# Hawking radiation of primordial black holes

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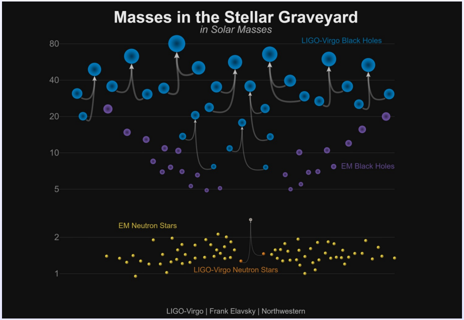
**Institut Henri Poincaré – December 13th, 2021**

# Observed black holes

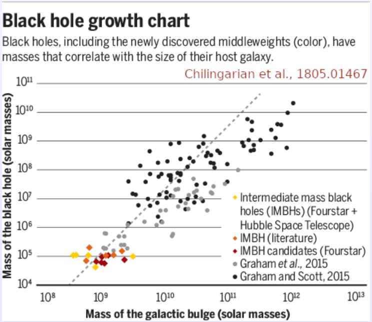
## Three types of black holes have been discovered

- Stellar black holes
  BHs originated in the explosion of massive stars/supernovae, $\sim 3 - 100\, M_\odot$
- Intermediate mass black holes (IMBH)
  New class of recently discovered BHs, $\sim 10^3 - 10^6\, M_\odot$
- supermassive black holes (SMBH)
  BHs at the center of galaxies, $\sim 10^6 - 10^9\, M_\odot$



Masses in the Stellar Graveyard
*in Solar Masses*

LIGO-Virgo | Frank Elavsky | Northwestern



**Black hole growth chart**

Black holes, including the newly discovered middleweights (color), have masses that correlate with the size of their host galaxy.

Chilingarian et al., 1805.01467

# Origin of primordial black holes

## Multiple inflationary origins

- collapse of large primordial overdensities
- phase transitions
- collapse of cosmic strings, domain walls

## Mass predictions

Assuming that one PBH can be formed in a Hubble volume in the early Universe, one gets

$$M_{\rm PBH} \sim M_{\rm Planck} \times \frac{t_0}{t_{\rm Planck}} \sim 10^{38}\ {\rm g}\ \times t_0({\rm s})$$

where $t_0$ is the creation time.

We get:

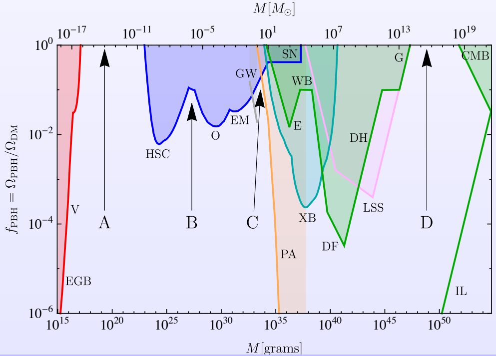
- $M \sim 10^{-5}$ g for $t_0 \sim 10^{-43}$ s $\rightarrow$ Planck black holes
- $M \sim 10^{15}$ g for $t_0 \sim 10^{-23}$ s $\rightarrow$ lightest black holes still (possibly) existing
- $M \sim 10^{5}\ M_\odot$ for $t_0 \sim 1$ s $\rightarrow$ IMHB? seeds for SMBH?

# Constraints on Primordial Black Holes

## Plausible dark matter candidates

- no need for Standard Model / General Relativity extension
- dynamically cold
- BH existence (somehow) proven
- mass ranges still available for BHs to represent all of dark matter

Constraints on PBHs – from Carr & Kuhnel, 2006.02838



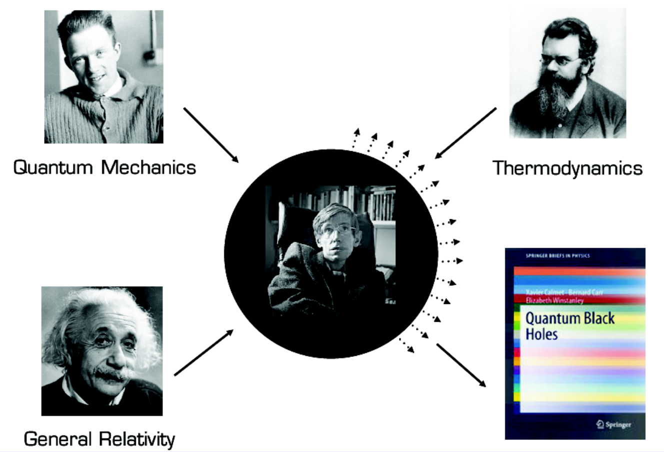
red: evaporation
blue: lensing
gray: gravitational waves
light blue: accretion
orange CMB distortions
green: dynamical effects
purple: large scale structure

**A-D: possible open windows**

# Hawking radiation

# Why are PBHs so special?

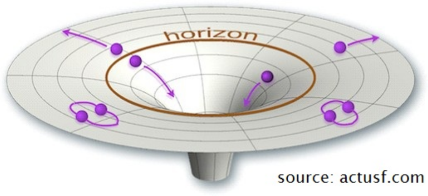
**Light PBHs cannot be described only with General Relativity...**



from B. Carr

**... because they emit Hawking radiation and evaporate!**

# Black hole Hawking radiation



source: actusf.com

## Fundamental equation for Kerr BHs

Rate of emission of Standard Model particles $i$ at energy $E$ by a BH of mass $M$ and spin parameter $a^*$:

$$Q_i = \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi} \sum_{\mathrm{dof.}} \frac{\Gamma_i(M, E, a^*)}{e^{E'/T(M,a^*)} \pm 1}$$

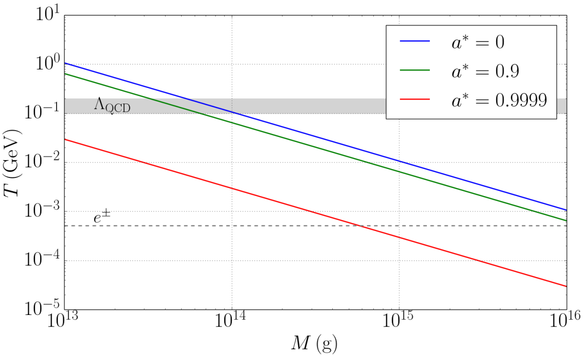
$\Gamma_i$ is the greybody factor ($\sim$ absorption coefficient in Planck's black-body law)

# Hawking temperature

## Hawking temperature for Kerr BHs

$$T(M,a^*) = \frac{1}{4\pi M}\left(\frac{\sqrt{1-(a^*)^2}}{1+\sqrt{1-(a^*)^2}}\right) \xrightarrow[a^*=0]{\text{Schwarzschild}} \frac{1}{8\pi M}$$

Comparison with the $e^\pm$ rest mass and QCD scale $\Lambda_{\rm QCD}$



BHs get warmer when they lose energy via Hawking radiation.

# Kerr Hawking radiation equations

## Kerr metric

$$\mathrm{d}s^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{4a^* M^2 r \sin^2\theta}{\Sigma^2}\mathrm{d}t\,\mathrm{d}\phi - \frac{\Sigma^2}{\Delta}\mathrm{d}r^2$$
$$- \Sigma^2\mathrm{d}\theta^2 - \left(r^2 + (a^*)^2 M^2 + \frac{2(a^*)^2 M^3 r \sin^2\theta}{\Sigma^2}\right)\sin^2\theta\mathrm{d}\phi^2$$

$\Sigma \equiv r^2 + (a^*)^2 M^2 \cos^2\theta$ and $\Delta \equiv r^2 - 2Mr + (a^*)^2 M^2$

## Equations of motion in free space

$$\mathrm{Dirac:}\ (i\partial\!\!\!/ - \mu)\psi = 0\ \mathrm{(fermions)}$$
$$\mathrm{Proca:}\ (\Box + \mu^2)\phi = 0\ \mathrm{(bosons)}$$

$\mu$ = rest mass

# Kerr Hawking radiation equations

## Teukolsky radial equation

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R}{\mathrm{d}r}\right)+\left(\frac{K^2+2i\,s(r-M)K}{\Delta}-4i\,sEr-\lambda_{slm}-\mu^2r^2\right)R=0$$

$R$ radial component of $\psi/\phi$
$K\equiv(r^2+a^2)E+a\,m$, $s$ = spin, $l$ = angular momentum and $m$ = projection

## Transformation into a Schrödinger equation

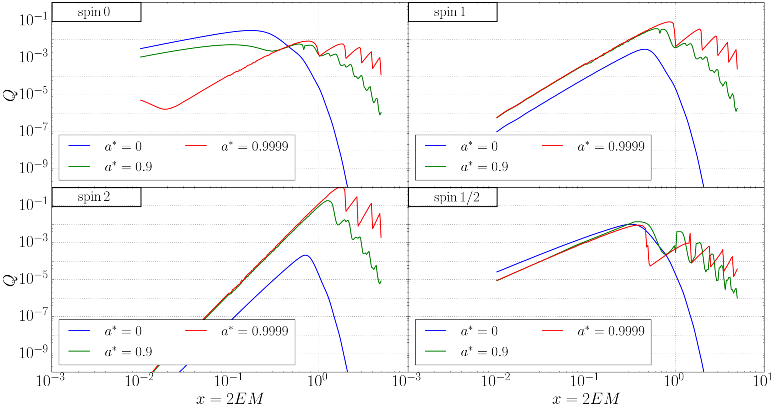
Change $\psi/\phi\longrightarrow Z$ and $r\longrightarrow r^*$ (generalized Eddington-Finkelstein coordinate system) (Chandrasekhar & Detweiler 1970s)

$$\frac{\mathrm{d}^2Z}{\mathrm{d}r^{*2}}+(E^2-V(r^*))Z=0$$

Solved with purely outgoing solution $Z\underset{r^*\to-\infty}{\longrightarrow}e^{-i\,Er^*}$

Transmission coefficient $\Gamma\equiv|Z_{\mathrm{out}}^{+\infty}/Z_{\mathrm{out}}^{\mathrm{horizon}}|^2$

# Hawking radiation of particles



All particles can be emitted by a black hole!

Including gravitons / gravitational waves... and even new physics particles!

Hawking radiation is enhanced for particles of spin 1 or 2.

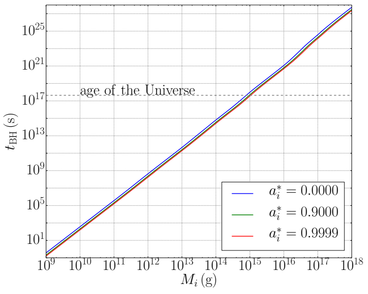
# Black hole lifetime

## Evolution equations

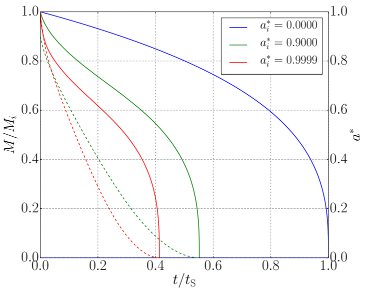
$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2}$$
$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3}$$

$$f \sim \int_E ener. \times emiss.$$
$$g \sim \int_E ang.\ mom. \times emiss.$$

BH lifetime



BH mass (solid) and spin (dotted) evolution



AA, J. Auffinger, J. Silk, MNRAS 494 (2020) 1257

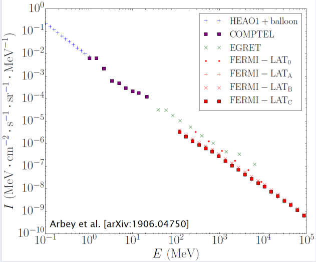
# Constraints on primordial black holes

# Isotropic gamma ray background (IGRB) constraints

## Origin

Diffuse background +

- Active galactic nuclei
- Gamma ray bursts
- DM annihilation/decay?
- Hawking radiation?



Arbey et al. [arXiv:1906.04750]

## Flux estimation for BHs

AA *et al.*, arXiv:1906.04750

$$I \approx \frac{1}{4\pi} E \int_{t_{\rm CMB}}^{t_{\rm today}} (1 + z(t))$$

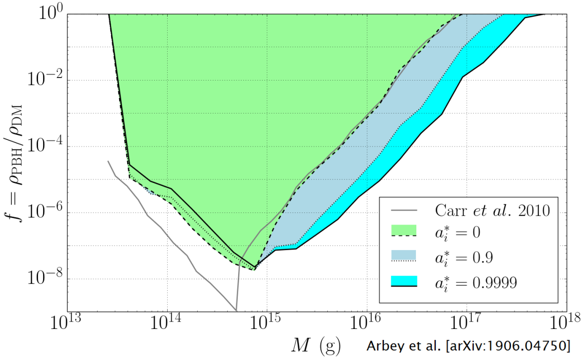
$$\times \int_M \left[ \frac{{\rm d}n}{{\rm d}M} \frac{{\rm d}^2 N}{{\rm d}t{\rm d}E}(M, (1 + z(t))E)\, {\rm d}M \right] {\rm d}t$$

# IGRB and Kerr PBHs: monochromatic mass distributions

## Main spin effects

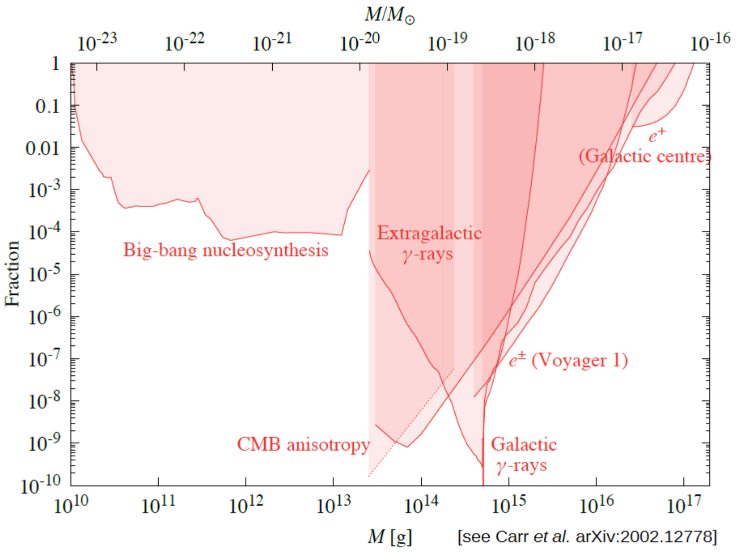
- enhanced luminosity ⇒ stronger constraints
- reduced temperature ⇒ reduced emission energy ⇒ weaker constraints

Monochromatic constraints from the IGRB



Arbey et al. [arXiv:1906.04750]

# Complementary constraints



[see Carr *et al.* arXiv:2002.12778]

# Non-standard black holes

# The case of static spherically-symmetric metrics

AA, J. Auffinger, M. Geiller, E. Livine, F. Sartini, arXiv:2101.02951 + 2107.03293

$$\mathrm{d}s^2 = -G(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + H(r)\mathrm{d}\Omega^2$$

Equations of motion:

$$\partial^2_{r^*} Z + \Big(\omega^2 - V\big(r(r^*)\big)\Big) Z = 0$$

where $\mathrm{d}r^*/\mathrm{d}r \equiv 1/\sqrt{FG}$ and the spin-dependent potentials $V$ are related to $F, G, H$.

Transmission coefficient: $\Gamma_i \equiv |Z^{+\infty}_{\rm out}/Z^{\rm horizon}_{\rm out}|^2$

Rate of emission of one degree of freedom $i$ per unit time $t$ and energy $E$:

$$\frac{\mathrm{d}^2 N_i}{\mathrm{d}t\,\mathrm{d}E} = \sum_{l,m} \frac{1}{2\pi} \frac{\Gamma_i(E, M, x_j)}{e^{E/T} - (-1)^{2s_i}} \,,$$

where $s_i$ is the spin of the particle $i$ and $T$ is its Hawking temperature given by

$$T = \frac{1}{4\pi} \left.\frac{F^{1/2} G'}{G^{1/2}}\right|_{\rm hor}$$

where “hor” denotes the horizon $r = r_{\rm H}$.

## Extra-dimensional PBHs

Metric:

$$F(r) = G(r) \equiv 1 - \left(\frac{r_{\rm H}}{r}\right)^{n+1}, \quad H(r) = r^2$$

where $n > 0$ is the number of extra dimensions.

Horizon radius is given by

$$r_{\rm H} = \frac{1}{\sqrt{\pi} M_*} \left(\frac{M}{M_*}\right)^{1/(n+1)} \left(\frac{8\Gamma\left((n+3)/2\right)}{n+2}\right)^{1/(n+1)}$$

where Γ is the Euler gamma function and the rescaled Planck mass is

$$M_{\rm Pl}^2 = M_*^{n+2} R^n$$

with $R$ the radius of the extra-dimension(s).

Temperature:

$$T_n = \frac{\kappa_n}{2\pi} = \frac{n+1}{4\pi r_{\rm H}}$$

Remark: Schwarzschild metric retrieved for $n = 0$.

# Extra-dimensional PBHs

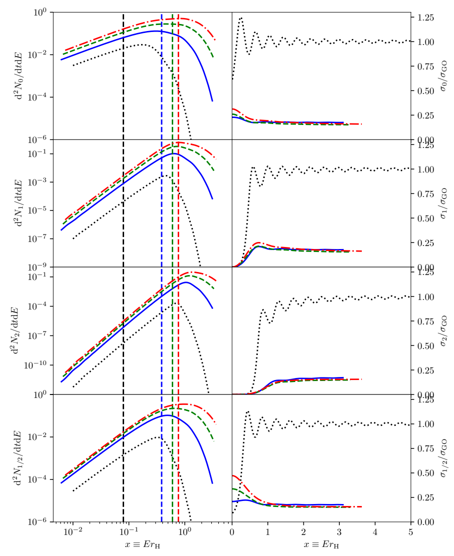
Hawking radiation spectra

dotted black: Schwarzschild

blue: $n = 2$

green: $n = 4$

red: $n = 6$



# LQG-inspired PBHs

Schwarzschild BHs are singular at the center → non-physical → quantum gravity?

Loop Quantum gravity (LQG): structure of spacetime described by spin networks.

Metric inspired by LQG, regularising the naked singularity at the center of the Schwarzschild metric:

$$G = \frac{(r-r_+)(r-r_-)(r+\sqrt{r_+ r_-})^2}{r^4+a_0^2}\,, \quad F = \frac{(r-r_+)(r-r_-)r^4}{(r+\sqrt{r_+ r_-})^2(r^4+a_0^2)}\,, \quad H = r^2 + \frac{a_0^2}{r^2}\,.$$

Two roots: $r_+ = \dfrac{2M}{(1+P)^2}$ and $r_- = \dfrac{2MP(\varepsilon)^2}{(1+P)^2}$

$M$: (ADM) mass

$P(\varepsilon) = \dfrac{\sqrt{1+\varepsilon^2}-1}{\sqrt{1+\varepsilon^2}+1}$: polymerization factor

$a_0$: minimal area in loop quantum gravity, typically of the Planck scale

$\varepsilon \geq 0$: typical scale of the geometry fluctuations (a priori independent parameter)

Temperature: $T_{\rm LQG} = \dfrac{r_+^2(r_+-r_-)}{4\pi(r_+^4+a_0^2)}$

## LQG-inspired PBHs

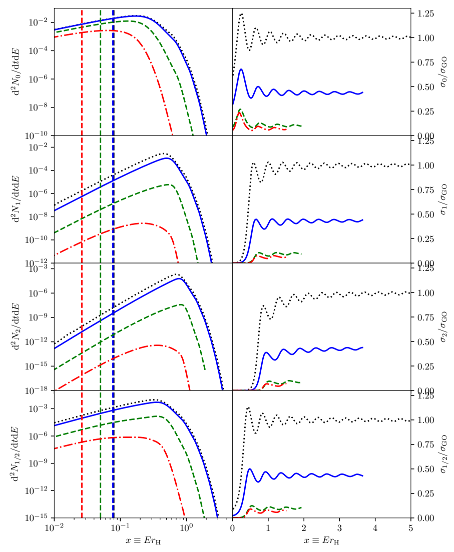
Hawking radiation spectra

dotted black: Schwarzschild

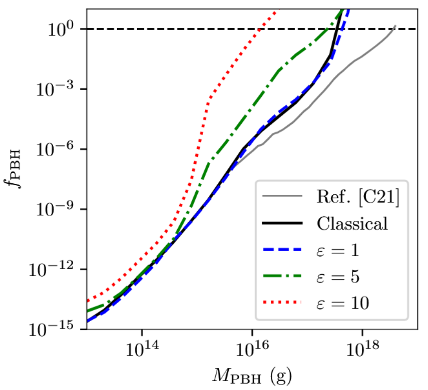
blue: $\epsilon = 1$

green: $\epsilon = 4$

red: $\epsilon = 10$



# LQG-inspired PBHs



AA, J. Auffinger, M. Geiller, E. Livine, F. Sartini, arXiv:2101.02951 + 2107.03293

Predictions for the sensitivity of the AMEGO gamma-ray experiment to classical and polymerised LQG-inspired PBHs

$\varepsilon$: scale of deformation of spacetime due to polymerisation in Loop Quantum Gravity

→ Theories beyond General Relativity predict very different signals of Hawking radiation.

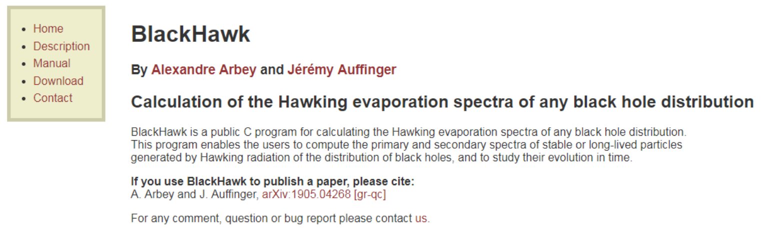
## BlackHawk

Public `C` code computing Hawking radiation:

- Schwarzschild & Kerr PBHs + non-standard BHs
- primary spectra of all Standard Model fundamental particles
- secondary spectra of stable particles
- extended mass and spin functions
- time evolution of the PBHs

**Download:** `http://blackhawk.hepforge.org`

**Manual: arXiv:1905.04268, Eur.Phys.J. C79, 693**
**+ arXiv:2108.02737, Eur.Phys.J. C81, 10**



# BlackHawk

**By Alexandre Arbey and Jérémy Auffinger**

### Calculation of the Hawking evaporation spectra of any black hole distribution

BlackHawk is a public C program for calculating the Hawking evaporation spectra of any black hole distribution. This program enables the users to compute the primary and secondary spectra of stable or long-lived particles generated by Hawking radiation of the distribution of black holes, and to study their evolution in time.

**If you use BlackHawk to publish a paper, please cite:**
A. Arbey and J. Auffinger, arXiv:1905.04268 [gr-qc]

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