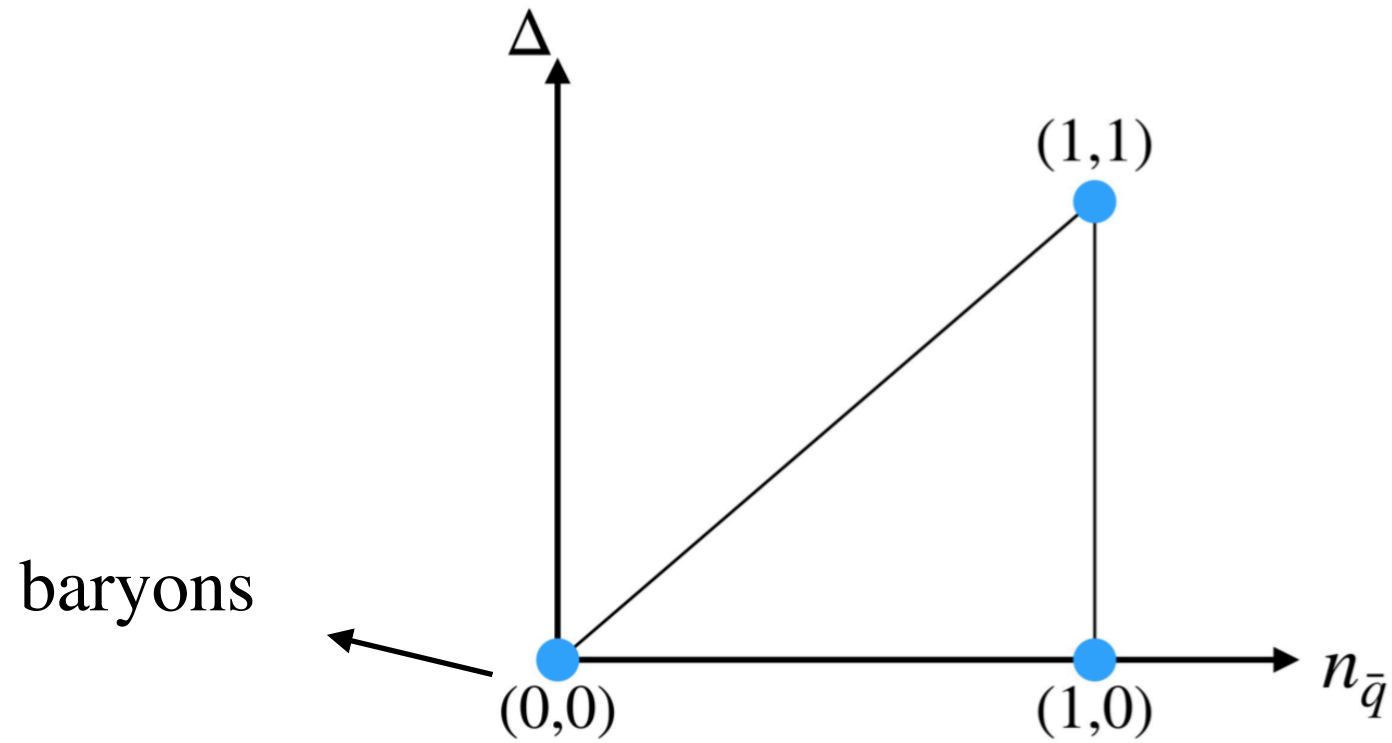


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\end{aligned}
$$

$n_{max} = bN_c + 1$

$$
\begin{aligned}
0 &= \ell\left(T^{\{n_{max}-1\}}_{\{0\};\Delta=1}, N_f - 1\right) + \ell\left(T^{\{n_{max}-1\}}_{\{0\};\Delta=0}, N_f - 1\right) \\
&= \ell\left(T^{\{n_{max}-1\}}_{\{0\};\Delta=1}, N_f\right) + \sum_{\{n_{max}\}} \ell\left(T^{\{n_{max}\}}_{\{1\};\Delta=0}, N_f\right) k(\{n_{max}\} \to \{n_{max} - 1\})
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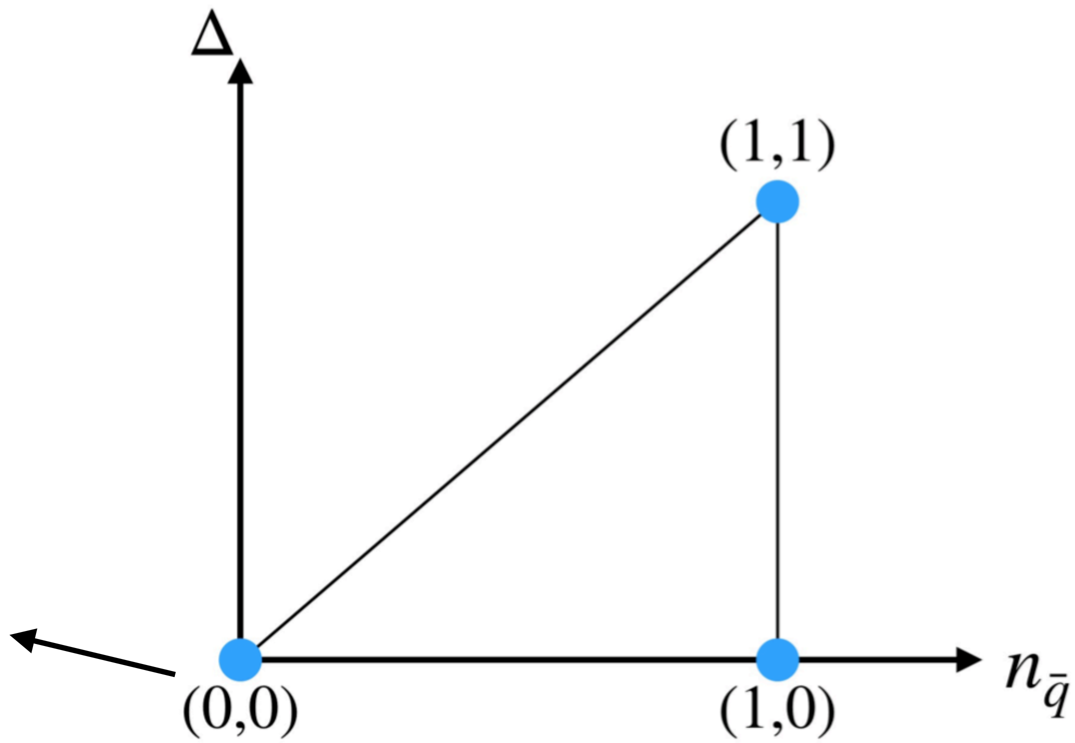


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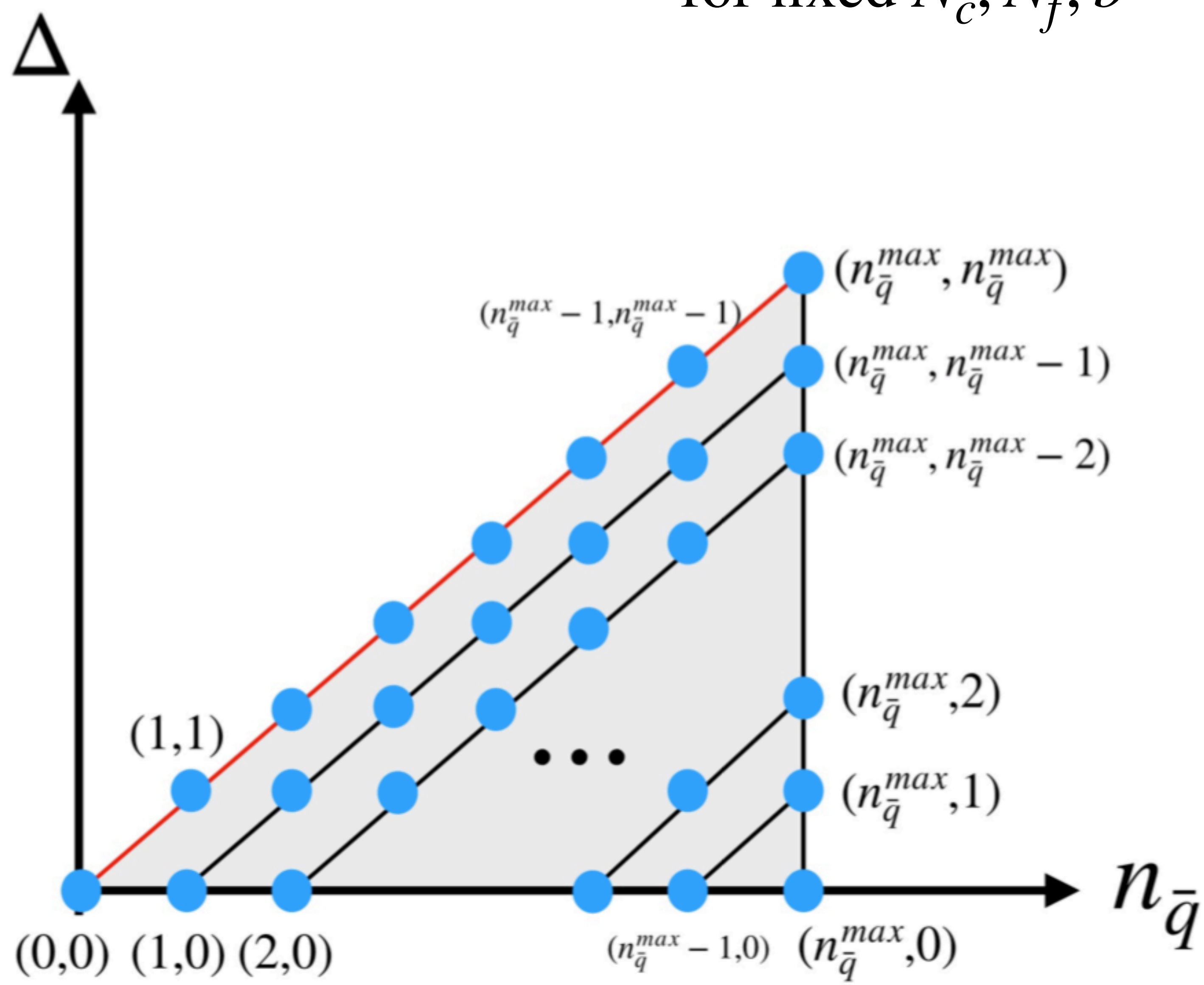
$$= \ell\left(T^{\{n_{max}-1\}}_{\{0\};\Delta=1}, N_f\right) + \sum_{\{n_{max}\}} \ell\left(T^{\{n_{max}\}}_{\{1\};\Delta=0}, N_f\right) k(\{n_{max}\} \to \{n_{max} - 1\})$$

$$\Rightarrow \quad \ell\left(T^{\{n_{max}-1\}}_{\{0\};\Delta=1}, N_f\right) = 0$$

# Generic Exotic States are vectorlike

for fixed $N_c, N_f, b$




$$\sum_{\Delta=1}^{i} \ell\left(T^{\{bN_c+n_{\bar{q}}^{max}-i\}}_{\{n_{\bar{q}}^{max}-i\};\Delta}, N_f\right) = 0\,.$$

# Implication from SUSY?

Real non-SUSY QCD can be embedded in SUSY QCD, with squarks and gluinos being completely decoupled

It is tempting to derive chiral symmetry breaking of real QCD from SUSY QCD
Recent literature argue that AMSB can do the job

However, it was surprising to us:

| SUSY QCD | Real QCD |
|---|---|
| moduli space (infinite many vacua) | How does the (unique) ChSB-vacuum emerge from the moduli space? |
| holomorphy | What are the massless states and condensates? |

# Near-SUSY QCD

- We derive the near-SUSY dynamics for $N_f \leq N_c + 1$ by carefully minimizing the full potential at tree-level AMSB

- We only find agreement with the usual chiral symmetry breaking pattern when $N_f < N_c$

- For $N_f = N_c$ and $N_f = N_c + 1$, we find that there is runaway direction of spontaneously-broken baryon number, which is in contradiction with real QCD in the non-SUSY limit. Therefore, phase transitions are more plausible.

hep-th/2202.01239 Andrea Luzio, L.X.X.

# Take-Home Messages

- For class A bound states, we give a complete and rigorous proof of chiral symmetry breaking of QCD-like theories combing together AMC and PMC.

- Phase transitions are more plausible between SUSY QCD and real QCD.