Dark matter subhalos

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with Julien Lavalle and Martin Stref

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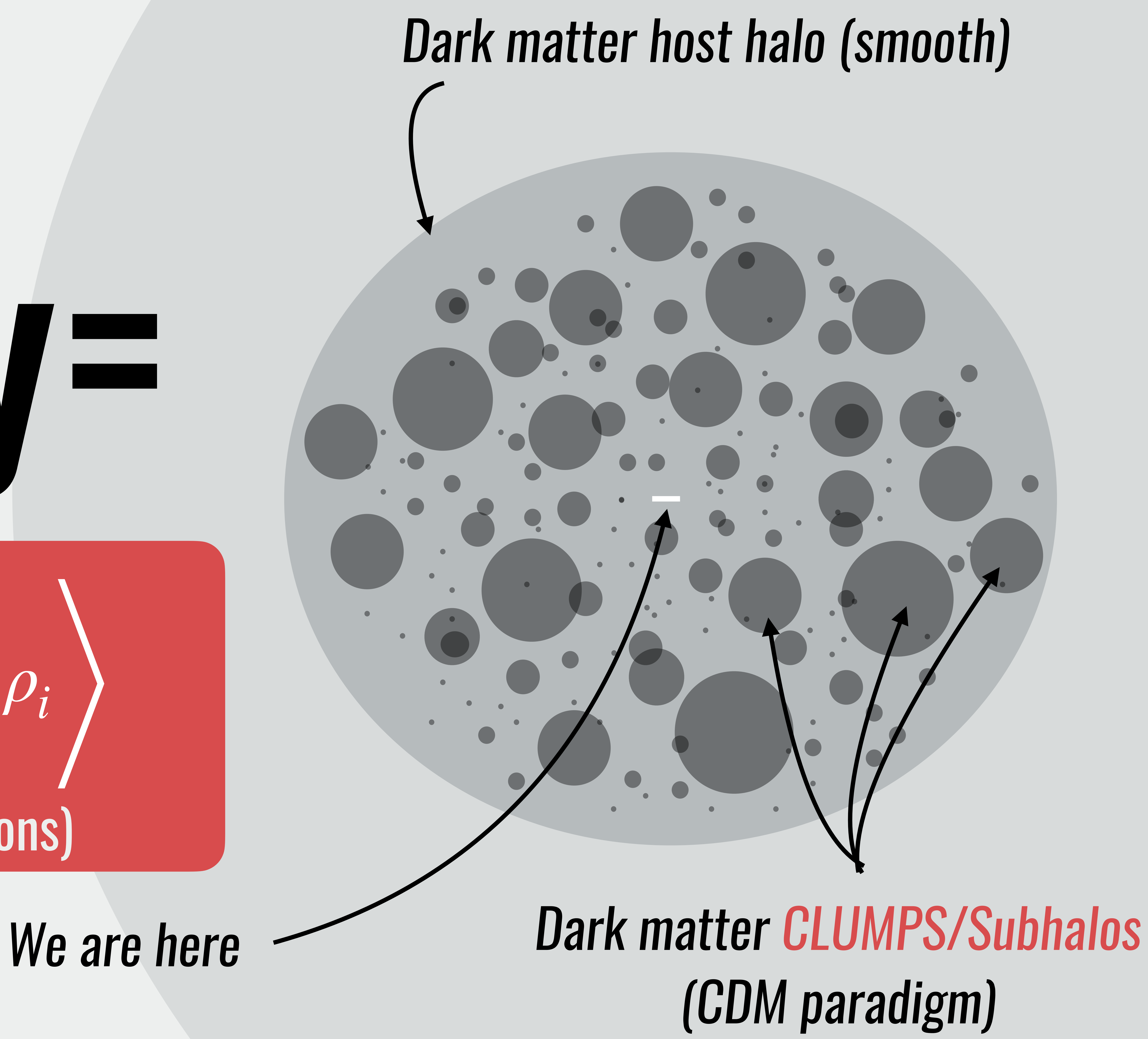
Galaxy=

$$\left\langle \rho_\chi = \rho_{\rm smooth} + \sum_{i=1}^{N_{\rm sub}} \rho_i \right\rangle$$

(Constrained by observations)



Why is looking for subhalos interesting?

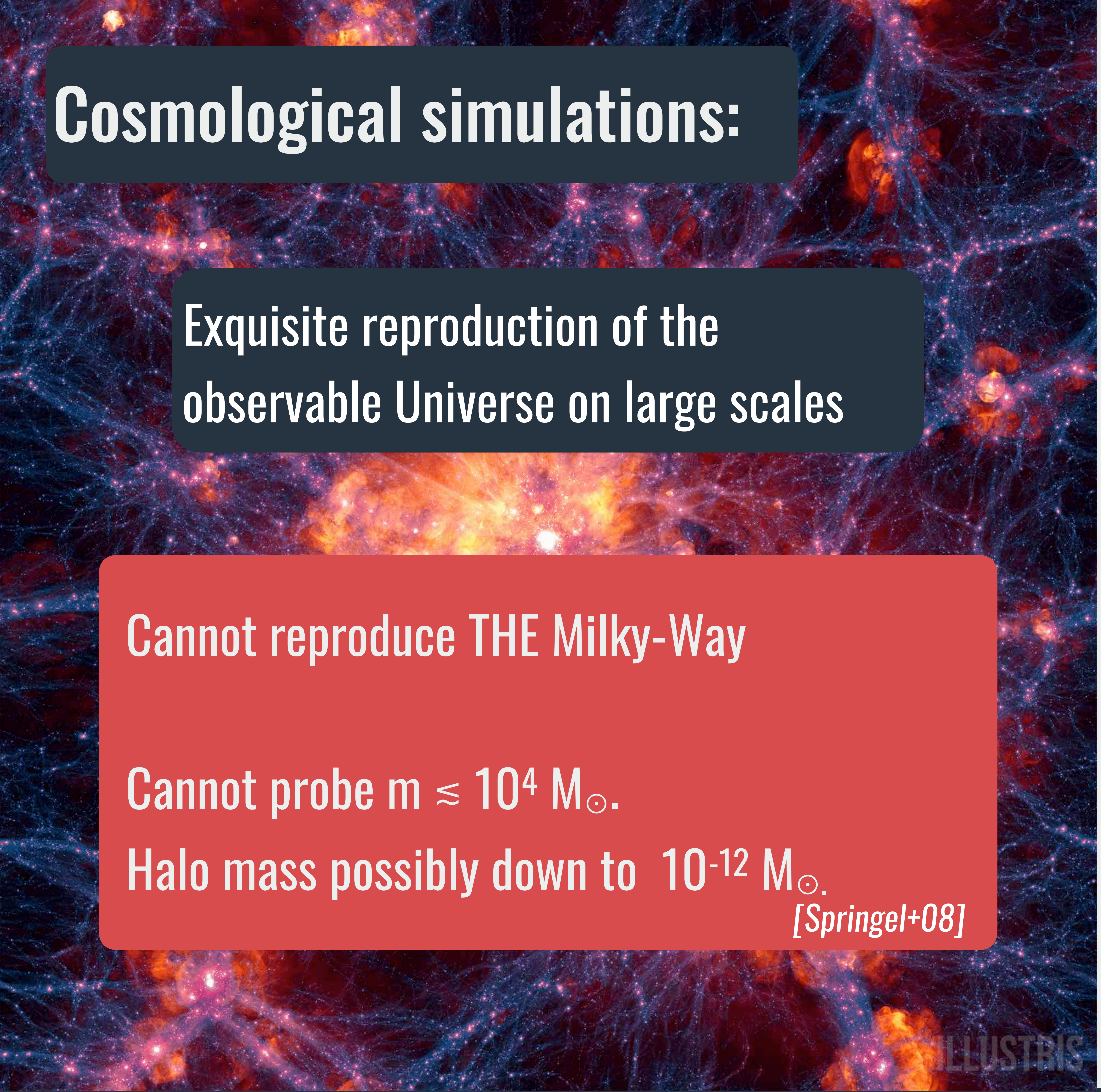
Nature of DM: Cold DM? Warm DM? Self Interacting DM? ...

Can be looked for with several strategies (DM annihilation, lensing, ...)

Need a reliable population model for Galactic searches

[Ibarra+19, Hütten+19, Calore+19, Hütten+16, Ando+19, ...]

[Facchinetti+20]



Cosmological simulations:

Exquisite reproduction of the observable Universe on large scales

Cannot reproduce THE Milky-Way

Cannot probe $m \lesssim 10^4\ M_\odot$.

Halo mass possibly down to $10^{-12}\ M_\odot$.

[Springel+08]

Analytical models:

Number of CDM subhalos in a MW-like halo: $N_{\rm sub} \gtrsim 10^6$

Evaluate the statistical distribution of halos

[Stref+17, Hiroshima+18, Bartels+15, Zavala+14, Benson+12, Van den Bosch+05, Peñarrubia+05, ...]

Two main ideas to describe the subhalo population

A dynamically constrained semi-analytical model for the subhalo population in the Milky Way (MW)

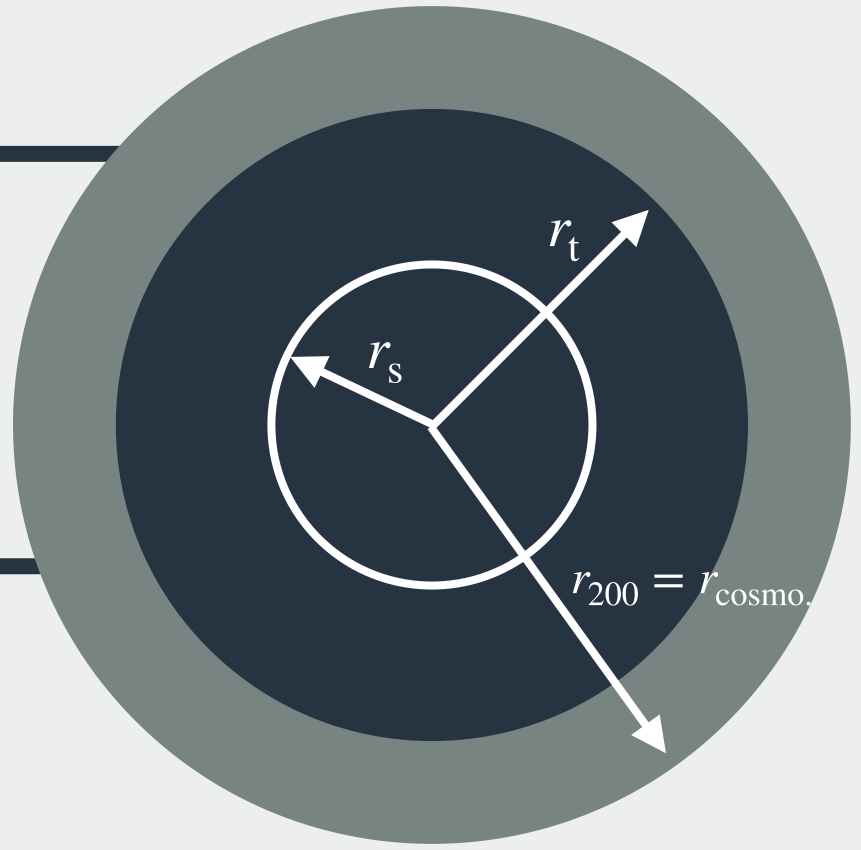
From [Stref and Lavalle (2017)] & [GF, Stref and Lavalle (2022, in prep.)]

Analytical initial/cosmological subhalo mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m} \propto m^{-\alpha}\Theta(m - m_{\mathrm{min}})$$

Dynamical/tidal effects in the host

Subhalo loose mass/shrink/may be disrupted



Analytically treated (consistent with the properties of the host)

Analytical evolved mass function (spatially dependent)

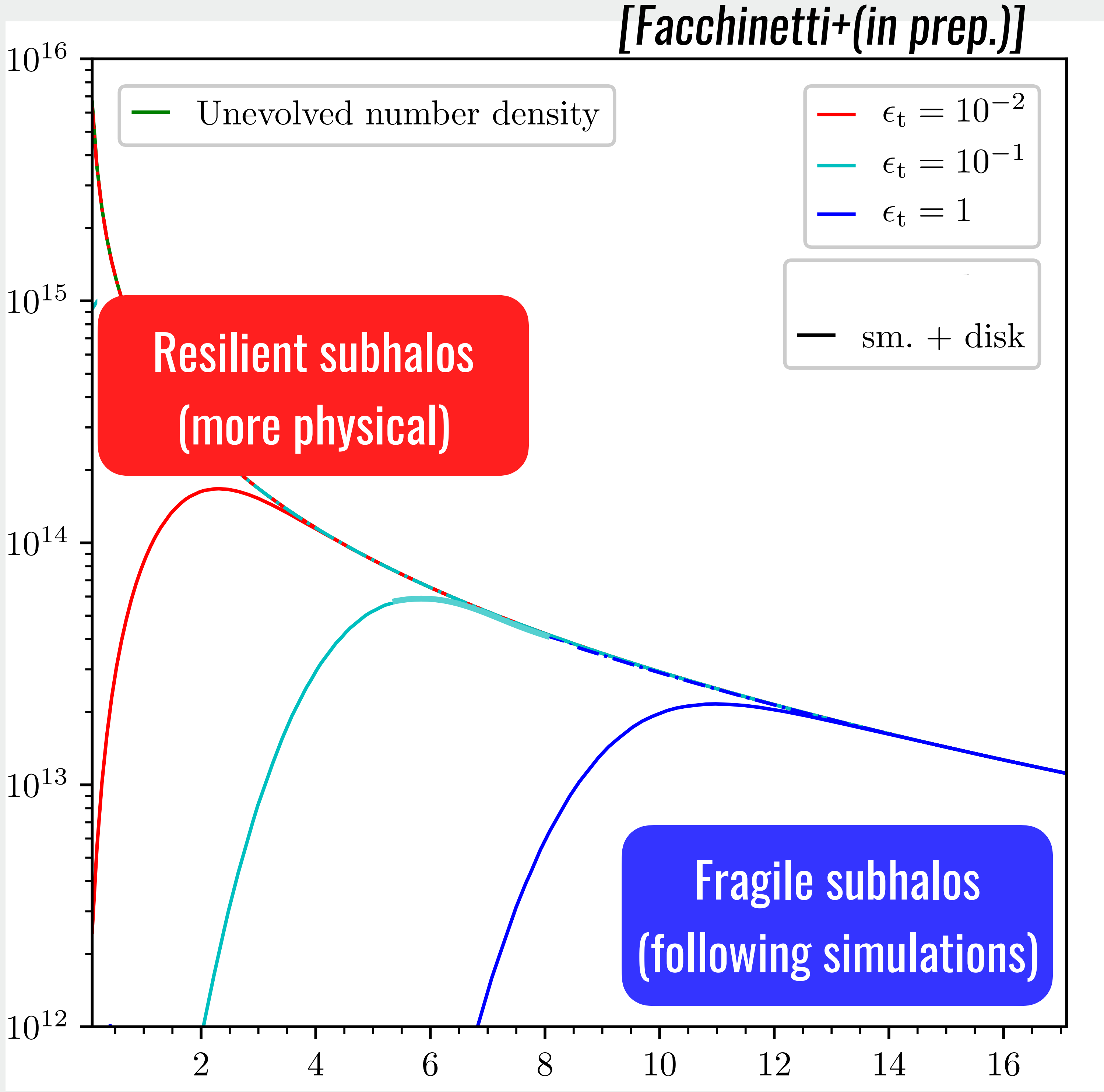
$$\frac{\mathrm{d}N_{\mathrm{sub}}(R)}{\mathrm{d}m_{\mathrm{t}}} = N_1 \iiint p_{\mathrm{sub}}^{\mathrm{late}}(m, c, R)\delta(m_t - m_t^\star(m, c, R))\mathrm{d}m\mathrm{d}c\mathrm{d}R$$

[Binney+08, Weinberg94, Gnedin+99, Stref+17]

[Tormen+98, Hayashi+03, Diemand+08, ...] [Van den Bosch+18, Errani+20]

Number density of subhalos [kpc^{-3}]

[Facchinetti+(in prep.)]



Distance from GC (MW) [kpc]

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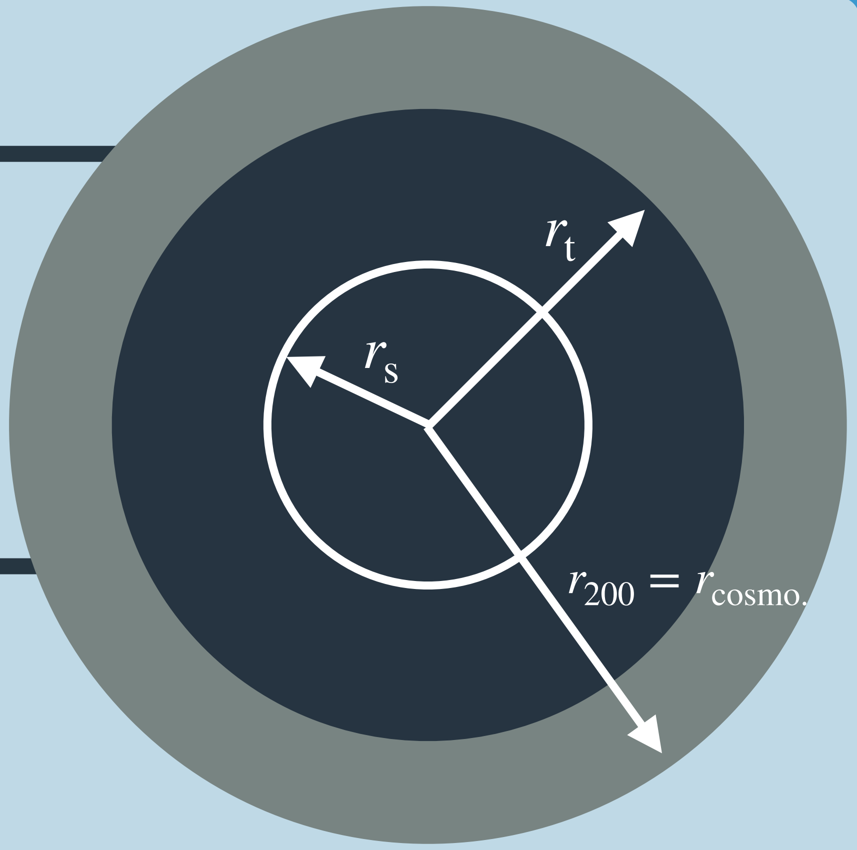

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[Binney+08, Weinberg94, Gnedin+99, Stref+17]

[Tormen+98, Hayashi+03, Diemand+08, ...] [Van den Bosch+18, Errani+20]

Part I

What is the value of m_{min} in a given particle model?

Part II

Imply the calibration of mass fraction in subhalos on DM only simulations.
How to avoid that?

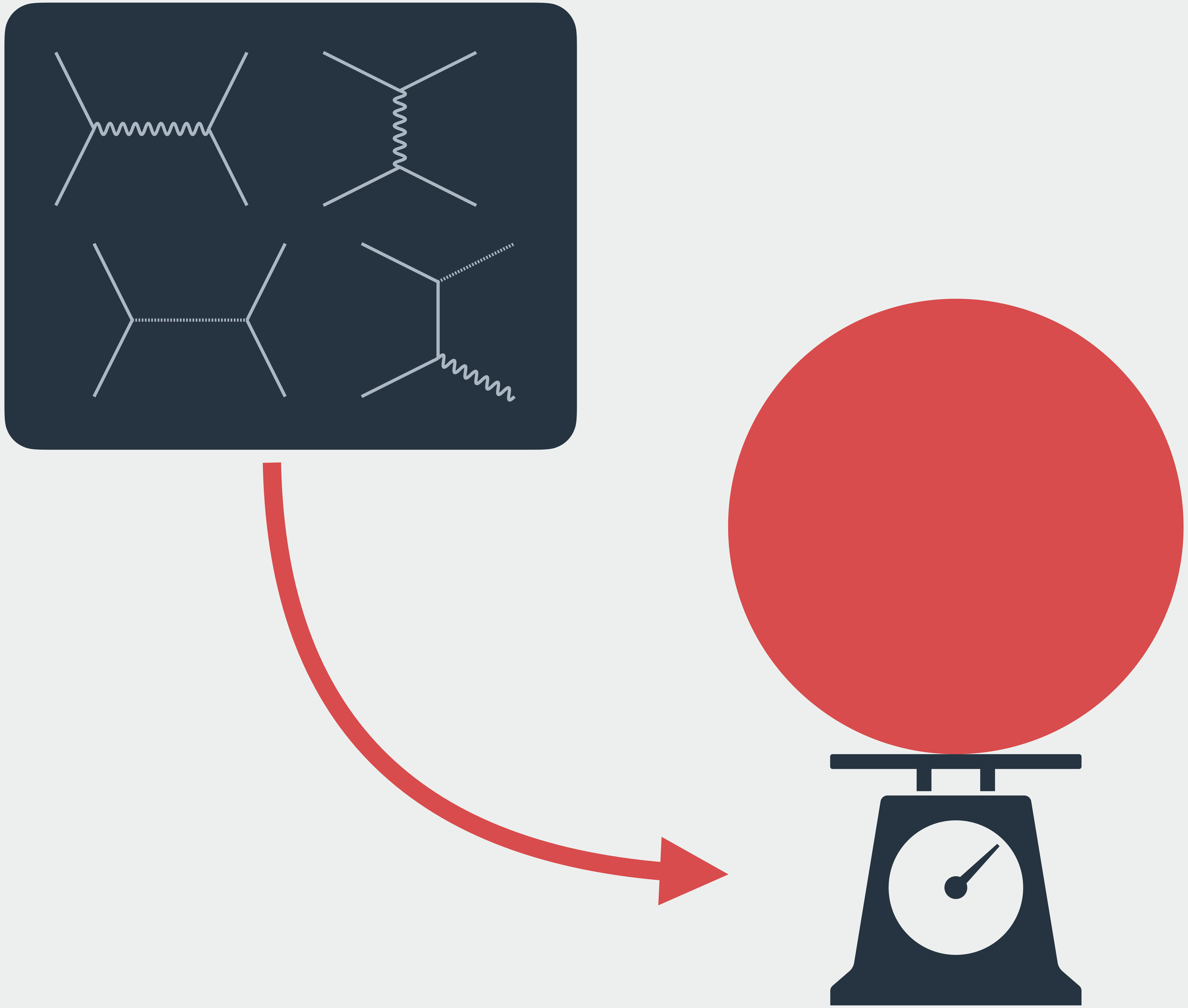
Part III

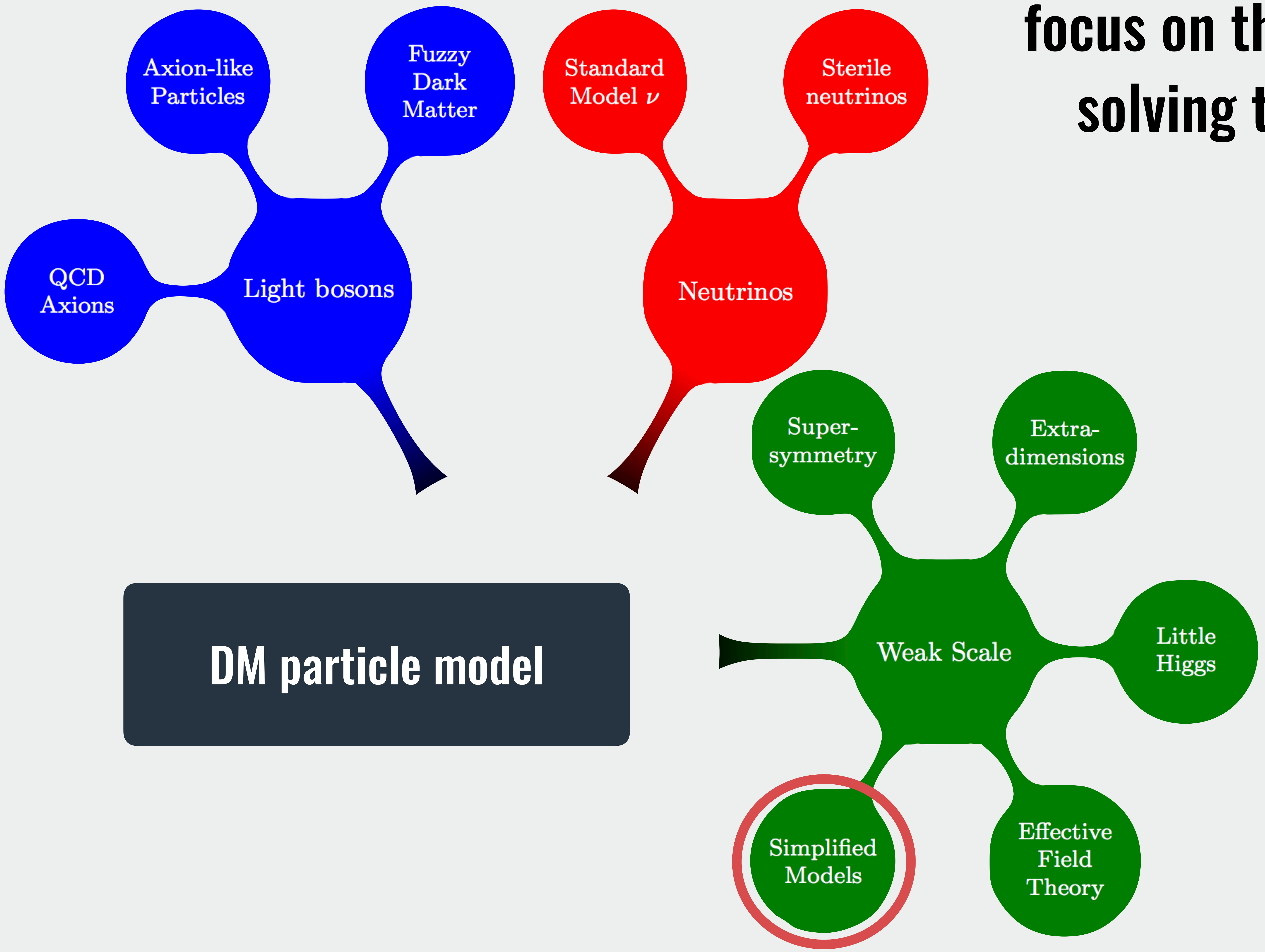
Impact of single star encounters
(Here for the Milky-Way)

Part I

Subhalo *minimal mass* in a simplified DM model

[arXiv:2203.xxxx]





« Historically »

focus on the phenomenology of particle models solving the electroweak hierarchy problem

top-down

No detection of new physics at LHC

Now

focus on the production mechanism

bottom-up (more generic)

[Cirelli+06, Abdallah+15, Abercrombie+15, Boveia+15, De Simone+16, Kraml+17, Arina+18, ...]

We work with the following model:

s-channel simplified model (for fermionc DM):

$$\mathscr{L} \ni -\bar{\chi}_i \delta_\chi (A_k^{ij} \phi_k + \imath\gamma^5 B_k^{ij} \varphi_k)\chi_j - \bar{\psi}_i (\mathscr{A}_k^i \phi_k + \imath\gamma^5 \mathscr{B}_k^i \varphi_k)\psi_i$$
$$+\bar{\chi}_i \gamma^\mu \delta_\chi (X_k^{ij} - \gamma^5 Y_k^{ij}) V_k^\mu \chi_j + \bar{\psi}_i \gamma^\mu \left(\mathscr{X}_k^i - \gamma^5 \mathscr{Y}_k^i\right) V_k^\mu \psi_i$$

Generic coupling DM-SM through
scalar, pseudoscalar,
vector and axial-vector mediators

In the literature, no generic connection between simplified models and subhalo minimal mass

Let's make this connection!

For thermally produced particles
with abundance fixed with **freeze-out** mechanism (WIMPs)
… and investigate its properties and features

Thermodynamical equilibrium

$n_\chi = n_\chi^{\mathrm{eq}}$ | $T_\chi = T$

Thermal equilibrium

$n_\chi \neq n_\chi^{\mathrm{eq}}$ | $T_\chi = T$

Free streaming

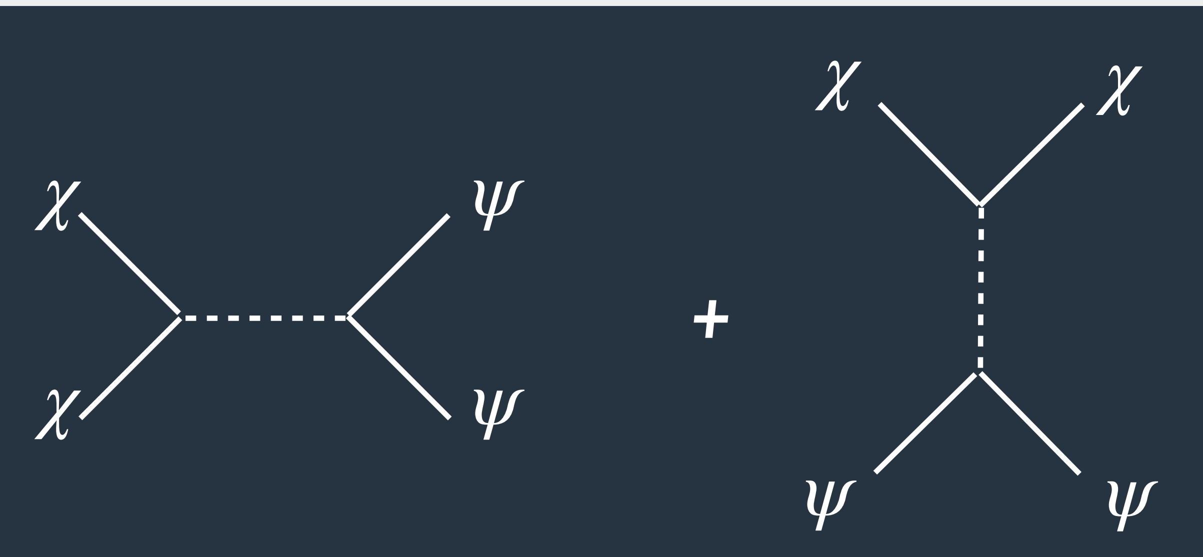
$n_\chi \neq n_\chi^{\mathrm{eq}}$ | $T_\chi \neq T$

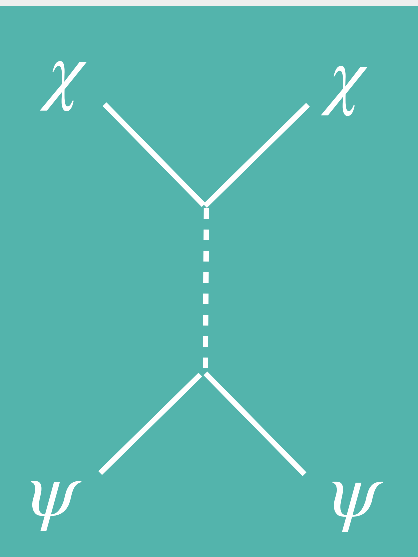
MD era

Perturbations growth

Chemical decoupling
Fixes the abundance

Kinetic decoupling





No interactions between DM and SM

Acoustic damping of modes:
(collisional damping)

$$k > k_{\mathrm{d}} \sim \frac{\sqrt{3}}{c} H(t_{\mathrm{kd}})$$

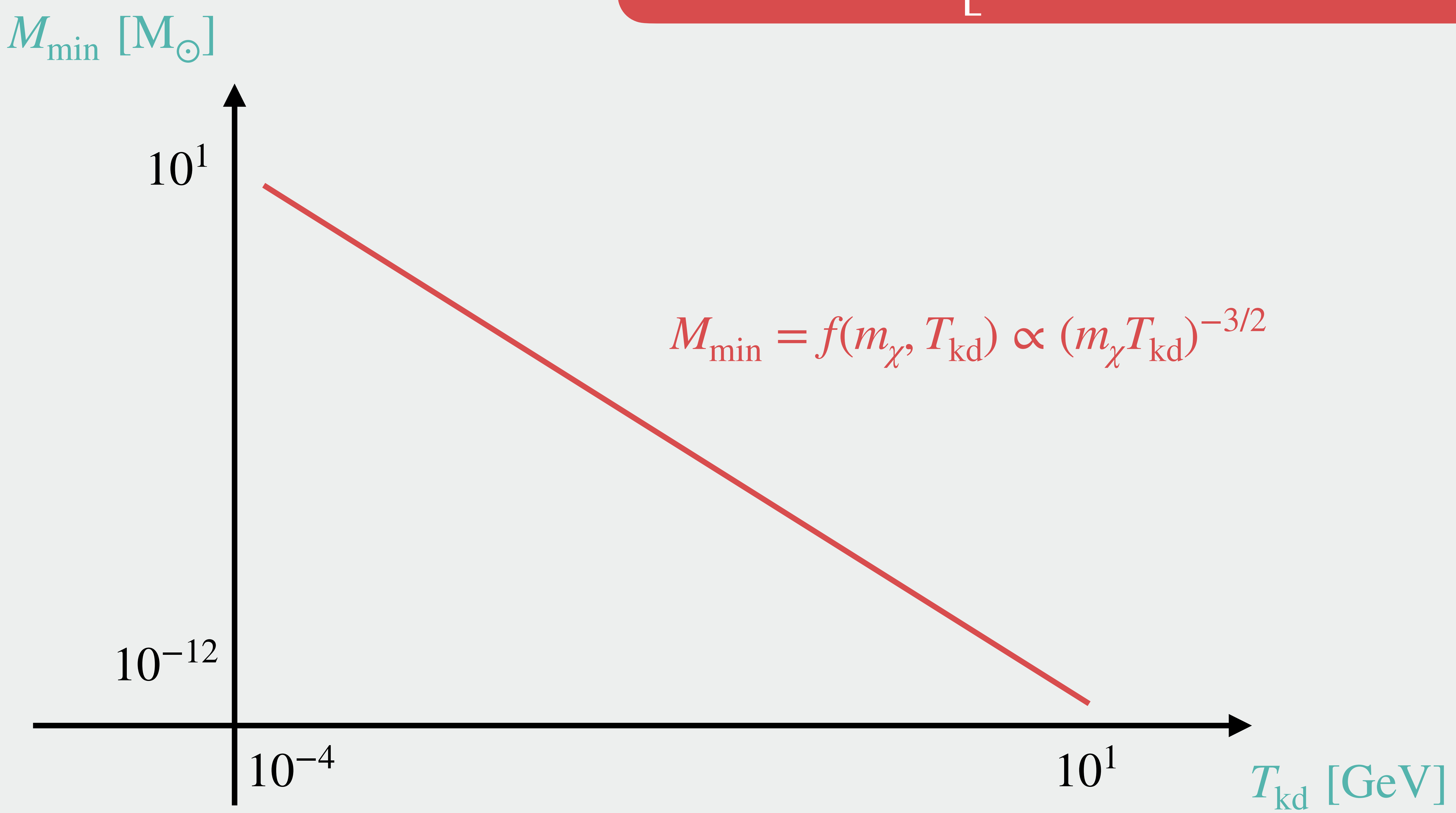
Free streaming damping of modes:
(collision-less damping)

$$k > k_{\mathrm{fs}} = \frac{2\pi}{\lambda_{\mathrm{fs}}} \sim \frac{2\pi}{a(t_{\mathrm{eq}})} \left(\int_{t_{\mathrm{kd}}}^{t_{\mathrm{eq}}} \frac{v(t)}{a(t)} \mathrm{d}t \right)^{-1}$$

$$M_{\mathrm{halo}} > \max \left[\frac{4\pi}{3} \bar{\rho}_m(t_{\mathrm{kd}}) \left(\frac{2\pi}{k_{\mathrm{d}}} \right)^3, \frac{4\pi}{3} \bar{\rho}_m(t_{\mathrm{eq}}) \left(\frac{2\pi}{k_{\mathrm{fs}}} \right)^3 \right]$$

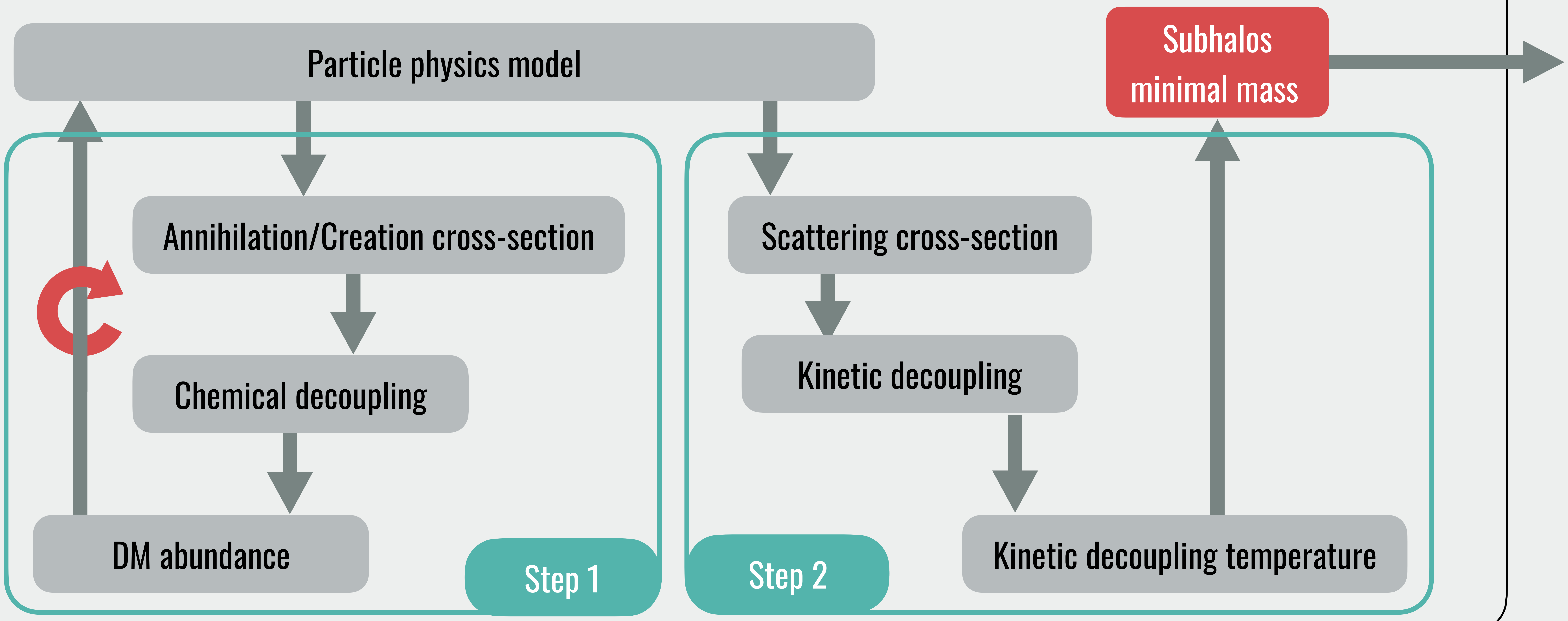
[Hofmann+01, Boehm+01, Green+05, Loeb+05, Bringmann+09, Gondolo+12]

$$M_{\rm halo} > \max\left[\frac{4\pi}{3}\bar{\rho}_m(t_{\rm kd})\left(\frac{2\pi}{k_{\rm d}}\right)^3, \frac{4\pi}{3}\bar{\rho}_m(t_{\rm eq})\left(\frac{2\pi}{k_{\rm fs}}\right)^3\right]$$



The minimal mass is directly related
to the kinetic decoupling temperature

Codes in C++ developed from scratch to optimise the computation speed



1 point phase-space distribution function: $f_\chi = f_\chi(t, |\mathbf{p}|)$

Boltzmann equation: $\hat{L}[f_\chi] = \hat{C}[f_\chi]$

[Lee+77, Binétruy+84, Bernstein+85, Srednicki+88, Gondolo+91, Griest+91, Edsjo+97 , Steigman+12]

[Hofmann+01, Bertschinger+06, Binder+16, Gondolo+12]

Step 1

0th moment:

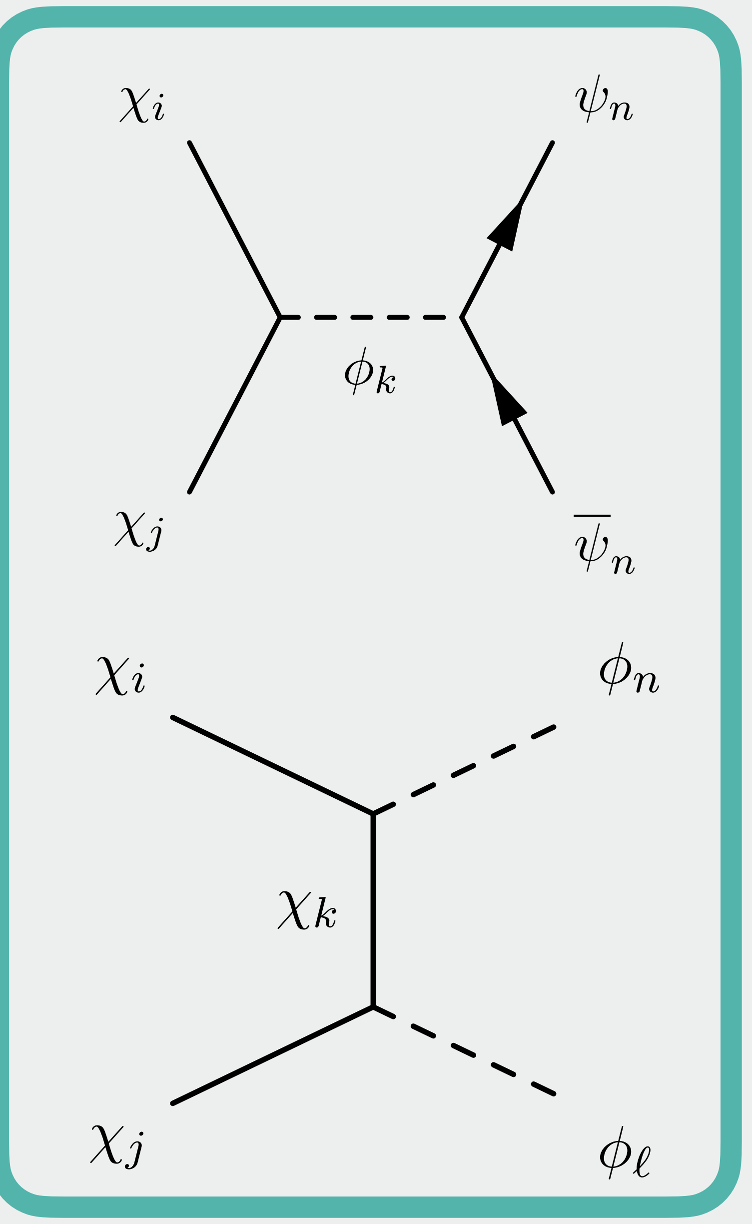
$$\int \hat{L}[f_\chi]\frac{1}{E_\chi}\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} = \int \hat{C}[f_\chi]\frac{1}{E_\chi}\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}$$

Equation on DM number density:

$$\frac{\mathrm{d}n}{\mathrm{d}t} + 3Hn = \langle \sigma_{\mathrm{ann}} v \rangle (n_{\mathrm{eq}}^2 - n^2)$$

Thermal cross-section

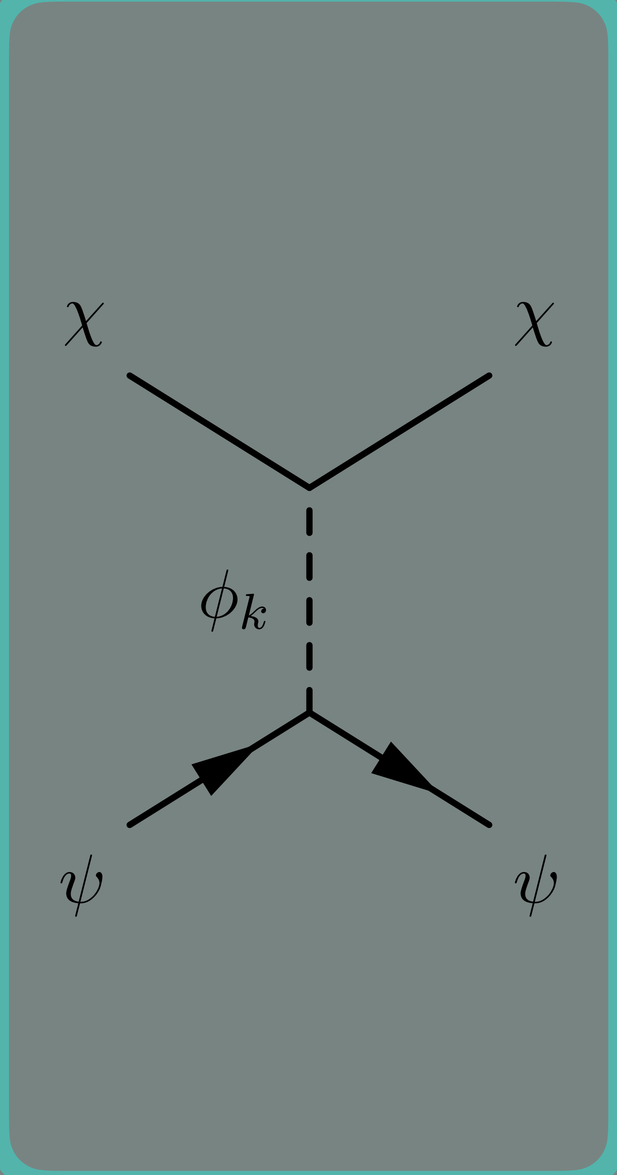
$$\langle \sigma_{\mathrm{ann}} v \rangle = \int \sigma_{\mathrm{ann}}(s) \ldots \mathrm{d}s$$



Step 2

2nd moment:

$$\int \hat{L}[f_\chi]\frac{|\mathbf{p}|^2}{E_\chi}\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} = \int \hat{C}[f_\chi]\frac{|\mathbf{p}|^2}{E_\chi}\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}$$



Equation on DM temperature:

$$\frac{\mathrm{d}T_\chi}{\mathrm{d}t} + 2HT_\chi = \gamma(T)(T - T_\chi)$$

Momentum relaxation rate

$$\gamma(T) \propto \sigma_T = \int \frac{\mathrm{d}\sigma_{\mathrm{scatt}}}{\mathrm{d}\Omega}(1 - \cos\theta)\mathrm{d}\Omega$$

The equations for chemical and kinetic decoupling are obtained from the Boltzmann equation

Let us treat the example of a single scalar/pseudoscalar mediator

$$\mathscr{L} \ni -\frac{1}{2}\lambda_\chi \bar{\chi}\phi\chi - \sum_\psi \lambda_\psi \bar{\psi}\phi\psi$$

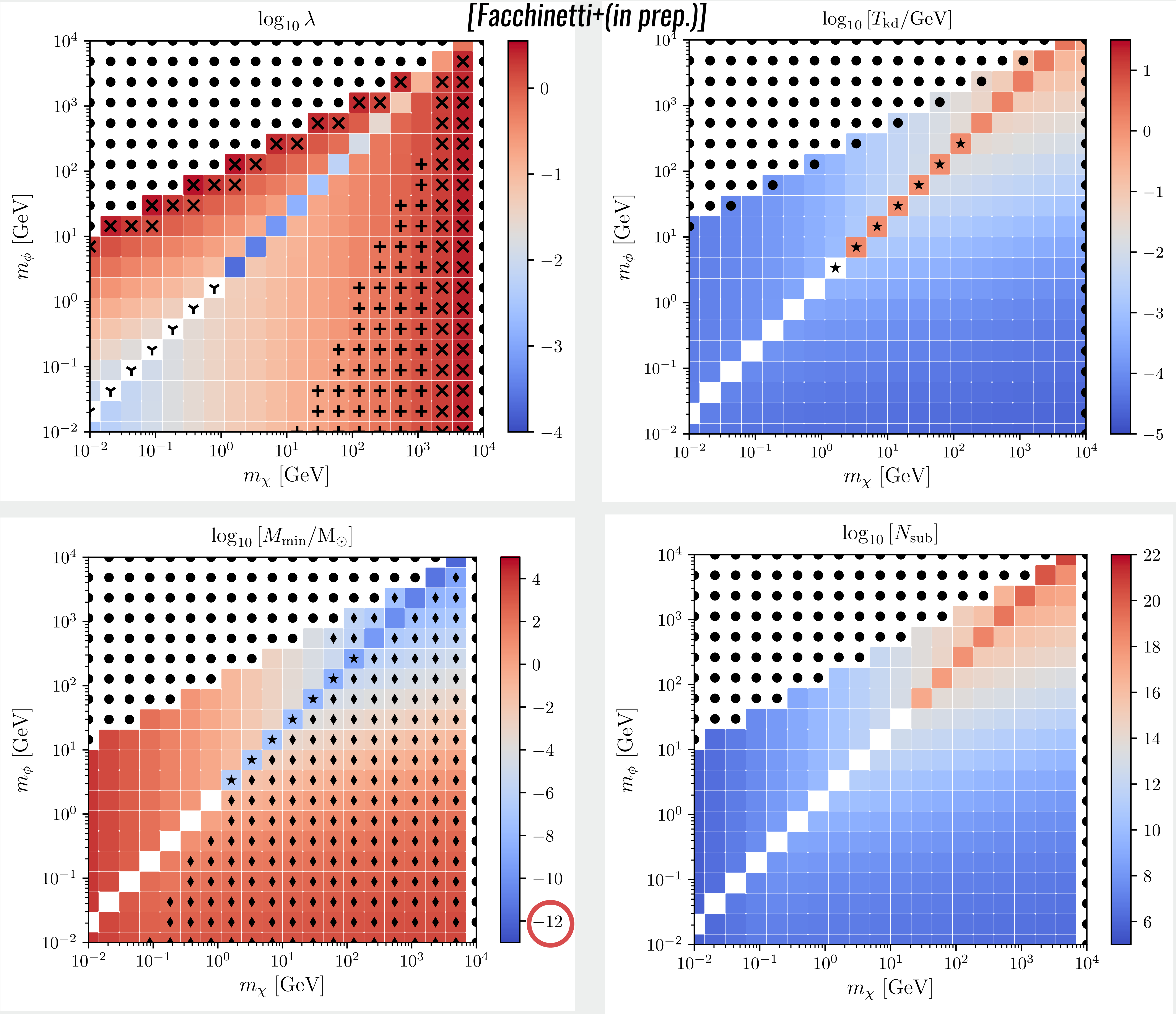
$$\mathscr{L} \ni -\frac{1}{2}\lambda_\chi \bar{\chi}\gamma^5\varphi\chi - \sum_\psi \lambda_\psi \bar{\psi}\gamma^5\varphi\psi$$

(Toy models)

Chemical decoupling + correct abundance: constraints on the factor $\lambda = \sqrt{\lambda_\chi \lambda_e}$

Constrained coupling constant

Temperature of Kinetic decoupling



Scalar

- \+ Sommerfeld effects
- x large decay width
- • large coupling
- ★ early kinetic dec.

◆ acoustic > free-stream.

$\mathrm{size} @ 10^{-12} M_\odot \sim 10^{-5}$ pc

solar system $\sim 10^{-4}$ pc

Minimal halo mass

Number of subhalos in the Milky-Way

From the coupling constant to the number of subhalos … and more

We derived several approximate scaling laws

	Annihilation (chemical decoupling/indirect searches)	Scattering (kinetic decoupling/direct searches)
Scalar	$\sigma^{\mathrm{scalar}}_{\chi\chi\to\psi\overline{\psi}} v_{\mathrm{rel}} \propto \lambda^4 \frac{(m_\chi^2 - m_\psi^2)^{3/2}}{m_\chi^a m_\phi^b} v_{\mathrm{rel}}^2$ (p-wave)	$\sigma^{\mathrm{scalar}}_{\chi\psi\to\chi\psi} \propto \lambda^4 \frac{m_\chi^2 m_\psi^2}{m_\phi^4 (m_\chi + m_\psi)^2}$ (v-indep.)
Pseudo-scalar	$\sigma^{\mathrm{pseudo-scalar}}_{\chi\chi\to\psi\overline{\psi}} v_{\mathrm{rel}} \propto \lambda^4 \frac{(m_\chi^2 - m_\psi^2)^{1/2}}{m_\chi^a m_\varphi^b}$ (s-wave)	$\sigma^{\mathrm{pseudo-scalar}}_{\chi\psi\to\chi\psi} \propto \lambda^4 \frac{m_\chi^4 m_\psi^4}{m_\varphi^4 (m_\chi + m_\psi)^6} v_{\mathrm{rel}}^4$ (v-dep.)

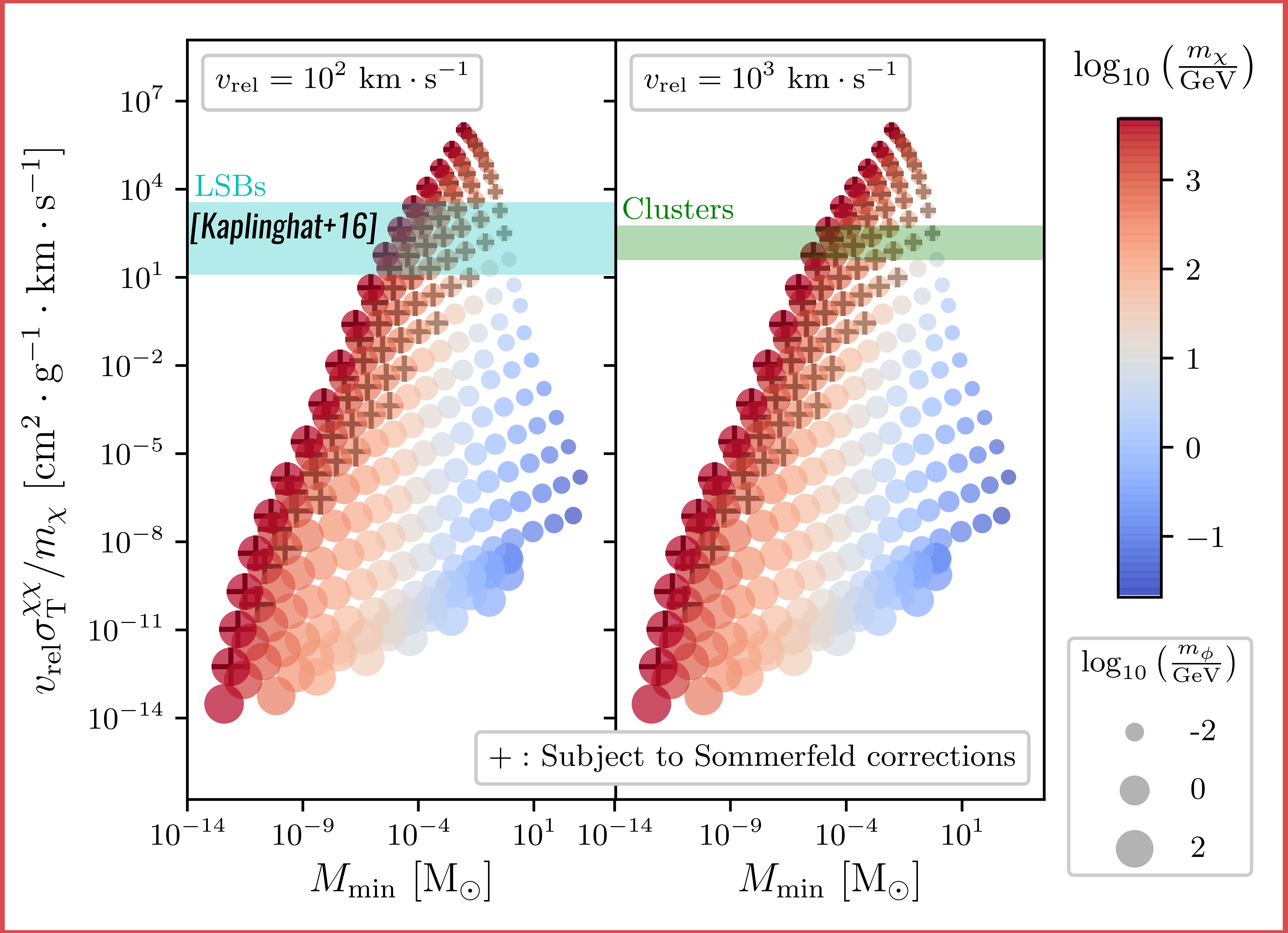
[Abdallah+15]

Couplings to have the right abundance

$$\lambda \propto \begin{cases} \sqrt{m_\chi} & \text{if} \quad m_\chi \gg m_\phi \\ m_\phi / \sqrt{m_\chi} & \text{if} \quad m_\chi \ll m_\phi \end{cases}$$

Minimal halo mass vs. self-interactions

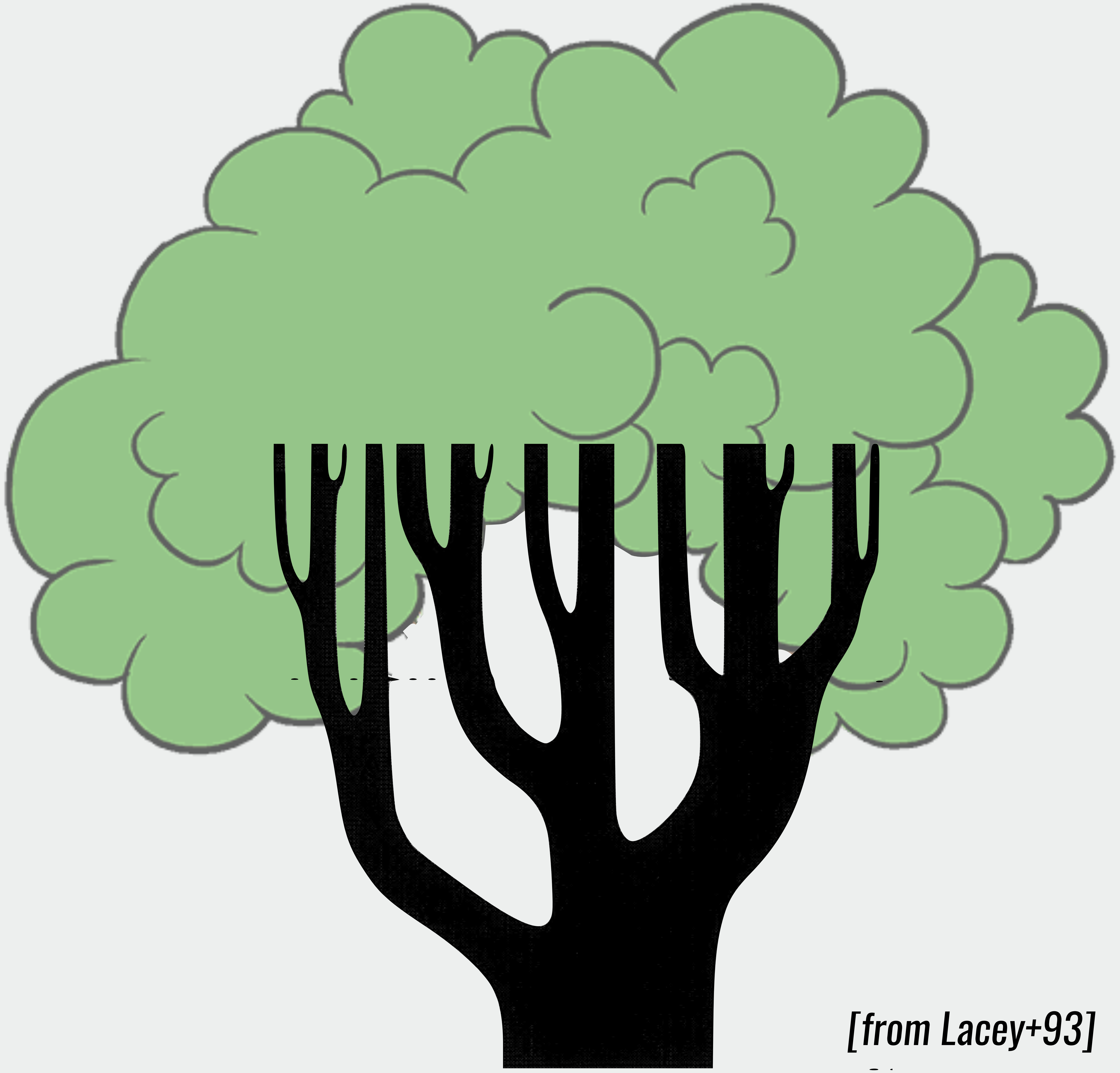
Scalar



[Facchinetti+(in prep.)]

The cosmological mass function from merger trees

Formalism used in [Lacroix, GF+(in prep.)]



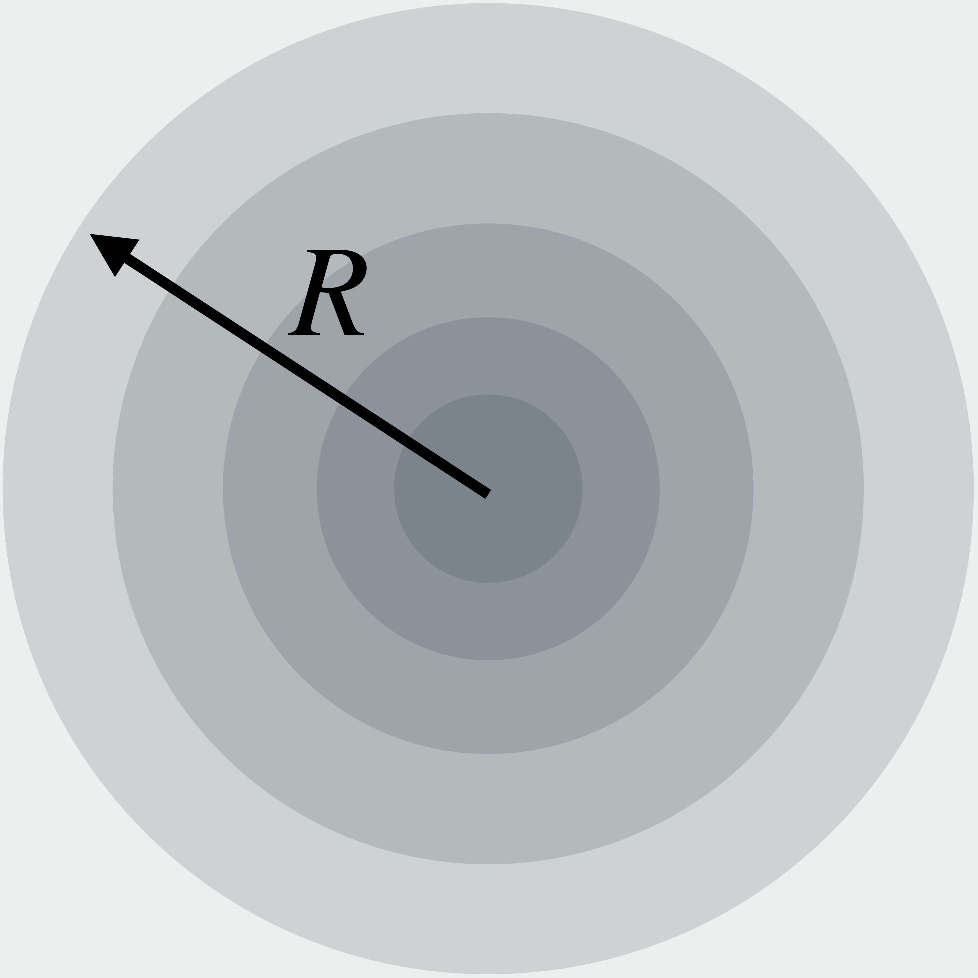
[from Lacey+93]

Recall:

Initial/cosmological mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m} \propto m^{-\alpha}\Theta(m - m_{\mathrm{min}})$$

Imply the calibration of mass fraction in subhalos on DM only simulations.
How to avoid that?

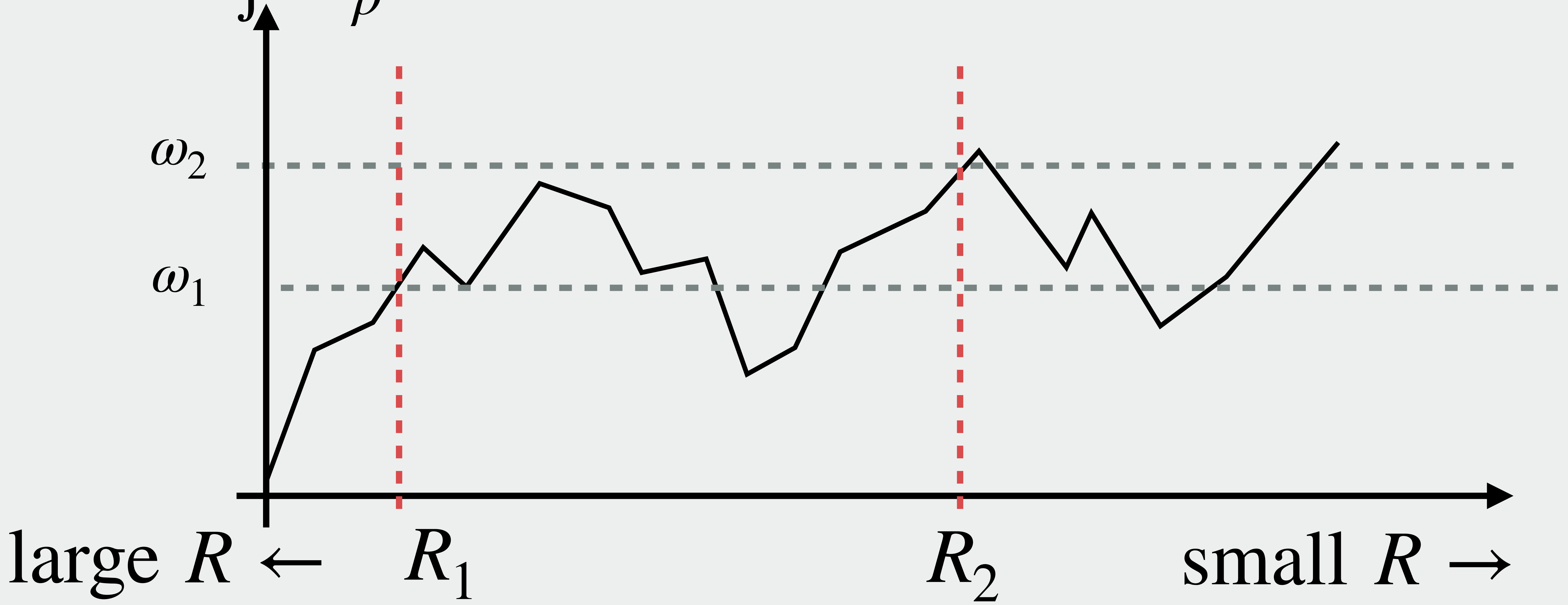


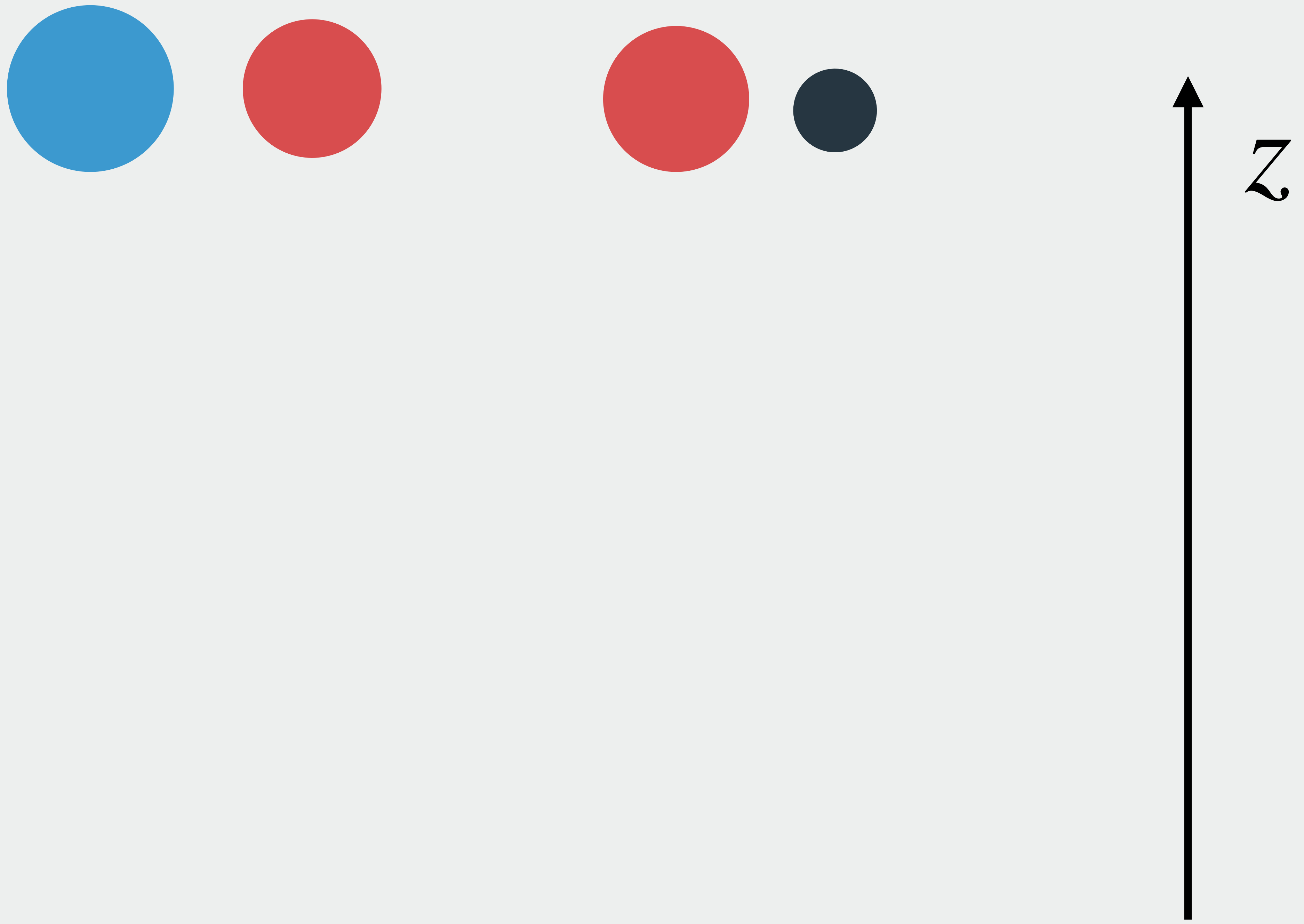
$$P_\mathrm{m}(k,z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{\mathrm{m},0}H_0^2}T(k)\right]^2 \mathscr{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$ *(matter power spectrum)*

$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_\mathrm{m}(k, z=0)k^2\mathrm{dk}$$ *(Smoothed variance)*

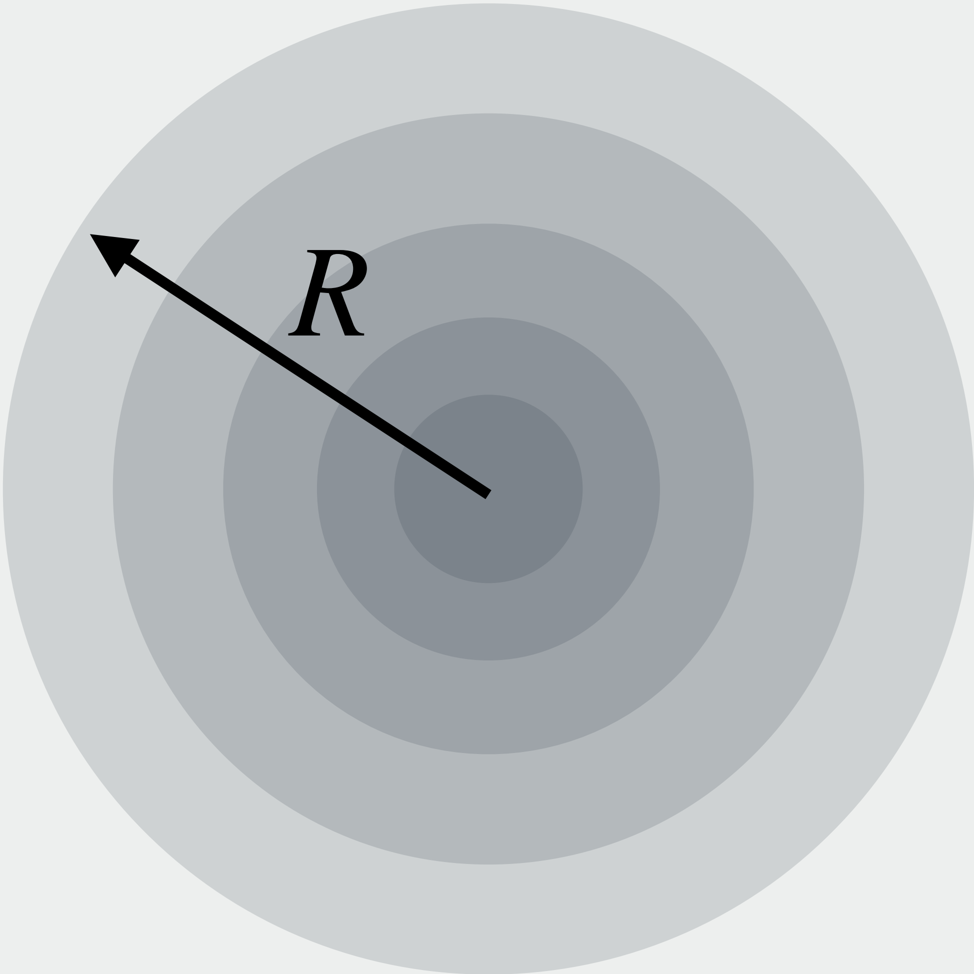
[Bond+91]

$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y}\frac{\delta\rho}{\bar{\rho}}W_R(|\mathbf{x}-\mathbf{y}|)$$ *(smoothed density contrast)*





From the excursion set theory to merger trees



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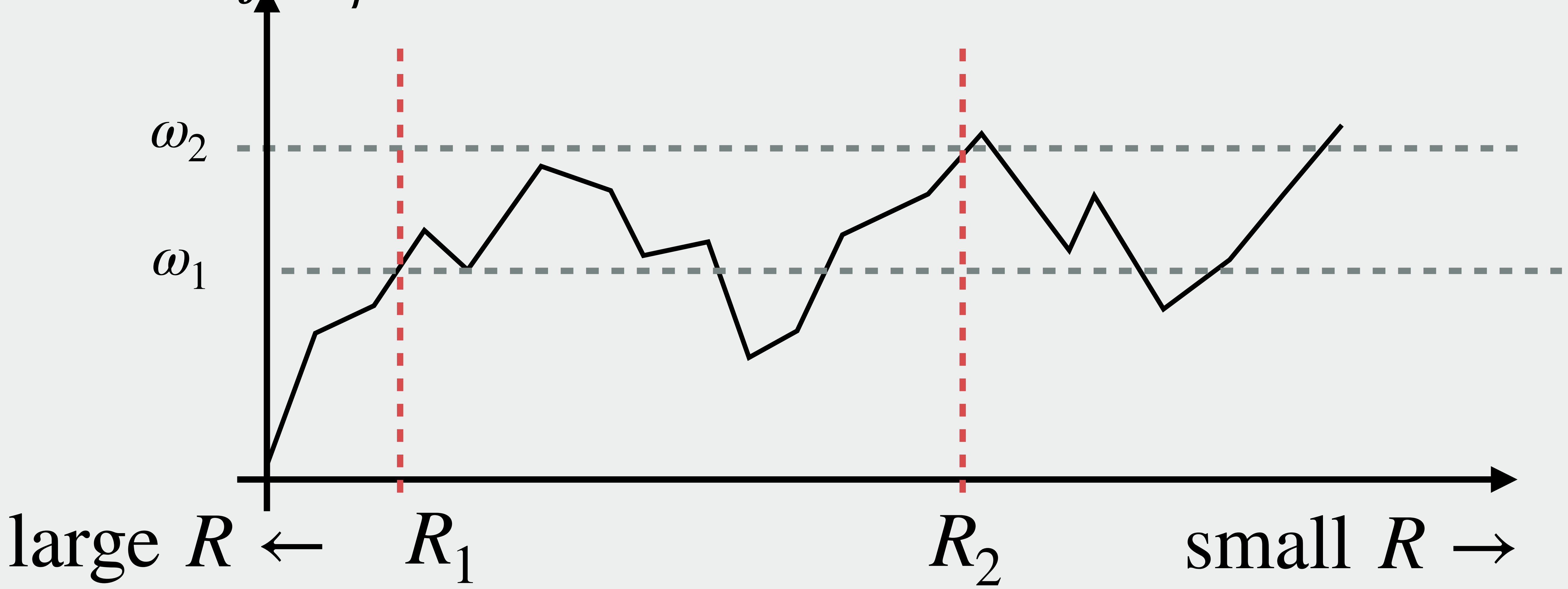

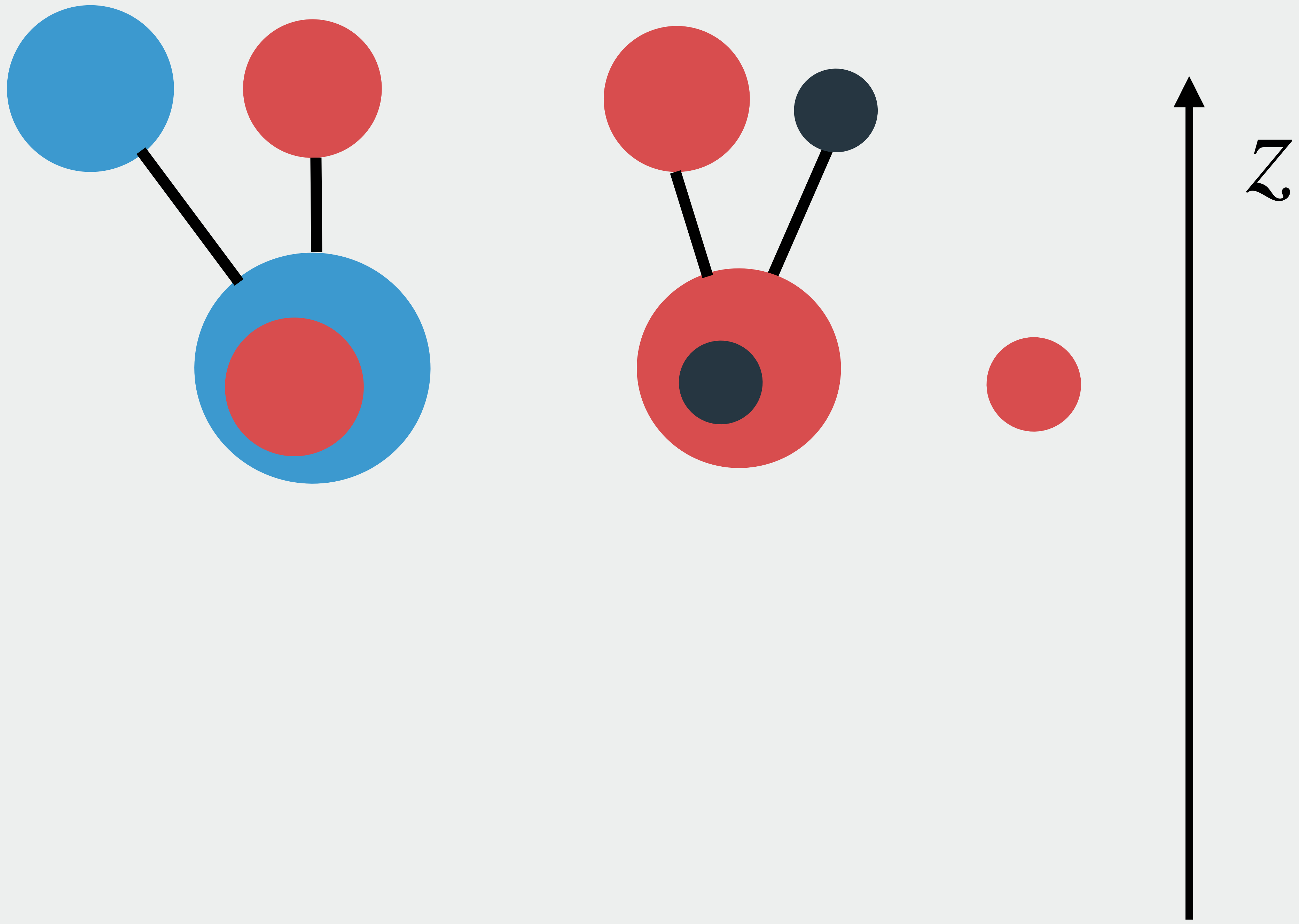

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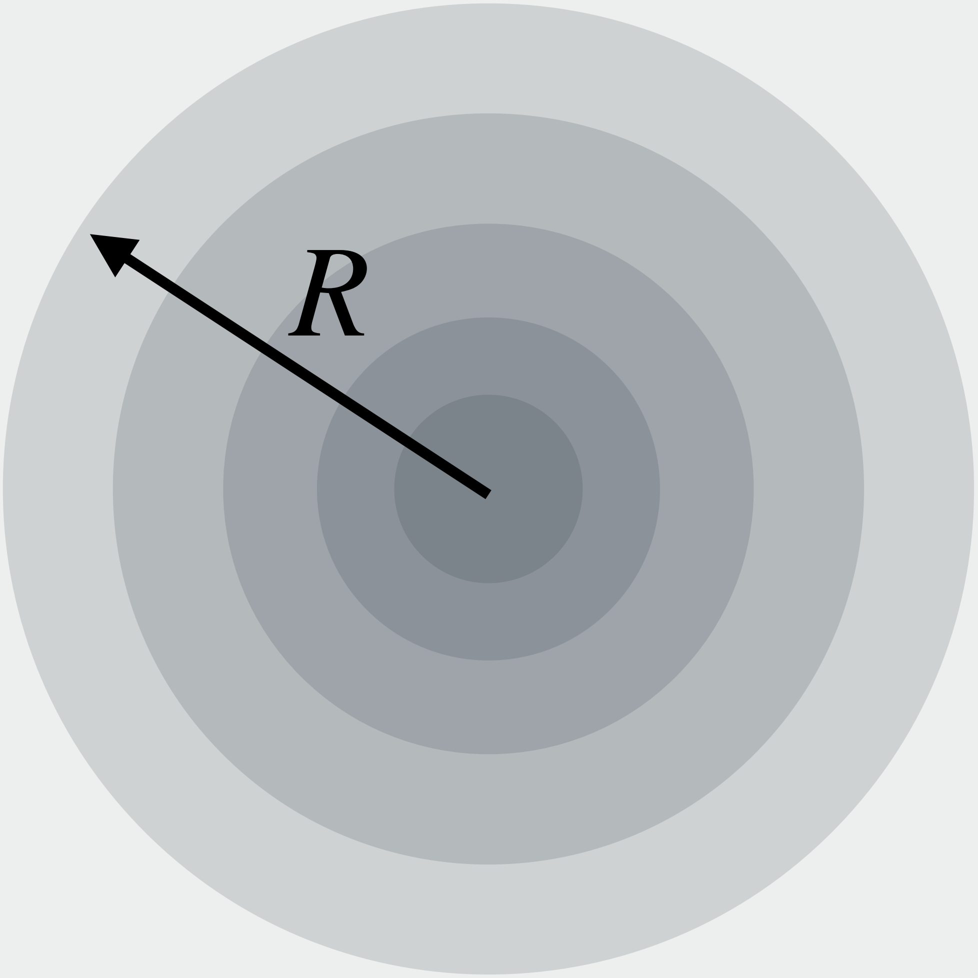

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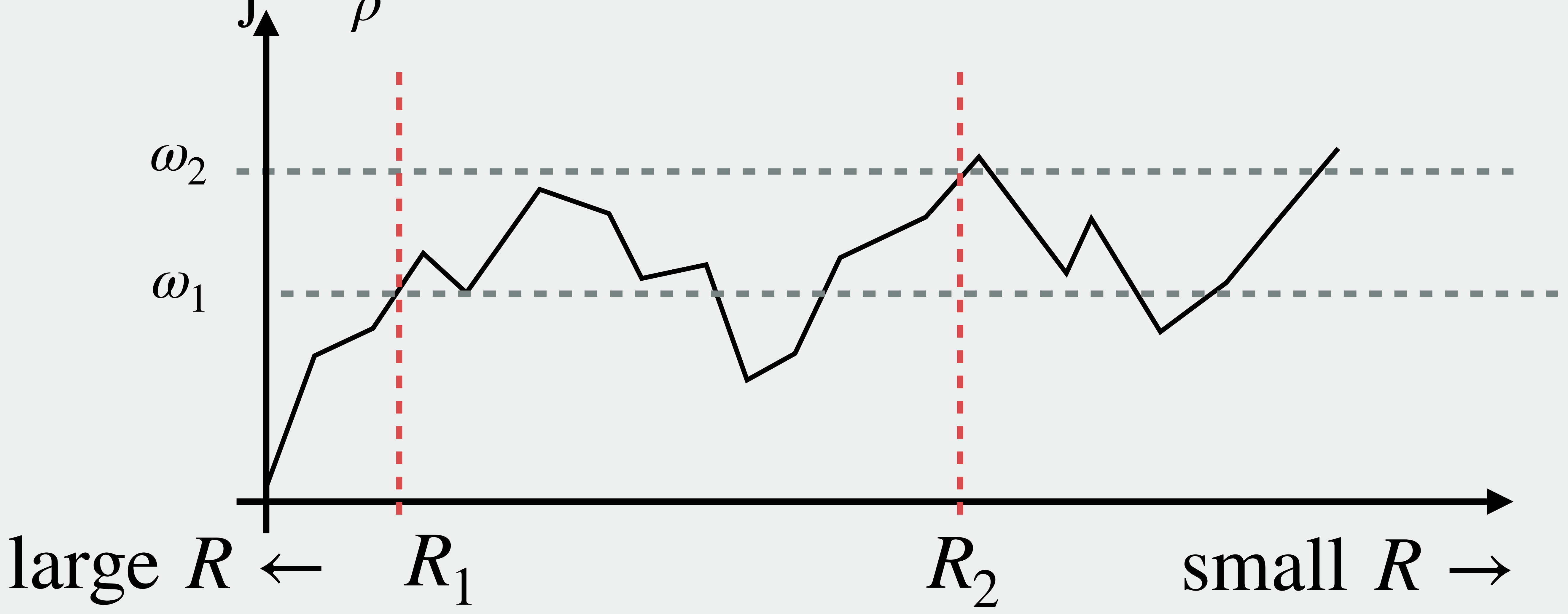

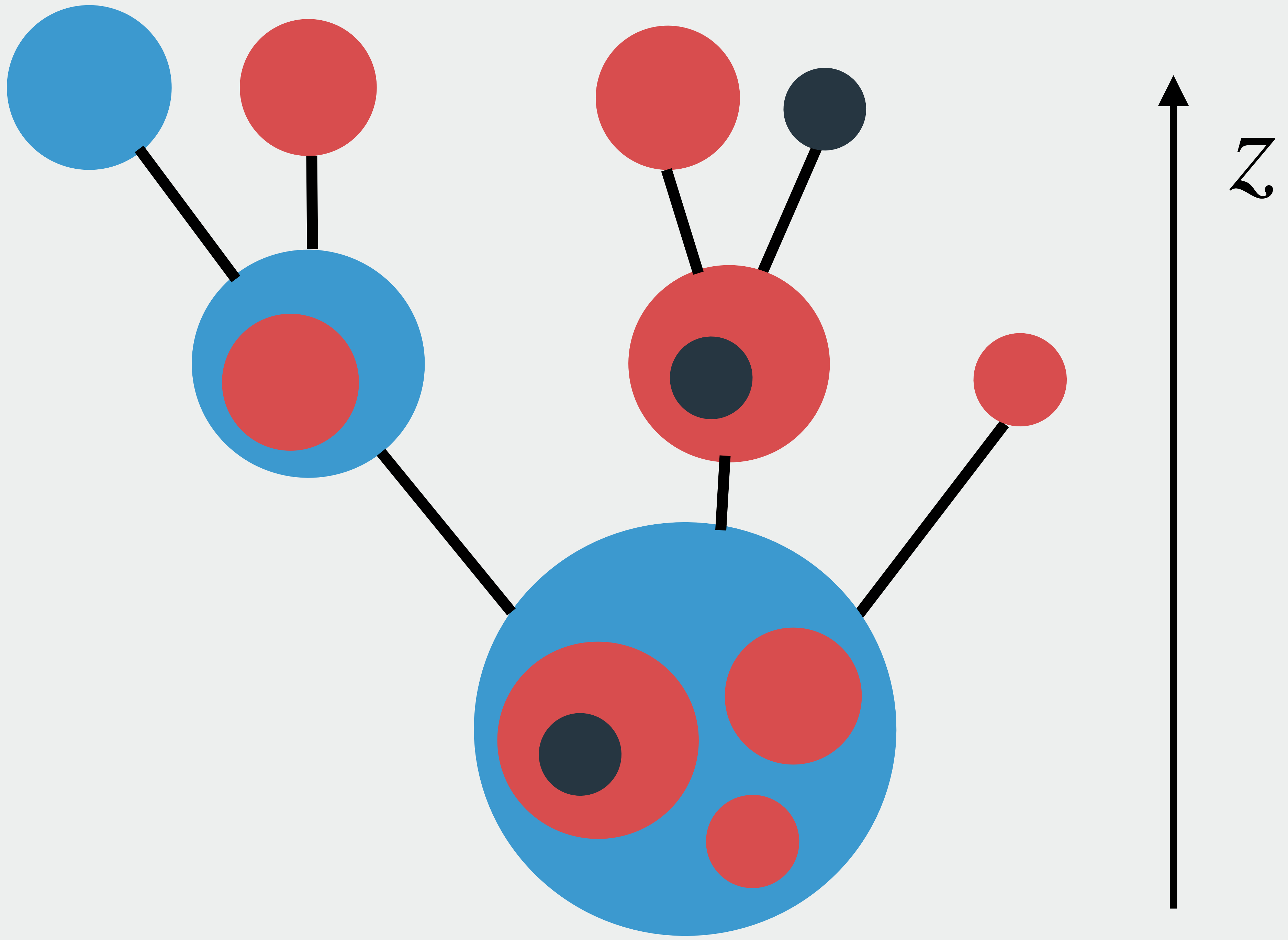

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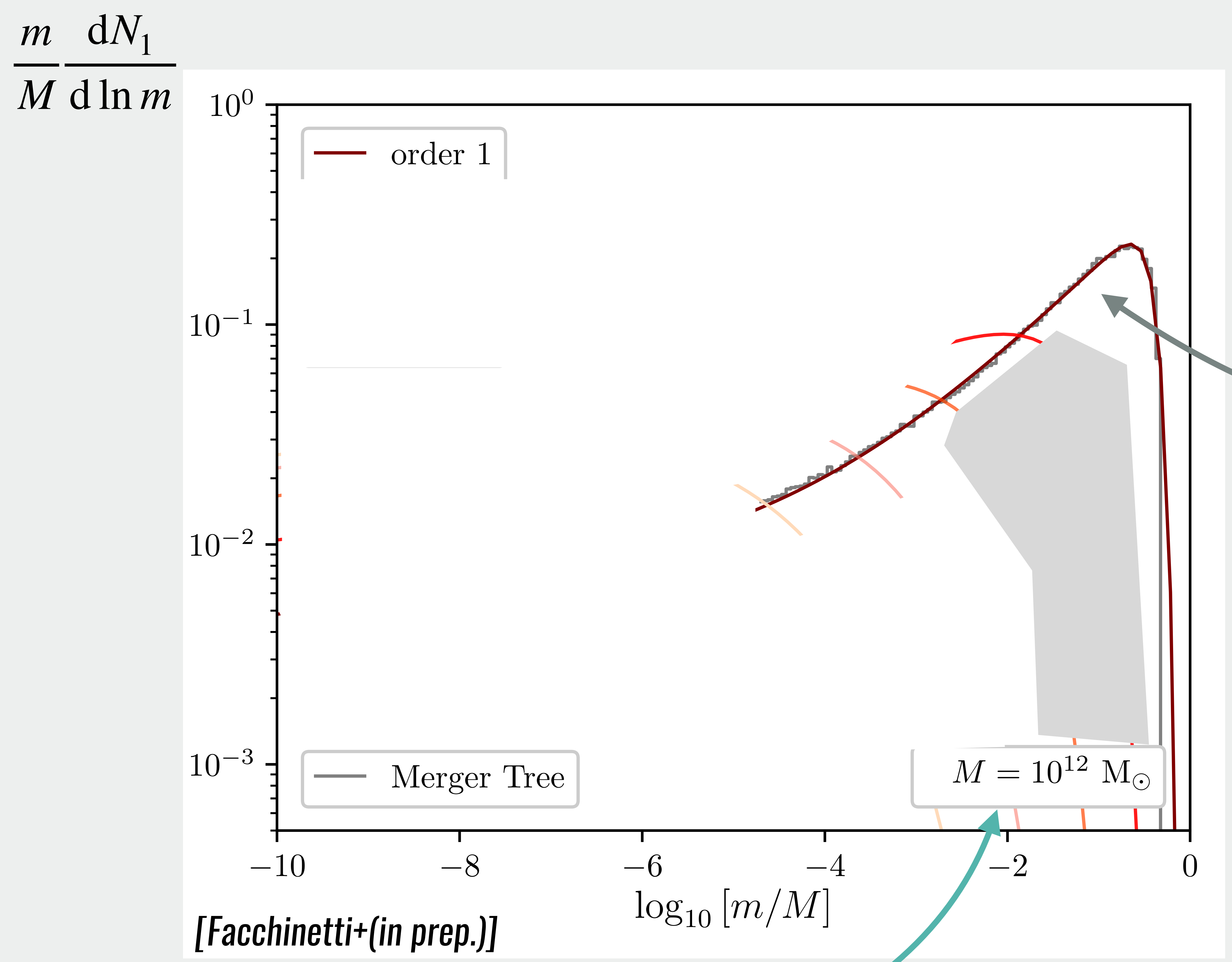


Host halo mass

Merger tree algorithm

[Cole+00]

Mass function (on large masses)

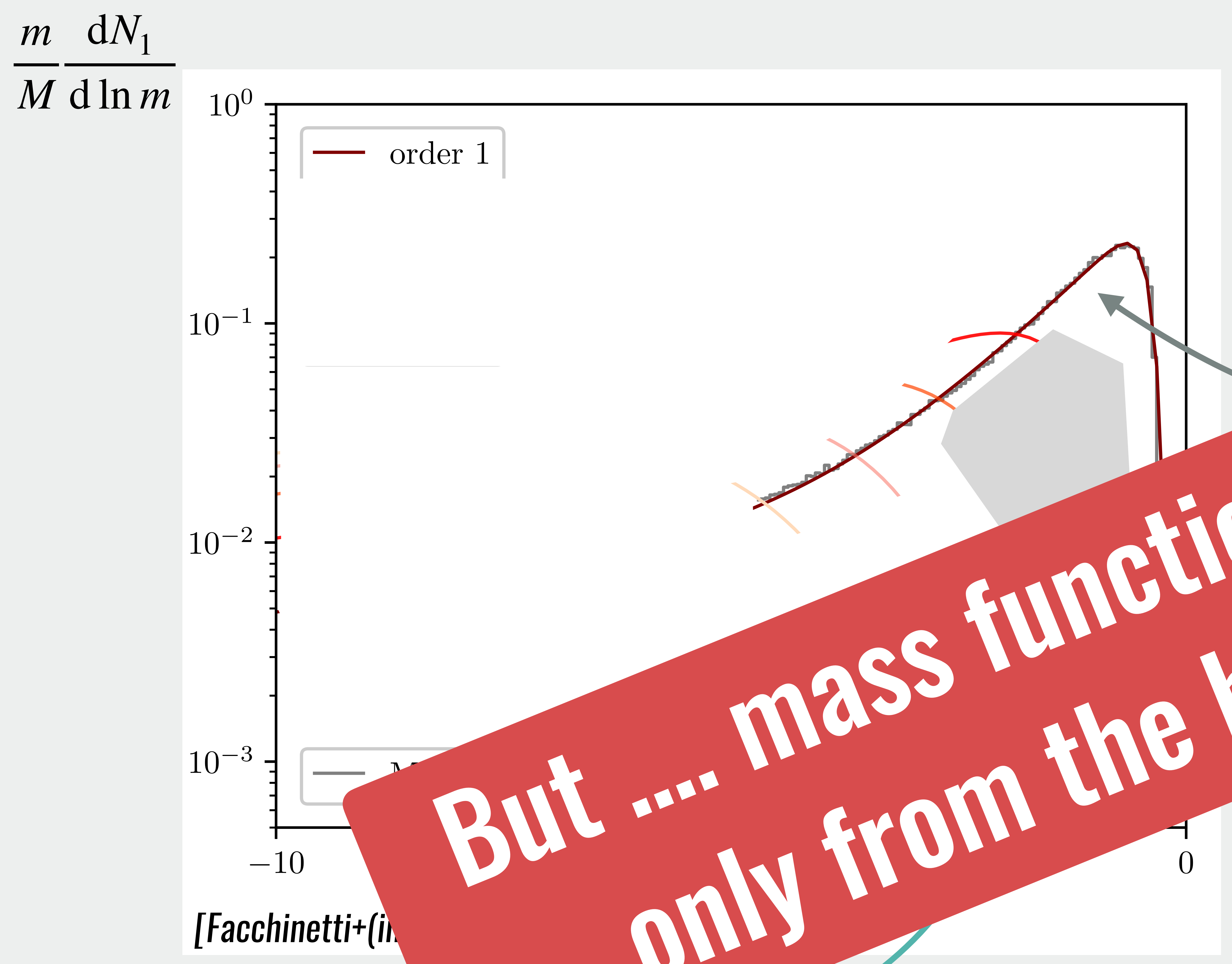
Fitting function (6 parameters)

$$f(m, M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

[Giocoli+08, Li+09, Jiang:+14]

Mass function (for all masses) $\frac{dN_1}{dm} = f(m, M)$

... it can be obtained from fits on the output of merger tree algorithms



[Facchinetti+(in

Host halo mass

Merger tree algorith…

…00]

…ses)

But …. mass function at small mass inferred only from the behaviour at large mass

…g function (6 parameters)

$$f(m, M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

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Mass function (for all masses) $\frac{\mathrm{d}N_1}{\mathrm{d}m} = f(m, M)$

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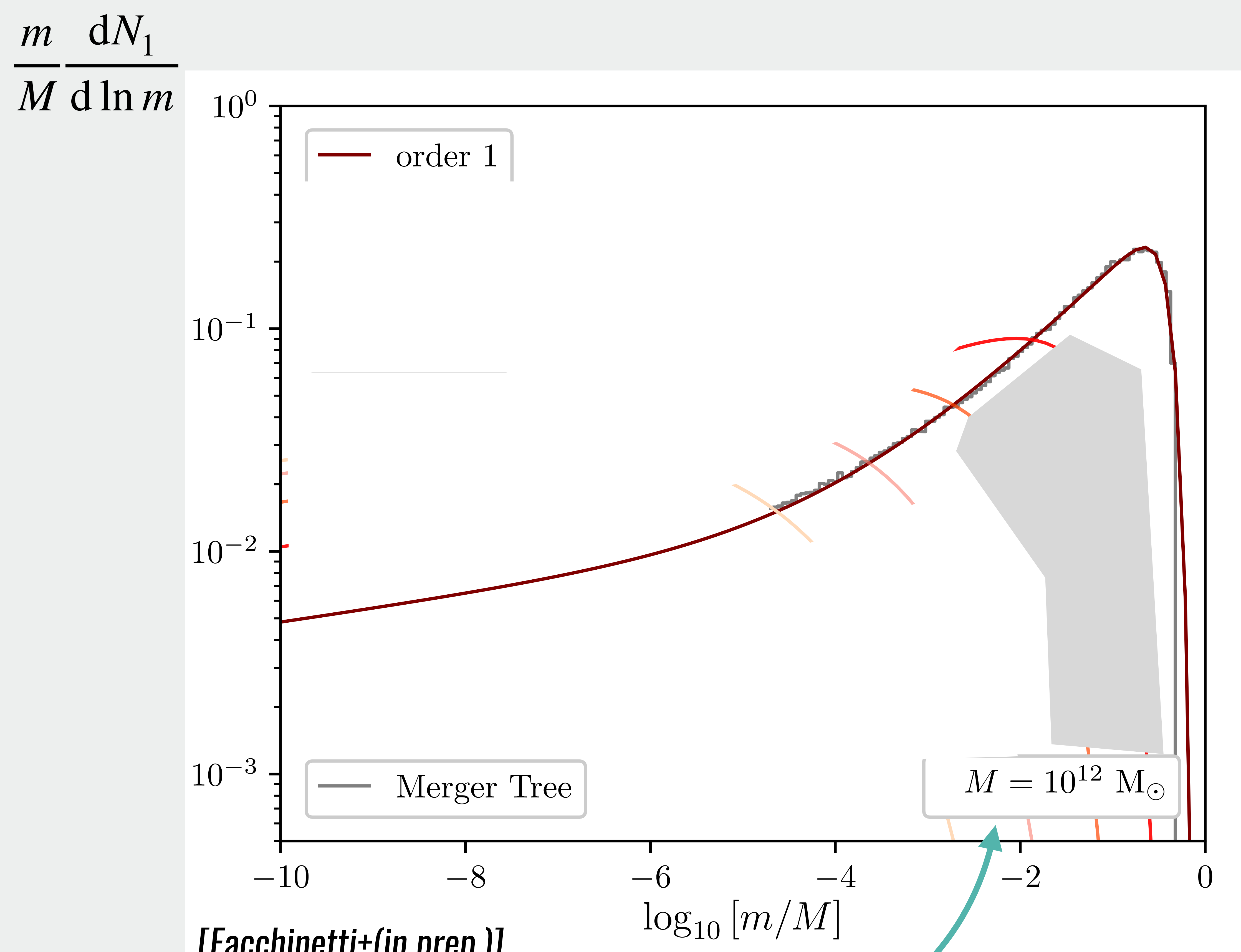


[Facchinetti+(in prep.)]

Host halo mass

New fitting procedure

Constraint on the shape by imposing the constraint

$$\frac{1}{M}\int_0^M m\frac{\mathrm{d}N_1}{\mathrm{d}m}\mathrm{d}m = 1$$

The host halo is entirely made of subhalos
Consistent with the fractal picture

Fixes the slope at small mass

$$\frac{\mathrm{d}N_1}{\mathrm{d}m} \sim \gamma m^{-\alpha} \quad \text{with} \quad \alpha \sim 1.95$$

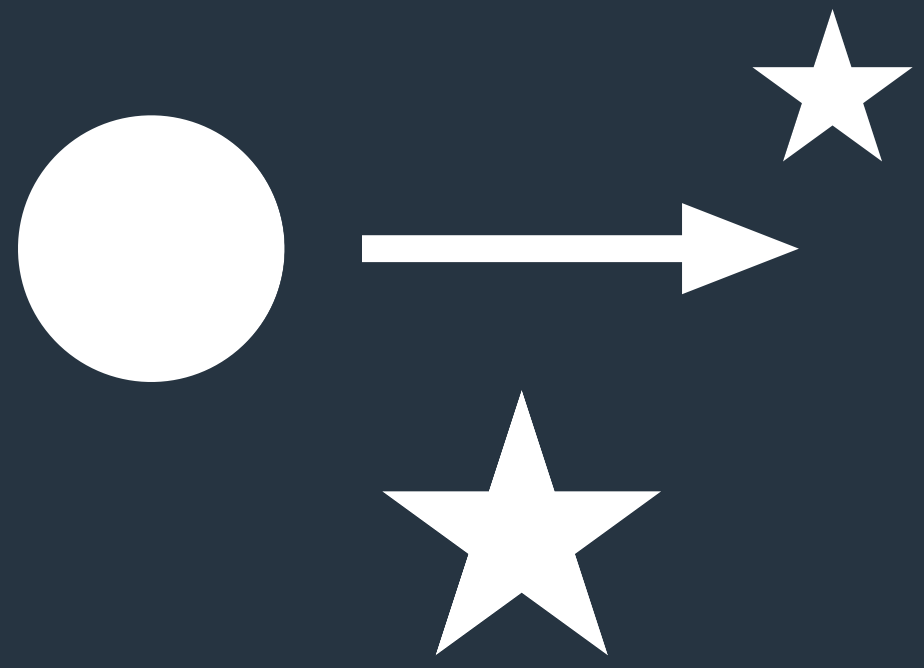
… it can be obtained from fits on the output of merger tree algorithms

$$\frac{\mathrm{d}N_1}{\mathrm{d}m}(m, M) = f(m, M) \quad \longrightarrow \quad \frac{\mathrm{d}N_1}{\mathrm{d}m}(m, M) = f(m, M)\Theta(m - m_{\mathrm{min}})$$

Total number of subhalos (before tidal disruption)

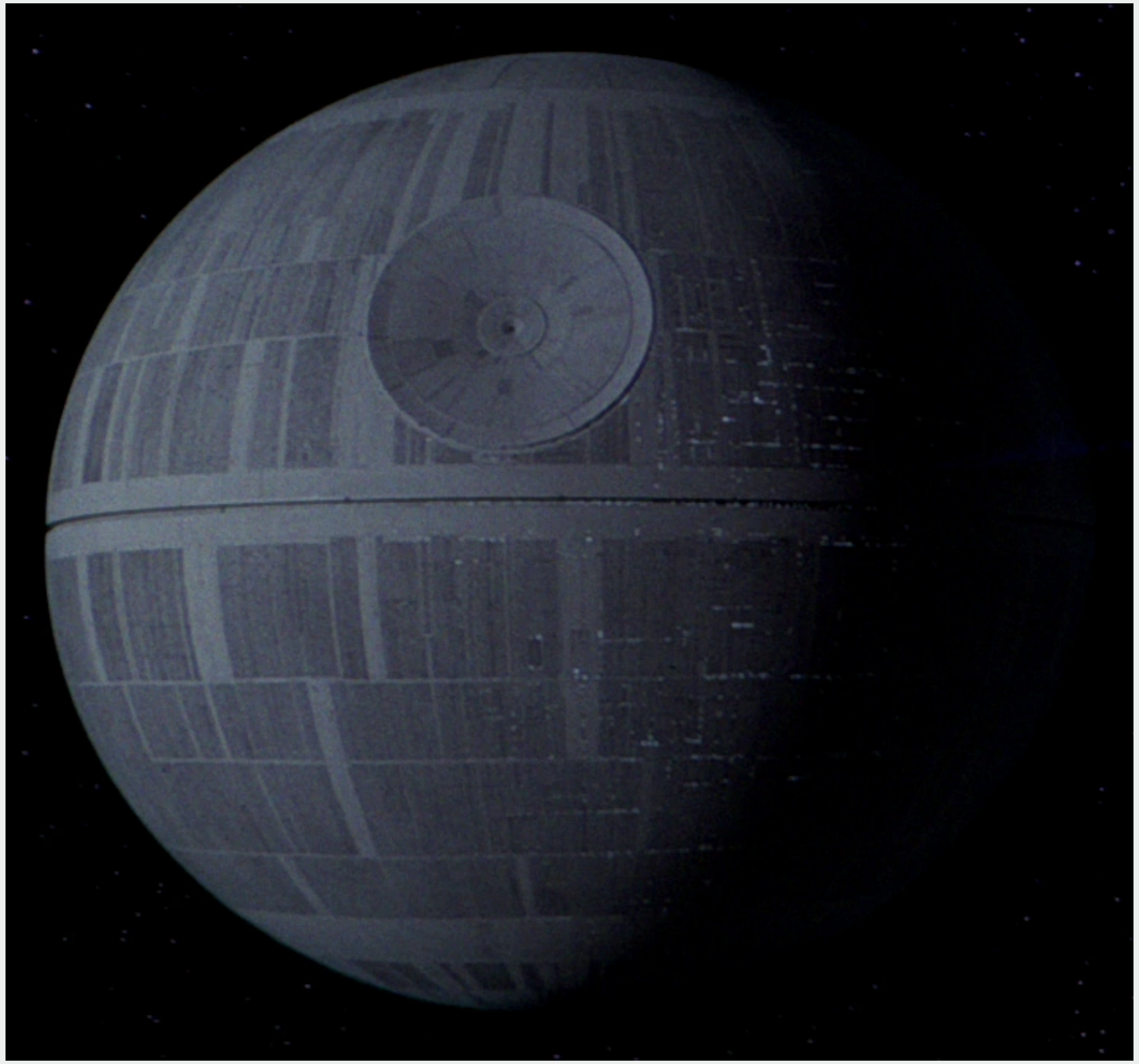
$$N_1(M) = \int_0^M f(m, M)\Theta(m - m_{\mathrm{min}})\mathrm{d}m$$

Cosmological simulations no longer needed!
Can be easily adapted to any host/cosmology

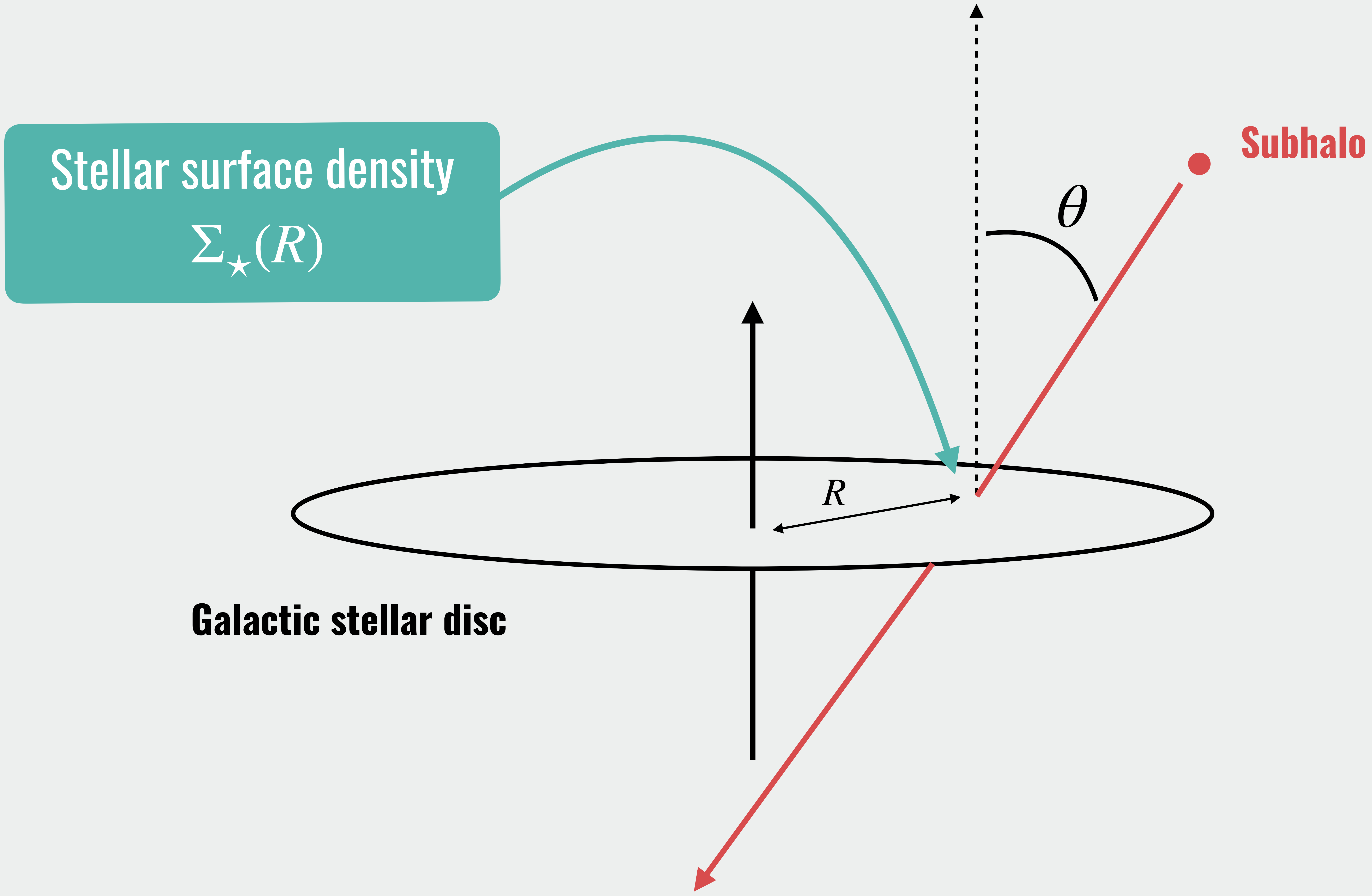


Stellar encounters in the Milky-Way (Snapshots)

[arXiv:2201.xxxx]



[Darth Vador+(a long time ago)]



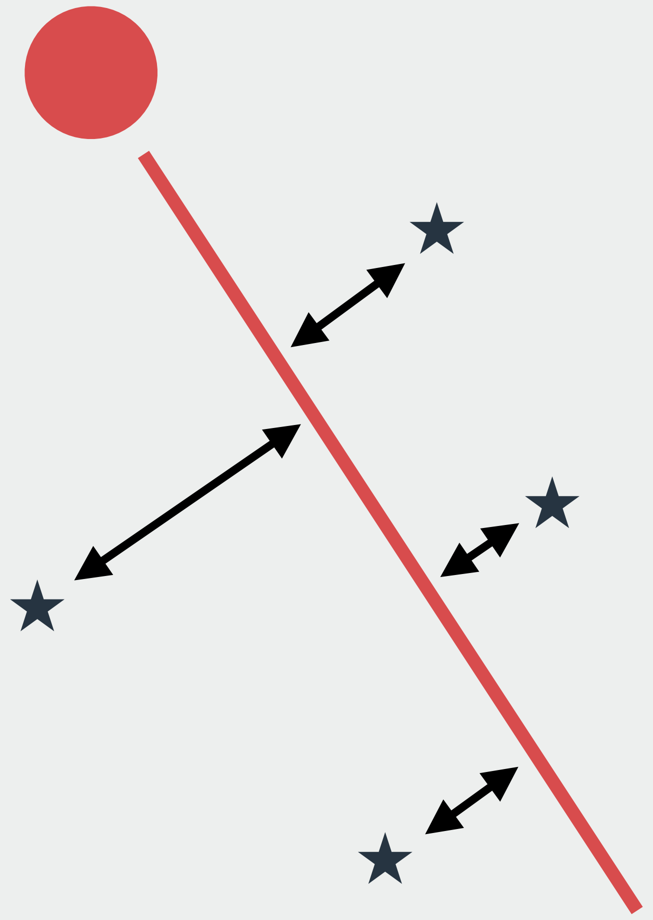
Stellar surface density
$\Sigma_\star(R)$
θ
Subhalo
R
Galactic stellar disc

Number of encountered stars

1) Evaluate the total energy/velocity kick received by the particles:

$$\Delta \mathbf{v} = \sum_{i=1}^{\mathcal{N}} \delta \mathbf{v}_i \qquad \Delta E = \frac{1}{2}(\Delta \mathbf{v})^2 + \mathbf{v} \cdot \Delta \mathbf{v}$$

Random walk in velocity space



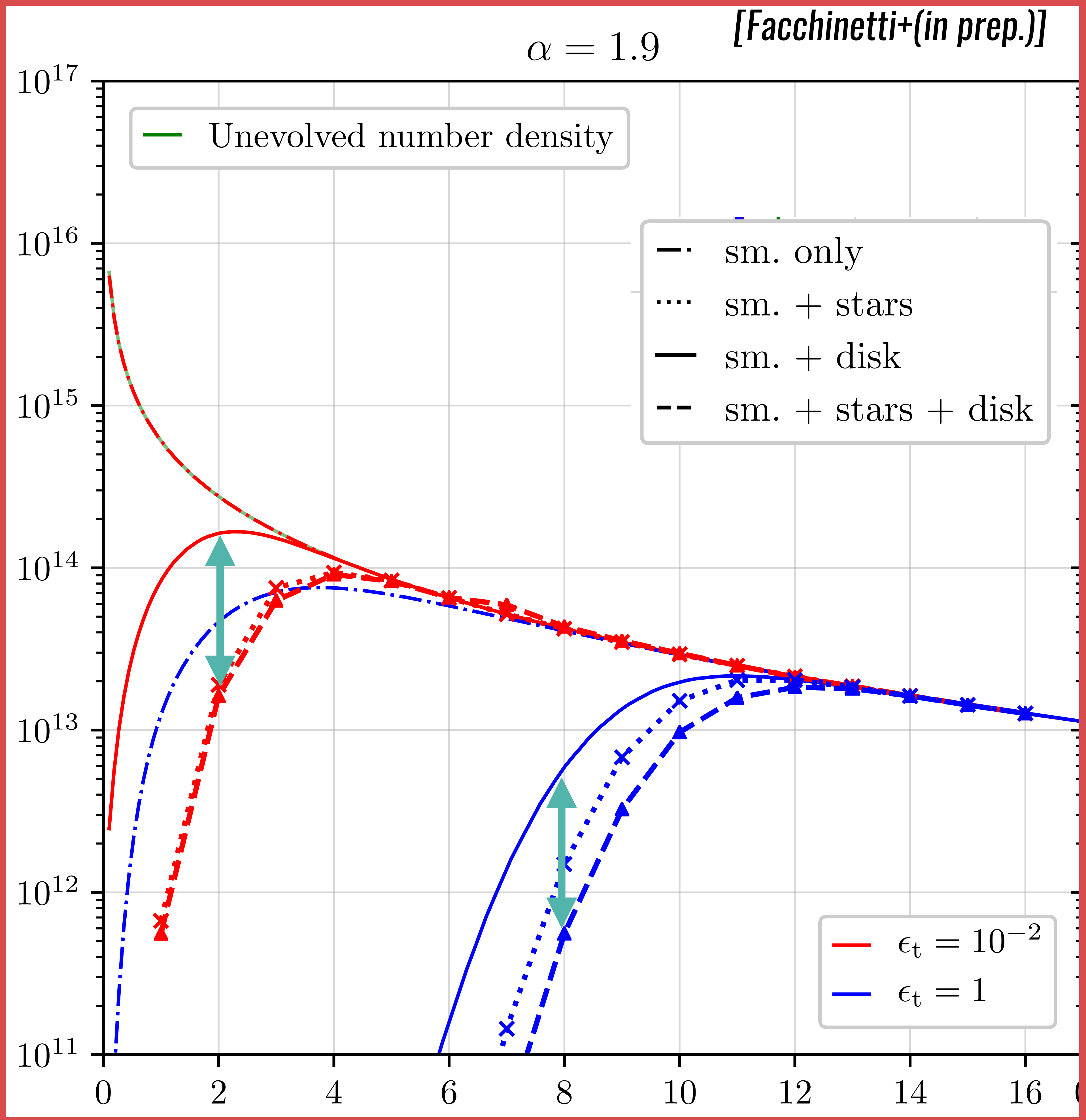
2) Ask whether the energy kick is high enough for the particles to be expelled: $''\Delta E(r) > |\Phi(r)|''$?

3) Evaluate it for the entire population of subhalo

Following/imrpoving on [Spitzer58, Gerhard+83, Carr+99, Green+07, Schneider+10, Delos19]

The total velocity kick is the result of a random walk

Number density
[kpc-3]



Distance from the Galactic center [kpc]

Star encounters have an important effect on the number density

Gaétan Facchinetti — gaetan.facchinetti@umontpellier.fr

Recent developments on analytical dark matter subhalo population models:

1.

In the WIMP scenario we made the connection between generic particle physics models and the distribution of subhalos.

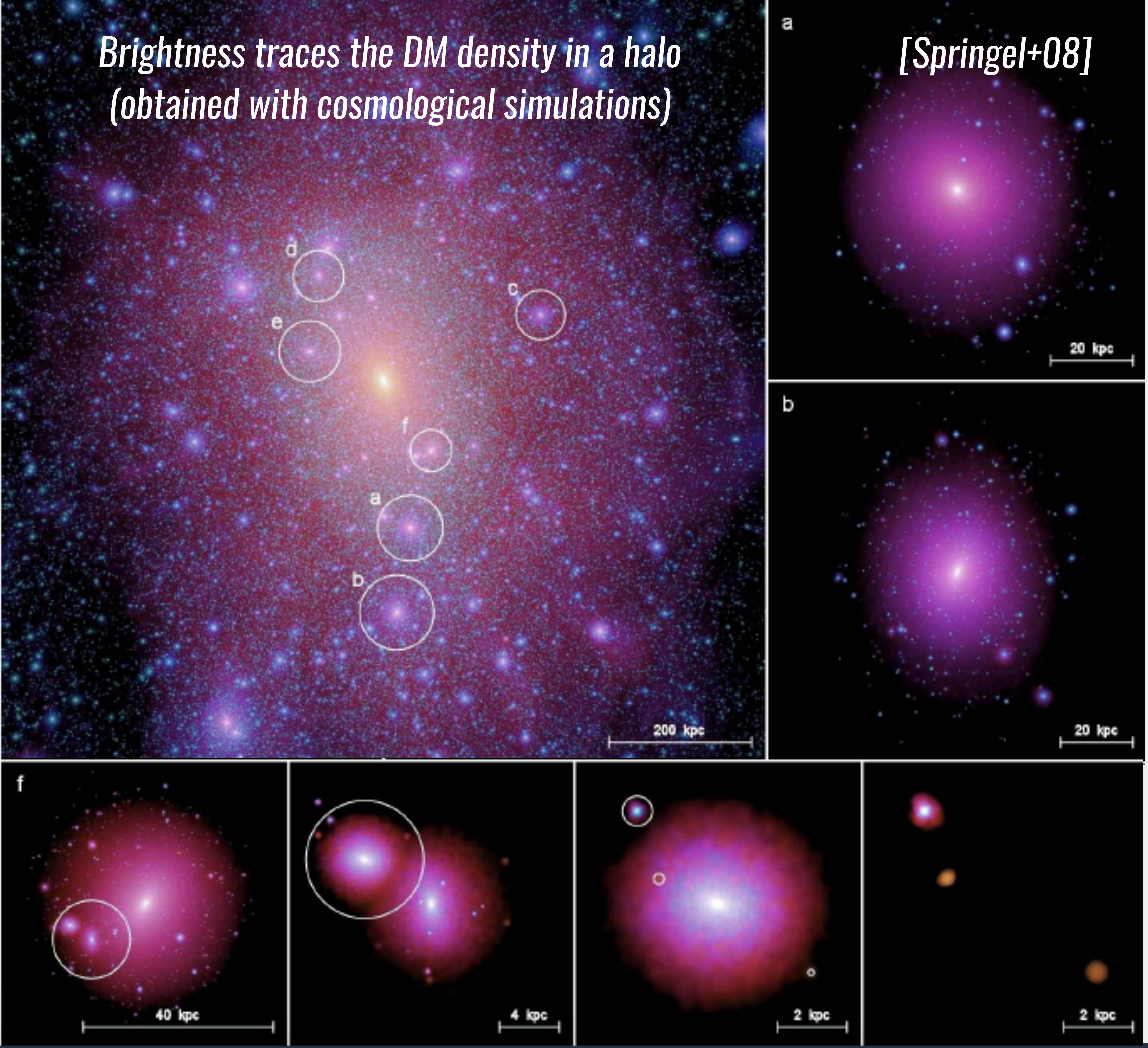
2.

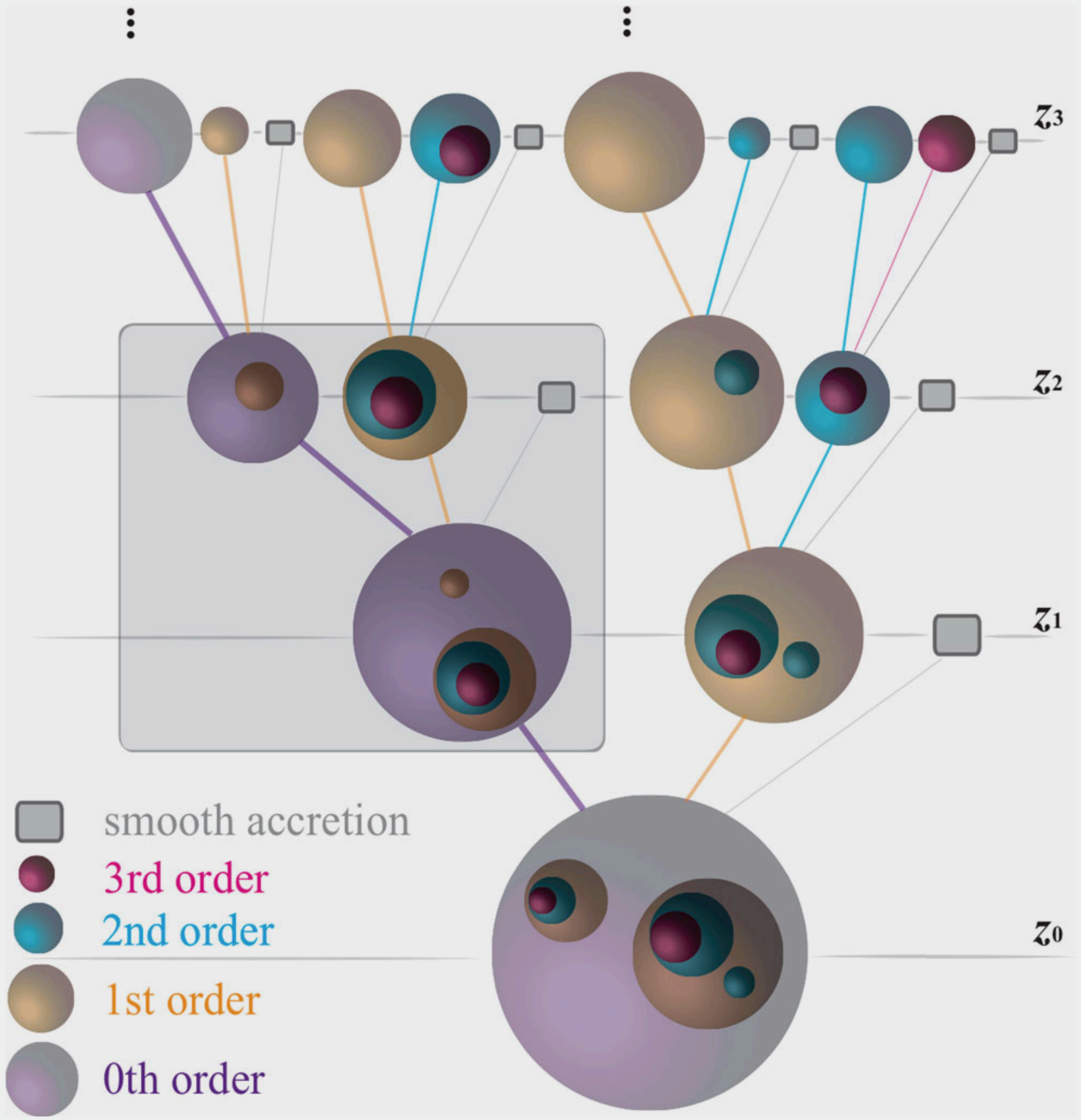
We developed a new method to derive a constrained cosmological subhalo mass function in any host and for any cosmology.

3.

We showed that encounters between stars and subhalos can significantly impact on their distribution.

Back-up slides

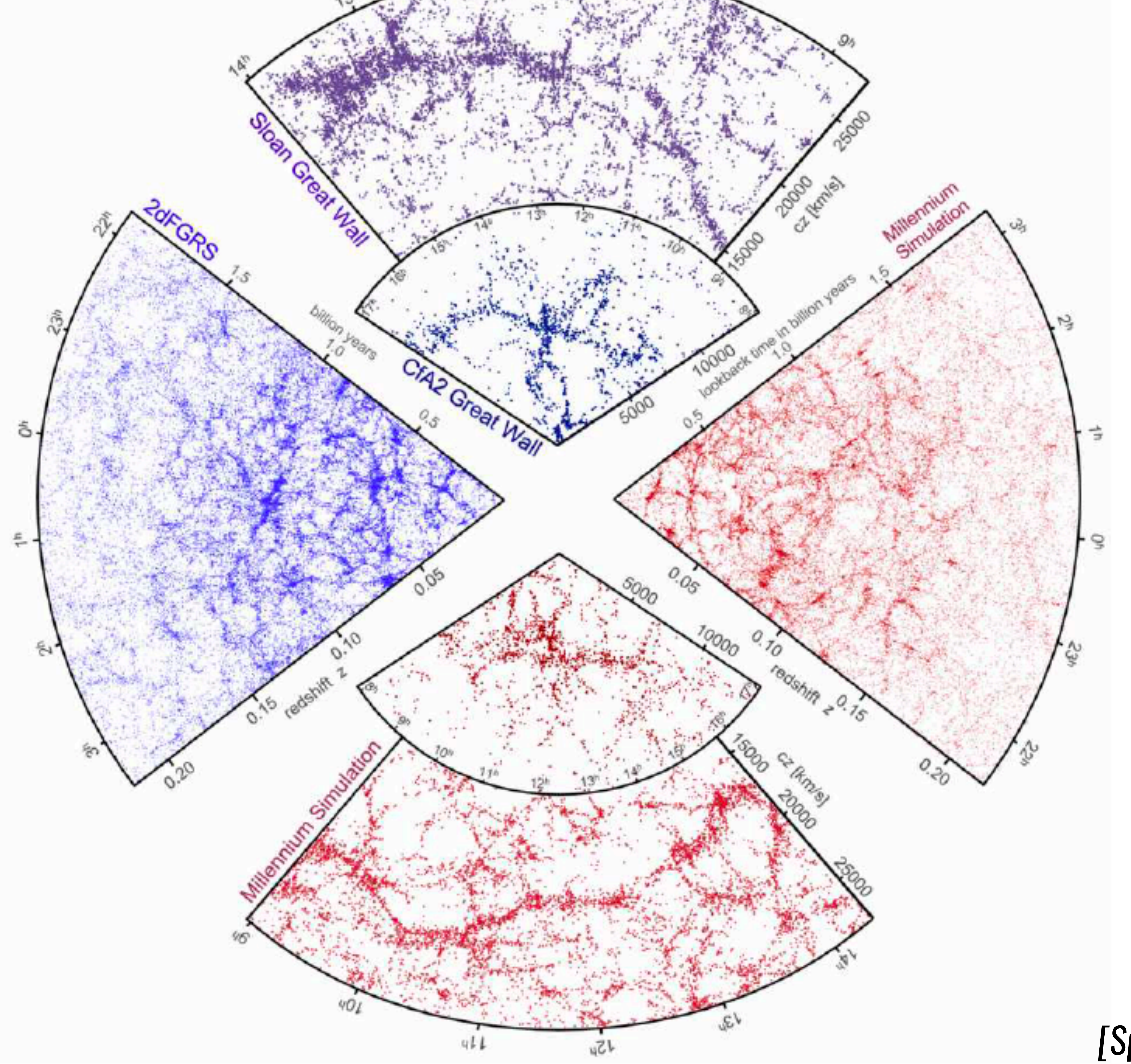




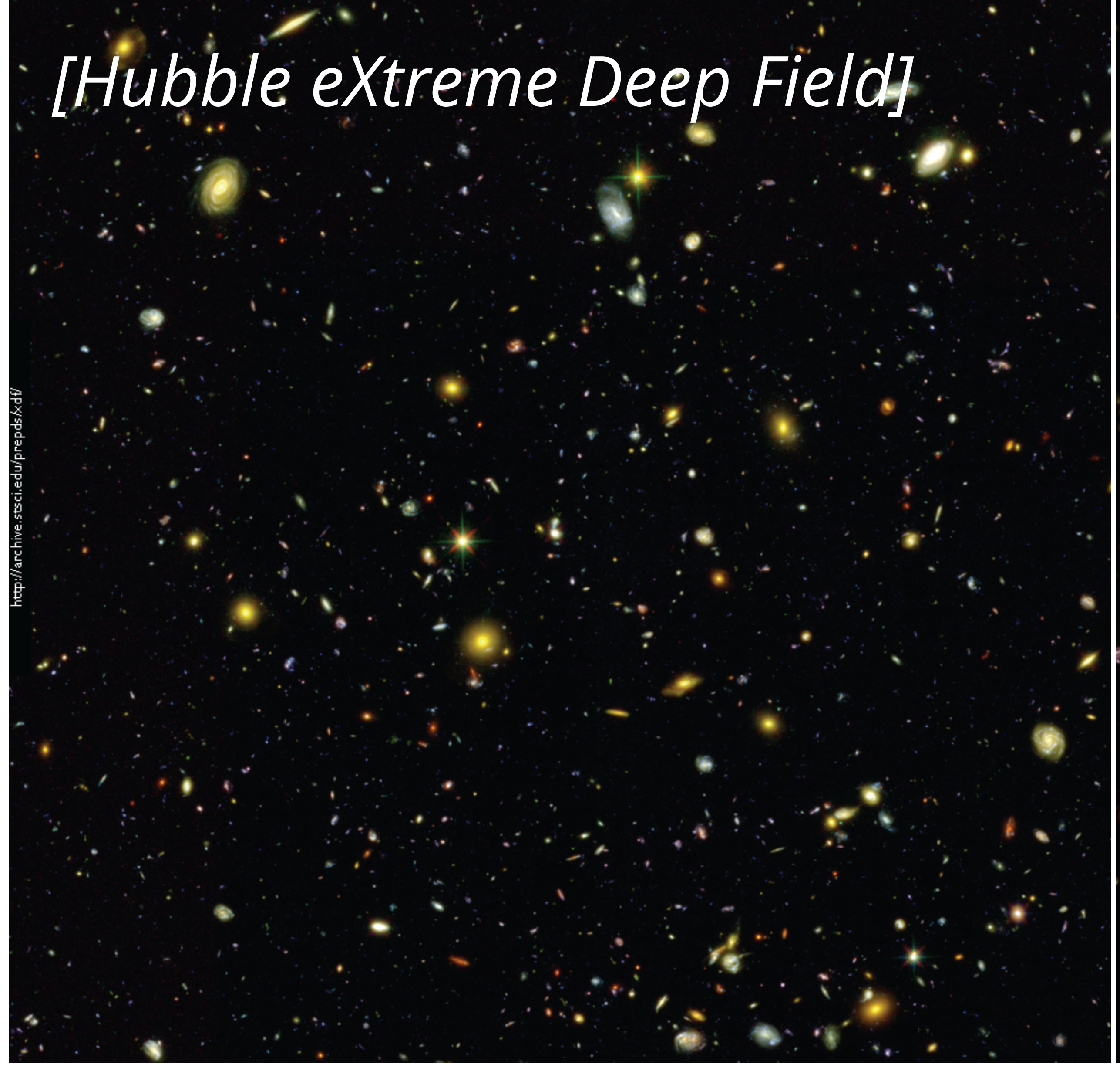
[Jiang+14]

Hierarchical formation leads to a fractal distribution

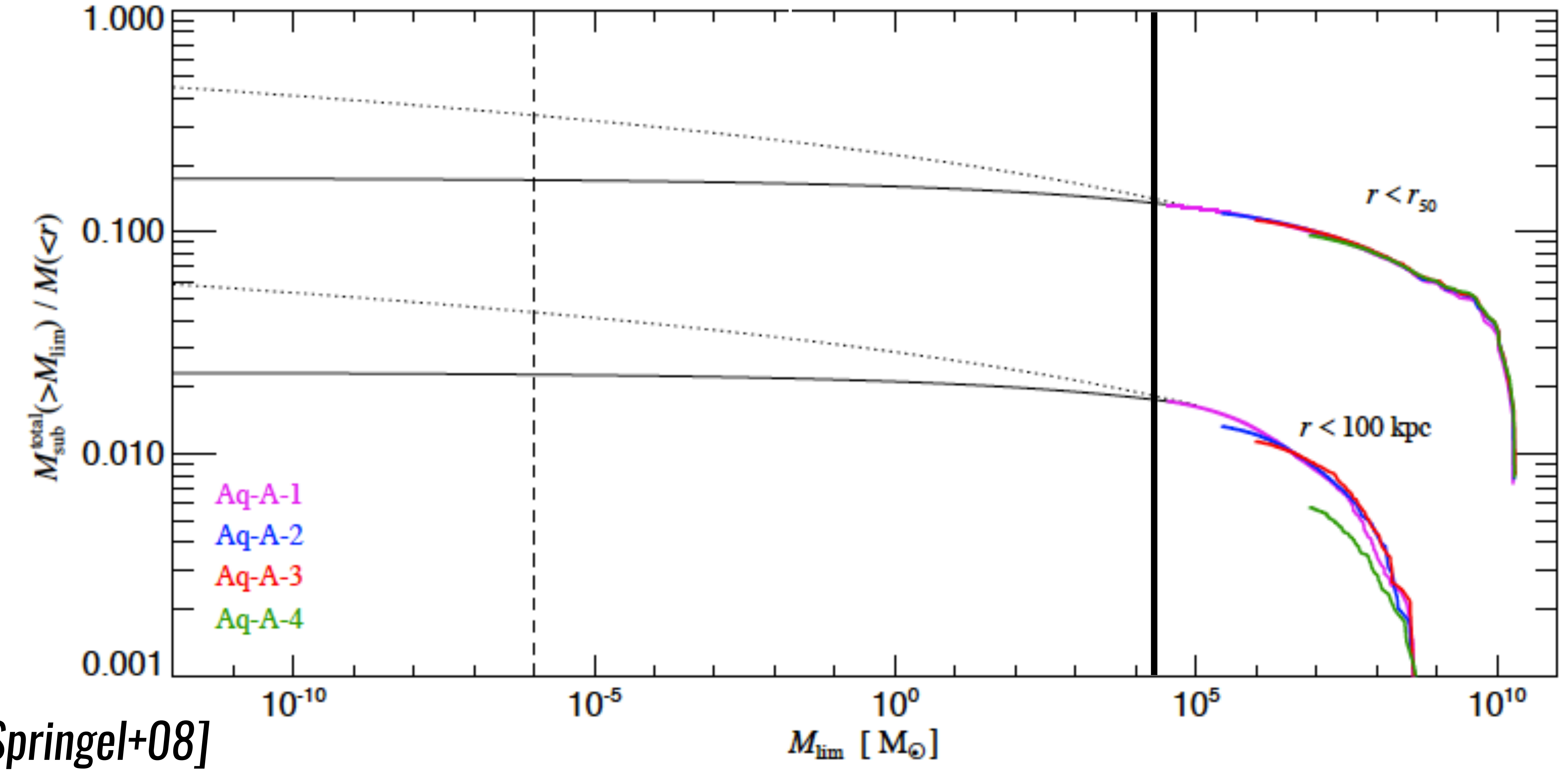
[Springel+06]



[Hubble eXtreme Deep Field]



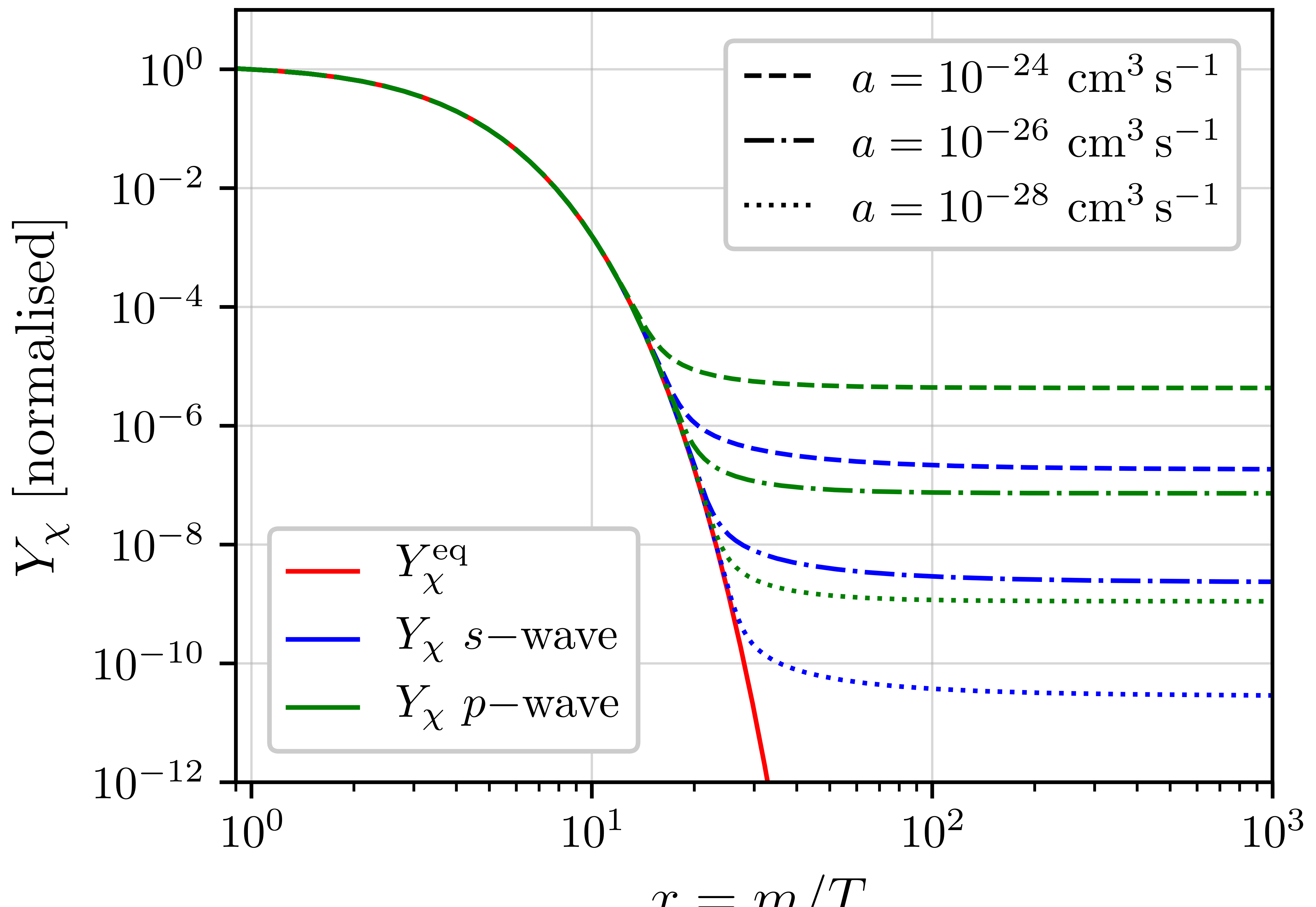
[Illustris collaboration]



[Springel+08]

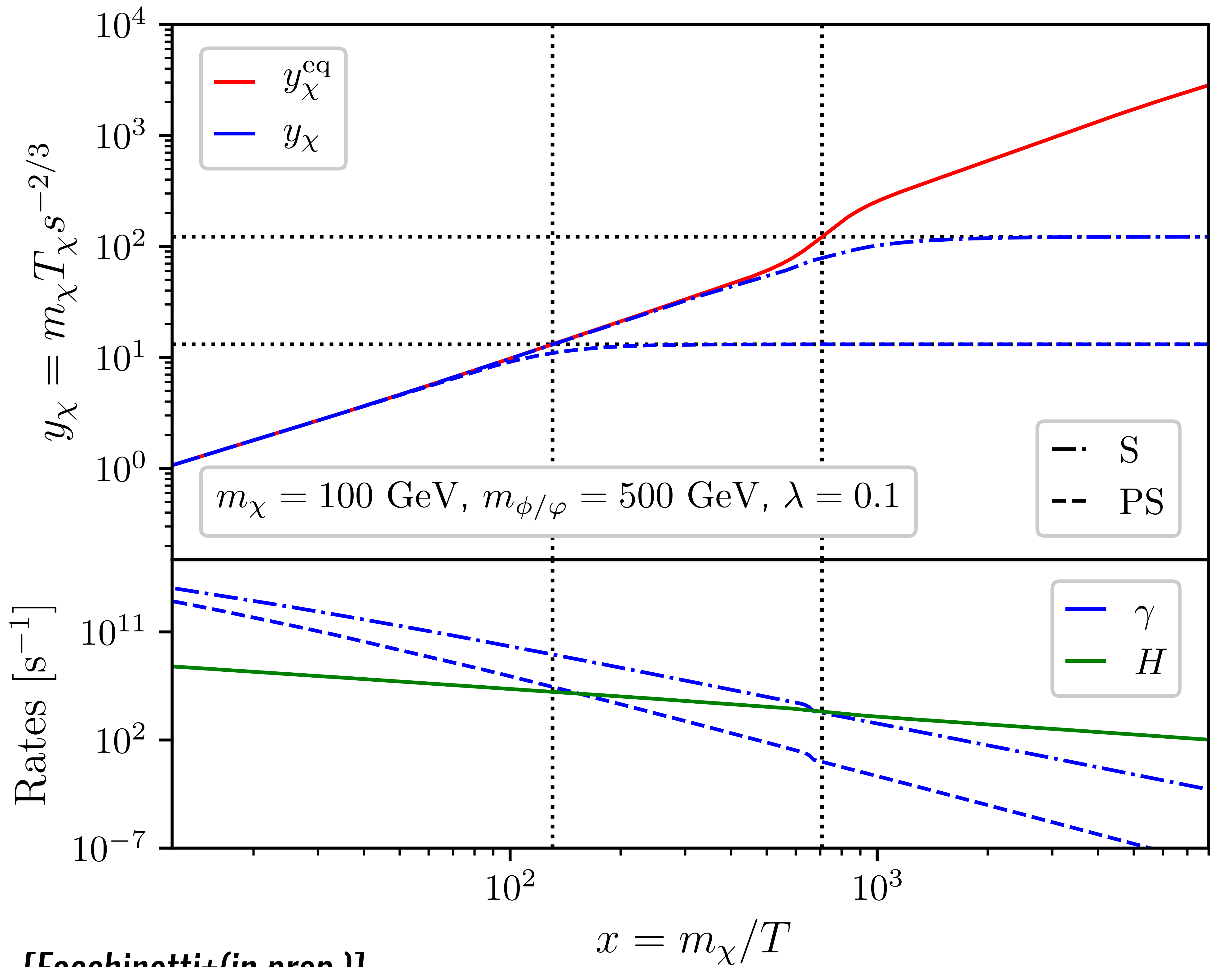
Cosmological simulations cannot probe very small scales

Chemical decoupling



[Facchinetti+(in prep.)]

Kinetic decoupling



[Facchinetti+(in prep.)]

Decoupling are characterized by a divergence from the equilibrium quantity

Initial distribution:
(without dynamics)

$(\rho_s, r_s) \leftrightarrow (m, c)$

Initial mass distribution
(cosmological mass function)

$$p_{\rm sub}^{\rm init}(m, c, R) = p_{\mathbf{R}}(R)\frac{1}{N_{\rm sub}}\frac{{\rm d}N_{\rm sub}}{{\rm d}m}p_c(c\,|\,m)$$

Spatial distribution
(follows potential of the host)

[McMillan+17]

Distribution in concentration

[Bullock+01,Sánchez-Conde+14]

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(without dynamics)

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Spatial distribution
(follows potential of the host)

[McMillan+17]

Distribution in concentration

[Bullock+01,Sánchez-Conde+14]

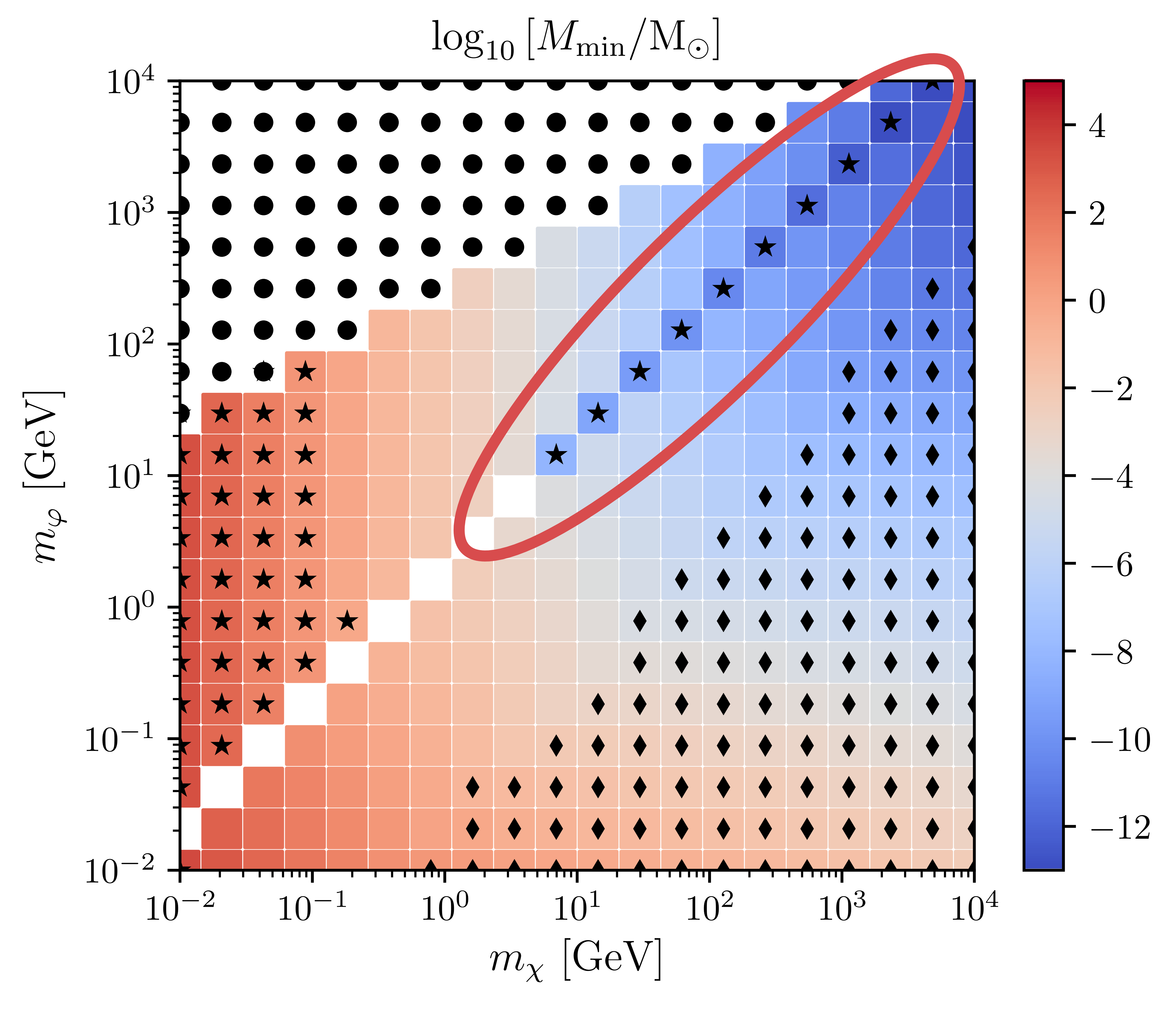
+ Constraints from dynamical effects $p_{\rm sub}^{\rm init}(m, c, R) \rightarrow p_{\rm sub}^{\rm late}(m, c, R)$

Minimal halo mass

Pseudo-scalar

+ Sommerfeld effects
x large decay width
• large coupling
★ early kinetic dec.

◆ acoustic > free-stream.



[Facchinetti+(in prep.)]

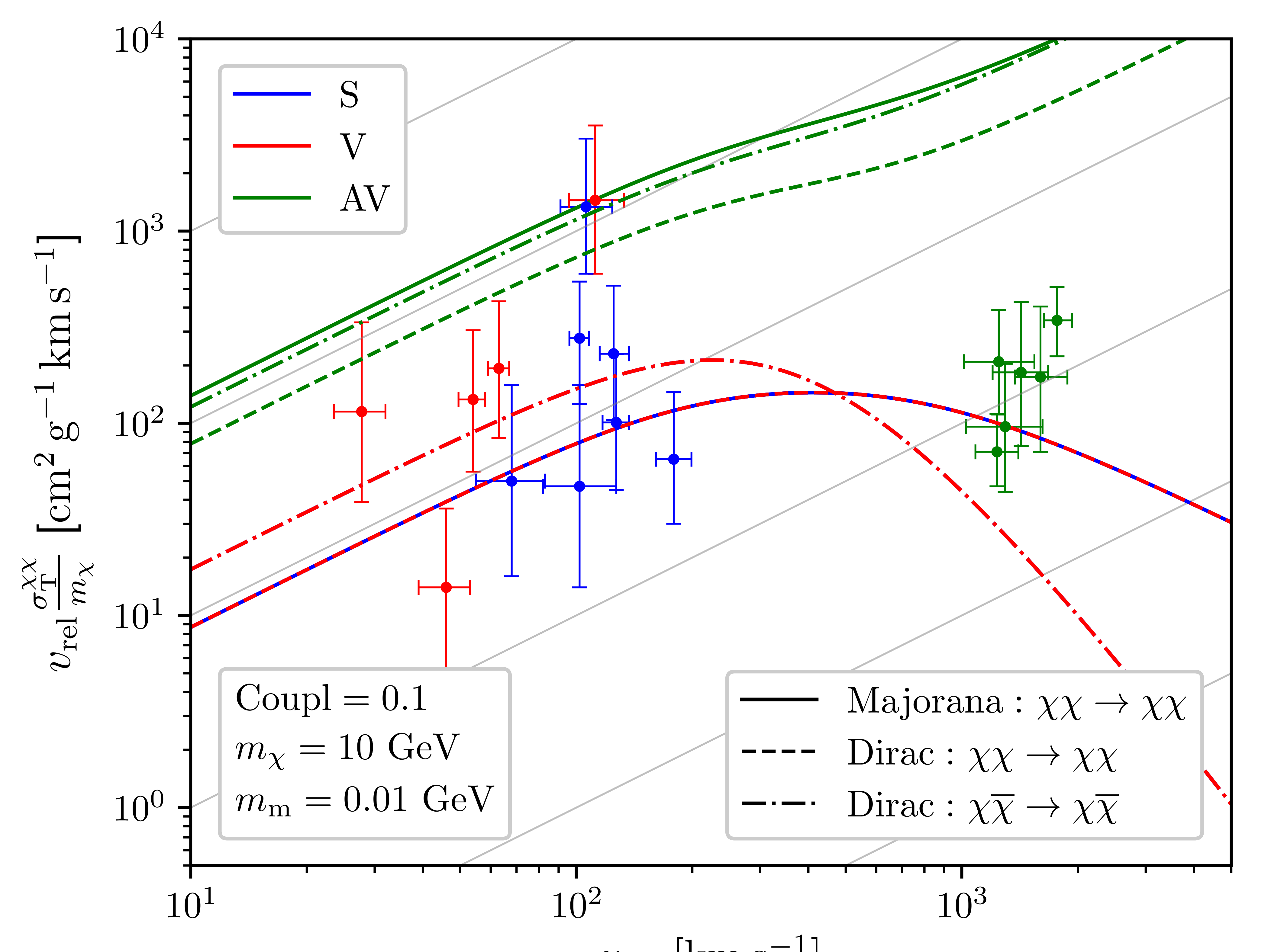
Annihilation on pole

Small couplings BUT Small minimal mass

Large number of subhalos

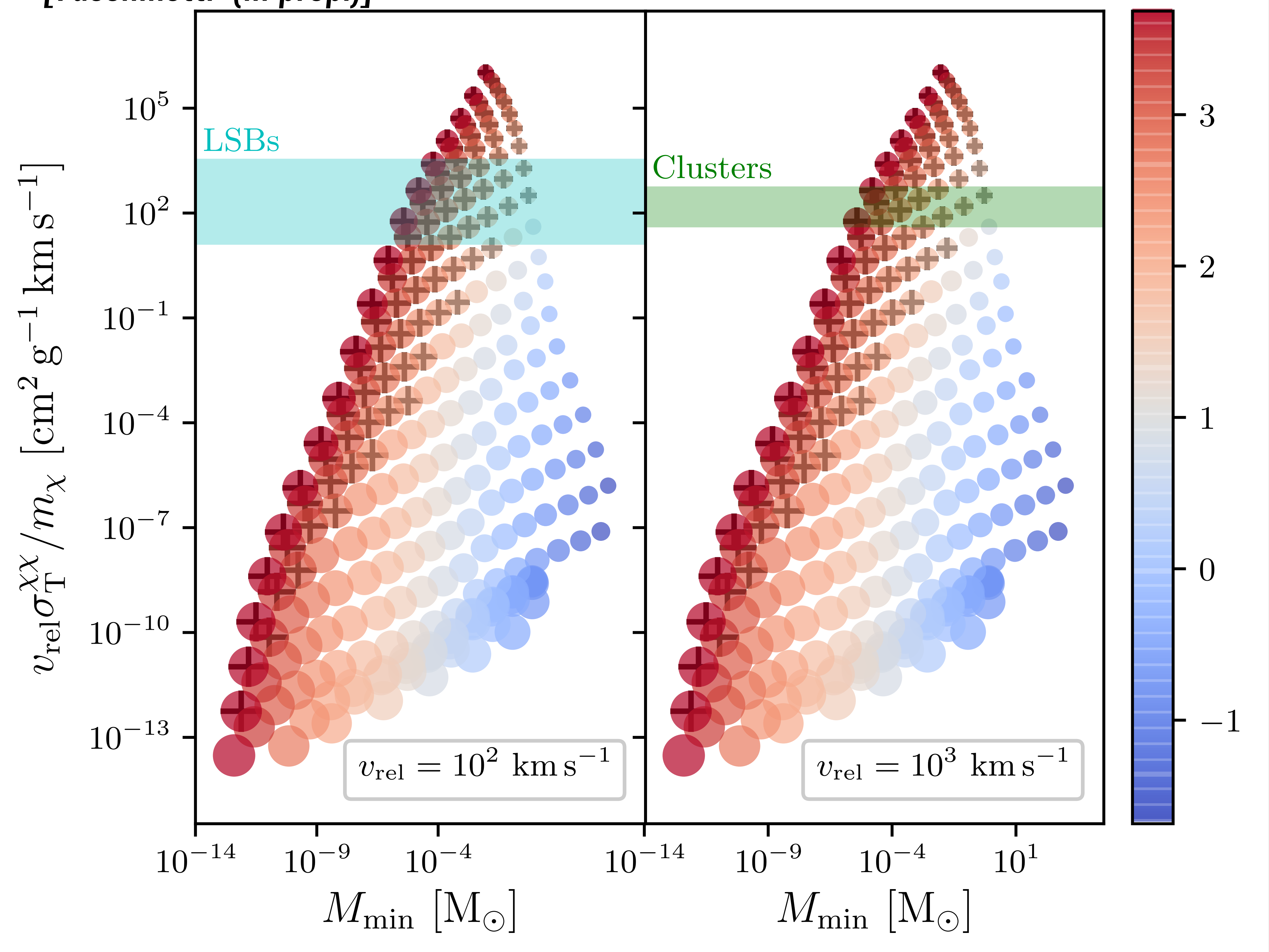
Enhanced annihilation for indirect detection

Scalar mediator



[Facchinetti+(in prep.)]

[Facchinetti+(in prep.)]



$$\mathscr{L} \ni -\frac{1}{2}\lambda\bar{\chi}\phi\chi - \lambda\bar{e}\phi e$$

Self-interaction

$$p_{\rm sub}^{\rm init}(\{m_i\}_i, \{c_i\}_i, \{\mathbf{R}_i\}_i) \simeq \left[p_{\rm sub}^{\rm init}(m, c, R)\right]^{N_{\rm sub}}$$

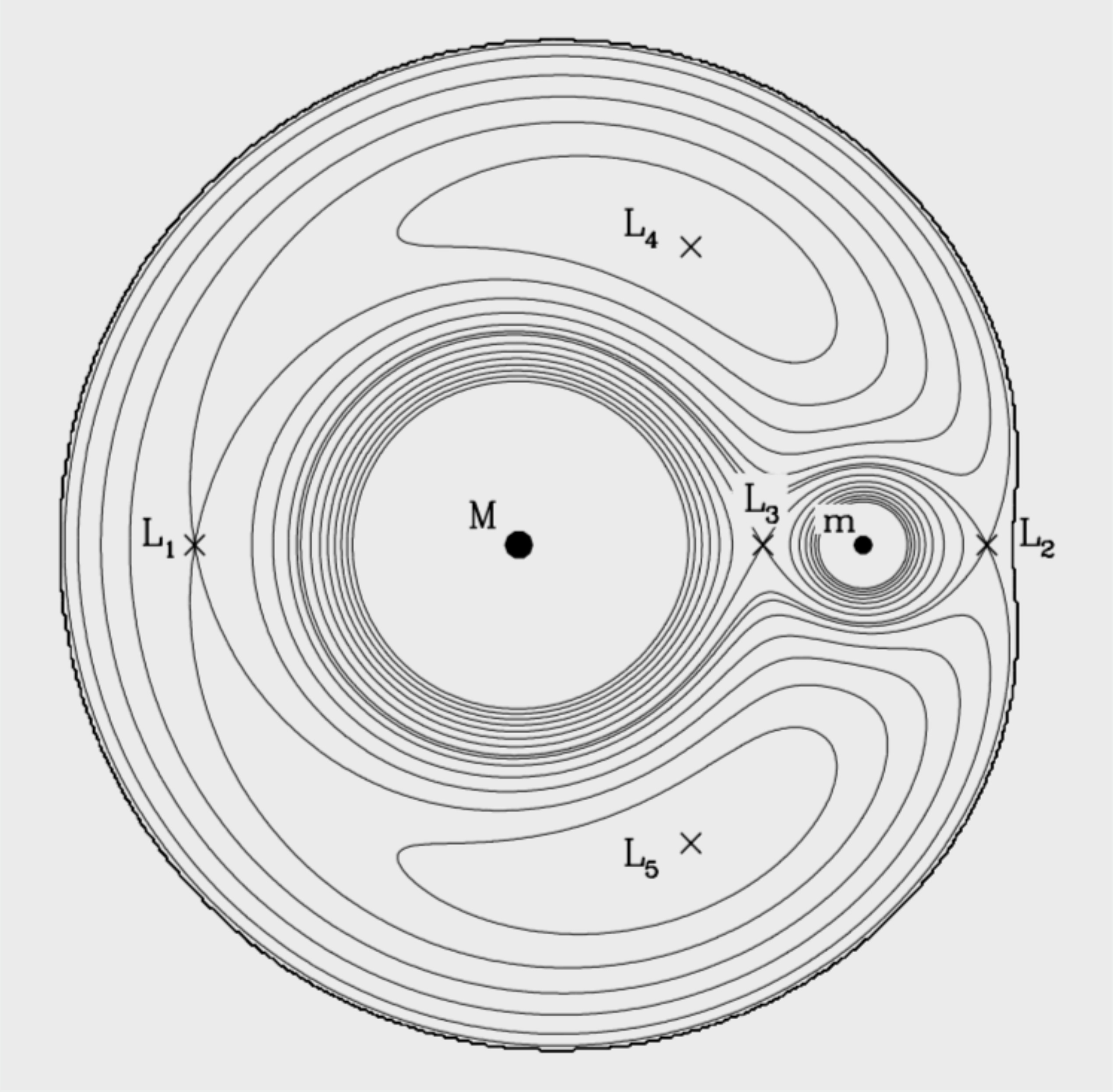
$$p_{\rm sub}^{\rm init}(m, c, R) = \frac{1}{N_{\rm sub}}\frac{{\rm d}N_{\rm sub}}{{\rm d}m}p_c(c\,|\,m)p_{\mathbf{R}}(R)$$

$$p_{\rm sub}^{\rm late}(m, c, R) = \frac{1}{K_{\rm t}}\frac{1}{N_{\rm sub}}\frac{{\rm d}N_{\rm sub}}{{\rm d}m}p_c(c\,|\,m)p_{\mathbf{R}}(R)\Theta[r_{\rm t}/r_{\rm s} - \epsilon_{\rm t}]$$

New number of subhalos $N_{\rm sub} \to K_{\rm t}N_{\rm sub}$

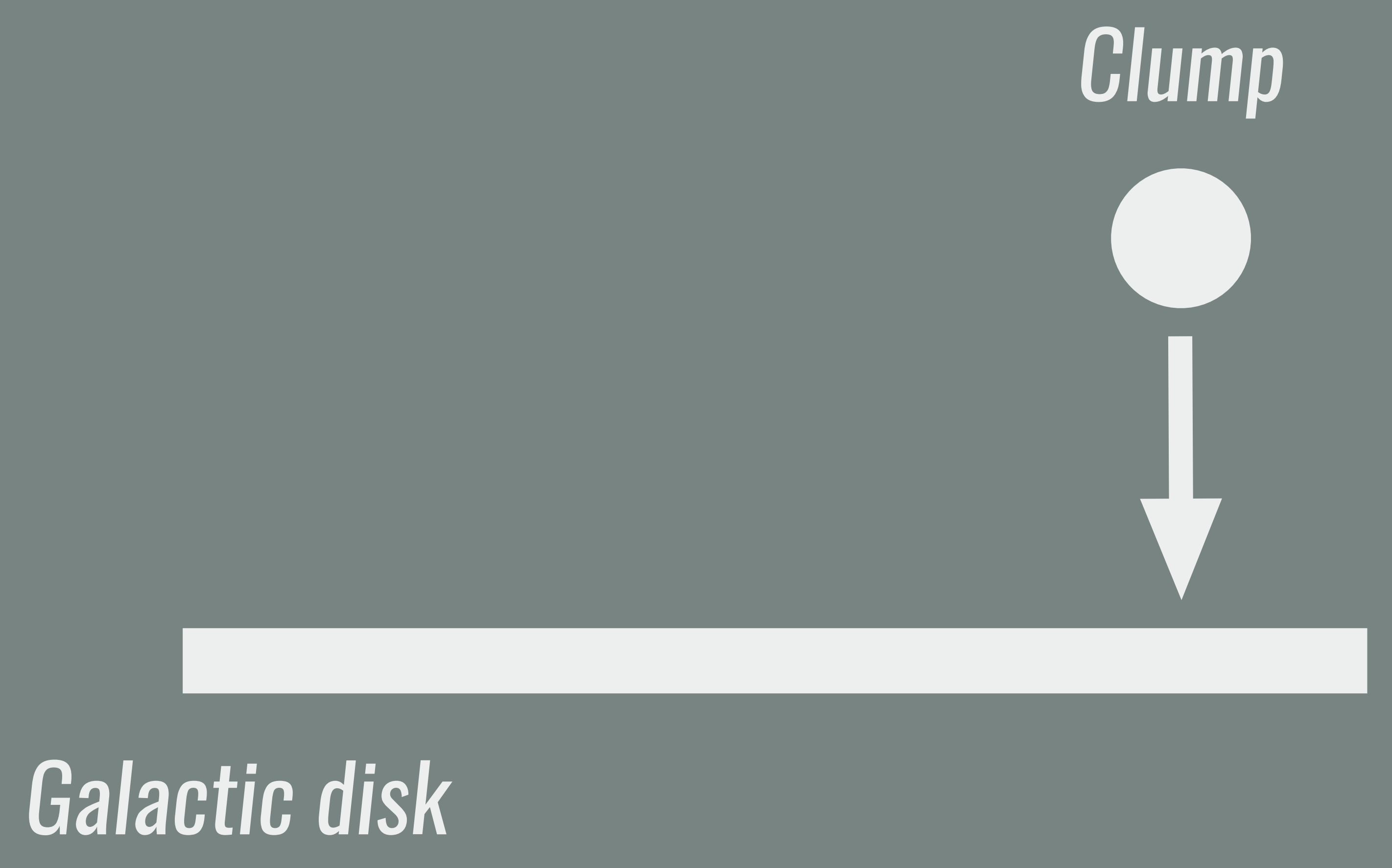
[Binney+08, Weinberg94, Gnedin+99, Stref+17]

$$r_t = R\left\{\frac{M_{\mathrm{int}}(R)}{3M(R)f[M(R)]}\right\}^{1/3}$$



Global tides

$$\left\langle\frac{\delta E}{m_\chi}\right\rangle = \frac{2}{3}\frac{g_{\mathrm{d}}^2}{V_z^2}A(\eta)r^2$$



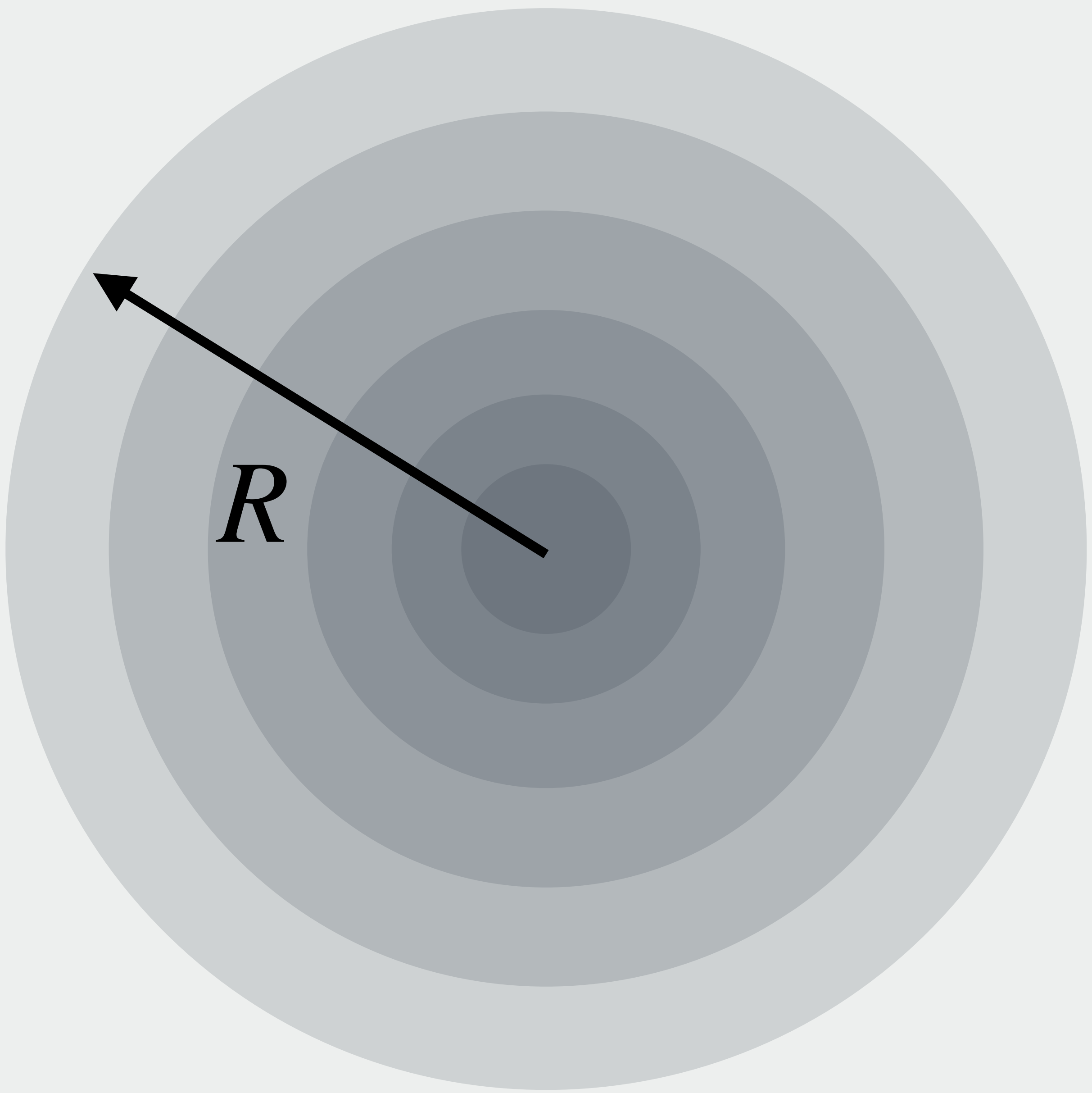
Disk shocking

Two sources of tidal stripping are considered and impact on the probability distribution

$$P_{\mathrm{m}}(k,z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{\mathrm{m},0}H_0^2}T(k)\right]^2 \mathcal{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$ *(power spectrum of density fluctuations)*

$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_{\mathrm{m}}(k, z=0)k^2\mathrm{dk}$$ *(smoothed variance)*

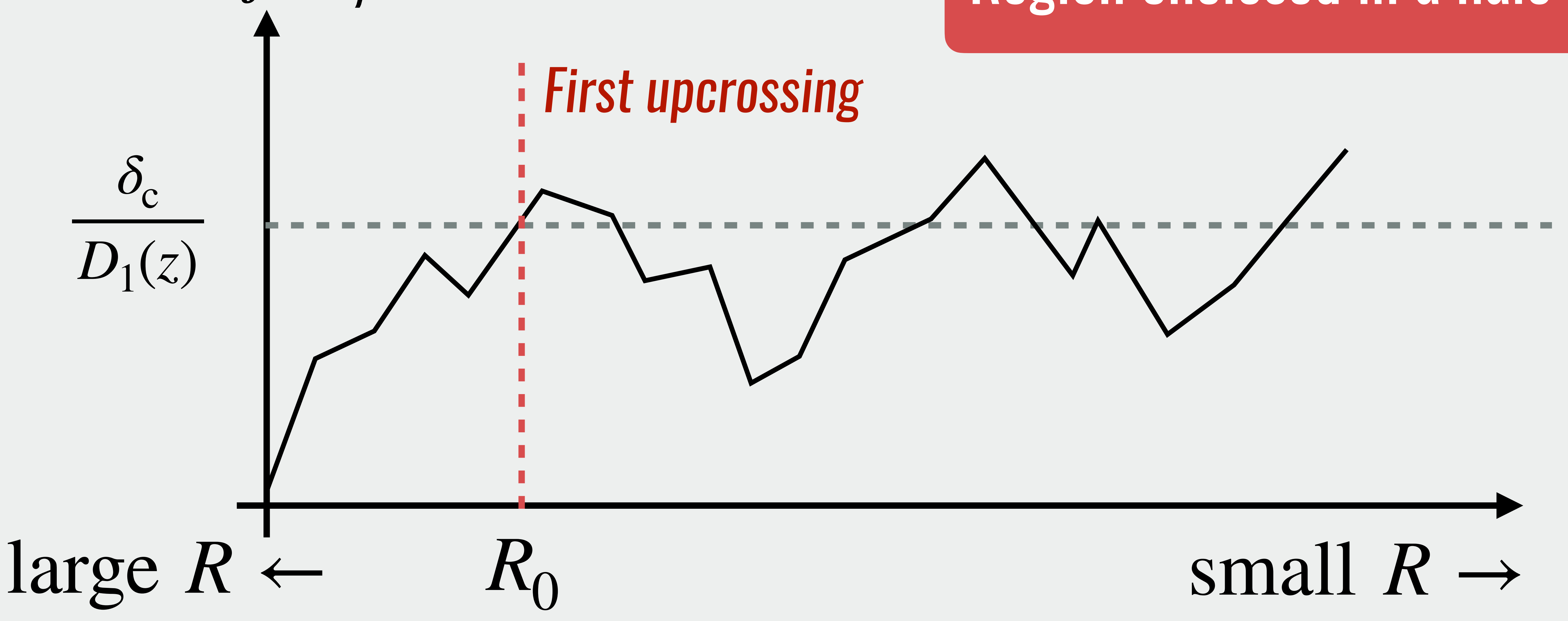
[Bond+91]



(smoothed density contrast)

$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y}\frac{\delta\rho}{\bar{\rho}}W_R(|\,\mathbf{x}-\mathbf{y}\,|)$$

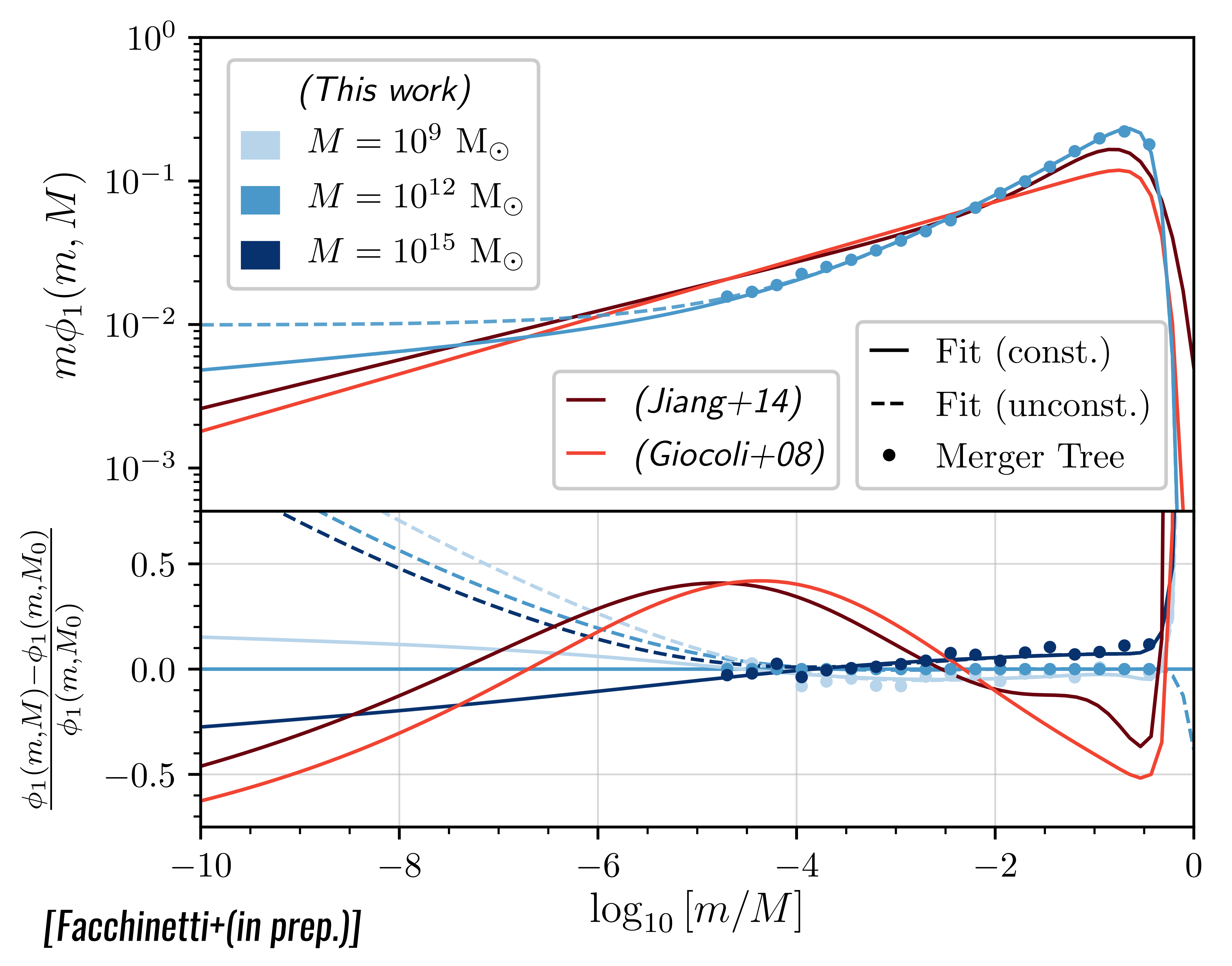
Region enclosed in a halo of size R_0

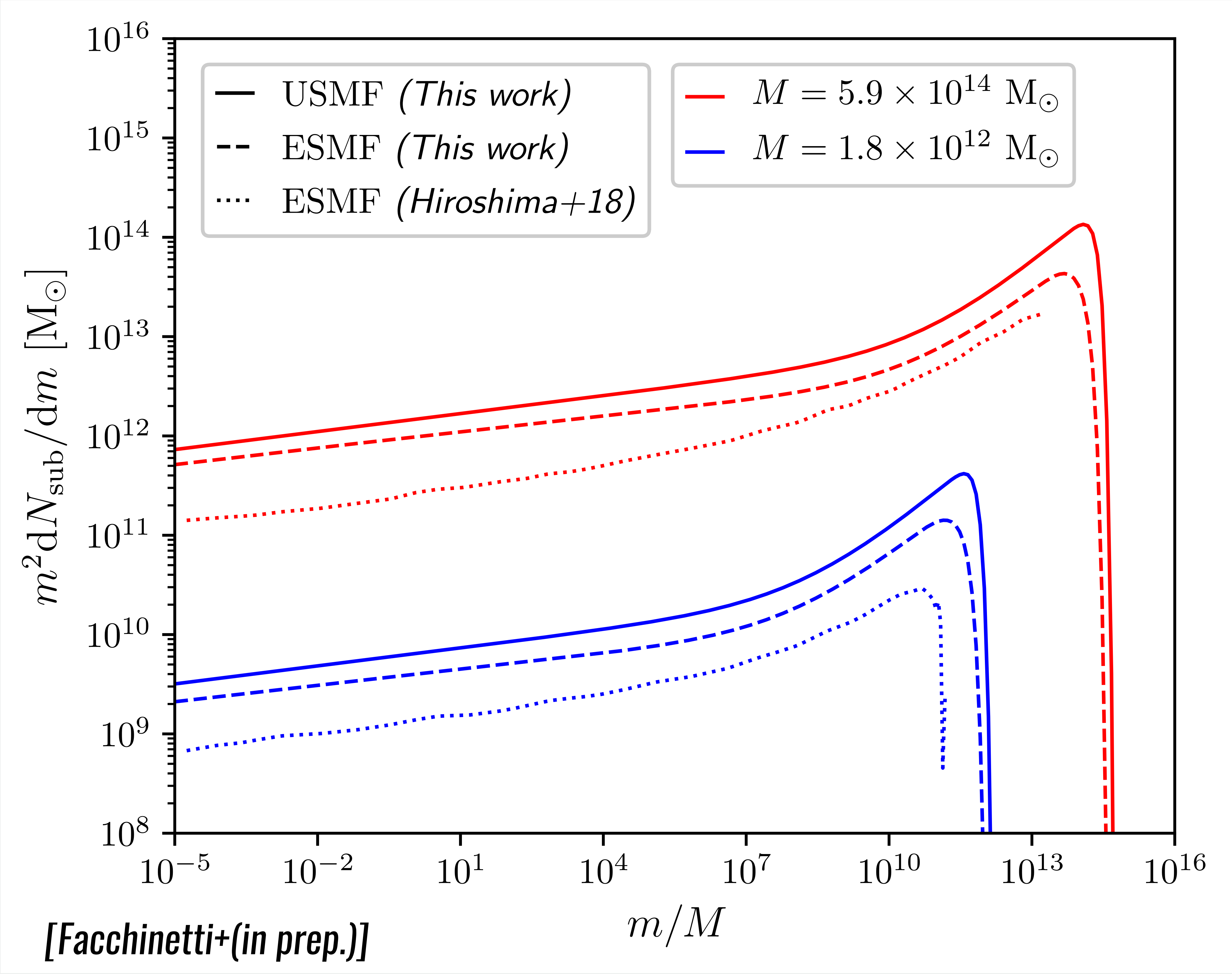


Fraction of mass in halos between M and M+dM

$$f(M)\left|\frac{\mathrm{d}S}{\mathrm{d}M}\right|\mathrm{d}M = \frac{\delta_c}{\sqrt{2\pi}S^{3/2}}\exp\left(-\frac{\delta_c}{2S}\right)\left|\frac{\mathrm{d}S}{\mathrm{d}M}\right|\mathrm{d}M$$

From the excursion set theory to merger trees





New calibration method

Let us finish part I with a small computation (preliminary)

Assume self-similarity

$$\frac{\partial N_p(m,M)}{\partial m} = \int_0^M \frac{\partial N_1(m,m')}{\partial m}\frac{\partial N_{p-1}(m',M)}{\partial m'}\mathrm{d}m' \qquad \frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m\mathrm{d}m = 1$$

Define the total mass function

$$\frac{\partial N_{\mathrm{tot}}(m,M)}{\partial m} = \sum_{p=0}^{\infty}\frac{\partial N_p(m,M)}{\partial m}$$

$$\frac{\partial N_{\mathrm{tot}}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m}\frac{\partial N_{\mathrm{tot}}(m',M)}{\partial m'}\mathrm{d}m'$$

Start with

$$\frac{\partial N_{\text{tot}}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m} \frac{\partial N_{\text{tot}}(m',M)}{\partial m'} \mathrm{d}m' \qquad \frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m \mathrm{d}m = 1$$

Change of variables
Assuming universality

$$\frac{\partial N_p(m,M)}{\partial m} = \frac{1}{m} g_p\left(-\ln\left(\frac{m}{M}\right)\right)$$

$$g_{\text{tot}}(x) = g_1(x) + \int_0^x g_1(y) g_{\text{tot}}(y-x)\mathrm{d}y \qquad \int_0^\infty g_p(x) e^{-x}\mathrm{d}x = 1$$

Laplace transform

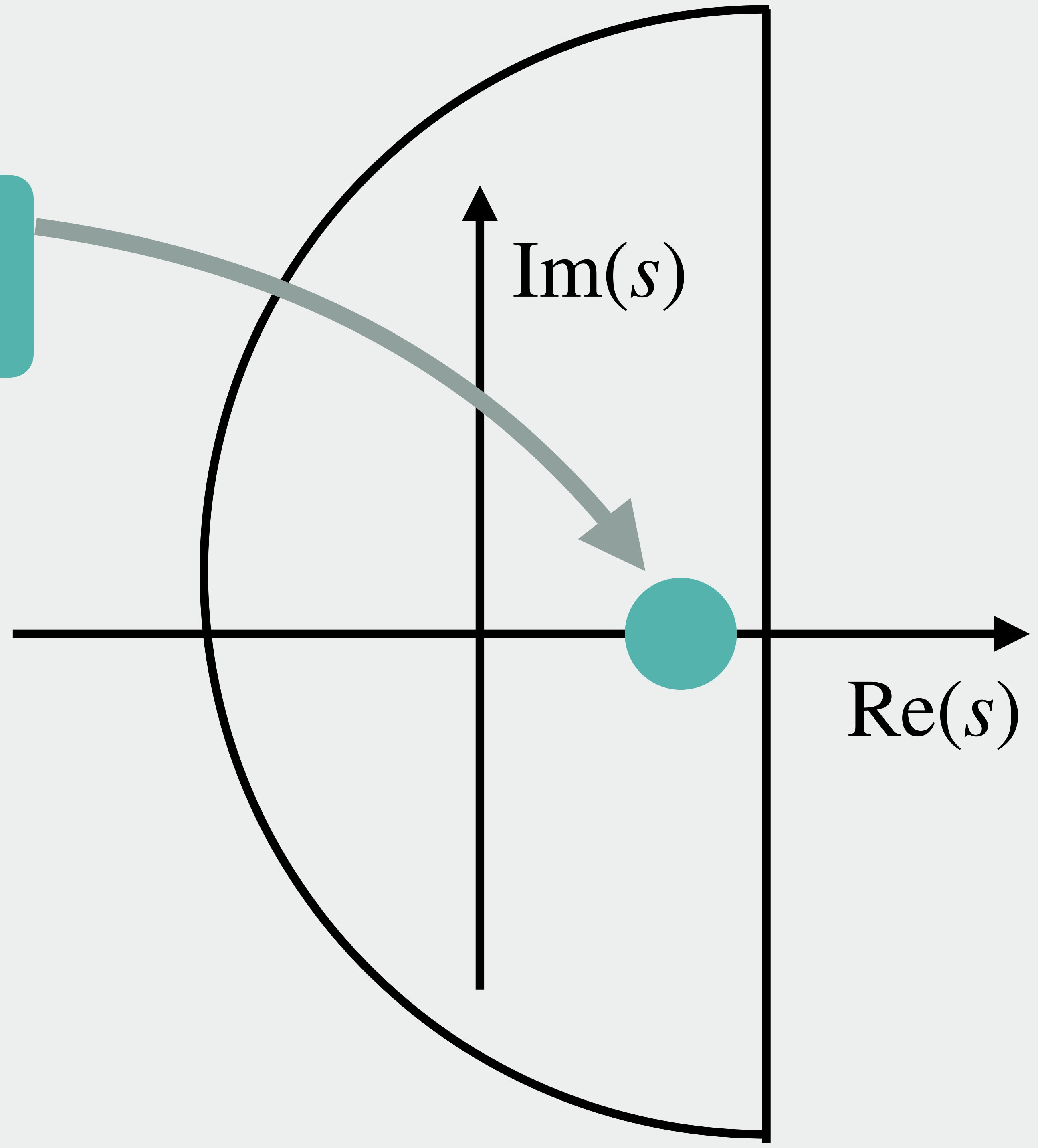
$$\hat{g}_p(s) \equiv \int_{[0,\infty[} g_p(x) e^{-sx} \mathrm{d}x$$

$$\hat{g}_{\text{tot}}(s) = \frac{\hat{g}_1(s)}{1-\hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Start with

$$\hat{g}_{tot}(s) = \frac{\hat{g}_1(s)}{1 - \hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Pole in s=1



Use residue theorem (assuming we can)

$$g_{tot}(x) = \sum_{i=0}^{n_{res}} c_i e^{s_i x} \qquad c_i \equiv \mathrm{Res}(\hat{g}_{tot}, s_i)$$

With the residue in s=1

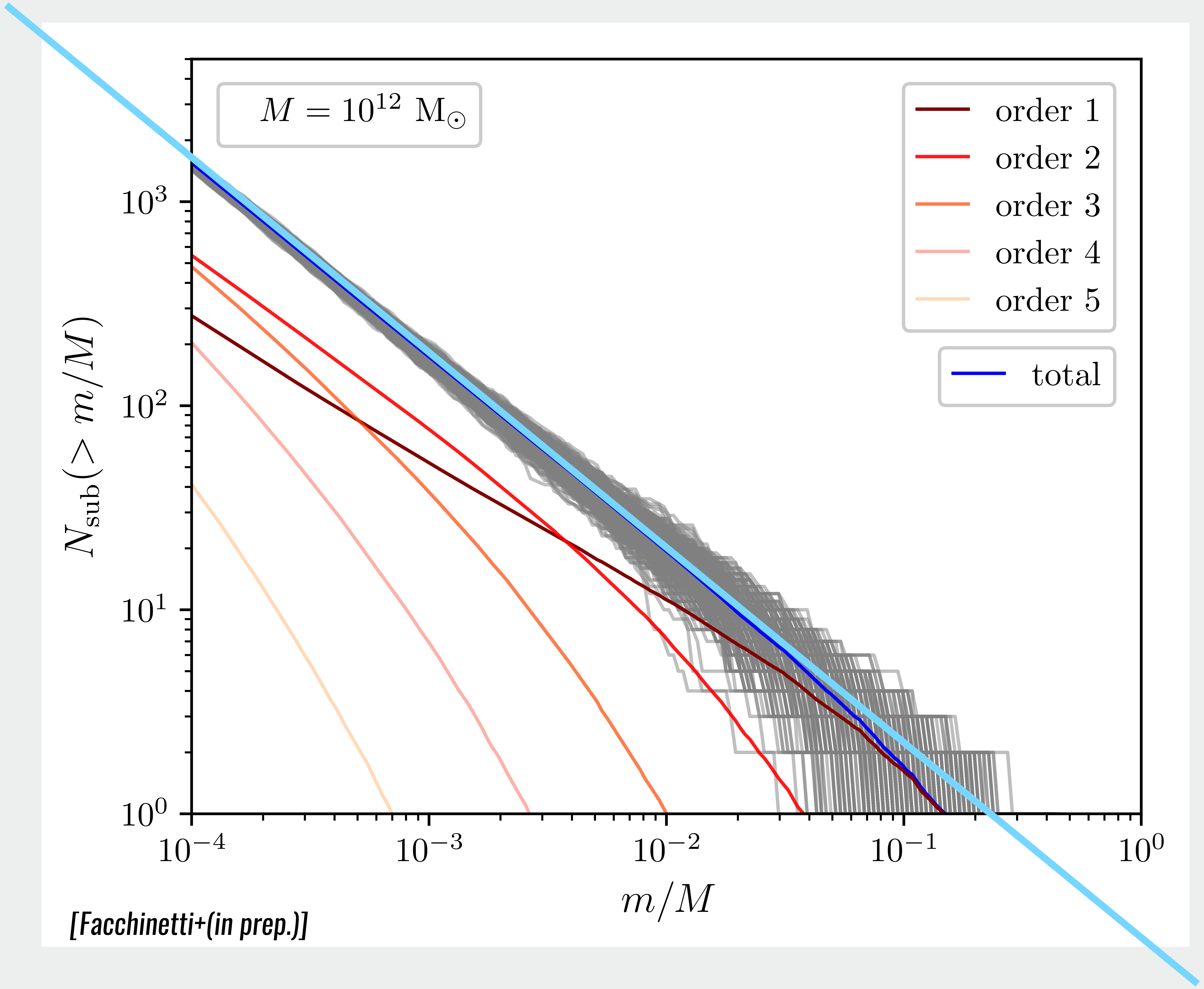
$$c_0 = \frac{1}{\hat{g}_1'(1)} \qquad s_0 = 1$$

$$g_{tot}(x) = \frac{1}{\hat{g}_1'(1)} e^x + \sum_{i>0} c_i e^{s_i x}$$

$$\frac{\partial N_{tot}(m, M)}{\partial m} = \frac{M}{\hat{g}_1'(1)} m^{-2} + \sum_{i>0} \frac{c_i}{m} \left(\frac{m}{M} \right)^{-s_i}$$

-2 is a critical exponent

$$\frac{\partial N_{tot}(m, M)}{\partial m} \underset{\sim}{\propto} m^{-2} \quad \text{if} \quad \mathrm{Re}(s_i) \ll 1 \ \forall i > 0$$



[Facchinetti+(in prep.)]

Merger Trees Monte Carlo results