Beyond Poisson equation in N-body simulations

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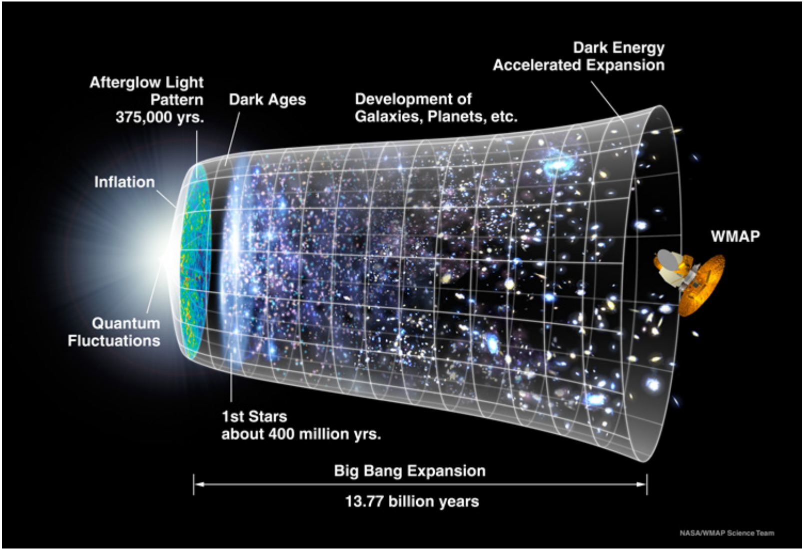
ThUG 21

Large Scale Structures (LSS) formation



Goal:

Propagate primordial non-gaussianities through cosmic history

Relativistic effects

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Propagate primordial non-gaussianities through cosmic history

- Bispectrum couples scales: $\langle\delta(\vec{k}_1,t)\delta(\vec{k}_2,t)\delta(\vec{k}_3,t)\rangle = (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(k_1,k_2,k_3,t)$.
- Future surveys will probe scales close to the Horizon scale $\rightarrow$ Newtonian physics is not enough
- Relativistic degree of freedom, allow to probe GR on the largest scales
- Relativistic species (neutrinos, cosmic strings, DDE, IDE.)
- Observations are made on the relativistic perturbed light cone.

Poisson:

$$\Delta\phi = 4\pi G a^2(t)\delta(\vec{x},t) \tag{1}$$

Relativistic Poisson:

$$e^{2\phi}\Delta\phi = 4\pi G a^2\delta - \frac{3}{2}e^{-2\psi}(\mathcal{H}-\phi')^2 + \frac{e^{2\phi}}{2}\phi_{,i}^2 \tag{2}$$

follows from metric in Poisson gauge

$$ds^2 = -a^2e^{2\psi}d\tau^2 + a^2e^{-2\phi}\delta_{ij}\left(dx^i+\beta^i d\tau\right)\left(dx^j+\beta^j d\tau\right). \tag{3}$$

Fundamental physics

Inflation models with energy scale below 10^{16} Gev have no observable primordial gravitational waves. Class these models using **primordial non-gaussianities**: complements GW searches (Meerburg 1903.04409).

Theorem: (Consistency relations), Maldacena 0210603

If only one light scalar field is active during inflation, the behavior of the three-point correlation function, in the squeezed limit, is entirely fixed by the two-point correlation function.

Single field predicts $f_{\mathrm{NL}} \simeq \frac{5}{12}(1-n_S) \simeq 0.02$. A detection of $f_{\mathrm{NL}} \gg 0.02$ rules out all single inflation.

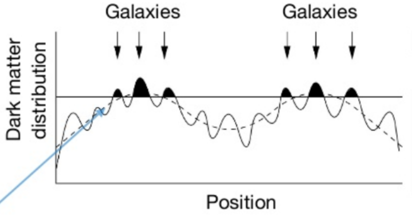
Way out of the theorem:

- Several fields active during inflation Sugiyama 1101.3636
- higher spin Arkani-Hamed 1503.08043
- 'modified' gravity Tahara 1805.00186
- anisotropic inflation Emaml 1511.01683
- electromagnetic field Chua 1810.09815 **Stahl** 1507.01686

These theorems also apply to the late universe (Creminelli 1309.3557)
$\rightarrow$ probe the early universe with LSS observables.

Signature of multi-field inflation for galaxies

Galaxies form at peaks of the dark matter distribution



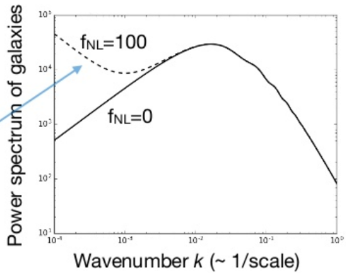
Multi-field inflation couples those peaks to the background potential

⇒ Galaxies are modulated by the background potential

$$\phi \propto \frac{\delta}{k^2}$$

⇒ Enhancement of the power spectrum of galaxies ~ f_{NL}/k^2

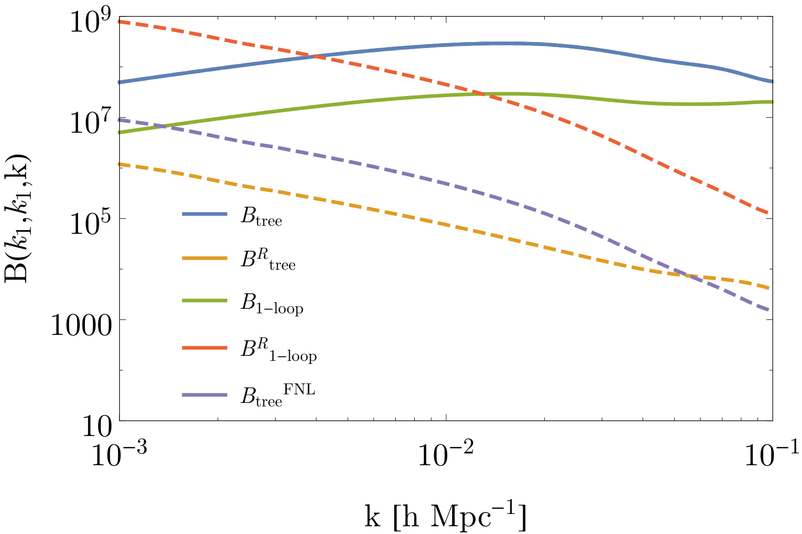
'Scale-dependent galaxy bias'



Kaiser (1984), Dalal et al. (2007), Top figure: J. Peacock

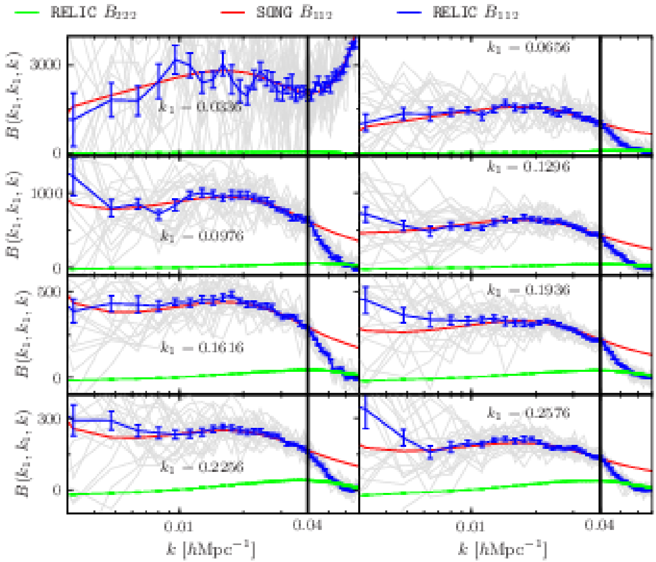
What we did

Analytic approach: Calculate the non-linear relativistic corrections to perturbation theory kernels. These relativistic effects are totally degenerated with primordial non-gaussianites of the local type (**Castiblanco** 1811.05452, **Calles** 1912.13034).



Non-gaussianities and relativistic corrections require non-linear initial conditions.

Numerical approach: Complement our analytical approach by introducing in the general relativistic N-body code `gevolution` (relativistic version of `Gadget`) non-gaussian initial conditions. (**Adamek** 2110.11249)



Conclusions

- Local non-gaussianites couple large (relativistic) scales and small (non-linear) scales.
- GR corrections to Poisson equation are totally degenerated with primordial non-gaussianities.
- We probe for the first time a bispectrum in a relativistic N-body simulation.
- Ray tracing exists within `gevolution`. Include the travel of the photons in a clumpy universe (cf. redshift space distortion, finger of god like effects).

Dirac-Milne: an invitation to reinterpret cosmological observations

- Ambitious framework without dark energy and dark matter.
- Exotic gravity with

$$\Delta\phi_1 = 4\pi G(\rho_1 - \rho_2) \tag{4}$$

$$\Delta\phi_2 = 4\pi G(-\rho_1 - \rho_2) \tag{5}$$

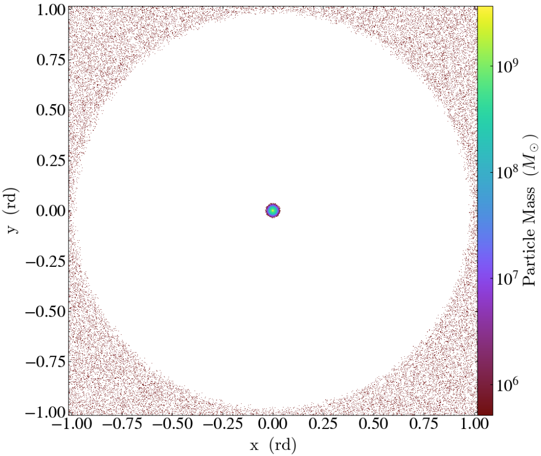
  Repulsion between matter and antimatter. Antimatter *antigravitates*.
- Solve in passing the matter-antimatter asymetry problem.
- Milne cosmology :

$$a(t) \propto t \tag{6}$$

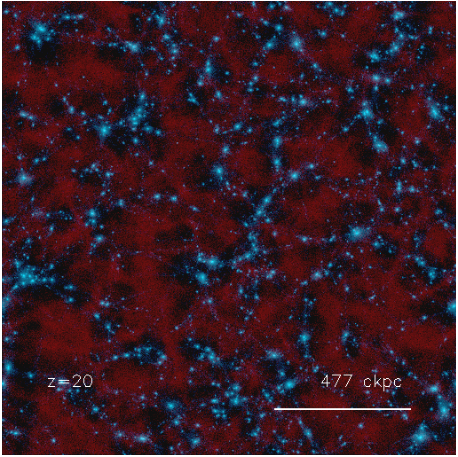
  On large scales: void universe: $\bar{\rho}(t) = 0$. Only coordinate expansion.

- See Levy 1110.3054 for a discussion on nucleosynthesis, the position of the first peak of the CMB and supernovae IA in Dirac-Milne cosmology.
- In Manfredi 1804.03067 and Manfredi 2010.07776 structure formation was studied using a N-body code which assumes spherical symmetry.

- Start from disjoint quasi homogeneous supports for Virialized matter and antimatter at redshift 1080. Density contrast between matter and antimatter are of order 1. Let the system evolve.
- Matter rapidly collapses into structures.
- Antimatter spreads around.
- But antimatter does not come very close from structures as it repels with matter. $\rightarrow$ creation of a *buffer* (depletion) zone which is totally void.



Our Simulation



Numerical setup:

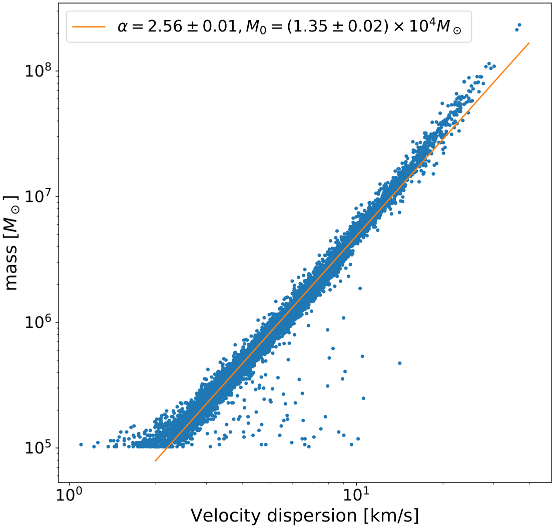
Grid: 256^3, BoxSize$= 1\ h^{-1}$ cMpc, Total mass in the box : $3.3 \times 10^{10} M_\odot$.

Assumption:

Matter is in the form of stars: particle sector of `RAMSES`, no hydro.

Main results: scaling relation (Tully-Fisher)

Tight scaling relation, slope closer from 3 than 4. See discussion in Lelli 1901.05966 or Ponomareva 1711.09112.



Main results: the Radial Acceleration Relation

- Define $\vec{g}_m = -\vec{\nabla}(\phi_1 - \phi_2)/2$ and $\vec{g}_+ = -\vec{\nabla}\phi_1$.
- The matter field feels an extra force due to the coupled Poisson equations.

Results and conclusions

Chardin 2102.08834, https://github.com/cspotz/RAMSES_Bi-Poisson, https://youtu.be/aqyuDYrwyBQ

- Explored scaling relation (Tully-Fischer) and find an exponent quite low (2.56)
- Currently the Radial Acceleration Relation is not explained in this model but the phenomenology goes into the right direction.
- Rich interesting phenomenology at galactic scales
- Prediction 1: acceleration scale of MOND is time dependant $a_0(z)$. $\rightarrow$ rotation curves of galaxies at high redshift.
- No known linear regime in Dirac-Milne cosmology. Need resolution of 100 ckpc. $\rightarrow$ computationally expensive to simulate the largest scales.
- Prediction 2: More small scales structures at high redshift (non-linear regime all the way from $z = 1080$). $\rightarrow$ observation of high redshift quasars.
- Prediction 3: Antimatter *antigravitates*. $\rightarrow$ Gbar, ALPHA-g and AEgIS @CERN

Dipolar dark matter

- Analogy with polarized medium in electromagnetism where permitvity is present in Maxwell equations.
- Blanchet 0901.3114 proposed gravitational polarization as a candidate for dark matter and dark energy.
- Polarized Poisson reads:

$$\Delta\phi = 4\pi G \left(\rho - \vec{\nabla} \cdot \vec{\Pi_{\perp}}\right), \tag{7}$$

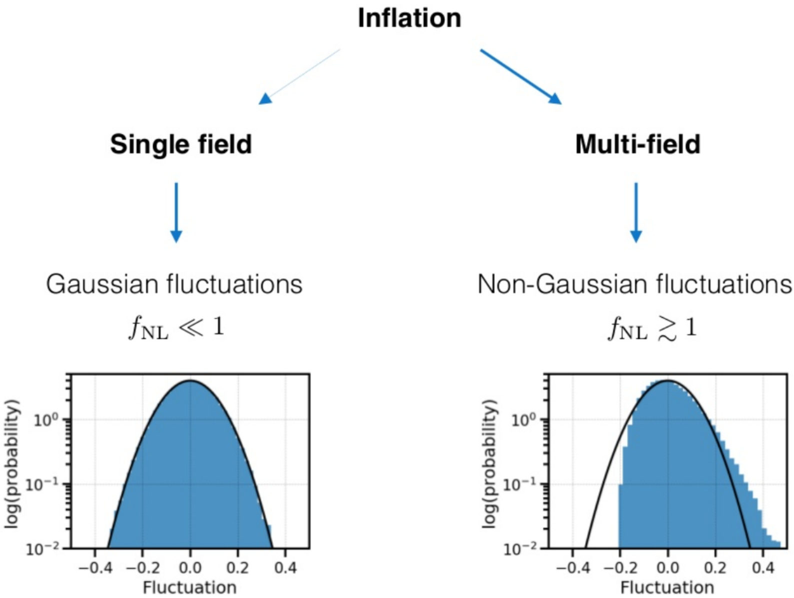
where Π follow a second order differential equation (see Eq. 3.5 of Blanchet 0901.3114)

- Equilibrium solutions for galaxies with secular instabilities.
- Trying to run a cosmological simulation.

Conclusions

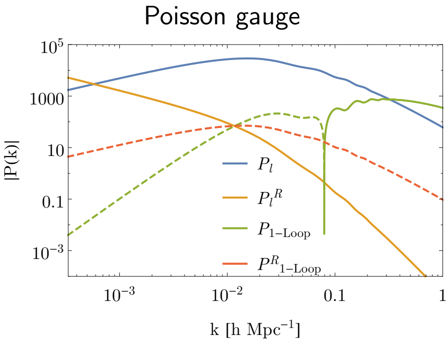
- My current main: extend the gravitational part of the N-body codes: $\Delta\phi = 4\pi G a^2 \delta$ upgrades into:
  1. $e^{2\phi}\Delta\phi = 4\pi G a^2 \delta - \frac{3}{2}e^{-2\psi}(\mathcal{H} - \phi')^2 + \frac{e^{2\phi}}{2}\phi_{,i}^2$
  2. $\Delta\phi_1 = 4\pi G(\rho_1 - \rho_2)$ and $\Delta\phi_2 = 4\pi G(-\rho_1 - \rho_2)$
  3. $\Delta\phi = 4\pi G\left(\rho - \vec{\nabla}\cdot\vec{\Pi_{\perp}}\right),$
- Link fundamental physics with cosmological observables:
  1. constrain inflation with LSS
  2. new cosmology
  3. nature of dark matter

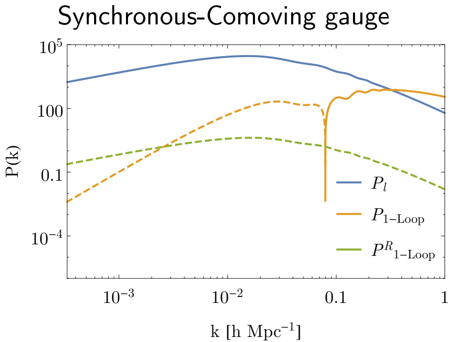
Thank you for your attention



Slide from M. Schmittfull.

More Results





IR behavior

We focus on the power spectrum (analogous results for the bispectrum).

$$P_{\text{1-loop}} = \langle \delta\delta \rangle = \langle \delta^{(2)}\delta^{(2)} \rangle + \langle \delta^{(1)}\delta^{(3)} \rangle = P_{22} + P_{13}. \quad (8)$$

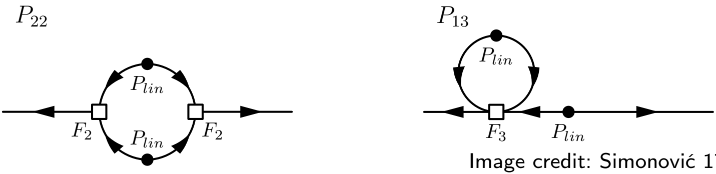


Image credit: Simonović 1708.08130

The IR behavior of the loop integrals is:

$$2 \lim_{q\to 0} P_{22}(t, \boldsymbol{k}) \sim \left(\frac{1}{6\pi^2} k^2 - \frac{1}{42\pi^2} H^2 a^2 \right) , \quad (9)$$

$$\lim_{q\to 0} P_{13}(t, \boldsymbol{k}) = \left(-\frac{1}{6\pi^2} k^2 - \frac{2521}{42\pi^2} H^2 a^2 \right) . \quad (10)$$

These divergences come from kinematic effects which change under a change of frame. Since Newtonian correlators are Galilean invariant (equivalence principle), these divergences cancel (Scoccimarro 9509047).
Rem: In GR, correlators are not invariant under a change of frame (Creminelli 1309.3557). The loop-integral depends on the IR cutoff, physically, it represents the size of the galaxy survey.

UV behavior

The UV behaviors of the loop integrals are:

$$\lim_{q\to\infty} P_{22} \sim a^4 k^4 \int dq \frac{P_L(q)^2}{q^2}\,, \tag{11}$$

$$\lim_{q\to\infty} P_{13} \sim a^4 k^2 P_L(k) \int dq P_L(q)\,. \tag{12}$$

Effective Field Theory (EFT):

Short scales physics 'unknown' (astrophysics) $\to$ compensate with counter-terms: $T_{ij}^{\text{ct}} = c^{22}(t)k^4$ (stochastic noise) and $T_{ij}^{\text{ct}} = c^{13}(t)k^2\delta$ (viscosity). Carrasco 1206.2926.

$$\lim_{q\to\infty} P_{22}^R \sim a^3 H_0^2 k^2 \int dq \frac{P_L(q)^2}{q^2}\,, \tag{13}$$

$$\lim_{q\to\infty} P_{13}^R \sim a^3 H_0^2 P_L(k) \int dq P_L(q)\,. \tag{14}$$

Other type of noise and relativistic correction to the speed of sound. Those terms cannot be predicted by the theory and need to be fitted with a relativistic N-body simulation such as Adamek 1604.06065.

Interlude (advertisement): Schwinger effect in the early universe

Schwinger effect

Above a critical value for an electric field: particle production occurs 'Schwinger effect' (Sauter 1930, Schwinger 1954). Not detected today.
Cosmology: if during inflation, a strong electric field is present: particle production Fröb 1401.4137, Kobayashi 1408.4141.
Their behavior depends on their spin (**Stahl** 1507.01686), on the spatial dimensions (**Bavarsad** 1602.06556) and changes if one adds a magnetic field (**Bavarsad** 1707.03975).

Produce non-gaussianities

'Cosmic-collider': to couple the pairs created to the inflaton leads to a unique signal Chua 1810.09815.

Impact primordial gravitational waves

Only for higher spin like SU(2) fields Lozanov 1805.09318.

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Stability of de Sitter space

backreaction to the inflation dynamics (**Bavarsad** 1602.06556)

Primordial magnetogenesis

Triggering particle production may help to generate the seed for the large scale magnetic field observed today **Stahl** 1603.07166, **Stahl** 1806.06692, Sobol 1807.09851.

Bispectrum for Fundamental physics: Inflation

The squeezed limit contains model independent information about the physics during *inflation*.

Image credit: Pablo Carlos Budassi

Inflation *explains* the origin of the primordial density perturbation. It predicts a Gaussian spectrum (nearly) scale invariant $P(k) = A_s k^{n_S}$.

The perturbations grow into the CMB anisotropies and eventually into the stars and galaxies we see around us.

We have a detection of a small departure from scale invariance, consistent with the expectations of simple inflationary models.

In inflationary paradigm, in the first fractions of second, the rapid expansion dillutes anything but quantum fluctuations which imprint into the *full* gravitational fields of the universe.

Bispectrum for Fundamental physics: Inflation

Successfull (and has no serious concurrent consistant with data) but... How did inflation occur? How did it begin? Are ground-state quantum fluctuations truly the source of density perturbations? What is the connection of inflation to the rest of physics? Are there observations that could falsify inflation?

Quite a zoology of inflation models (Encyclopaedia Inflationaris, Martin 1303.3787, 368 pages, 192 figures)

Large Scale Structures (LSS) formation

In LSS, split between large scales *background*: $\bar{\rho}(t)$ (expanding universe, well defined mean density) and intermediate scales *perturbations* $\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)-\bar{\rho}(t)}{\bar{\rho}(t)}$ (density differs little from background).

Perturbation theory

- Assumptions: Linear fluids mechanics in an expanding universe.
- Boltzmann-Poisson : $\mathcal{L}[f] = 0$ and $\Delta\phi = 4\pi G a^2(t)\delta(\vec{x},t) \rightarrow$ $\ddot{\delta}(\vec{x},t) +$ (expansion) $\dot{\delta}(\vec{x},t) -$ (gravity) $\delta(\vec{x},t) = 0$

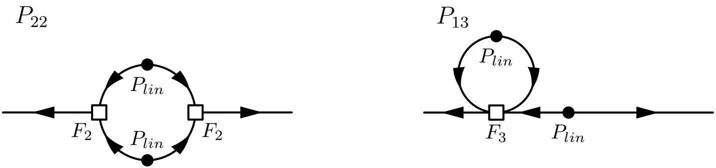
Statistical properties

- One universe = one realization. $\delta(\vec{x},t)$ not very useful $\rightarrow$ statistical properties.
- $\langle\delta(\vec{x},t)\rangle = 0$ by construction
- Variance $\langle\delta(\vec{x},t)^2\rangle$ useful but there is more..
- 2-point correlation function $\langle\delta(\vec{x},t)\delta(\vec{y},t)\rangle$ or in Fourier space power spectrum P: $\langle\delta(\vec{k}_1,t)\delta(\vec{k}_2,t)\rangle = (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2)P(k_1,t)$ contain most information
- Bispectrum B $\langle\delta(\vec{k}_1,t)\delta(\vec{k}_2,t)\delta(\vec{k}_3,t)\rangle = (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(k_1,k_2,k_3,t)$ and higher-order correlations, also very interesting.

- Cosmic structures grow out of tiny initial fluctuations $\delta_l(\vec{k},t)$ and become non-linear.
- Linear perturbation theory fails big time for $k > 0.1\ h$/Mpc Possible to push perturbation theory to obtain some non-linear term :

$$\delta(\vec{k},t) = \sum_{n=1}^{\infty} a^n(t) \int_{\vec{k}_1..\vec{k}_n} F_n(\vec{k}_1,..,\vec{k}_n)\delta_l(\vec{k}_1)..\delta_l(\vec{k}_n) \equiv \sum_{n=1}^{\infty} \delta^{(n)} . \tag{15}$$

$$P_{\text{1-loop}} = \langle\delta\delta\rangle = \langle\delta^{(2)}\delta^{(2)}\rangle + \langle\delta^{(1)}\delta^{(3)}\rangle = P_{22} + P_{13}. \tag{16}$$



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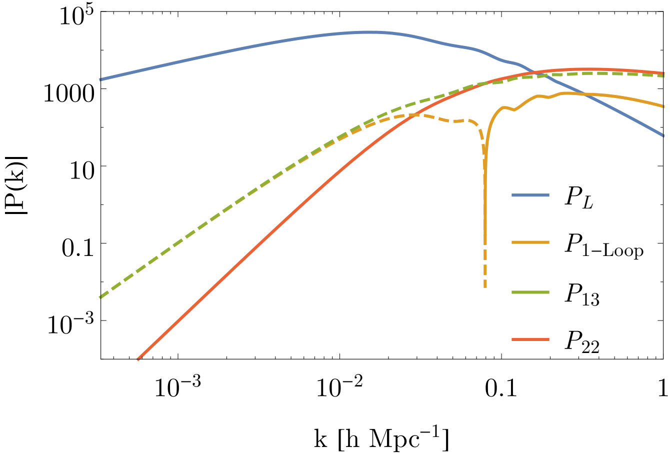

- Recursive relations for kernels F_n (Bernardeau 0112551, eq. 43), but still need to calculate multi dimensional integrals in an efficient way (see Simonović 1708.08130 for a technique).
- Import techniques from field theory to improve the convergence properties of perturbation theory: regularized perturbation theory, path integral formalism, renormalization group flow, coarse grained perturbation theory, effective field theory, kinetic field theory... (see **Castiblanco** 1910.03931 for references).