Gravitational Causality

and the photon self-stress

2108.05896

[B. Bellazzini, GI, M. Lewandowski, F. Sgarlata]

Giulia Isabella

IPhT, CEA

14 December 2021

Introduction

Causality in gravity is not well understood

S- matrix

Introduction

Causality in gravity is not well understood

S- matrix

Dominated by gravity

$$GE_{CM}^2 = Gs \gg 1$$ ⟷ Transplanckian scattering

Large impact parameter

$$b$$

Introduction

Causality in gravity is not well understood

$\Leftrightarrow$

S- matrix

Amati, Ciafaloni, Veneziano
90's

Transplanckian
eikonal approximation

Dominated by gravity

$$GE_{CM}^2 = Gs \gg 1$$ $\Leftrightarrow$ Transplanckian scattering

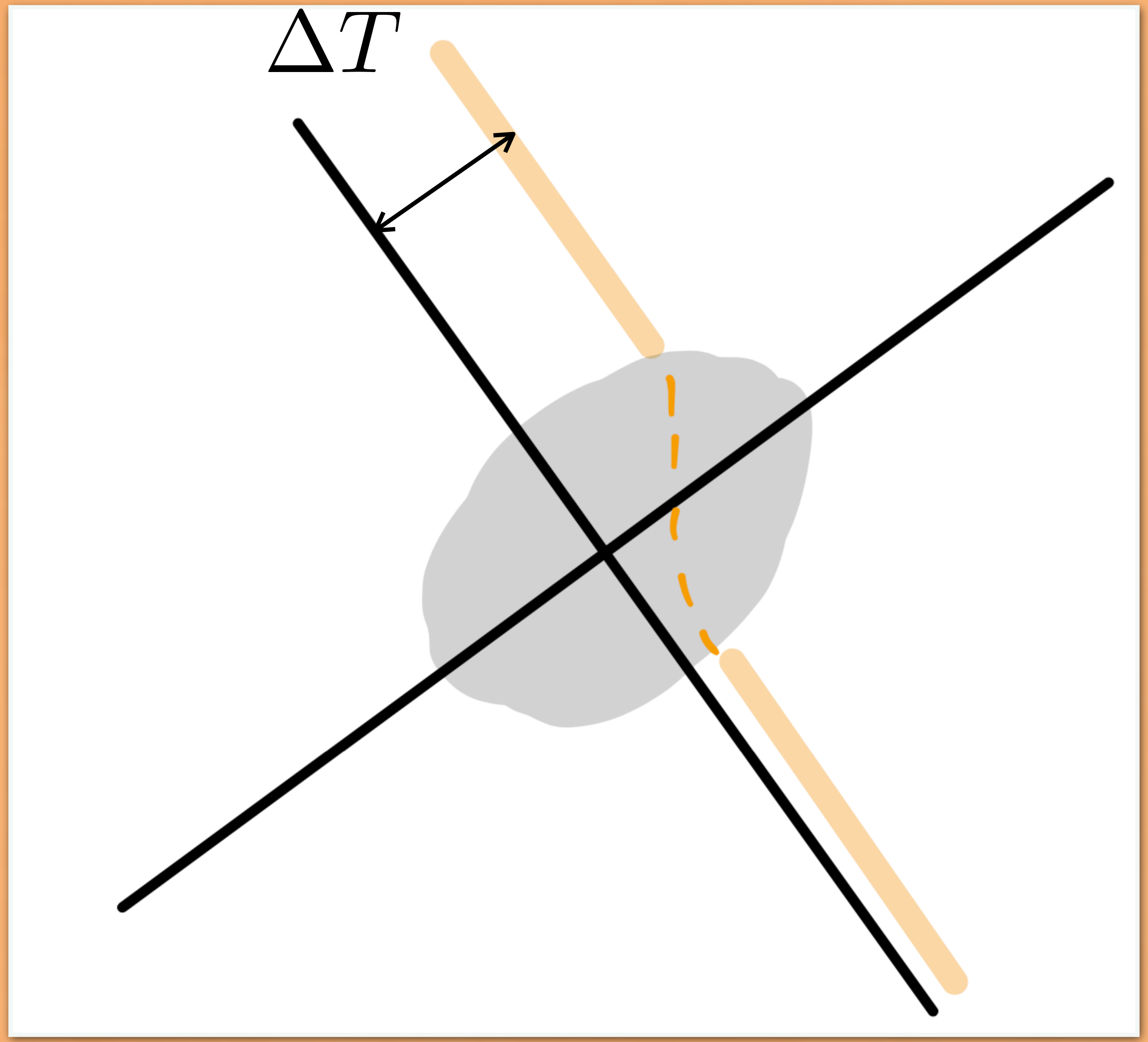
Large impact parameter

$$b$$

$$S_{eik}(s, \vec{b}) = e^{2i\delta(s, \vec{b})}$$

$$\delta(s, \vec{b}) = \frac{1}{4s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \mathcal{M}_{eik}(s, \vec{q}_\perp)$$

Asymptotic Causality



Wald & Gao
gr-qc/0007021

$$\Delta T = \frac{\partial \delta(s, \vec{b})}{\partial E} \geq 0$$

For all species

Resolvability

$$|\Delta T| \gg 1/E \quad \rightarrow \quad \delta \gg 1$$

Transplanckian scattering

A tale of scales

What regime are we looking at?

Kinematic scales

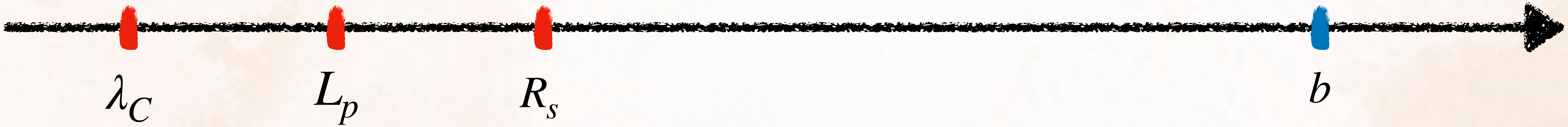
$$\lambda_C = \frac{1}{\sqrt{s}} \quad , b$$

Planck length

$$L_p \sim \frac{1}{M_{Pl}}$$

Schwarzschild Radius

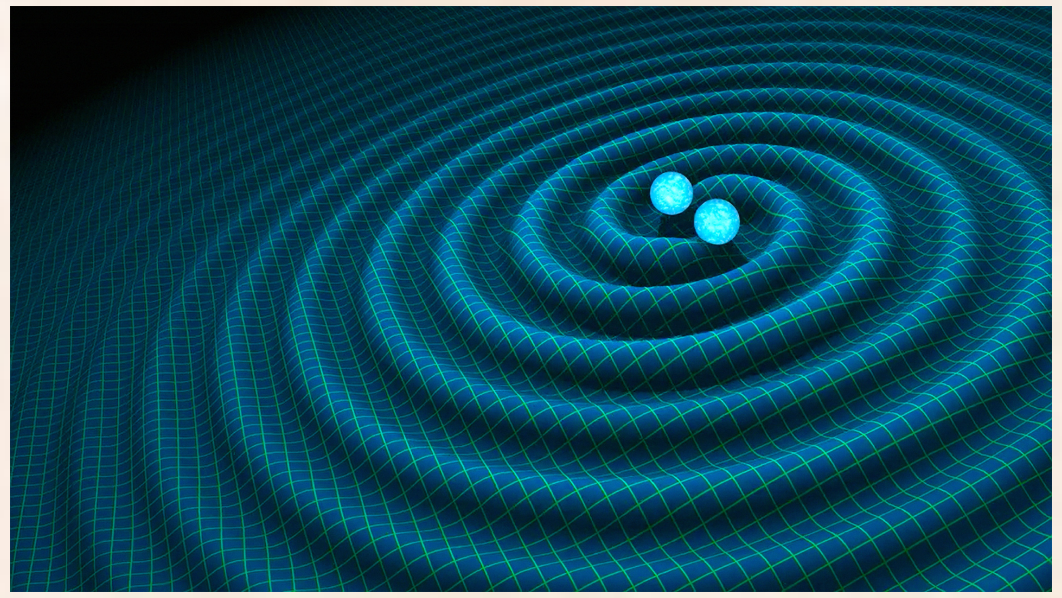
$$R_s = 2G\sqrt{s} = \frac{\sqrt{s}}{M_{Pl}} L_p$$



$$S_{eik}(s, \vec{b}) = e^{2i(\delta_0(s,\vec{b})+\delta_1(s,\vec{b})+\delta_2(s,\vec{b})+\ldots)}$$

Dimensionless small parameters

$$\left(\frac{R_s}{b}\right)^n \quad \left(\frac{L_p}{b}\right)^n$$

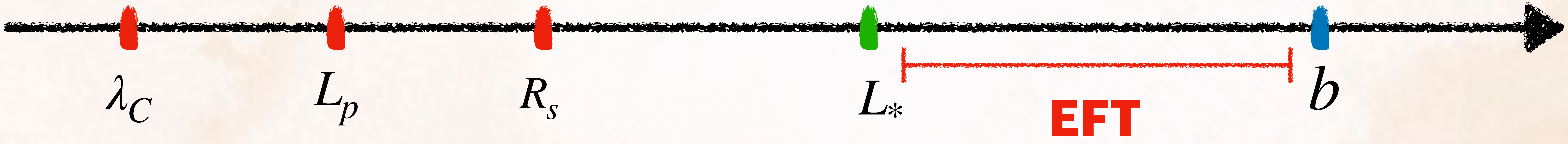


A tale of scales

What about new physics?

New d.o.f.

$$L_* \sim \frac{1}{M_*}$$



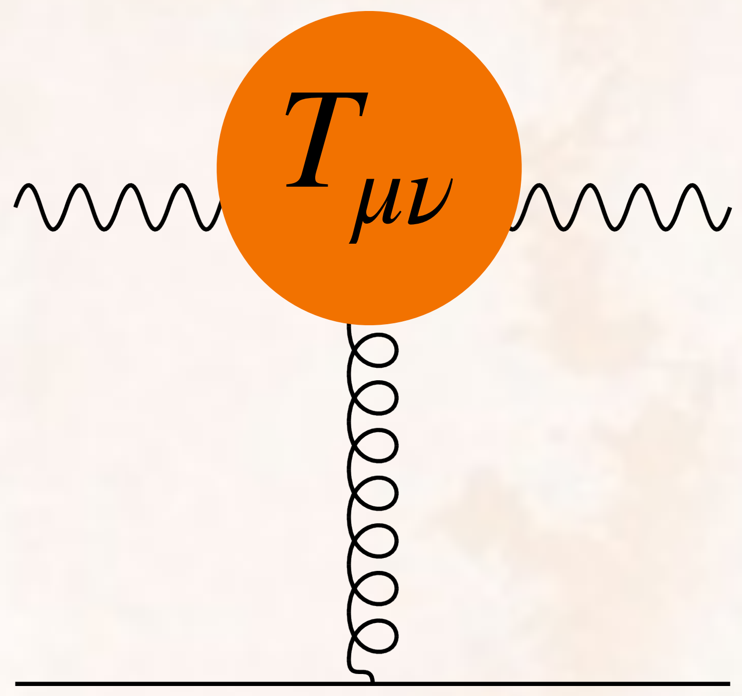
$$S_{eik}(s, \vec{b}) = e^{2i(\delta_0(s,\vec{b})+\delta_1(s,\vec{b})+\delta_2(s,\vec{b})+\dots)}$$

Dimensionless small parameters

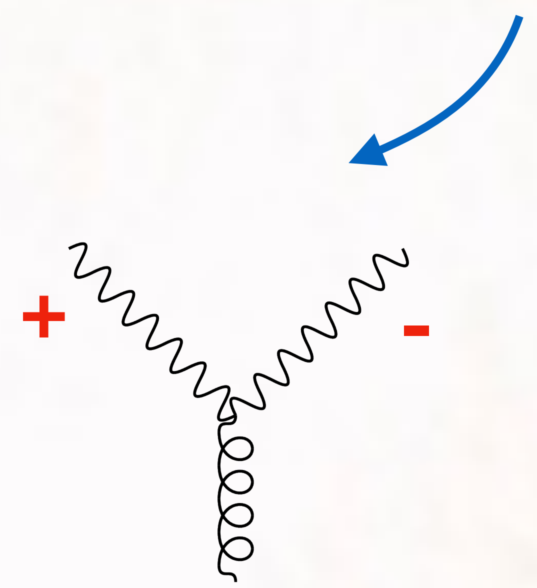
$$\left(\frac{R_s}{b}\right)^n \quad \left(\frac{L_p}{b}\right)^n \quad \left(\frac{L_*}{b}\right)^n$$

New physics effects

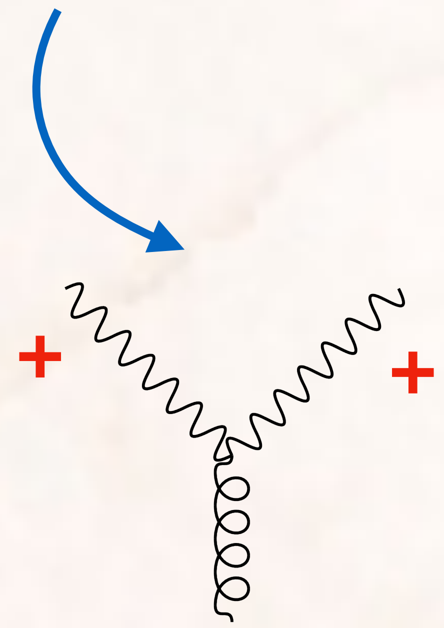
Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597



$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}^2 + \alpha_3 F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}\right]$$



Helicity preserving

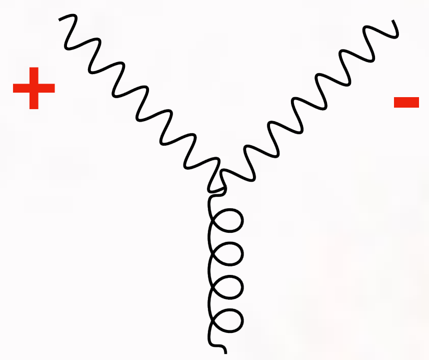


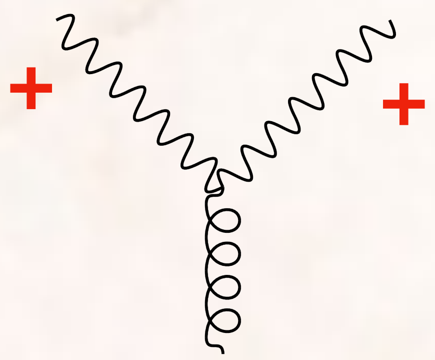
Helicity flipping

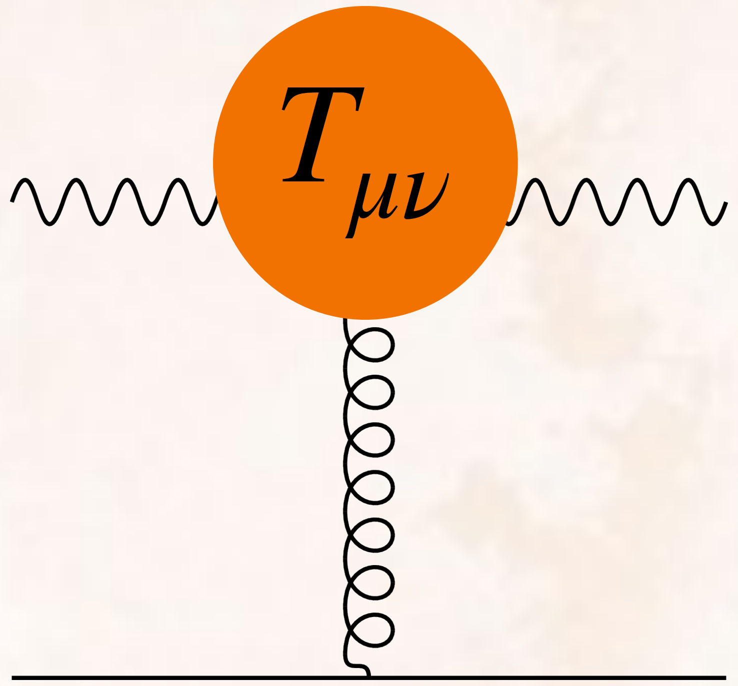
New physics effects

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}^2 + \alpha_3 F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}\right]$$







\+ -   + +

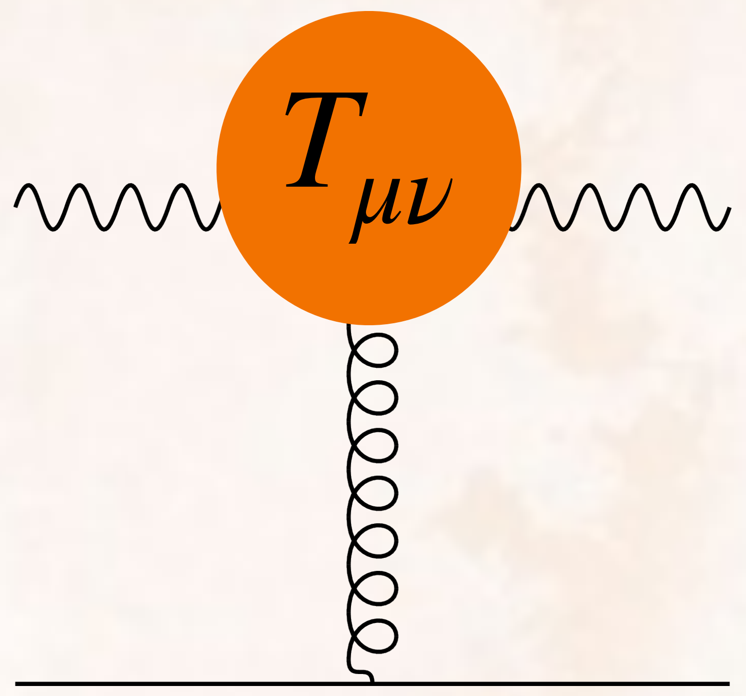
$$\mathscr{M}_{eik} = \frac{s^2}{M_{pl}^2 t}\begin{pmatrix} 1 & -4\alpha q_+^2 \\ -4\alpha q_-^2 & 1 \end{pmatrix}$$

$$q_\pm \propto q_1 \pm i q_2$$

New physics effects

Camanho, Edelstein, Maldacena and Zhiboedov 1407.5597

$$\delta(s, \vec{b}) = \frac{1}{4s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \mathcal{M}_{eik}(s, \vec{q}_\perp)$$



+ -   + +

$$\mathcal{M}_{eik} = \frac{s^2}{M_{pl}^2 t} \begin{pmatrix} 1 & -4\alpha q_+^2 \\ -4\alpha q_-^2 & 1 \end{pmatrix}$$

$q_\pm \propto q_1 \pm i q_2$

$$\delta_\pm(s, \vec{b}) = Gs \left(-\log \frac{b}{b_{IR}} \pm \frac{\alpha}{b^2} \right)$$

It exists an impact parameter b such that asymptotic causality is violated*

What have we learnt?

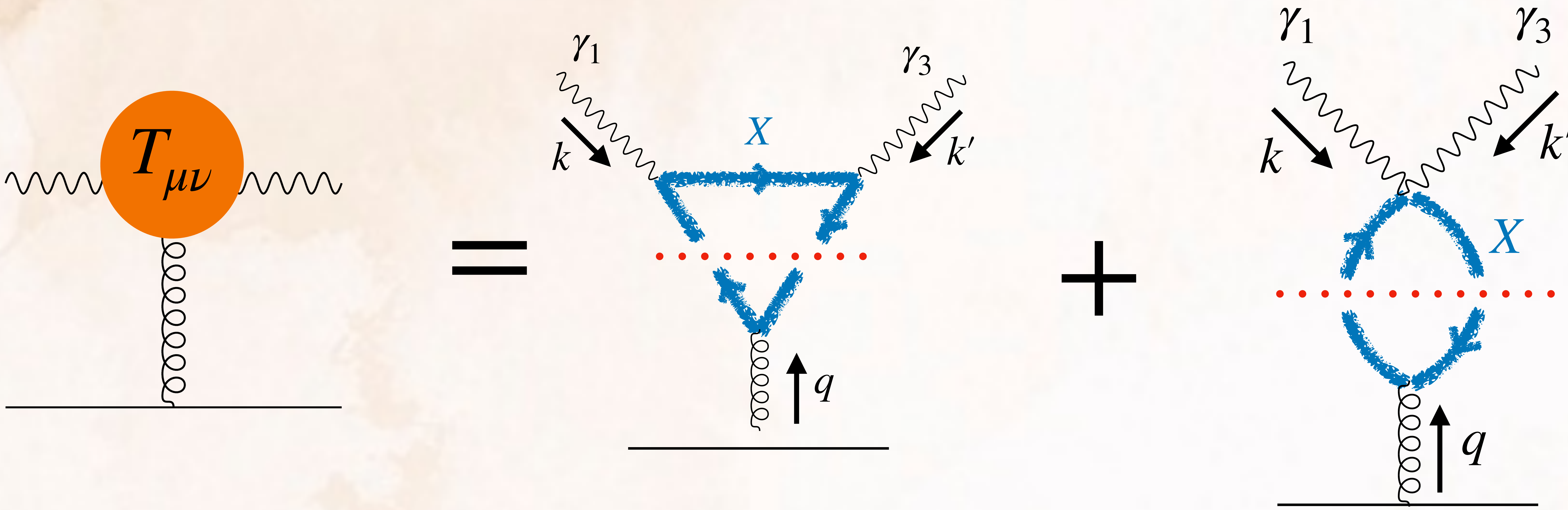
Asymptotic causality correctly predicts the breakdown of the theory

→ **EFT**

(1407.5597) shows that tree-level solution to causality issue requires tower of higher spin

How things change with loops ?

A loop solution to causality problems ?

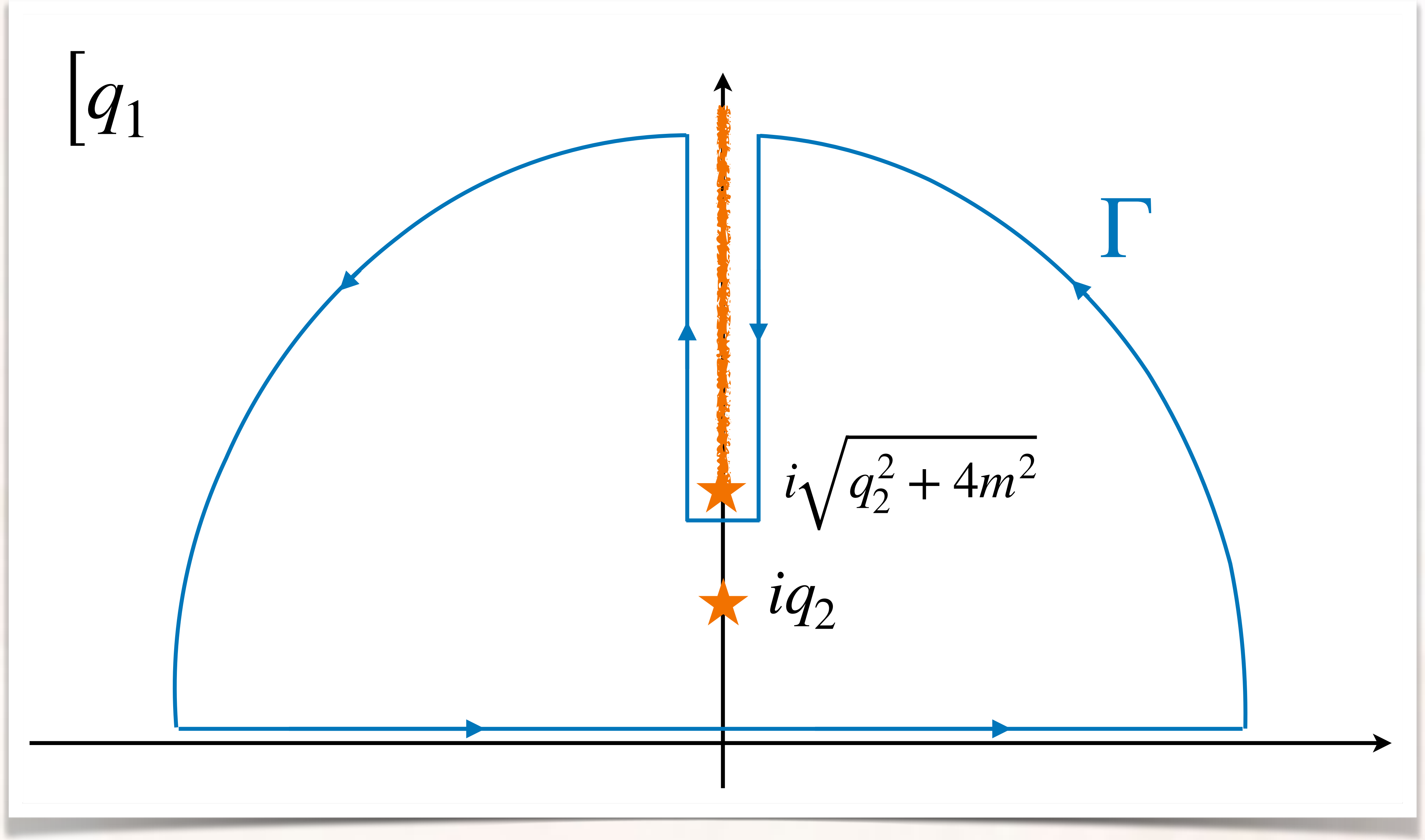


$$\rightarrow \quad \mathscr{M}_{eik} = \frac{s^2}{M_{pl}^2 t} \begin{pmatrix} F_1(t) & -4q_+^2 F_3(t) \\ -4q_-^2 F_3(t) & F_1(t) \end{pmatrix}$$

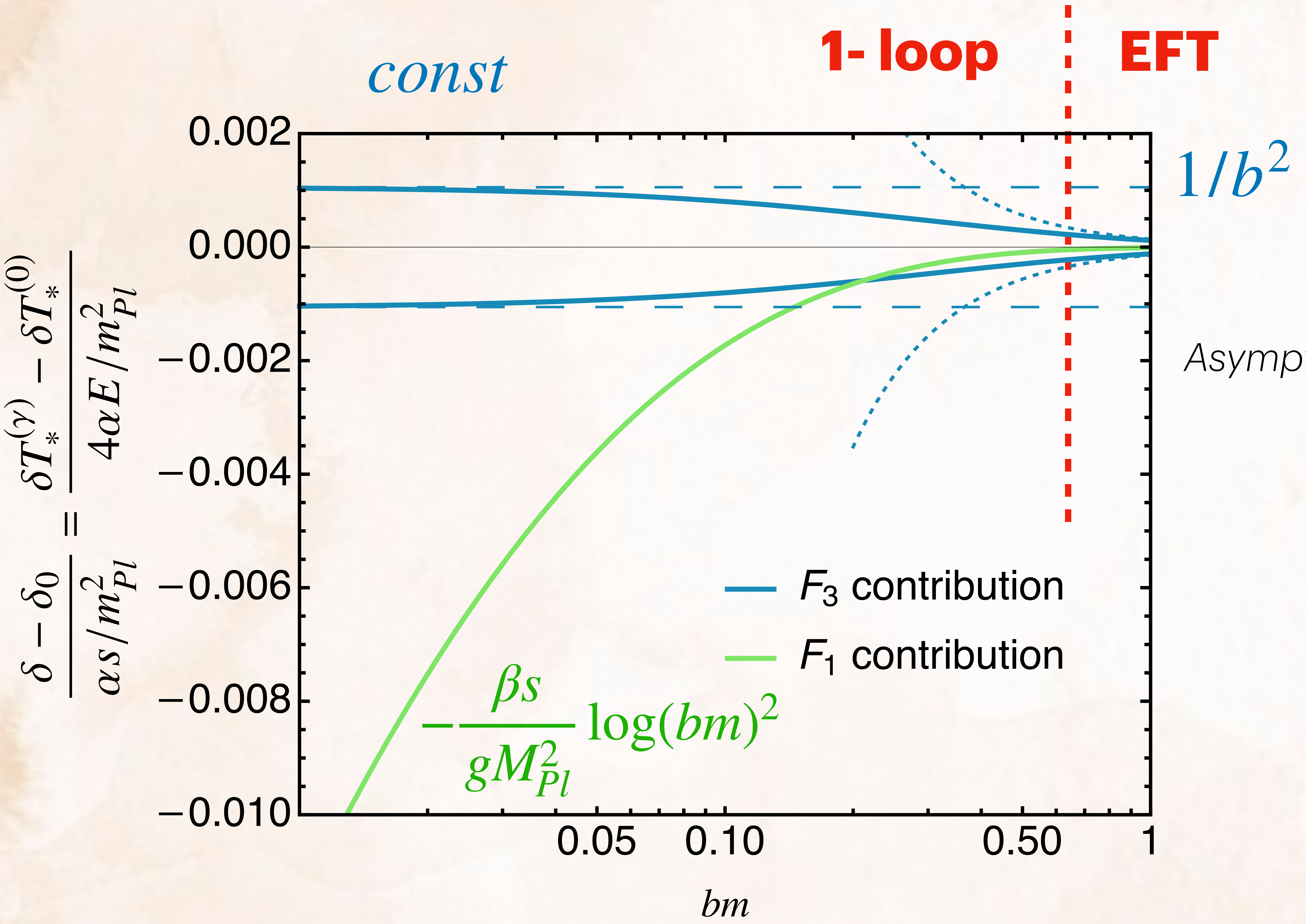
$q_\pm \propto q_1 \pm i q_2$

A loop solution to causality problems ?

$$\delta_\pm(b,s) = \frac{s}{4M_{Pl}^2}\left[\underbrace{-\frac{1}{2\pi}\left(F_1(0)\log\frac{b}{b_{IR}} \mp \frac{8}{b^2}F_3(0)\right)}_{\text{EFT}} + \underbrace{\frac{i}{(2\pi)^2}\int_{4m^2}^{\infty}\frac{dt}{t}\left(\text{Disc}F_1(t)K_0(b\sqrt{t}) \pm 4t\,\text{Disc}F_3(t)K_2(b\sqrt{t})\right)}_{\text{loop}}\right]$$



A loop solution to causality?



Asymptotic causality is only violated at

$$b_L \sim \frac{1}{m} e^{-g/\beta}$$

Conclusions

Asymptotic Causality (Wald & Gao)

$$\Delta T \geq 0$$ For all species

Seems to be a robust definition

- Correctly detects the breakdown of the theory both at tree and loop-level
- It does not predict the details of the UV completion

Conclusions

Asymptotic Causality (Wald & Gac

$$\Delta T \geq 0$$ For all sp

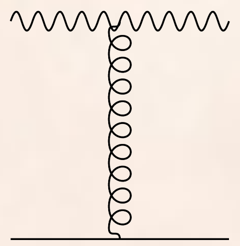
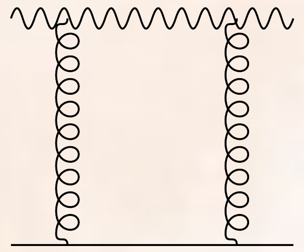
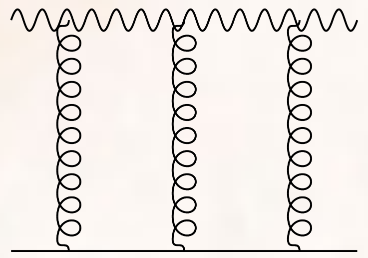
Seems to be a robust definition

- Correctly detects the breakdown of the theory both at tree and loop-level

- It does not predict the details of the UV completion

Backup

Exponentiation in the eikonal

$\mathcal{A}_{eik}(s,t) =$  $+$  $+$  $+ \ \dots$

$$= \frac{1}{2s}\mathcal{M}_0 \otimes \mathcal{M}_0(\vec{Q}) + \ldots = \frac{1}{2s}\sum \frac{1}{n!}\mathcal{M}_0 \otimes \mathcal{M}_0 \ldots \otimes \mathcal{M}_0(\vec{Q})$$

$$\mathcal{A}_{eik}(s,\vec{b}\,) = e^{2i\delta_0(s,\vec{b}\,)}$$

Born approximation

Phase-shift

$$\delta_0(s,\vec{b}\,) = \frac{1}{4s}\int \mathcal{M}_0\, e^{i\vec{Q}\cdot\vec{b}} = -\frac{s}{8\pi M_{Pl}^2}\log\left(\frac{b}{b_{IR}}\right)$$

$$\Delta T = 2\frac{\partial \delta(s,\vec{b}\,)}{\partial E}$$

Beta function

Form factors control the coupling's running

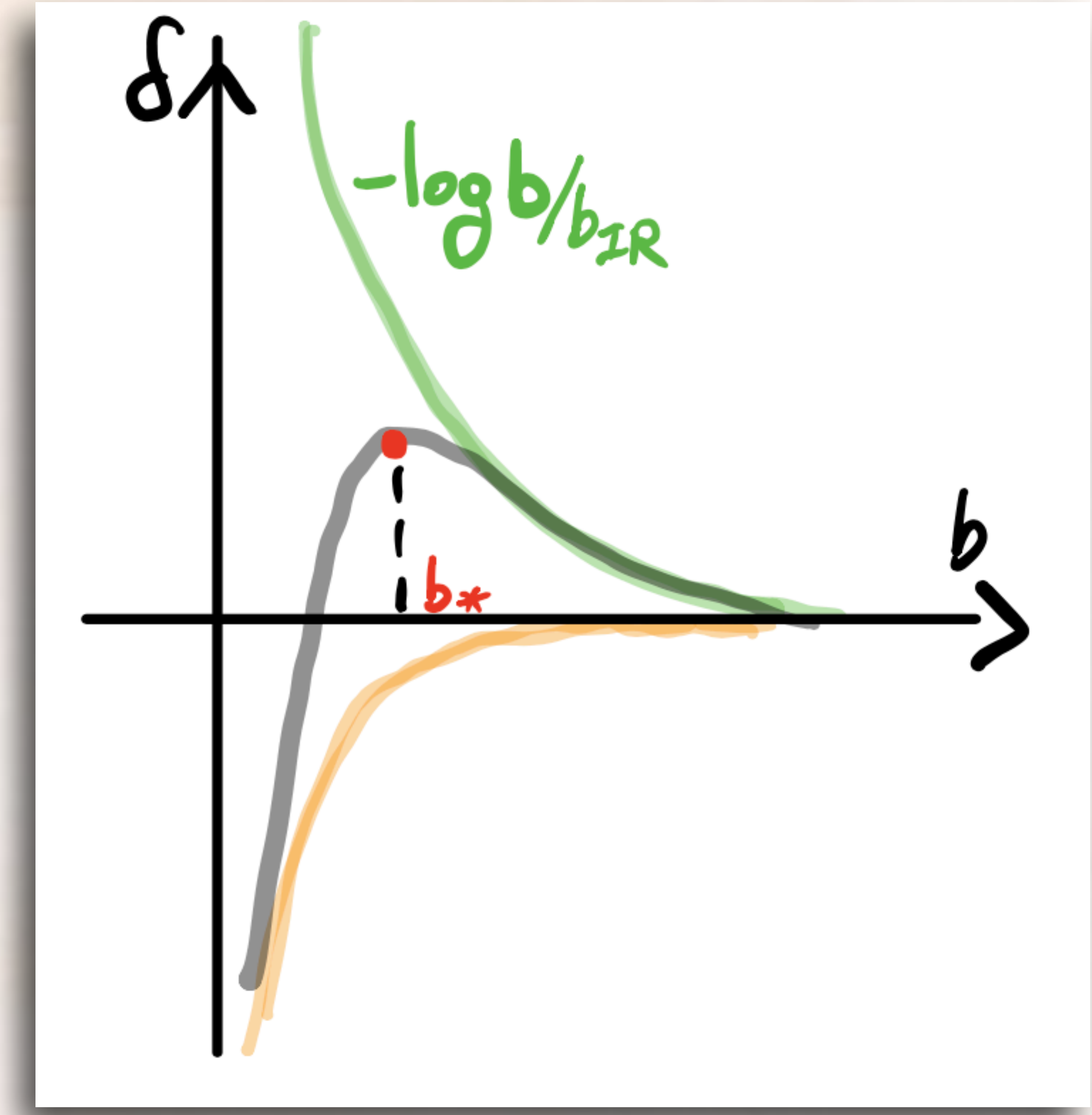
$$S \supset \int d^4x \sqrt{-g} \left(M_{Pl}^2 R - \frac{1}{g^2(\mu)} F_{\mu\nu} F^{\mu\nu} + \dots \right)$$

$$\supset \int d^4x \, h_{\mu\nu} \frac{1}{g^2(\mu)} < 0 | T^{\mu\nu} | \gamma\gamma >$$

$$\beta = -\frac{g}{2} \frac{d}{d\log\mu} F_1(t = -\mu^2) \Big|_{m \ll \mu} = \lim_{t/m^2 \to \infty} \frac{g}{\pi} \frac{\mathrm{Disc} F_1(t)}{2i}$$

IR cutoff

Form factors control the coupling's running



b^* is where gravity becomes repulsive, and it is independent of b_{IR}