Testing theories of gravity with asymptotic symmetries ?

Simone Speziale

TUG @IHP, 14-12-21

based on :

- *A note on dual gravitational charges* (arXiv:2010.01111)
- *The Weyl BMS group and Einstein's equations* (arXiv:2104.05793)
- *Extended corner symmetry, charge bracket and Einstein's equations* (2104.12881)

with Laurent Freidel, Roberto Oliveri and Daniele Pranzetti

Centre de Physique Théorique

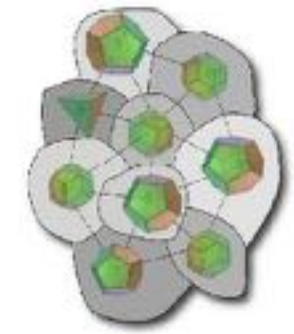
Outline:

- Flux-Balance laws for GW at scri can be derived from Noether's theorem

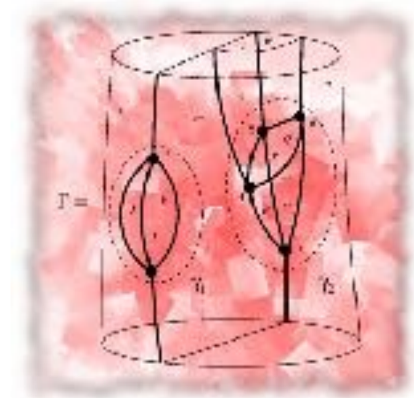
- This derivation gives further motivation to investigate enlargements of the asymptotic symmetry group of gravity

- These enlargements can be studied with covariant phase space methods, and could potentially lead to new observations

Relevance for Loop Quantum Gravity

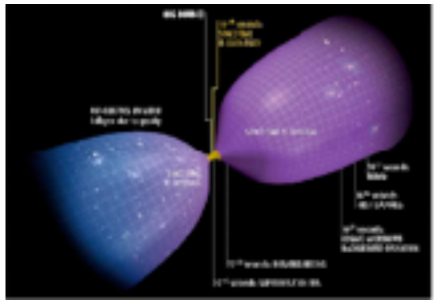


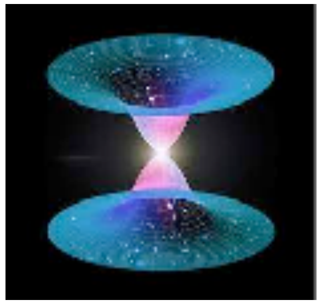
- Loop quantum gravity offers a consistent theory in which a notion of quantum spacetime can be precisely constructed, and includes non-commutativity and discreteness of geometrical quantities



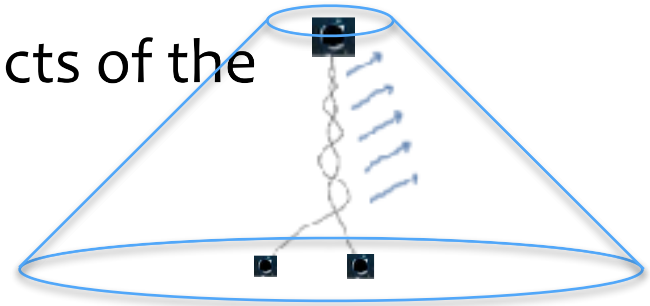
- Dynamical consequences of this discreteness could be derived in principle, but in practise we haven't been able to so far

- Symmetry reduced dynamics have shown great potential to deduce physical effects but they are model-dependent and not really falsifiable predictions of the theory





- The recent developments on a deeper understanding of the link between gravitational waves and asymptotic symmetries suggest that key aspects of the foundations of loop quantum gravity, like the choice of tetrad variables instead of the metric and the inclusion of a dual term in the action, can potentially be discriminated in future observations

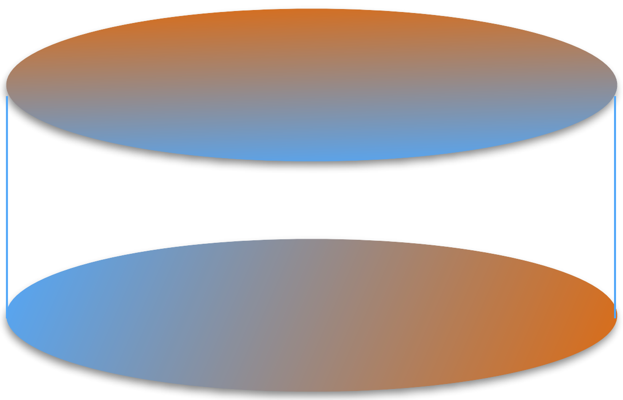


Noether's theorem

If a Lagrangian has a continuous symmetry, then:

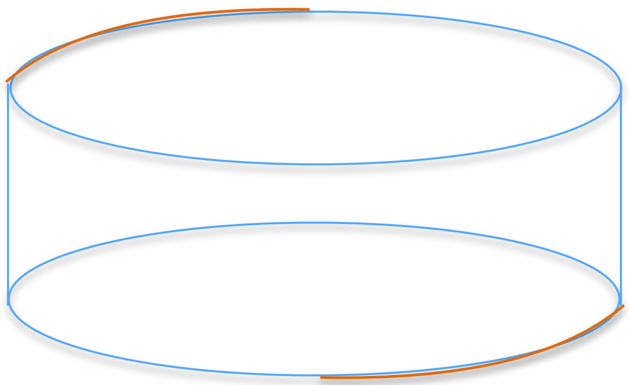
1. there exist a current which is conserved on shell,
2. the integral of the current defines a charge
   - conserved in time
   - canonical generator of the symmetry

Typical examples:



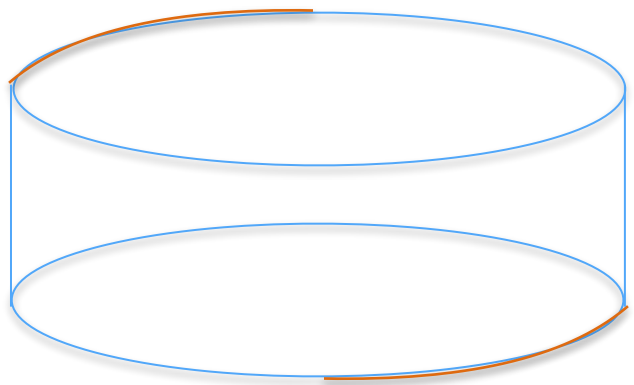
- ***Global*** U(1) invariance ⇨ conservation of electric charge
- Poincaré invariance ⇨ conserved energy-momentum tensor

Less known examples:



- ***Local*** U(1) invariance ⇨ conservation laws for *surface* charges
- Diffeomorphisms ⇨ conservation laws for *surface* charges

Surface charges in gravity



Less known examples:

- Diffeomorphisms ⇨ conservation laws for *surface* charges
- ***Local*** U(1) invariance ⇨ conservation laws for *surface* charges

In gravity, there is no local energy density: the Hamiltonian is exactly zero on-shell

Surface charges is all that there is

- Without boundaries, or with trivial gauge transformations at the boundary, no charges for the e-m field nor for gravity
- With boundaries and non-trivial gauge transformations there, we have access to a priori infinitely many surface charges and their conservation laws

In which physical context are we interested in non-vanishing gauge transformations at the boundary?

Modelling an isolated gravitational system: at large distances, we posit the flat Minkowski metric

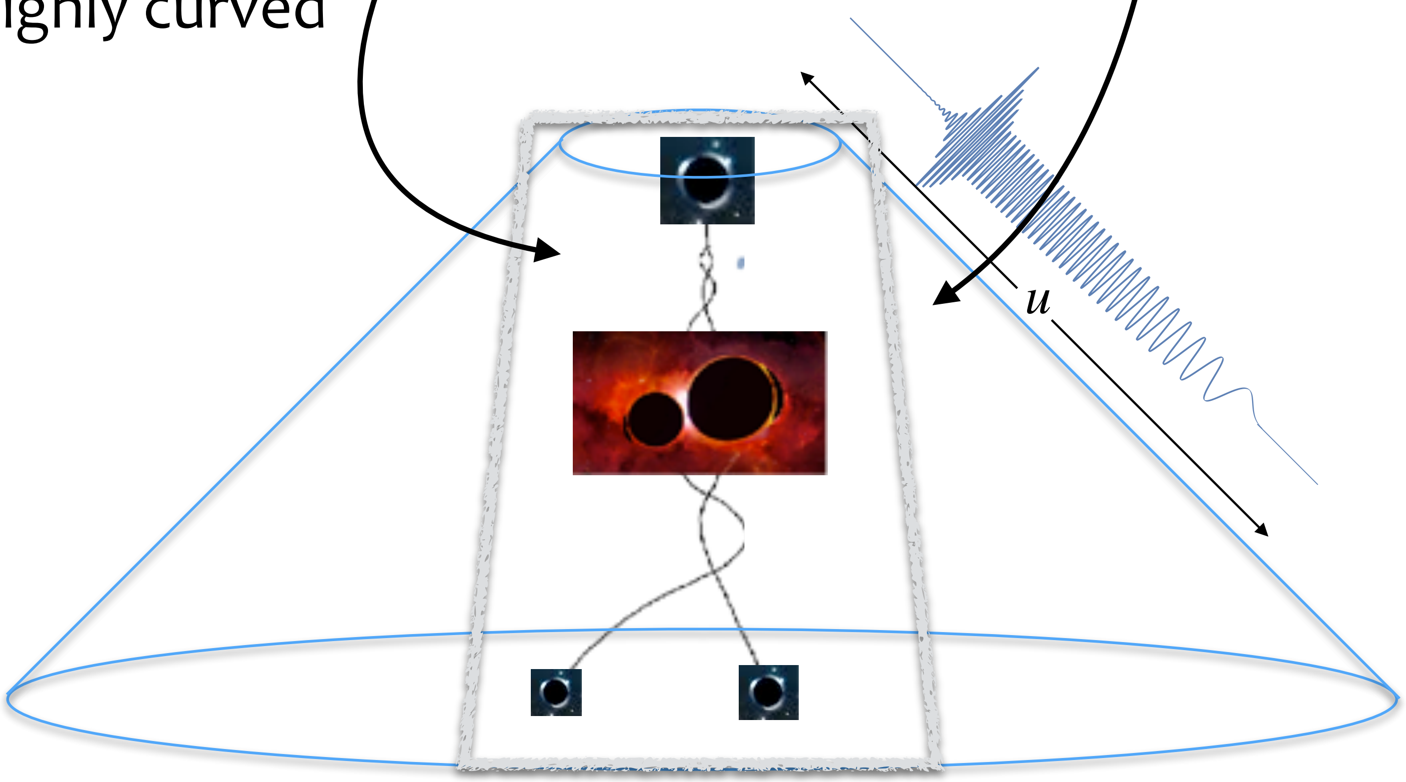
The diffeomorphisms which asymptote to Poincaré transformations are isometries of the asymptotic metric and non-vanishing gauge transformations at the boundary

Modelling an isolated gravitational system

Noether's energy as a surface charge at the boundary of space can be used to understand GWs

weak-field region: spacetime becomes *asymptotically flat*

strong-field region: highly curved



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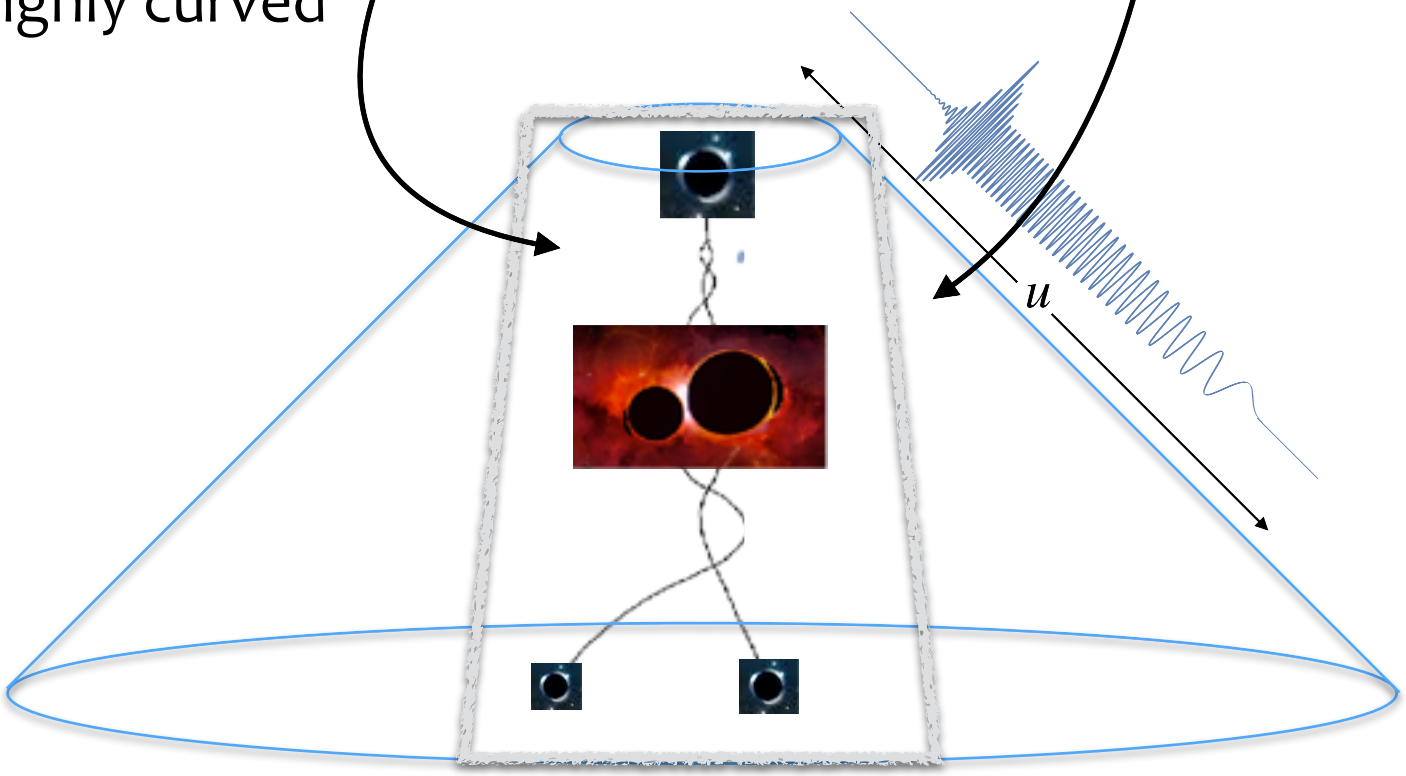


can use asymptotic Cartesian frames and inertial observers

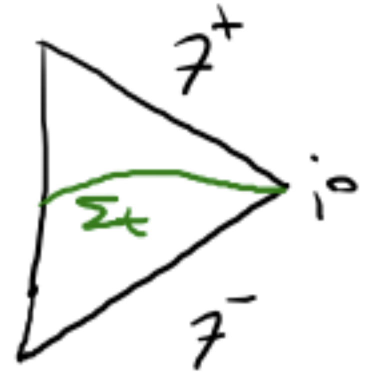
The relation between the mass and angular momentum of the system and the flux of GW can be derived as an application of Noether's theorem

$$Q_\xi[S'] - Q_\xi[S] = f(g, T, S, S')$$

e.g. the Bondi energy loss formula: usually read off the EEs, can be derived from Noether theorem

$$M[u_1] - M[u_0] = \int_{S^2} \int_{u_0}^{u_1} \left[\frac{\partial}{\partial u} m = \frac{1}{4} D_A D_B N^{AB} - \frac{1}{8} N_{AB} N^{AB} \right]$$

Two ways to go to infinity

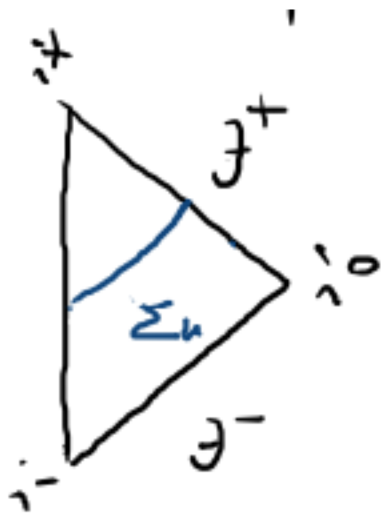


Space-like infinity:

there is a natural choice of boundary conditions: $q_{ab} \overset{i^0}{\mapsto} \eta_{ab}$

Residual diffeos: the isometries of the flat metric: The Poincaré group, absent as global symmetry group of GR, emerges as asymptotic symmetry group of asymptotically flat metrics

Surfaces charges at spatial infinity: mass, angular momentum and more in general all Poincaré charges (ADM, Regge-Teitelboim, Beig-Murchadha, Ashtekar-Hansen, …)



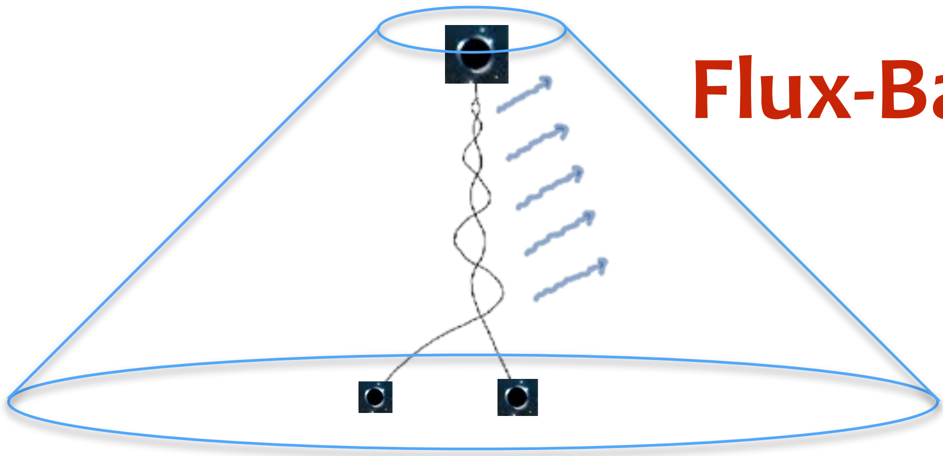
Null infinity:

the same choice can be done, $q_{ab} \overset{\mathcal{I}}{\mapsto} \eta_{ab}$

but does *not* select the Poincaré group

Residual diffeos: the isometries of a null slicing of the flat metric: This gives an infinite-dimensional group, known as the BMS group, which contains infinitely many copies of the Poincaré group, one for each cut of null infinity

Surfaces charges at null infinity: mass, angular momentum and more in general all BMS charges (Bondi-Metzer-Sachs, Newman-Penrose, Ashtekar-Streubel, Wald, …)



Flux-Balance laws at null infinity

Bondi-Metzer-Sachs, Newman-Penrose,
Thorne, Ashtekar, Damour, Blanchet…

By going to null infinity (and picking up an enlarged infinite-d symmetry along the way) we are able to define surface charges that are independent of the surface and coordinate used, and that can be related to the gravitational flux

Caveat: for some diffeomorphisms (those not tangent to *S*), there is a shift between the Noether charge and the canonical generator of the symmetry

We can then:

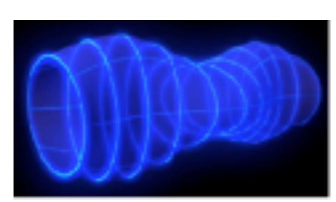
- identify those surface charges that in the stationary case reproduce the mass and angular momentum of black holes

$$M = \int_S m \qquad\qquad J = \int_S (\partial_\phi)^A P_A$$

- derive flux-balance laws directly from the Einstein's equations

$$\dot{m} = \frac{1}{4} D_A D_B \dot{C}^{AB} - \frac{1}{8} \dot{C}_{AB} \dot{C}^{AB} \qquad\qquad \dot{P}_A = \frac{1}{8} D_A (C_{BC} \dot{C}^{BC}) + \ldots$$

Bondi energy-loss formula

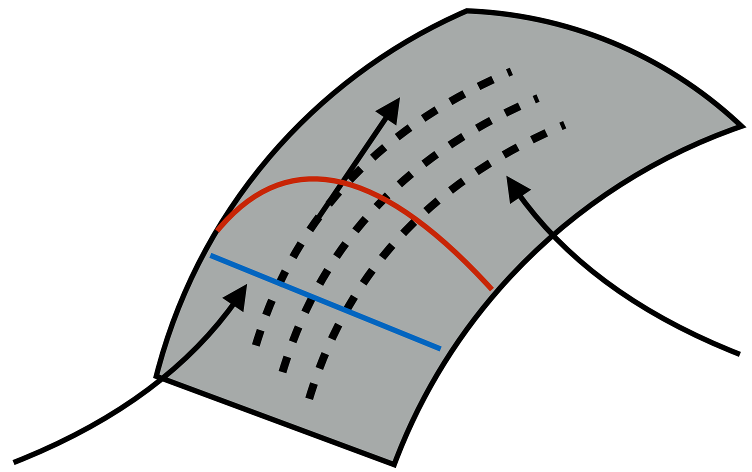


Angular momentum flux-balance law

These flux-balance laws contribute to reconstructing the physics of the source (i.e. the merging of BHs) from the observed signals (the GWs)

Why an infinite-dimensional symmetry group?

The big difference is that a null hyperplane has a degenerate metric:
there is no distinguished notion of Cartesian coordinate in the degenerate direction



transverse directions:
distances measured
by the round 2-sphere metric

zero distances along the null directions

the blue cut and the red cut are equivalently good observers

The extension is the freedom of making the translations of the Poincaré group not rigid, but depending on the point of the sphere: **supertranslations** $T(\theta, \phi)$

The 10-parameter Poincaré group is extended to the infinite-d BMS group

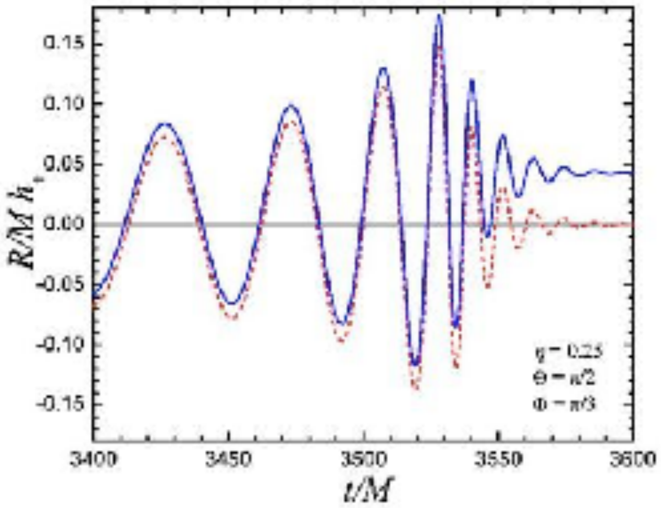
$$P^4 = \mathrm{SL}(2,\mathbb{C}) \ltimes T^4 \qquad \to \qquad BMS = \mathrm{SL}(2,\mathbb{C}) \ltimes R^S$$

(6-param. + a free function on the sphere)

(Pick a frame on the sphere, then the first 4 harmonics of T provide the translations wrt that frame)

What physical meaning have the extra charges?

The supertranslations charges have physical consequences
they are related to memory effects:
permanent displacements of the detector
(Christodoulou, Blanchet-Damour '90s,
Teukolsky, Nichols, Favata, …)



GW with memory

GW w/o memory

Favata '10

Caveat! This plot is an idealized situation
the typical signal is way too small to be seen in the LIGO-Virgo detectors
Possibly with LISA and enough statistics

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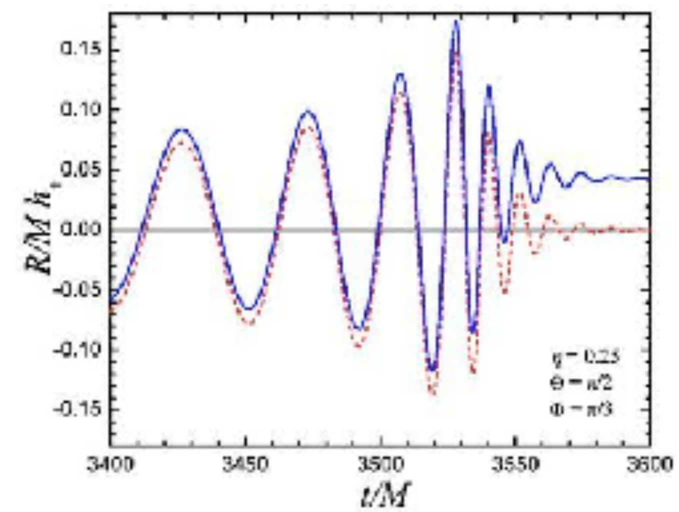

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Favata '10

The electromagnetic memory effect:

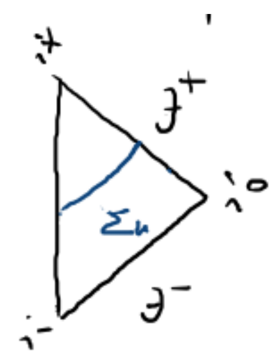
$$\int_{\mathcal{I}} j = \underbrace{\int_{\mathcal{I}} \lambda J_\psi}_{\text{'hard'}} + \underbrace{\int_{S^2} \partial\lambda \cdot \int du \vec{E}}_{\text{'soft'}}$$

- soft term: e-m memory (the physical effect of a velocity kick to the particles in the detector)

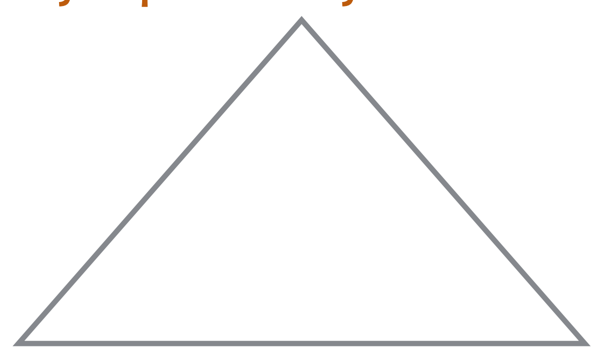
$$m\ddot{\vec{x}} = q\vec{E} \qquad \rightarrow \qquad \Delta\vec{v} = \frac{q}{m}\int_{-\infty}^{u} \vec{E}$$

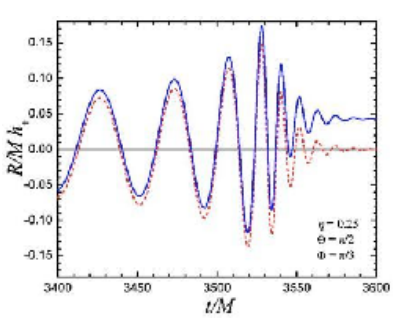
Strominger's infrared triangle

The relation between asymptotic symmetries and memory effects is one side of the now famous Strominger IR triangle



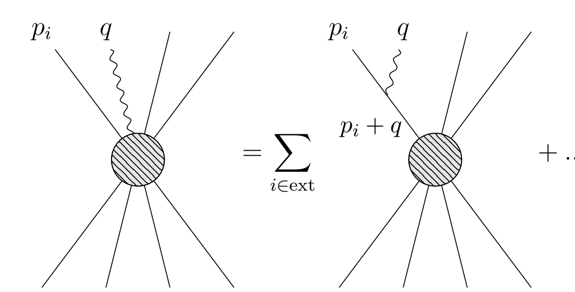
asymptotic symmetries





memory effects

soft theorems



The larger the asymptotic symmetry, the more of such triangles we have

What is the largest possible symmetry group of gravity?

Motivations to look for larger asymptotic symmetries

Is the BMS symmetry the end of the story? Many reasons to think not!

- subleading soft theorems (Strominger, …) ⇨ gen. BMS group (Campiglia-Laddha, Compere-Fiorucci-Ruzziconi ’18)
- holography (Barnich, …) ⇨ extended BMS group (Barnich-Troessaert ’11)
- larger symmetry algebra gives more control on quantization (Barnich, Grumiller, Freidel, …)
- the algebra of quasi-local observables is actually larger than BMS (Flanagan et al, Ciambelli et al, Freidel et al)
- … various further ideas (Hawking-Strominger-Perry, …)

A hierarchy of asymptotic symmetries

Enlargements of the BMS groups have been found in the last ten years:

Asymptotic background structure		*Symmetry parameters*	*Algebra*
Original BMS Bondi-Metzer-Sachs '62	round 2-sphere	$T(\theta,\phi)\,,\ Y^A(\theta,\phi)$ CKV	$\mathrm{SL}(2,\mathbb{C}) \ltimes R^S$
Extended BMS Barnich-Troessaert '10	round 2-sphere	$T(\theta,\phi)\,,\ Y^A(\theta,\phi)$ mero CKV	$\mathrm{Virasoro} \ltimes R^S$
Generalized BMS Laddha-Campiglia '14, Compere et al. '18	scale factor	$T(\theta,\phi) \quad Y^A(\theta,\phi)$	$\mathrm{Diff}(S) \ltimes R^S$
Weyl BMS Freidel-Oliveri-Pranzetti-S '21	foliation of *scri*	$T(\theta,\phi) \quad Y^A(\theta,\phi) \quad W(\theta,\phi)$	$(\mathrm{Diff}(S) \ltimes R^S) \ltimes R^S$

These extensions are quite challenging to achieve, and one has to deal with various technical points: holographic renormalization, charge-integrability, covariant phase space with anomalies, etc.

Some technical details of the BMSW group

We relax the original BMS conditions   coordinates: (u, r, θ^A)

$$g_{ur} = -1 + \mathcal{O}(r^{-2}), \quad g_{uA} = \mathcal{O}(1), \quad g_{uu} = -1 + \mathcal{O}(r^{-1}), \quad q_{AB} = \mathring{q}_{AB} + \mathcal{O}(r^{-1})$$

$$\bar{\xi}_{T,Y} := T\partial_u + Y^A\partial_A + \frac{1}{2}D\cdot Y(u\partial_u - r\partial_r) \qquad BMS = \mathrm{SL}(2,\mathbb{C}) \ltimes R^S$$

to:

$$g_{ur} = -1 + \mathcal{O}(r^{-2}), \qquad g_{uA} = \mathcal{O}(1), \qquad g_{uu} = \underline{\mathcal{O}(1)}, \qquad q_{AB} = \underline{\mathcal{O}(1)}$$

$$\bar{\xi}_{(T,W,Y)} := T\partial_u + Y^A\partial_A + W(u\partial_u - r\partial_r) \qquad (\mathrm{Diff}(S) \ltimes R^S) \ltimes R^S$$

$$\pounds_\xi g_{AB} = r^2(\pounds_Y \bar{q}_{AB} - 2W\bar{q}_{AB}) + O(r)$$

$$\pounds_\xi g_{uu} = (\pounds_Y + 2W)\bar{R} + 2\bar{\Delta}W + O(r^{-1})$$

In particular this shows that the BMSW vectors are **not** asymptotic Killing vectors of the flat Minkowski metric

They provide asymptotic symmetries in a more general sense, wrt a *smaller* background structure

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Recovering BMS as a subgroup:

$$\pounds_\xi g_{AB} = r^2(\pounds_Y \bar{q}_{AB} - 2W\bar{q}_{AB}) + O(r)$$ (→ 0)

⇨ Y CKV

$$\pounds_\xi g_{uu} = (\pounds_Y + 2W)\bar{R} + 2\bar{\Delta}W + O(r^{-1})$$ (→ 0)

⇨ $W = \frac{1}{2}D\cdot Y$

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Advantages of the BMSW extension

- includes BMS, eBMS and gBMS as subalgebroids (subalgebras via redefinitions of the parameters)
- matches the algebra of quasi-local observables $\left(\text{diff}(S) \oplus \mathfrak{sl}(2,\mathbb{R})^S\right) \oplus (\mathbb{R}^2)^S$

(Chandrasekaran et al '18, FOPS '21, Ciambelli-Leigh '21)

- disentangles diffeos and Weyl rescalings of the 2-sphere

(explaining for instance why there is flux of ang. mom. even though tang. diffeos are integrable)

Provides a new motivation to enlarge the asymptotic symmetry:
there is a deep interplay between dynamics and the symmetry algebra

$$\{Q_\xi, Q_\chi\} = Q_{[\xi,\chi]} + \int_S \xi^\mu \chi^\nu G_{\mu\nu}$$

- The surface charges provide a representation of the algebra on-shell of the field equations
- Conversely, requiring a representation of the algebra imposes the Einstein's equations

But how many of them? ***the larger the algebra, the more of the asympt. equations can be derived***

Original BMS : only 1 asymptotic equation! (The Bondi energy loss)
Generalized BMS : 3 asympt. equations (The energy and angular momentum loss formulas)
Our new Weyl BMS : 8 asympt. equations

Some of the technical results

- Relaxing boundary fall-offs to find larger algebra can introduce divergences in the limit

⇨ renormalization based on the freedom available in the construction of the phase space
(or in the more modern terms of FGP '19, freedom in taking an adequate boundary Lagrangian)
Making the "mixture of art and science" under the control of an organizing principle
Reproduces the Brown-York subtraction scheme

- "Non-integrability" of the charges for vector fields leaking symplectic flux

⇨ prove that one can consistently use the Noether split
In particular, the Ashtekar-Dray-StreubelWald-Zoupas/Barnich-Troessaert/Flanagan-Nichols expressions
have been shown to be Noether charges for a specific choice of boundary Lagrangian

- Anomalies in the transformation laws due to the presence of non-covariant background structures such as the expansion parameter or gauge-fixing used

⇨ extend the covariant phase space formalism to deal with anomalies
A purely technical result, but with conceptual consequences

… and a "non-result"

The Weyl charge carries no additional information

Its flux-balance law is not independent from the energy and momentum ones

⇨ *possibly no new memory effects expected from the BMSW enlargement;*

however see Freidel-Pranzetti-Raclariu '21 for relations to the sub-sub-leading soft graviton theorem

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Having established this more general framework for asymptotic symmetries with weaker boundary conditions, we can look for further new effects

For instance, effects associated with different fundamental variables for gravity, as the tetrad variables described in Geiller's previous talk

New charges

GR in tetrad variables:

$$S[g] = \frac{1}{16\pi G}\int \sqrt{-g}(g^{\mu\nu}R_{\mu\nu}(g) - 2\Lambda) \quad \rightarrow \quad S[e,\omega] = \frac{1}{32\pi G}\int \epsilon_{IJKL}e^I \wedge e^J \wedge (F^{KL}(\omega) - \frac{\Lambda}{3}e^K \wedge e^L)$$

$$+\frac{1}{\gamma}\int e_I \wedge e_J \wedge F^{IJ}(\omega)$$

Changing variables and adding the dual term does not affect the classical field equations but is crucial to construct loop quantum gravity,
and **it turns out to give non-trivial contribution to the charges** $\not\delta \widetilde{H}^e_\xi = \int_{\partial\Sigma} 2\pounds_\xi e_I \wedge \delta e^I$

Oliveri-S '19, Freidel-Geiller-Pranzetti '20, …
Godazgar-Perry '19, Oliveri-S '20

Seraj-Oblak last week,
Oblak's talk yesterday

Effects are not so easy to see! The new contributions can show up in **subleading** gravitational effects, and even the leading memory effect is not guaranteed to be observed by LISA

So it may be a long shot to observe such effects, but it is a concrete proposal, that can be made quantitative, and offers us a good path to deepen our understanding of gravitational waves with a potential for physical predictions that directly affect the premises of loop quantum gravity

Conclusions

- Strominger's question *What is the largest symmetry group of gravity?* is a powerful approach to dive deeper into the structure of gravitational dynamics

- the use of covariant phase space methods with anomalies and (holographic) renormalization puts the "*mixture of art and science*" on more controlled ground

- strong relation between asymptotic symmetries and quasi-local corner symmetries gives new input to the program of quasi-local observables

- possibility of discriminating formulations of gravity which are classically bulk-equivalent but not boundary equivalent, like metric vs. tetrad, with implications for models of quantum gravity