# Krein space regularization of IR-UV divergences in de Sitter space

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## Divergences in QFT- negative norm states

- The appearance of singularities (IR-UV divergences) in QFT is a manifestation of the existence of anomalies in this theory.

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- In 1950, Gupta applied the idea of indefinite metric quantization to the QED for obtaining a covariant formalism [3].

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## Divergences in QFT- quantum metric fluctuation

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- The negative energy solutions of the field equations are discarded for avoiding negative probability states, but then the symmetrical properties of the field solutions are broken as was mentioned by Dirac. This fact can be easily seen in the quantization of the massless minimally coupled scalar field in de Sitter space-time.


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## mmc scalar field

- The “massless” minimally coupled (mmc) field in de Sitter space obeys

$$\Box_H \Phi_{\text{mmc}} = 0 = Q_0 \Phi_{\text{mmc}}$$

where $\Box_H$ is the Laplace-Beltrami operator on dS space-time

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- As proved by Allen, the covariant canonical quantization procedure with positive norm states must fail in this case [1]. The Hilbert space generated by any complete set of positive norme modes is not de Sitter invariant,

$$\mathscr{H} = \{\sum_k \alpha_k \phi_k ; \sum_k |\alpha_k|^2 < \infty\},$$

  where $k = \{(L, l, m) \in \mathbb{N} \times \mathbb{N} \times \mathbb{Z};\ 0 \leq l \leq L,\ -l \leq m \leq l\}$.

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- This means that the positive norme states are not closed under the action of the de Sitter group generators.

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## Krein space quantization

- The appearance of infrared divergence and breaking of de Sitter invarient:

$$\mathscr{W}_p(x,x') = \frac{H^2}{8\pi^2}[\frac{1}{1-\mathscr{Z}} - \ln(1-\mathscr{Z}) + \ln 2 + f_{AB}(\eta,\eta')], \quad (1)$$

where $f_{AB}(\eta,\eta')$ is a function of the conformal time [1].

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- Nevertheless, Gazeau et al obtained a fully covariant quantum field by adopting a new construction [1]. They added all the modes $L < -1$ or negative norm states:

- The field operator in Krein space quantization is:

$$\phi(x) = \frac{1}{\sqrt{2}}\left[\phi_p(x) - \phi_n(x)\right],$$

  where

$$\phi_p(x) = \sum_k a_k \phi_k(x) + H.C., \;\; \phi_n(x) = \sum_k b_k \phi^*(x) + H.C..$$

  The positive mode $\phi_p(x)$ is the scalar field as was used by Allen.

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## Two-point function

- Then we have the following comutation relations

$$a_k|0>=0,\ [a_k,a^\dagger_{k'}]=\delta_{kk'},\ b_k|0>=0,\ [b_k,b^\dagger_{k'}]=-\delta_{kk'},\ [a_k,b^\dagger_{k'}]=0\,.$$

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$$\mathscr{W}(x,x')=<0\mid\phi(x)\phi(x')\mid 0>=\frac{1}{2}[\mathscr{W}_p(x,x')+\mathscr{W}_n(x,x')],$$

where $\mathscr{W}_n(x,x')=-\mathscr{W}^*_p(x,x')$ and $\mathscr{W}_p(x,x')$ is the two-point function for the positive modes Equation (1).

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- The two-point function in Krein space quantization is [1]:
$$\mathscr{W}(x,x')=\frac{iH^2}{8\pi}\varepsilon(x^0-x'^0)[\delta(1-\mathscr{Z}(x,x'))-\theta(\mathscr{Z}(x,x')-1)],$$
where
$$\varepsilon(x^0-x'^0)=\begin{cases}1 & x^0>x'^0\\ 0 & x^0=x'^0\\ -1 & x^0<x'^0.\end{cases},$$
and $\theta$ is the Heaviside step function.
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## Generalization

- Therefore the theory or the two-point function is 1) de Sitter invariant, 2) free of any infrared divergence, and 3) only the delta function singularity exist and the other ultraviolet divergences disappear.

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- Does one can generalize Krein space quantization to the interaction case and flat space?

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- While the Krein space quantization is applied in a rigorous mathematical way to the free field theory, the description of an interaction field theory in terms of the Krein-space quantization approach remains an open mathematical question [1]. It was proved that the negative norm states disappear in the one-loop approximation.

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- While the Krein space quantization is applied in a rigorous mathematical way to the free field theory, the description of an interaction field theory in terms of the Krein-space quantization approach remains an open mathematical question [1]. It was proved that the negative norm states disappear in the one-loop approximation.
- We can use the Krein space quantization including quantum metric fluctuation as a new method of quantum field regularisation whereas this property holds at all orders of perturbation theory [2].
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## Green function

- The singularity in QFT appears due to the coincident points and also the multiplication of the Feynman Green functions [1]:

$$G_F^p(x,x') = -\mathrm{i} < 0 \mid T\phi_p(x)\phi_p(x') \mid 0 >= \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{e^{-\mathrm{i}k\cdot(x-x')}}{k^2 - m^2 + \mathrm{i}\varepsilon}$$

$$= -\frac{1}{8\pi}\delta(\sigma) + \frac{m^2}{8\pi}\theta(\sigma)\frac{J_1\left(\sqrt{2m^2\sigma}\right) - \mathrm{i}N_1\left(\sqrt{2m^2\sigma}\right)}{\sqrt{2m^2\sigma}} - \frac{\mathrm{i}m^2}{4\pi^2}\theta(-\sigma)\frac{K_1\left(\sqrt{-2m^2\sigma}\right)}{\sqrt{-2m^2\sigma}}, \quad (2)$$

where $2\sigma = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$ is the square of geodesic distance.

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- The time-ordered product propagator in Krein space quantization is [2]:

$$G_T(x,x') = \int \frac{\mathrm{d}^4k}{(2\pi)^4} e^{-\mathrm{i}k\cdot(x-x')} \mathscr{P}\frac{1}{k^2-m^2} = -\frac{1}{8\pi}\delta(\sigma) + \frac{m^2}{8\pi}\theta(\sigma)\frac{J_1\left(\sqrt{2m^2\sigma}\right)}{\sqrt{2m^2\sigma}},$$

  where $\mathscr{P}$ is principal part symbol.

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- Due to the quantum metric fluctuation a unique and definite light cone does not exist and then the Dirac delta singularity disappears.
- We are primarily interested in the behavior of functions near the perturbed light cone:

$$G_F(x-x':m=0)=\frac{-1}{8\pi}\delta(\sigma)=\frac{-1}{16\pi^2}\int_{-\infty}^{\infty}\mathrm{d}\alpha e^{\mathrm{i}\alpha\sigma_0}e^{\mathrm{i}\alpha\sigma_1}.$$

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- In the presence of the perturbation $h_{\mu\nu}$, ($g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$) we have $2\sigma=g_{\mu\nu}(x^\mu-x'^\mu)(x^\nu-x'^\nu)$, where $\sigma=\sigma_0+\sigma_1+O(h^2)$. $\sigma_1$ is the first-order shift in $\sigma$ (an operator in the linear quantum gravity).

## KSQ and QMF

- By taking the average over the quantum metric fluctuations, the Green function is replaced by its quantum expectation value [1]:

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This integral converges only if $\langle\sigma_1^2\rangle$ is greater than zero ($\langle\sigma_1^2\rangle > 0$).

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  This integral converges only if $\langle\sigma_1^2\rangle$ is greater than zero ($\langle\sigma_1^2\rangle > 0$).

- Then by considering the QFT in Krein space quantization with quantum metric fluctuations included, all singular behaviors of the free scalar Green functions are completely removed:

$$\langle G_T(x-x')\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1\left(\sqrt{2m^2\sigma_0}\right)}{\sqrt{2m^2\sigma_0}}. \quad (3)$$

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- In the case of $\sigma_0 = 0$, due to the quantum metric fluctuation, $h_{\mu\nu}$, $\langle\sigma_1^2\rangle \neq 0$, and we obtain:

$$\langle G_T(0)\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}} + \frac{m^2}{8\pi}\frac{1}{2}.$$

  It should be noted that $\langle\sigma_1^2\rangle$ is related to the density of gravitons [1].

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## Regularization method

- The singularity in QFT appears due to the coincident points and also the multiplication of the Feynman Green functions:

$$G_F^p(x_i, x_i)\,,\ \left[G_F^p(x_i, x_j)\right]^n\,,\ \cdots.$$

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- Krein regularization method can be completed in two simple steps:
- (a) *Replacing the Feynman Green functions (2) with the time-ordered product propagator (3).*
- (b) *Using the same renormalization conditions as the usual method.*
- From step (a), it is clear that the theory is completely finite and there is no appearance of any singularity since the two-point function is finite and free of any divergences.
- The other step (b) guarantees that the physical result does not change. The effect of the negative norm states and quantum metric fluctuations for the internal lines is the elimination of the singularity.

## Renormalization

- For the $\lambda\phi^4$ theory, the classical Lagrangian density and $S$-matrix operator are given respectively by:

$$\mathscr{L}_c(\phi) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4, \quad S = Te^{\mathrm{i}\frac{\lambda}{4!}\int\phi^4(x)d^4x}.$$

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- The effective Lagrangian or "quantum Lagrangian" can be established through the loop correction to the classical Lagrangian:

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$$\mathscr{L}_q = \mathscr{L}_c + \hbar\mathscr{L}_1 + \hbar^2\mathscr{L}_2 + \cdots.$$

- The renormalization conditions are imposed through:

$$\lambda_\mu = -\left.\frac{\delta^4\mathscr{L}_q}{\delta\phi^4}\right|_{\phi=\mu}, \quad m_\mu^2 = -\left.\frac{\delta^2\mathscr{L}_q}{\delta\phi^2}\right|_{\phi=\mu}, \quad 1 = \left.\frac{\delta}{\delta\partial^\nu\phi}\frac{\delta\mathscr{L}_q}{\delta\partial_\nu\phi}\right|_{\phi=\mu}. \tag{4}$$

The parameter $\mu$ is the energy scale according to which the mass and the coupling constant are measured.

## Beta function

- $\lambda_\mu$ is a function of a parameter $\mu$ which is the energy scale of the interaction [1]:

$$\lambda_\mu = \lambda - \frac{\lambda^2}{(8\pi)^2}\left[6\ln\frac{\mu^2}{m^2} + 19 + 12\ln 2\right] + O(\lambda^3).$$

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- For calculating the running coupling constant a scale of energy must be chosen as $\phi = \mu = e^{-t}$. Then the running coupling constant is defined as $\bar{\lambda}(t,\lambda)$.
- The Beta function can be calculated as well [1]:

$$\beta = \frac{d\bar{\lambda}(t,\lambda)}{dt} = \frac{3\lambda^2}{16\pi^2},$$

which is the same as in the usual results in the one-loop approximation.

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## Conclusion

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- The problem of non-renormalizability of linear quantum gravity can be solved and then **the linear quantum gravity is renormalizable in KSQ**.
- We now have all of the building blocks needed to build a unitary super-gravity in de Sitter space-time.

**Thank you**