Dalitz analysis section

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Dalitz section: goals

Overview:

- Formalism description
- Treatment of experimental effects
- Tips and tricks for efficient analysis

What to include and what not:

- Include features used in the analyses from more than one section.
- Features specific to only one or two analysis will go to physics sections.

Inter-relations

- Physics sections:
  - Charmed and charmless B decays,
  - Charm decays
  - Charmonia
  - D mixing
  - CKM angles
- Methods and tools:
  - Maximum likelihood fits
  - Angular analysis
  - Vertexing (kinematic fits)

List of modes used for Dalitz analyses

- Chamless B decays
  - $B^+ \to K^+\pi^-\pi^+$ [Babar, Belle]
  - $B^+ \to \pi^+\pi^+\pi^-$ [Babar]
  - $B^+ \to K^+K^+K^-$ [Babar, Belle]
  - $B^0 \to \pi^+\pi^-\pi^0$ [Babar, Belle]
  - $B^0 \to K_S K^+ K^-$ [Babar]
  - $B^0 \to K^+\pi^-\pi^0$ [Babar]
  - $B^0 \to K_S\pi^+\pi^-$ [Babar, Belle]
- D decays
  - $D^0 \to K_S K^+ K^-$ [Babar, Belle]
  - $D^0 \to K_S\pi^+\pi^-$ [Babar, Belle]
  - $D^0 \to \pi^0\pi^+\pi^-$ [Babar]
  - $D^0 \to K^+K^-\pi^0$ [Babar]
  - $D_s \to \pi\pi\pi^0$ [Babar]
- Chamed B decays
  - $B^+ \to D^-\pi^+\pi^+$ [Babar]
  - $B^0 \to D^-K^0\pi^+$ [Babar]
  - $B^0 \to D^0\pi^+\pi^-$ [Belle]
  - $B^- \to \Lambda_c^+\bar{p}\pi-$ [Belle]
  - $B^0 \to K^-\pi^+\chi_{c1}$ [Belle]
  - $B \to K\pi^+\psi'$ [Belle]
  - $B^+ \to D^0\overline{D}^0K^+$ [Belle]

Introduction

3-body decay phase space:

$$dN \propto \delta^4\left(p_B - \sum_{i=1}^{3} p_i\right)\prod_{i=1}^{3}\frac{d^3p_i}{E_i} = \delta\left(m_B - \sum_{i=1}^{3} E_i\right)\frac{p_1^2dp_1p_2^2dp_2d\Omega_1d\Omega_2}{E_1E_2E_3}$$

$$\propto \; dE_1dE_2$$

$$\propto \; dm_{12}^2dm_{23}^2$$

Boundaries, kinematic constraints:

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = m_B^2 + m_1^2 + m_2^2 + m_3^2 \, ,$$

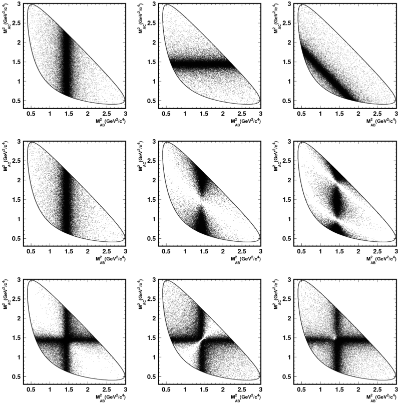
$$\left(m_{\pi^+\pi^-}^2\right)_{\max} = (E_{\pi^+} + E_{\pi^-})^2 - (p_{\pi^+} - p_{\pi^-})^2$$

$$\left(m_{\pi^+\pi^-}^2\right)_{\min} = (E_{\pi^+} + E_{\pi^-})^2 - (p_{\pi^+} + p_{\pi^-})^2$$

Examples of usage:

- Searches for new states
- Measuring resonance properties
- CP violation
- B and D mixing
- Resolving ambiguities

Illustrations:



Various channels, spins, interference phases

Amplitude description: dynamics

Content can be similar to PDG review [D. Asner]

- Breit-Wigner amplitudes:
  - Corrections: running width, BW factors
  - Limitations: narrow resonances, no interference, otherwise breaks unitarity
- Gounaris-Sakurai amplitude for $\rho \to \pi\pi$
- K-matrix formalism
  - Unitarity by construction, identical to BW for 1 pole
  - Typical parametrizations of K matrix and production vector
  - K matrix fixed from scattering experiments, production vector P is a free parameter
- Flatte formalism
  - Case of K matrix with 2 poles: useful to describe the amplitude near threshold ($f_0(980) \to \pi\pi$)

$$\Gamma = \Gamma_r \left(\frac{q}{q_r}\right)^{2L+1} \left(\frac{m_r}{m_{ab}}\right) B'_L(q, q_0)^2 ,$$

L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$

where $z = (|q|\,d)^2$ and $z_0 = (|q_0|\,d)^2$

$$\hat{F}_i = (I - i\hat{K}\rho)^{-1}_{ij}\hat{P}_j = (\hat{T}\hat{K}^{-1})_{ij}\hat{P}_j ,$$

$$K_{ij}(s) = \left[\sum_\alpha \left(\frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s}\right) + f_{ij}^{sc}\frac{1 - s_0^{sc}}{s - s_0^{sc}}\right] \left[\frac{(s - s_A m_\pi^2/2)(1 - s_{A0})}{(s - s_{A0})}\right] . \quad (13)$$

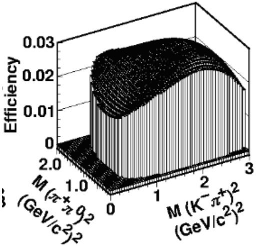
$$P_j(s) = \left[\sum_\alpha \left(\frac{\beta_\alpha g_j^{(\alpha)}}{m_\alpha^2 - s}\right) + f_{1j}^{pr}\frac{1 - s_0^{pr}}{s - s_0^{pr}}\right] \left[\frac{(s - s_A m_\pi^2/2)(1 - s_{A0})}{(s - s_{A0})}\right] . \quad (14)$$

Amplitude description

- Angular terms: [Angular analysis]
  - Explicit formulas for spin J
  - Warning: phase conventions ($\phi \to \phi + \pi$)
  - Non-scalar final states: several helicity amplitudes may contribute [Charm analyses]
- Nonresonant description

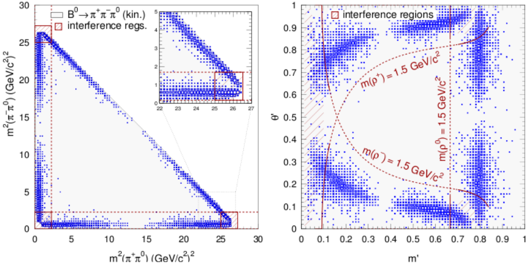
$J \to L + l$	Angular distribution
$0 \to 0 + 0$	uniform
$0 \to 1 + 1$	$(1+\zeta^2)\cos^2\theta$
$0 \to 2 + 2$	$\left(\zeta^2 + \frac{3}{2}\right)^2 (\cos^2\theta - 1/3)^2$

- Backgrounds:
  - Extraction: MC, sidebands, etc.
  - Parameterization
  - Nonparametric description: kernel estimation of the pdf, KEYS function.
- Efficiency:
  - Typical behavior: falloff in corners due to acceptance
  - Parametrization
  - Nonparametric treatment: MC integration using distribution after full detector MC.
- Momentum resolution, self cross-feed.



Technical details

- Modifications of phase space:
  - Square Dalitz plots: mass and helicity angle.
    - Easier binned analysis
    - Factorization of efficiency
    - Transformation Jakobian needed



  - Identical particles in the final states: symmetry

- Fitting:
  - Binned fits: adaptive binning, χ^2 tests [ML fits]
  - Unbinned fit: likelihood [ML fits]

$$-2\log L = -2\left[\sum_{i=1}^{n}\log p(m^2_{+,i}, m^2_{-,i}) - n\log\int_D p(m^2_+, m^2_-)dm^2_+ dm^2_-\right],$$

  - Time-dependent analyses
  - Normalization: analytic, numerical, MC
  - Improving fit performance:
    - Complex expansion of the $|\text{amplitude}|^2$: faster calculation
    - Polar vs. Cartesian parameters
- Fit fractions, interference fractions
  - Quantities independent of amplitude conventions. Useful to compare different analyses.

$$FF_j = \frac{\int\int_{DP}\left|c_j e^{i\theta_j} F_j(m^2_{K^+\pi^-}, m^2_{\pi^+\pi^-})\right|^2 dm^2_{K^+\pi^-} dm^2_{\pi^+\pi^-}}{\int\int_{DP}\left|\sum_j c_j e^{i\theta_j} F_j(m^2_{K^+\pi^-}, m^2_{\pi^+\pi^-})\right|^2 dm^2_{K^+\pi^-} dm^2_{\pi^+\pi^-}}$$

Model uncertainties

- Estimation
- Model-independent analyses [γ/ϕ_3 section]
  - Dalitz plot binning, optimization
  - Relation with studies at charm threshold
- Model-independent partial wave analysis [$\rightarrow$ charm section]
  - Scalar amplitude interpolated on a set of points. Interference with other channels provides information about amplitudes and phases.

In total — 15-20 pages

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Need to define partitioning and responsible persons for each part.