What to do with structure@horizon ?

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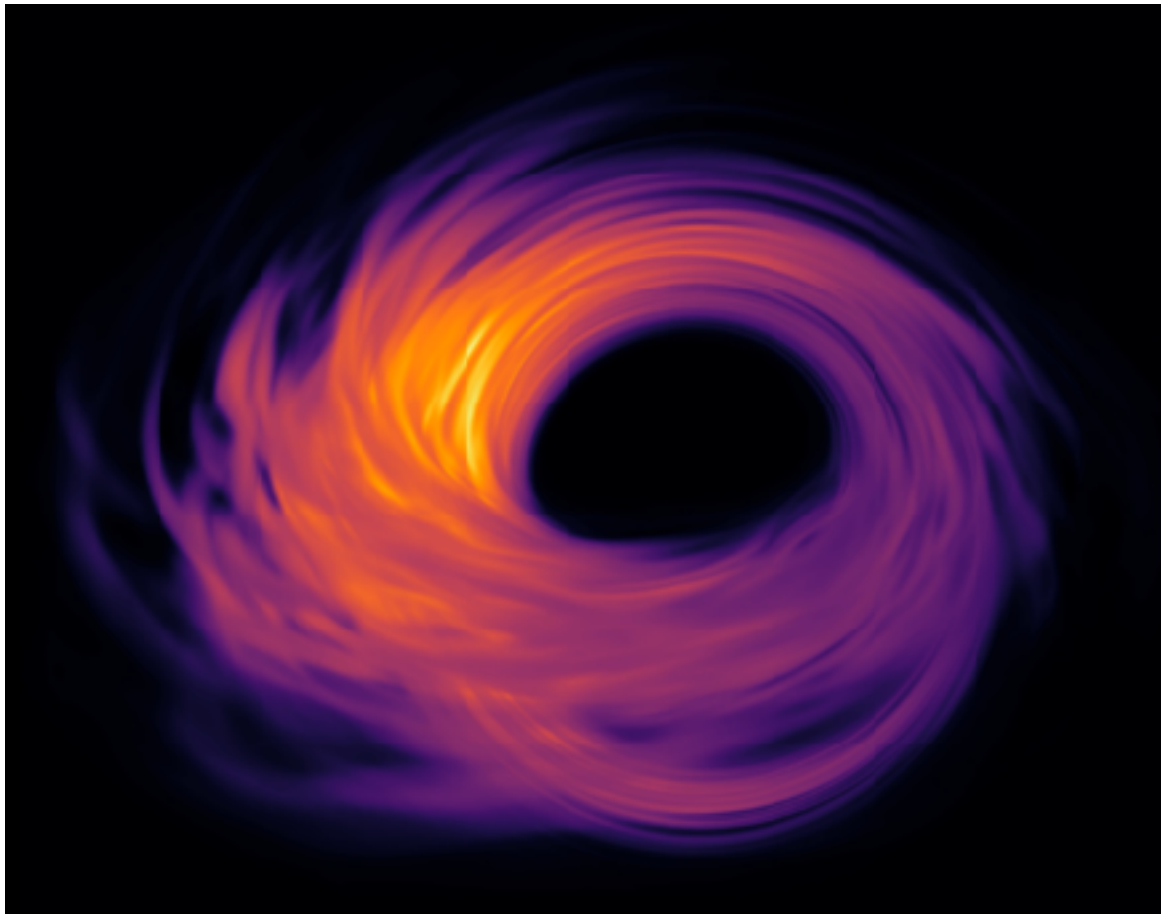

BPS Multipoles *(w/ Daniel Mayerson)*

- *Short paper - 2006.10750, accepted in PRL*
- *Long paper - 2007.09152*

Almost-BPS multipoles *(w/ Ibou Bah, Pierre Heidmann, Yixuan Li, and Daniel Mayerson)*

- *2104.10686*

Tidal Love Numbers *(w/ Pierre Heidmann and Daniel Mayerson)*

- *22xx.xxxxx*

erc

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Quantum Mechanics vs General Relativity

Hawking's Information Paradox → ***conflict***

black hole of TON618 quasar: $S_{\text{Bekenstein-Hawking}} \sim 10^{98}$

Quantum Mechanics: ⇒ $e^{10^{98}} = e^{10000000\ \ldots\ 00000}$ states

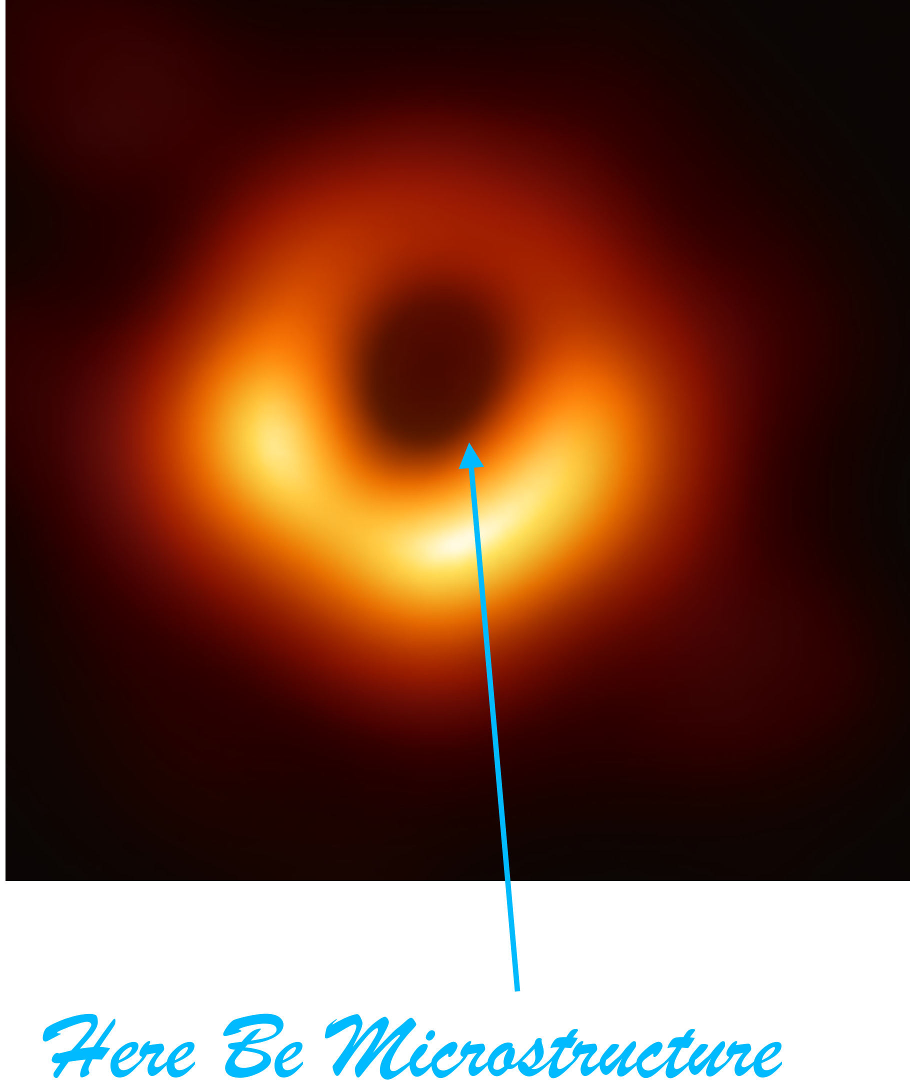
General Relativity

⇒ 1 big fat state

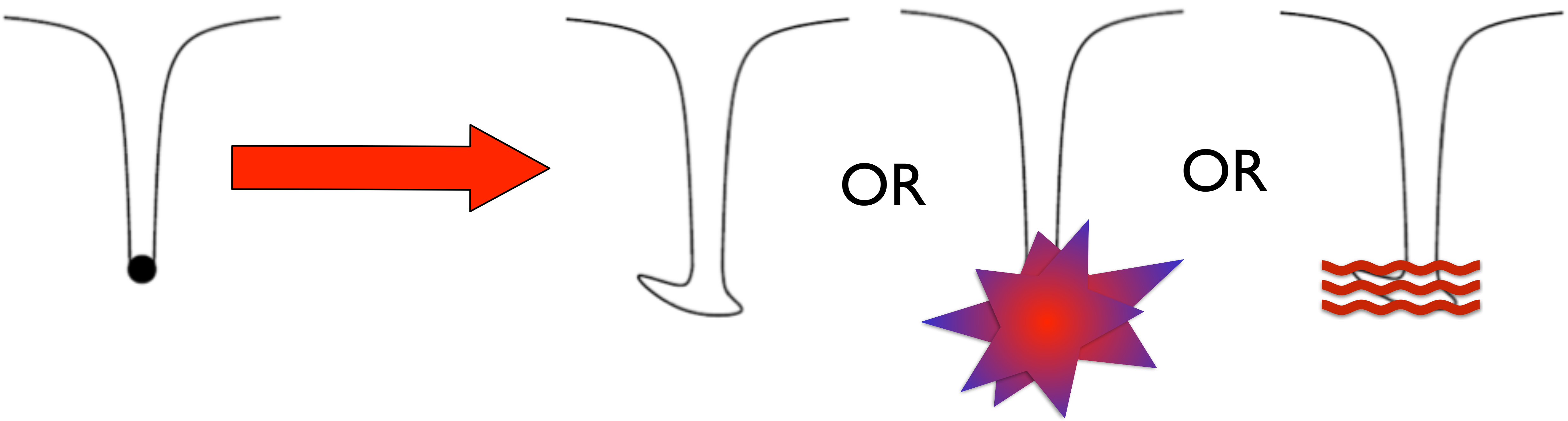
Biggest unexplained number of physics !

The resolution:
There must exist structure@horizon

Mathur 2009,
Almheiri, Marolf, Polchinski, Sully 2012



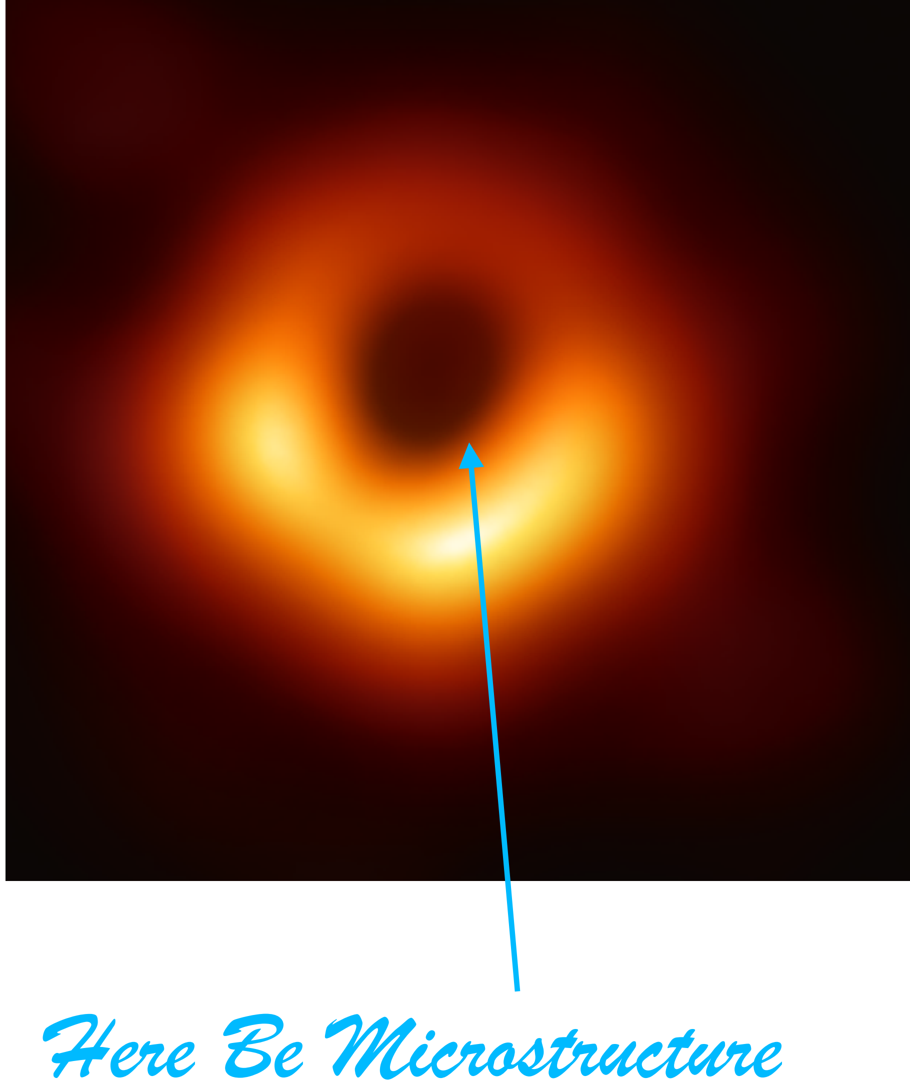
Only other viable alternative: ER=EPR, Islands
⇒ wormholes over megaparsec distances



How does it look like ?

Structure@horizon in vogue these days (ECO)

- Gravastars
- Quark-stars
- Boson-stars
- Gas of wormholes (ER=EPR)
- Quantum Black Boxes
- BMS / Soft hair & horizon
- Mirrors floating on Pixie Dust
- Modified gravity
- Bose-Einstein condensate of gravitons
- Infinite density firewall hovering just above horizon



Not so easy

1. Growth with G_N ↔ BH size for all masses

Horowitz

- Normal objects shrink; BH horizon grows
- **microstate geometries** have BH size for all masses
- D-branes = solitons, $m \sim 1/g_s$ lighter as $G_N = g_s^2$ increases

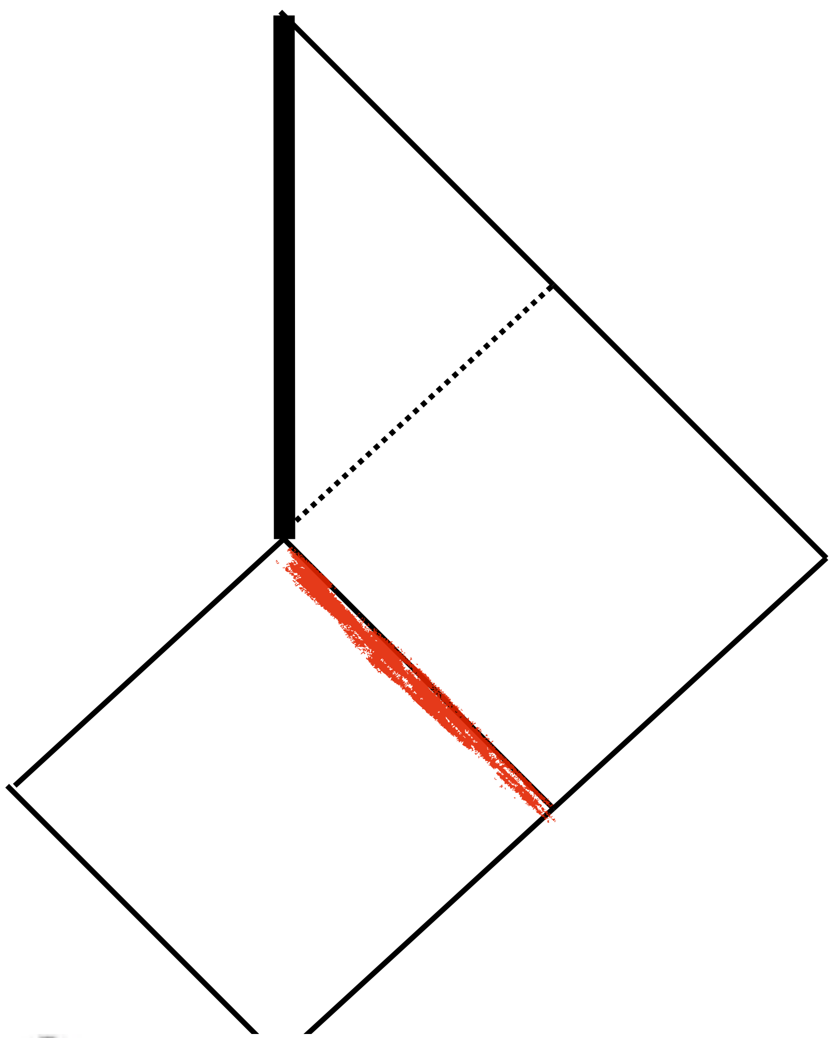
To build structure@horizon, non-perturbative degrees of freedom you must use !

- Boson stars need scalar fields of different masses to replace various BH's: One field for $M_\odot$, another for 30 $M_\odot$, etc.
- String theory non-perturbative d.o.f. ➙ fields whose mass decreases for larger BH

toy models at most

2. Mechanism not to fall into BH

Very difficult !!!



GR Dogma:

Thou shalt not put anything at the horizon !!!

- Null ➔ speed of light.
- If massive: ∞ boost ➔ ∞ energy
- If massless: dilutes with time

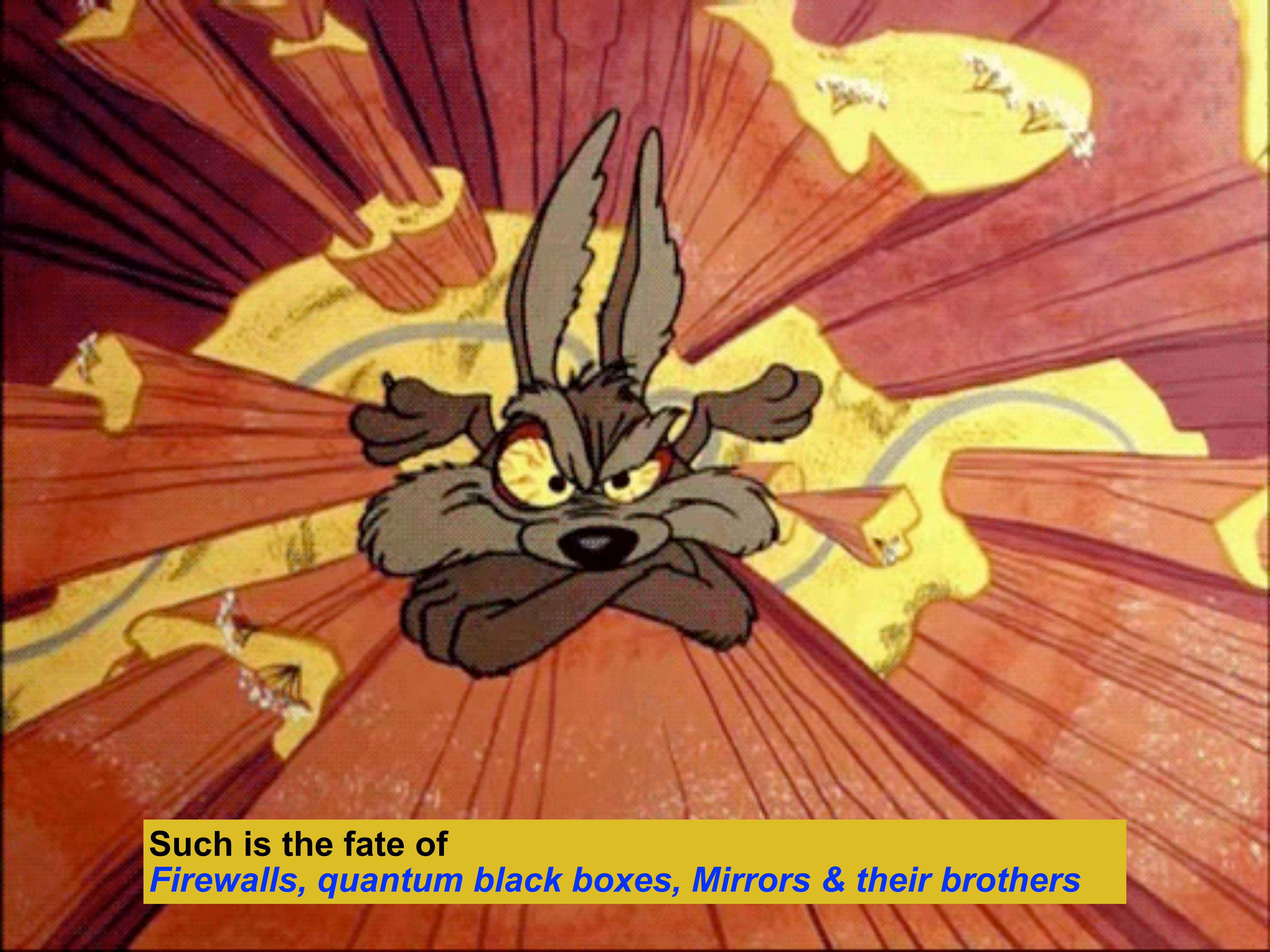
- Nothing can live there !
  (or carry degrees of freedom)
- No membrane, no spins, no "quantum stuff"
- No (fire)wall

If support mechanism have you not,
go home and find one

"Quantum Coyote principle"

Quantum Coyote Principle

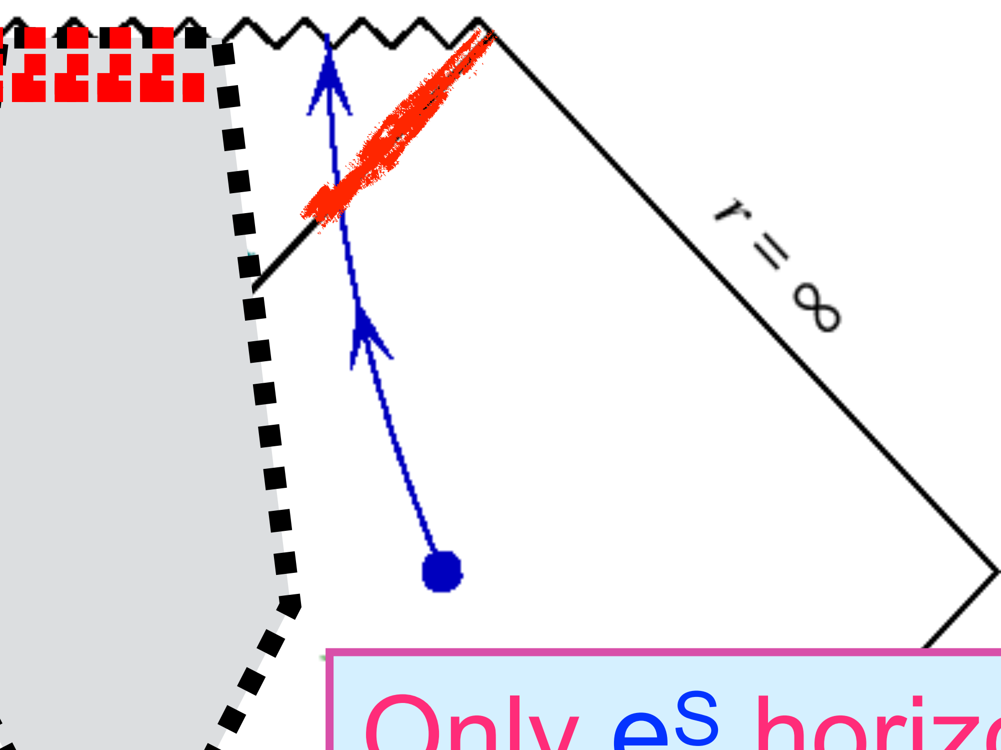
GRAVITY DOES NOT WORK `TILL YOU LOOK DOWN ….



Such is the fate of
Firewalls, quantum black boxes, Mirrors & their brothers

3. Avoid forming a horizon

– Collapsing shell forms horizon @ low curvature
  Oppenheimer and Snyder (1939)
– By the time shell becomes curved-enough for quantum effects to become important, horizon in causal past (180 hours for TON618 BH)



Backwards in time - illegal !

BH has e^S microstates with no horizon
Small tunneling probability = e^{-S}
Shell tunnels with probability **ONE** !!!
Kraus, Mathur; Bena, Mayerson, Puhm, Vercnocke

Only e^S horizon-sized microstates can do it !

Black hole entropy the structure must have

Rules out gravastars & almost everything else

Microstate (Fuzzball) Geometries:

- Only construction with all three properties
- Top-down
- Largest family of solutions known to mankind

Arbitrary fns. of 3 variables: ∞ X ∞ X ∞ parameters !

Cohomogeneity-5 !

Bena, Giusto, Russo, Shigemori, Warner, 2015
Heidmann, Mayerson, Walker, Warner, 2019

$$
\begin{aligned}
ds_{10}^2 &= \frac{1}{\sqrt{\alpha}}\, ds_6^2 + \sqrt{\frac{Z_1}{Z_2}}\, d\hat{s}_4^2\,,\\
ds_6^2 &= \frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right]+\sqrt{\mathcal{P}}\, ds_4^2\,,\\
e^{2\Phi} &= \frac{Z_1^2}{\mathcal{P}}\,,\\
B &= -\frac{Z_4}{\mathcal{P}}(du+\omega)\wedge(dv+\beta)+a_4\wedge(dv+\beta)+\delta_2\,,\\
C_0 &= \frac{Z_4}{Z_1}\,,\\
C_2 &= \frac{Z_2}{\mathcal{P}}(du+\omega)\wedge(dv+\beta)+a_1\wedge(dv+\beta)+\gamma_2\,,\\
C_4 &= \frac{Z_4}{Z_2}\,\widehat{\mathrm{vol}}_4-\frac{Z_4}{\mathcal{P}}\gamma_2\wedge(du+\omega)\wedge(dv+\beta)+x_3\wedge(dv+\beta)+\mathcal{C}\,,\\
C_6 &= \widehat{\mathrm{vol}}_4\wedge\left[-\frac{Z_1}{\mathcal{P}}(du+\omega)\wedge(dv+\beta)+a_2\wedge(dv+\beta)+\gamma_1\right]\\
&\quad -\frac{Z_4}{\mathcal{P}}\,\mathcal{C}\wedge(du+\omega)\wedge(dv+\beta)\,,
\end{aligned}
$$

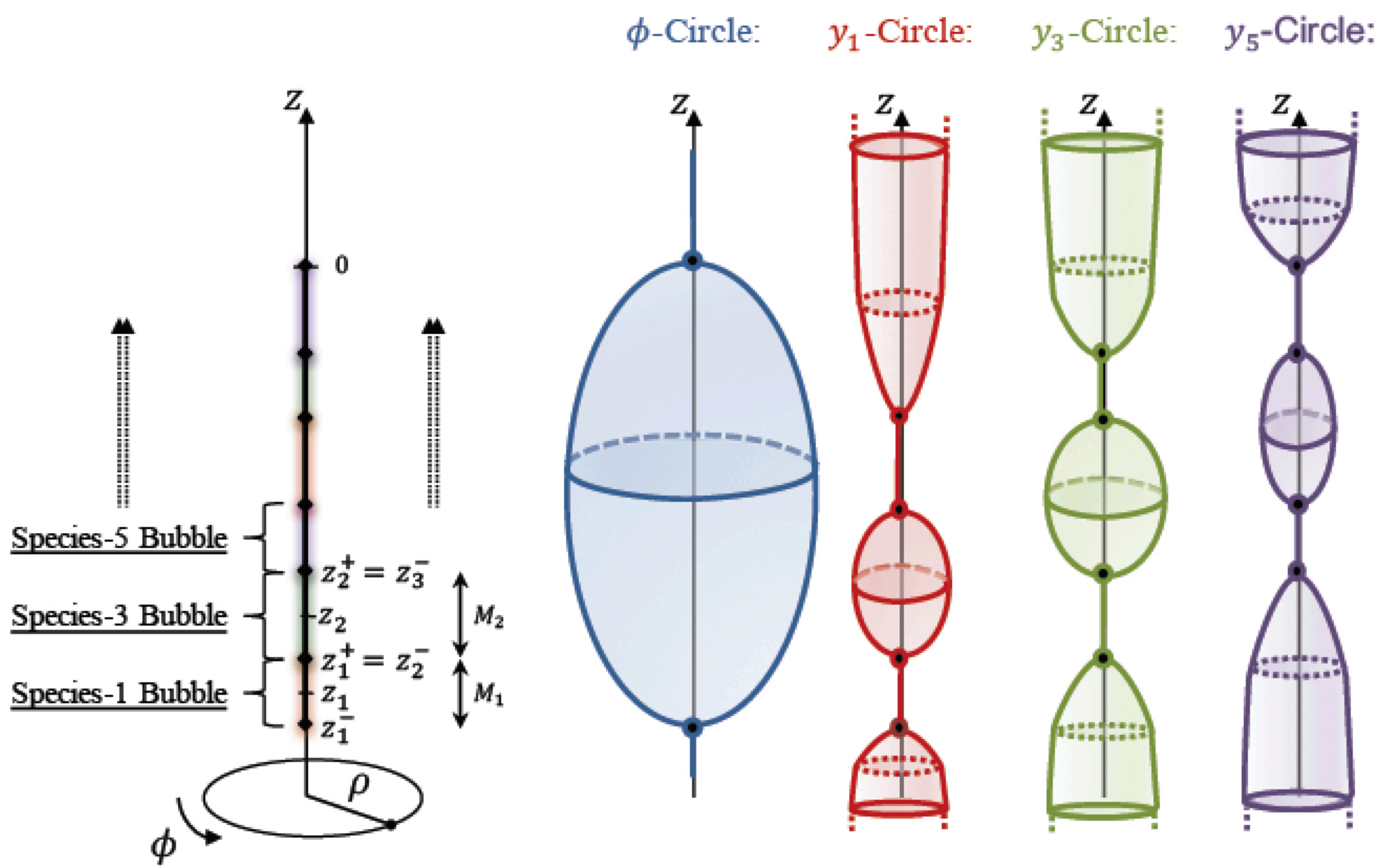
$$
\begin{aligned}
\omega_r^{(2)} &= -\frac{R\, r}{\sqrt{2}\,k_2(m_1^2-1)}\,\frac{m_1(k_2+m_1+1)\Delta_{k_2+m_1-1,m_1-1}+(k_2+m_1-1)\Delta_{k_2+m_1-3,m_1-1}}{(r^2+a^2)^2}\,,\\
\omega_\theta^{(2)} &= \frac{R}{\sqrt{2}\,k_2(m_1^2-1)a^2\sin\theta\cos\theta}\Big[2(m_1-1)\Delta_{k_2+m_1-3,m_1-1}\\
&\quad +(m_1-1)(m_1-2)\Delta_{k_2+m_1-1,m_1-1}+m_1(k_2-2)\Delta_{k_2+m_1-1,m_1+1}\\
&\quad -m_1(m_1-1)\Delta_{k_2+m_1+1,m_1-1}+(m_1^2(k_2-1)+1)\Delta_{k_2+m_1+1,m_1+1}\Big]\,,\\
\omega_\phi^{(2)} &= -\frac{R}{\sqrt{2}}\,\frac{\Delta_{k_2+m_1+1,m_1+1}}{\Sigma}\sin^2\theta-\frac{R}{\sqrt{2}\,k_2(m_1^2-1)a^2}\Big[2(m_1-1)\Delta_{k_2+m_1-3,m_1-1}\\
&\quad +(m_1^2-2m_1+k_2-1)\Delta_{k_2+m_1-1,m_1-1}+m_1(k_2-2)\Delta_{k_2+m_1-1,m_1+1}\\
&\quad +m_1(k_2-m_1-1)\Delta_{k_2+m_1+1,m_1-1}+(k_2(m_1^2+m_1-1)-m_1(m_1+1))\Delta_{k_2+m_1+1,m_1+\cdots}\\
\omega_\psi^{(2)} &= \frac{R}{\sqrt{2}}\,\frac{\Delta_{k_2+m_1+1,m_1+1}}{\Sigma}\cos^2\theta-\frac{R}{\sqrt{2}\,k_2(m_1^2-1)a^2}\Big[(k_2-1)(m_1-1)\Delta_{k_2+m_1+1,m_1+3}\\
&\quad -2(m_1-1)\Delta_{k_2+m_1-3,m_1-1}-(m_1-1)(m_1-2)\Delta_{k_2+m_1-1,m_1-1}\\
&\quad (m_1-1)(k_2-3)\Delta_{k_2+m_1-1,m_1+1}+m_1(m_1-1)\Delta_{k_2+m_1+1,m_1-1}\\
&\quad -(m_1-1)(m_1(k_2-1)+1)\Delta_{k_2+m_1+1,m_1+1}\Big]\,.
\end{aligned}
$$

Issue: only for SUSY and extremal black holes

NonExtremal - first progress in 10 years

- Heidmann:

  Non-BPS Floating Branes and Bubbling Geometries, 2112.03279



First bubbling geometry for non-extremal BH !

Does this have any consequence ?

Ideal world: construct 4D Kerr non-extremal microstate geometries, compare to data

Real world: 3 options

1. Charged microstates → universal features

2. Some non-extremal microstates

Heidmann solutions

3. Compare uncharged Kerr BH's with charged microstates Bianchi, Consoli, Grillo, Morales, Pani

apples and oranges ?

better toy model than using boson stars ?

Some properties I believe to be universal

Multipole moments - the big idea

- Kerr BH with vacuum at horizon
- spinning ball of dust
- spinning ball of liquid
- spinning solid shell
- boson star
- other cockamamie BH replacements

} Different gravitational multipoles

What about spinning *STRUCTURE@horizon ?*

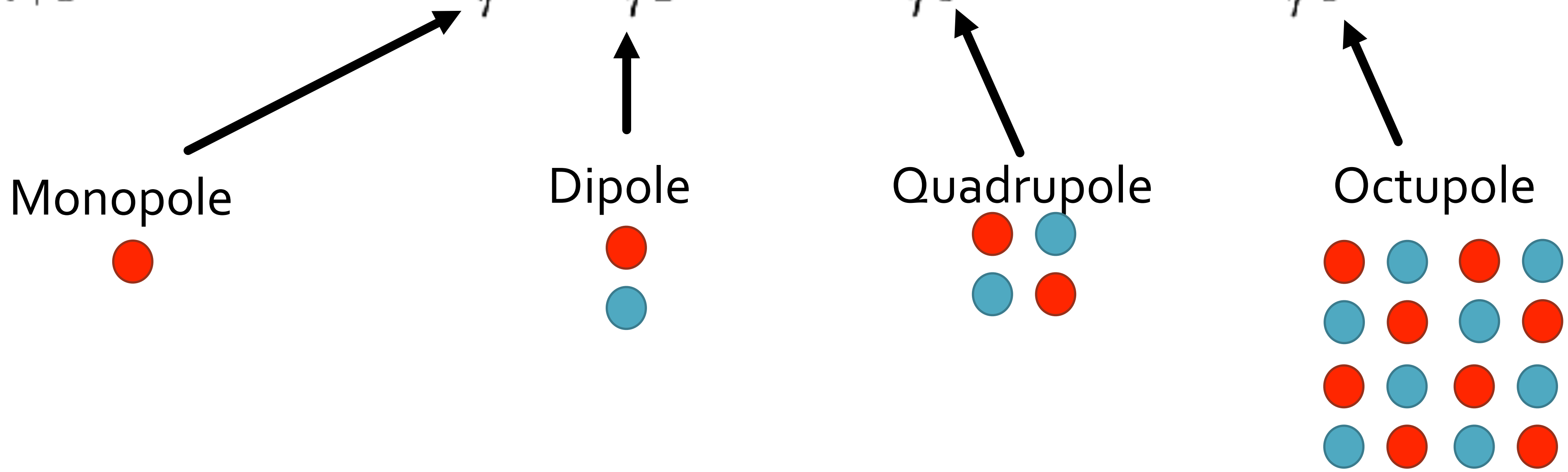
Gravity waves from Extreme Mass-Ratio Inspiral (EMRI) with non-aligned spins can measure multipoles

Need 3 moments to rule out Kerr, 4 to rule out spinning boson star (Ryan '95)

Gravitational multipoles

Reminder: multipoles in electrodynamics

$$V = \sum_{l \geq 0} \frac{1}{r^{l+1}} M_l P_l(\cos\theta) = \frac{M_0}{r} + \frac{M_1}{r^2}\cos\theta + \frac{M_2}{r^3} P_2(\cos\theta) + \frac{M_3}{r^4} P_3(\cos\theta) + \dots$$



GR - coordinate transformations: naive multipoles not well defined

Geroch-Hansen formalism (conformal compactification) $\leftrightarrow$

Thorne formalism (ACMC-N coordinates)

Thorne formalism: ACMC-N coordinates

asymptotically Cartesian and mass-centered to order N

Mass multipoles: Coordinate-invariant

Mass

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{l \geq 2}^{N} \frac{2}{r^{l+1}} \left(M_l P_l + \sum_{l' < l} c_{ll'}^{(tt)} P_{l'} \right)$$

$$+ \frac{2}{r^{N+2}} \left(M_{N+1} P_{N+1} + \sum_{l' \neq N+1} c_{(N+1)l'}^{(tt)} P_{l'} \right) + \mathcal{O}\left(r^{-(N+3)} \right) ,$$

Coordinate-dependent "harmonics"

No dipole!

Similar: $g_{t\phi} \sim S_l$ (current multipoles)

constrained expansion of space-space components

4d Kerr BH

- Solution depends on M, a
- Multipoles: $M_\ell + iS_\ell = M(ia)^\ell$

or $M_{2n} = M(-a^2)^n \ , \quad S_{2n+1} = Ma(-a^2)^n$

$$M_{2n+1} = S_{2n} = 0 \ !!!$$

- Mass: $M_0 = M$
- Angular momentum: $J \equiv S_1 = Ma$
- Kerr-Newman the same (independent of Q)

4d Kerr BH - multipole ratios

- Remember Kerr: $M_{2n+1} = S_{2n} = 0$
- Ratio of vanishing multipoles = $\frac{0}{0}$
- Embed it in String Theory + deform it to big fat STU black hole
  10 parameters: 4 electric + 4 magnetic, mass, angular momentum
- Multipoles: functions of 4 parameters: $\mathscr{M}(M, J, a, D)$
- Compute ratios. Take back Kerr limit.

For example: $\mathscr{R}_{\text{Kerr}} = \dfrac{M_2 S_2}{M_3 S_1} = 1$!!!

ratio naively undefined (vanish for Kerr)

$$\frac{M_{l+1}M_{l+2}}{M_l M_{l+3}} = \frac{S_l S_{l+1}}{S_{l-1}S_{l+2}} = \frac{M_{l+2}S_l}{M_l S_{l+2}} = 1 - \frac{4}{3+(-1)^l(2l+1)}$$

$$\frac{S_{l+1}S_{l+2}}{M_l M_{l+3}} = -\frac{1+(-1)^l(2l+3)}{3+(-1)^l(2l+1)},$$

$$\frac{M_{l+1}M_{l+2}}{S_l S_{l+3}} = -\frac{1-(-1)^l(2l+1)}{3-(-1)^l(2l+3)}.$$

Mayerson numbers

Independent of deformation direction inside 56-dim space of charged String Theory BH's !

Constraining small deviations from Kerr

Small deviations: $M_l = (M_l)_{\text{Kerr}} + m_l\,\epsilon, \qquad S_l = (S_l)_{\text{Kerr}} + s_l\,\epsilon$

Use Ratios: $$\frac{M_2 S_{2n}}{M_{2n+1} S_1} = 1 = -\,a\frac{s_{2n}}{m_{2n+1}} + \mathcal{O}(\epsilon)$$

Constrains all perturbative deviations away from Kerr ! $\delta(\text{Kerr}) \sim \epsilon$

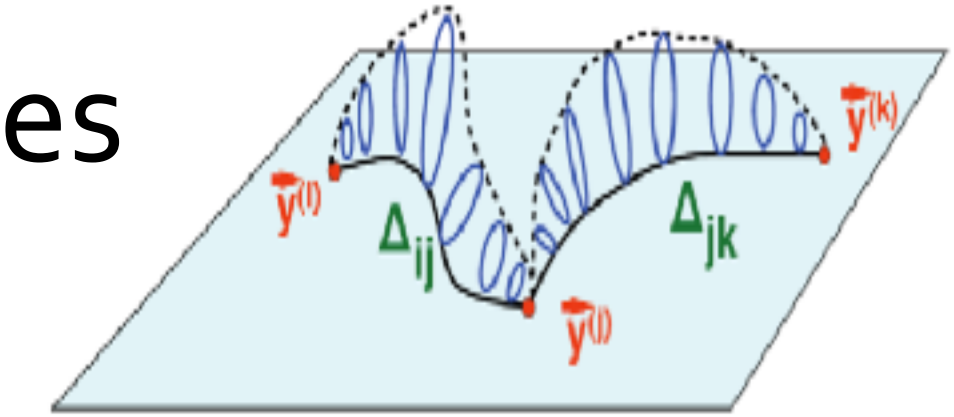
$$S_{2n} = -nM(-a^2)^n \epsilon,$$
$$M_{2n+1} = nMa(-a^2)^n \epsilon,$$
$$M_{2n} - (M_{2n})_{\text{Kerr}} = -n^2 M(-a^2)^n \left(\frac{2n-3}{4n}\right)\epsilon^2,$$

Calculated from String-Theory embedding
Prediction of String Theory ! ? !

Homework:

Calculate ratios in **modified-gravity Kerr** ! Same results ?

Structure@horizon multipoles Big deviation from BH



$$ds^2 = -(\mathcal{Q}(H))^{-1/2}(dt+\omega)^2 + (\mathcal{Q}(H))^{1/2}\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right)$$

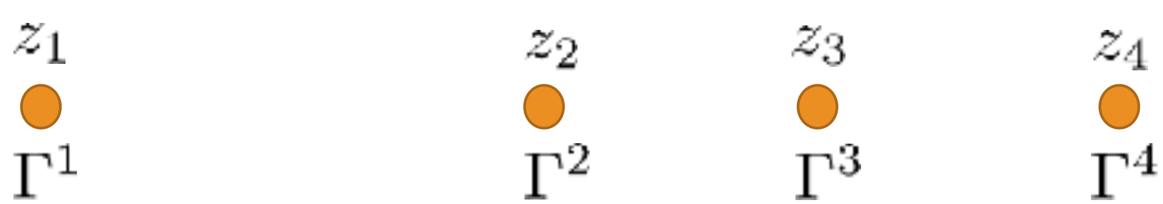
$\mathcal{Q}(V, K^I, L_I, M)$
$I = 1, \cdots, 3$

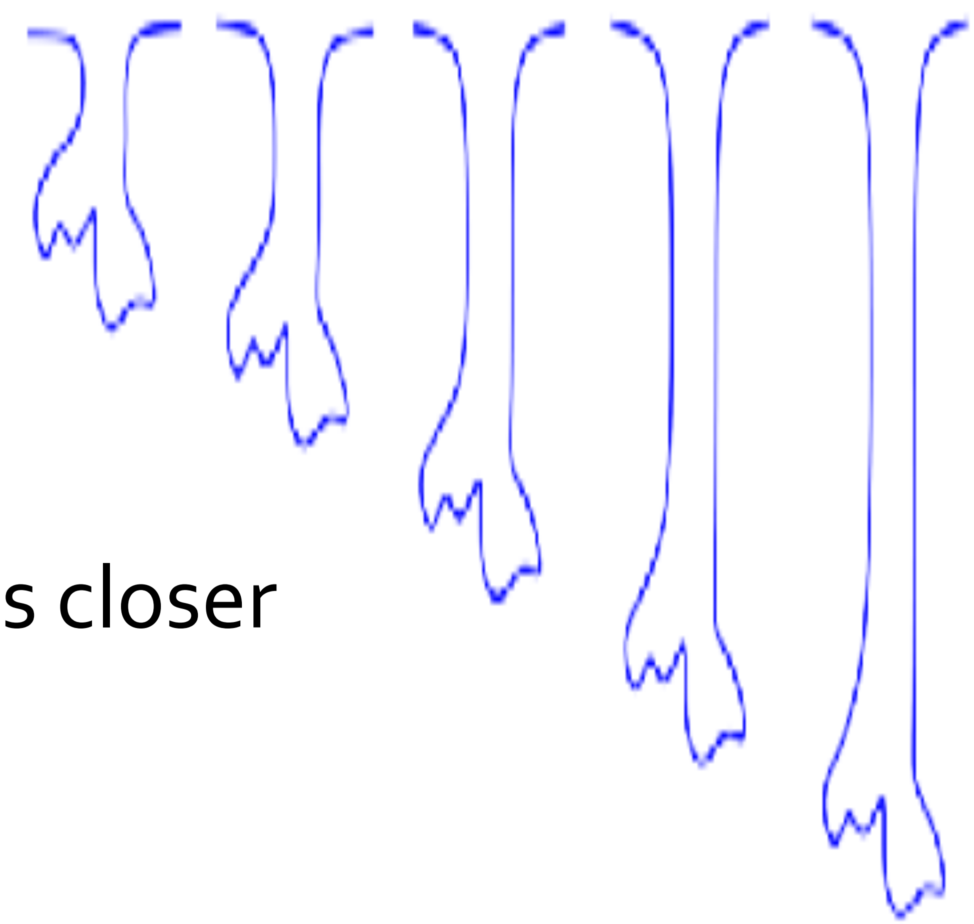
$\mathbb{R}^3$ Harmonic functions: $H = h + \sum_{i=1}^{N} \frac{\Gamma^i}{r_i}$

Charge of center i

moduli

Distance from center i





- Different from BPS BH
- Approach BH multipoles as structure gets closer and closer to the horizon

Non-susy but extremal

Ibou Bah, I.B., Pierre Heidmann, Yixuan Li, and Daniel Mayerson

- Much more general BH ⇒ lots of nontrivial multipoles
- Build almost-BPS microstates (method by Heidmann '15)
- Compute multipoles of microstates

$$4\tilde{M}_l = l^1_\infty l^2_\infty l^3_\infty Q_0 z_0^l + v_\infty \frac{|\varepsilon_{IJK}|}{2} l^I_\infty l^J_\infty \sum_{j=0}^{n} l^K_j z_j^l + v_\infty Q_0 \frac{l^J_\infty l^K_\infty}{2} \sum_{j,k=1}^{n} k^J_j k^K_k \frac{q_l^{(2)}(z_i,z_j)}{z_j z_k}$$
$$- 2m_\infty \sum_{j=0}^{n} m_j z_j^l - 2m_\infty \sum_{j=0}^{n} \alpha_j\, l z_j^{l-1} - m_\infty v_\infty l^I_\infty \sum_{j=1}^{n} k^I_j z_j^l$$
$$- m_\infty Q_0 \sum_{j=1}^{n} l^I_j k^I_j \frac{4^{l-1}}{\binom{2(l-1)}{l-1}} \frac{l}{2l-1} z_j^{l-2}$$
$$- 2m_\infty Q_0 \sum_{1\le i\neq j\le n} l^I_i k^I_j \left(\frac{q_l^{(2)}(z_i,z_j)}{2z_j(z_i-z_j)} - \frac{l}{2l-1} \frac{q_{l-1}^{(2)}(z_i,z_j)}{z_i - z_j} \right)$$
$$- 6m_\infty Q_0^2 \sum_{1\le i,j,k\le n} k^1_i k^2_j k^3_k \frac{l}{2l-1} \frac{q_{l-1}^{(3)}(z_i,z_j,z_k)}{z_i z_j z_k}.$$

Bivariate **Li** polynomial

$$q_n^{(2)}(z_i,z_j) = \frac{1}{\binom{2n}{n}} \sum_{p+q=n} \binom{2p}{p}\binom{2q}{q} z_i{}^p z_j{}^q$$

Trivariate **Li** polynomial

$$q_n^{(3)}(z_i,z_j,z_k) = \frac{1}{\binom{2n}{n}} \sum_{p+q+s=n} \binom{2p}{p}\binom{2q}{q}\binom{2s}{s} z_i{}^p z_j{}^q z_k{}^s$$

$$\tilde{S}_l = \frac{h_\infty}{2} \sum_{j=1}^{n} l^I_\infty k^I_j z_j{}^l - \frac{q}{2} \sum_{j=1}^{n} l^I_j k^I_j \frac{l}{2l-1} \frac{4^{l-1}}{\binom{2(l-1)}{l-1}} z_j{}^{l-2} - q \sum_{1\le i\neq j\le n} l^I_i k^I_j \left(\frac{l}{2l-1} \frac{q_{l-1}^{(2)}(z_i,z_j)}{z_j - z_i} - \frac{q_l^{(2)}(z_i,z_j)}{2z_j(z_j-z_i)} \right)$$
$$- 3q^2 \sum_{1\le i,j,k\le n} k^1_i k^2_j k^3_k \frac{l}{2l-1} \frac{q_{l-1}^{(3)}(z_i,z_j,z_k)}{z_i z_j z_k} - \sum_{j=0}^{n} m_j z_j{}^l - \sum_{j=0}^{n} \alpha_j\, l\, z_j{}^{l-1}.$$

Punchline

- Microstate multipoles different from BH multipoles
- Approach BH multipoles as structure gets closer and closer to the horizon

- Is this **obvious** of **highly nontrivial** ?
- Geometry looks the same as the BH geometry in the scaling limit
- Structure@horizon could have given different multipoles

- maybe extremal microstates have same multipoles as BH, but non-extremal microstates do not ?!?
- or maybe spinning structure@horizon always gives same result as BH ?!?

Exactly the same story with
Tidal Love Numbers

Conclusions

- We can build self-supporting **structure@horizon**
  - Topology and fluxes. Non-perturbative d.o.f.
- Only kosher construction of ECO. Susy, extremal, and since a few days ago also non-extremal.
- Ratios of vanishing multipoles = new window in BH
  - String Theory prediction (other theories different ?)
  - Kerr multipole ratios: constrain small deformations !

- Multipoles and Tidal Love Numbers different from BH.
- They approach BH values as structure approaches BH
- Average may differ from BH value. Nontrivial signature.