INSTITUT POLYTECHNIQUE DE PARIS

ÉCOLE POLYTECHNIQUE

REVIEW:

CELESTIAL AMPLITUDES AND ASYMPTOTIC SYMMETRIES

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THEORIE, UNIVERS ET GRAVITATION, IHP, 13 DECEMBRE 2021

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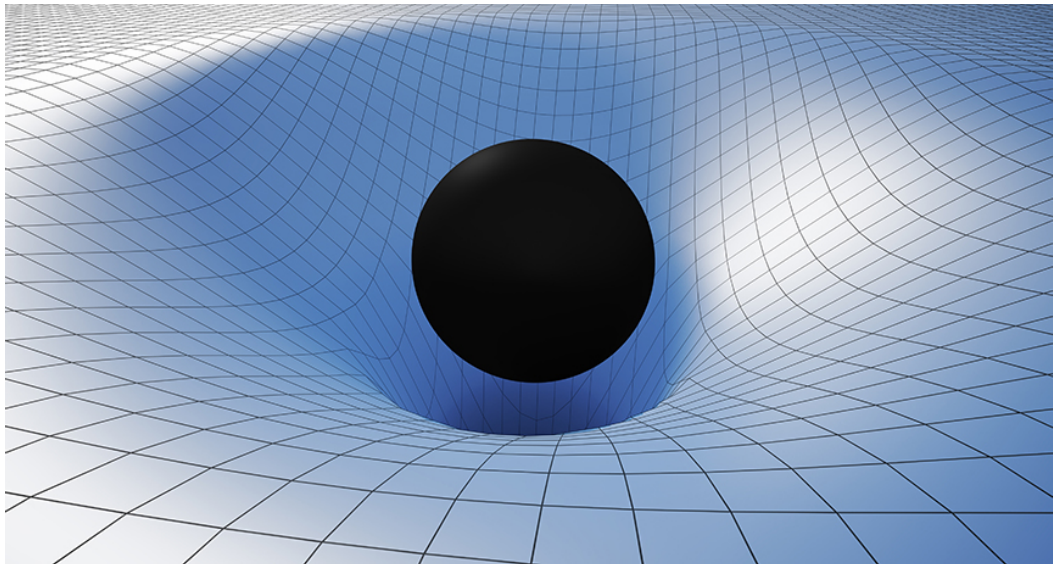
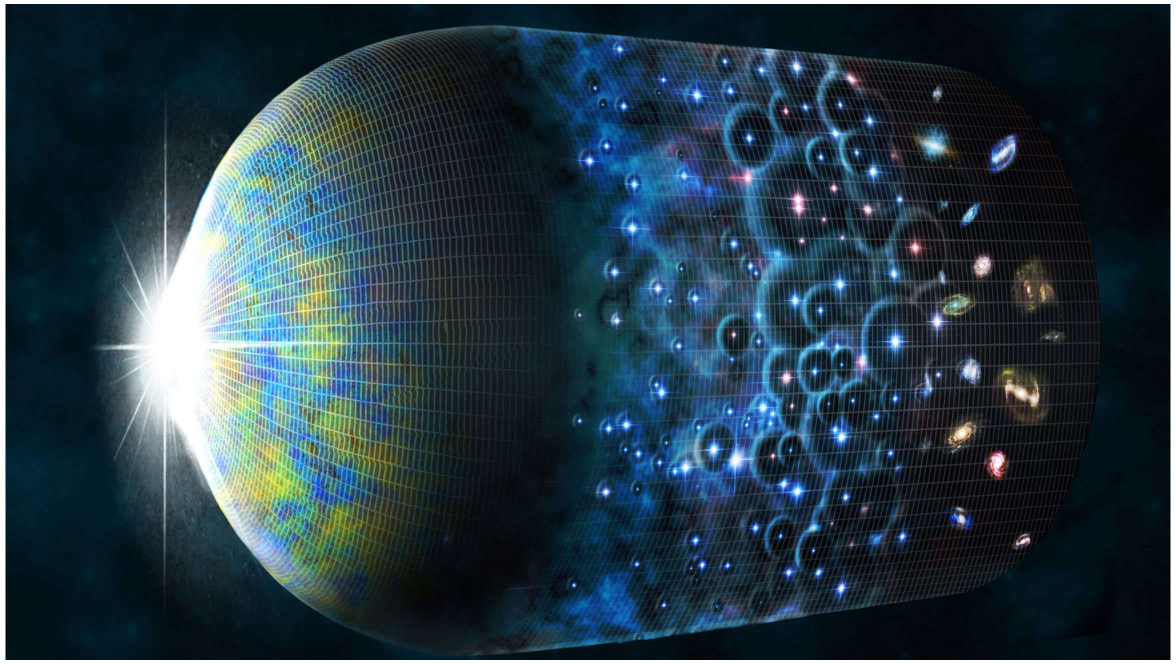
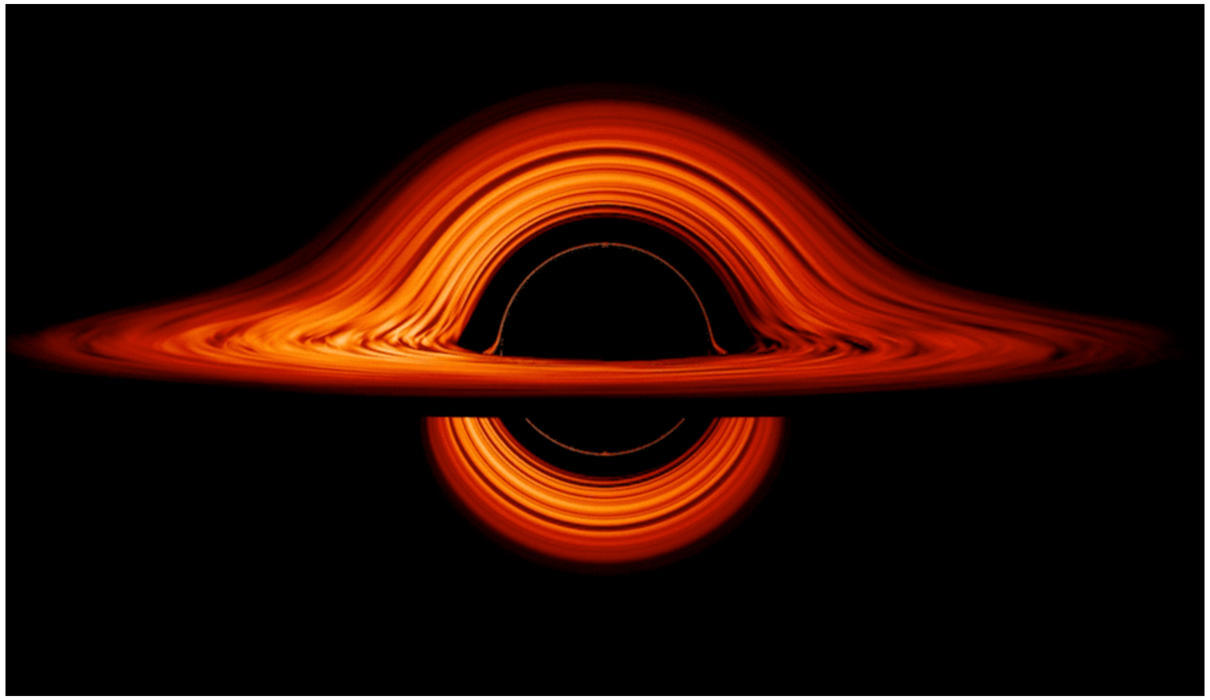
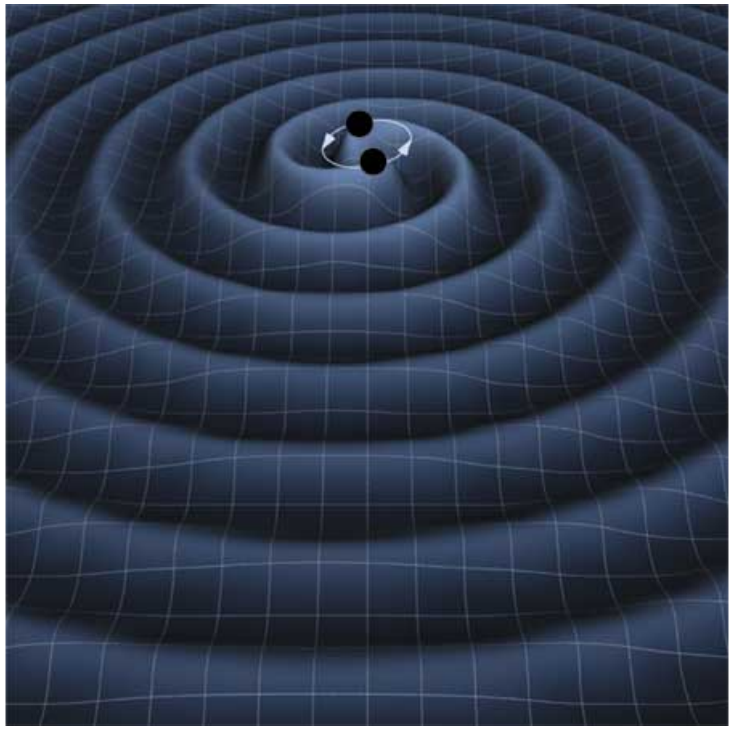
Gravity in our Universe





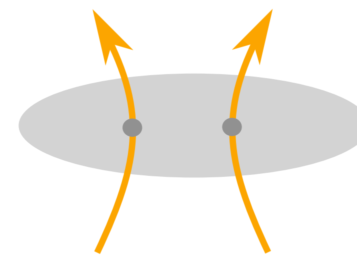


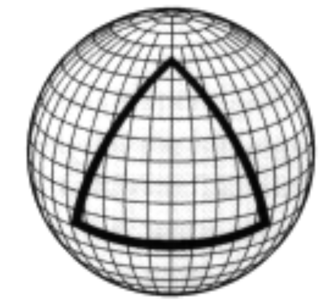


Sources: ScienceNews, NASA, Harvard Gazette , LIGO

Modelling our Universe

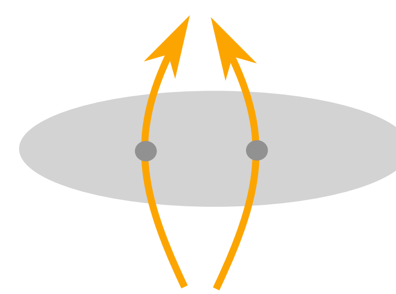
- $\Lambda > 0$ **de Sitter:** spacetime @ cosmological scale

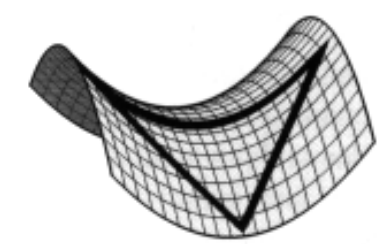




Positive Curvature

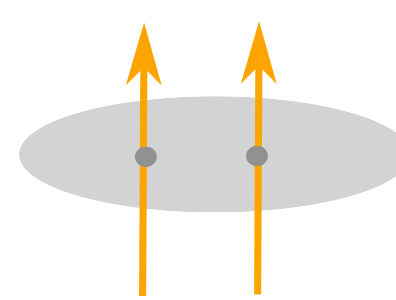
- $\Lambda < 0$ **Anti de Sitter:** spacetime @ throat of highly rotating or charged black holes

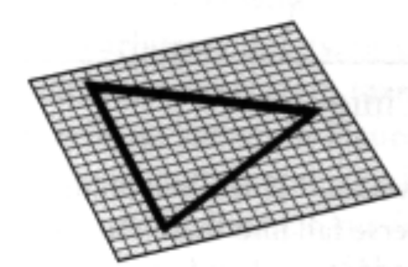




Negative Curvature

- $\Lambda = 0$ **Minkowski:** spacetime @ intermediate scales



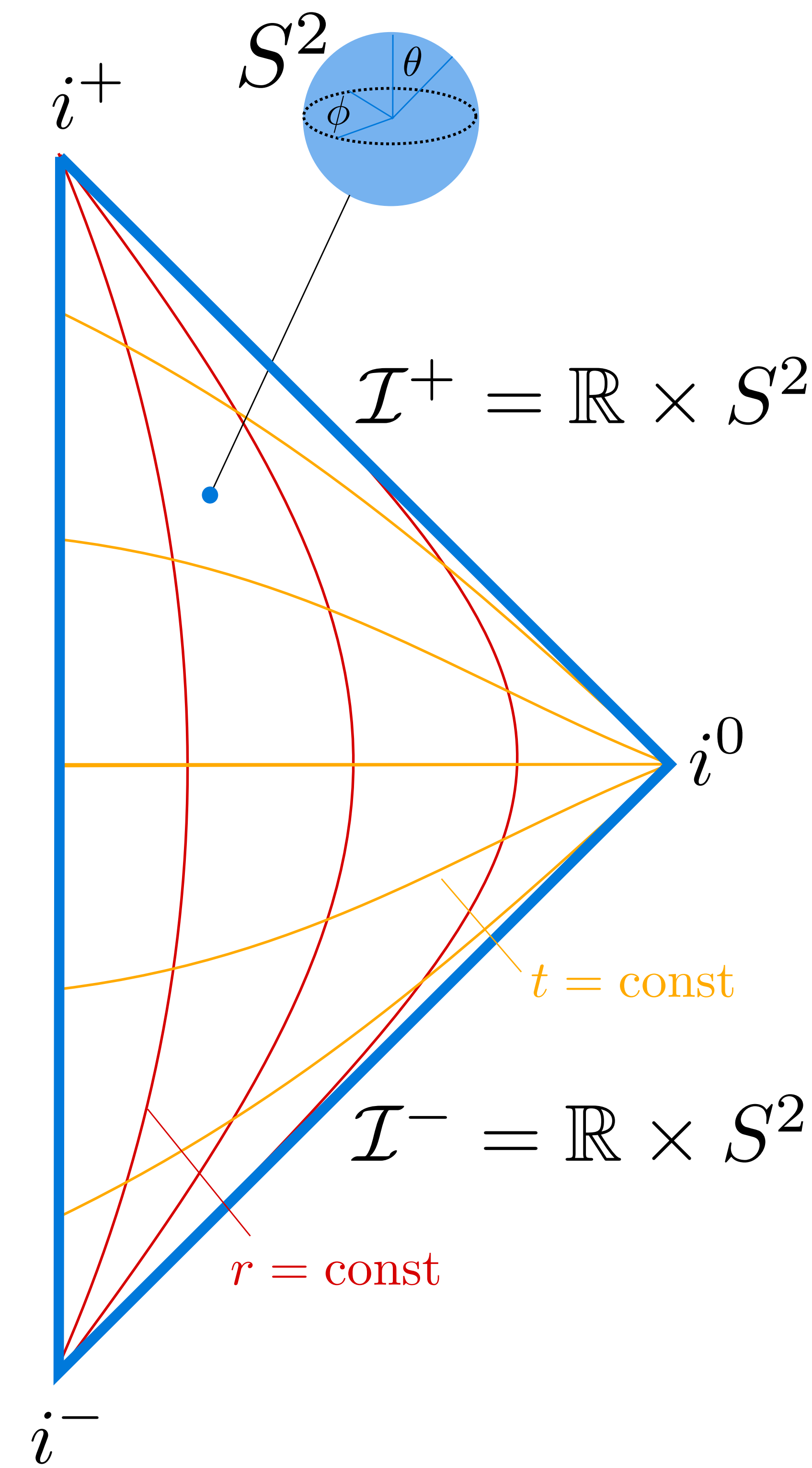


Flat Curvature

$\Lambda = 0$ quantum gravity

All Λ *Holography* conference image

Quantum gravity: metric fluctuates
but causal structure of asymptotically
flat spacetime same as Minkowski.



Outline

Quantum gravity in *asymptotically flat* spacetimes.

What are the (**asymptotic**) **symmetries**?

Ward identities

classical scattering

quantum scattering

universal behavior in infrared

Poincaré

=

Translations & Lorentz

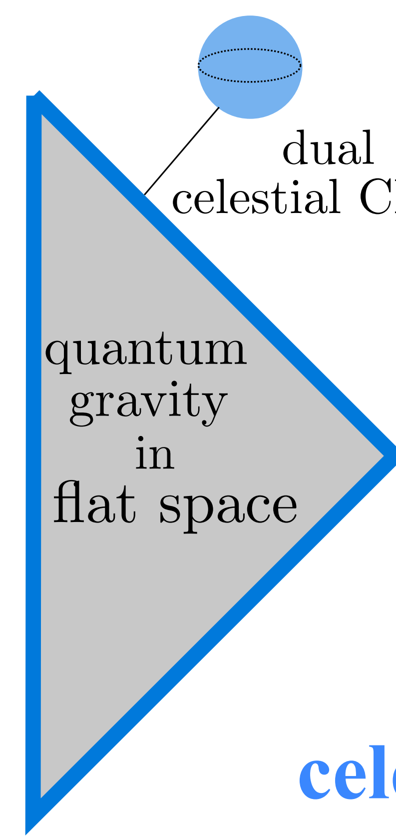
act on celestial sphere as

global conformal

∞ enhancement

BMS

local conformal



celestial holography programme

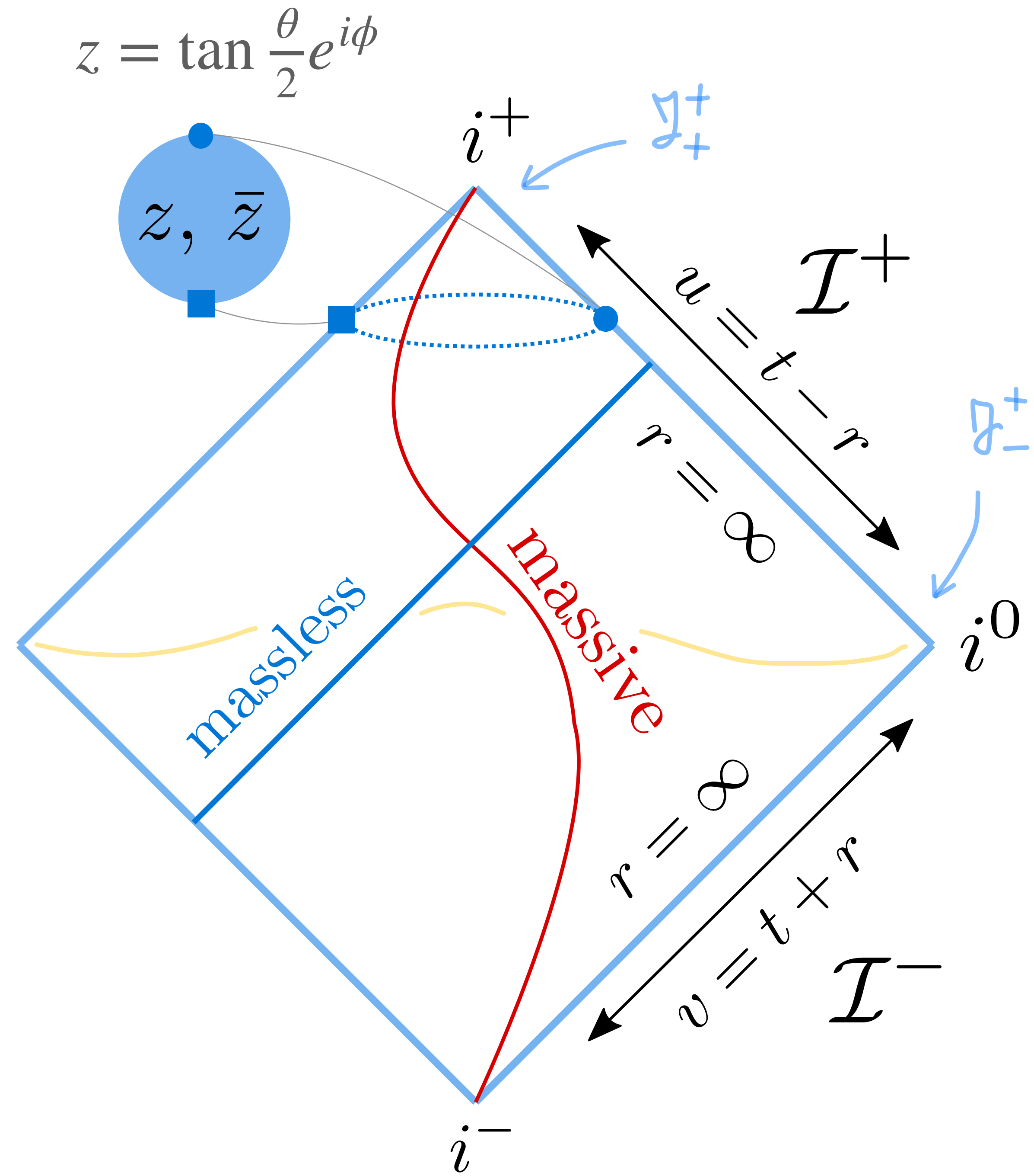
celestial amplitudes

Causal structure of flat space

Massive particles enter at past timelike infinity (i^-) and exit at future timelike infinity (i^+).

Spacelike geodesics extend to spatial infinity (i^0).

Massless particles enter at past null infinity ($\mathscr{I}^-$) and exit at future null infinity ($\mathscr{I}^+$).

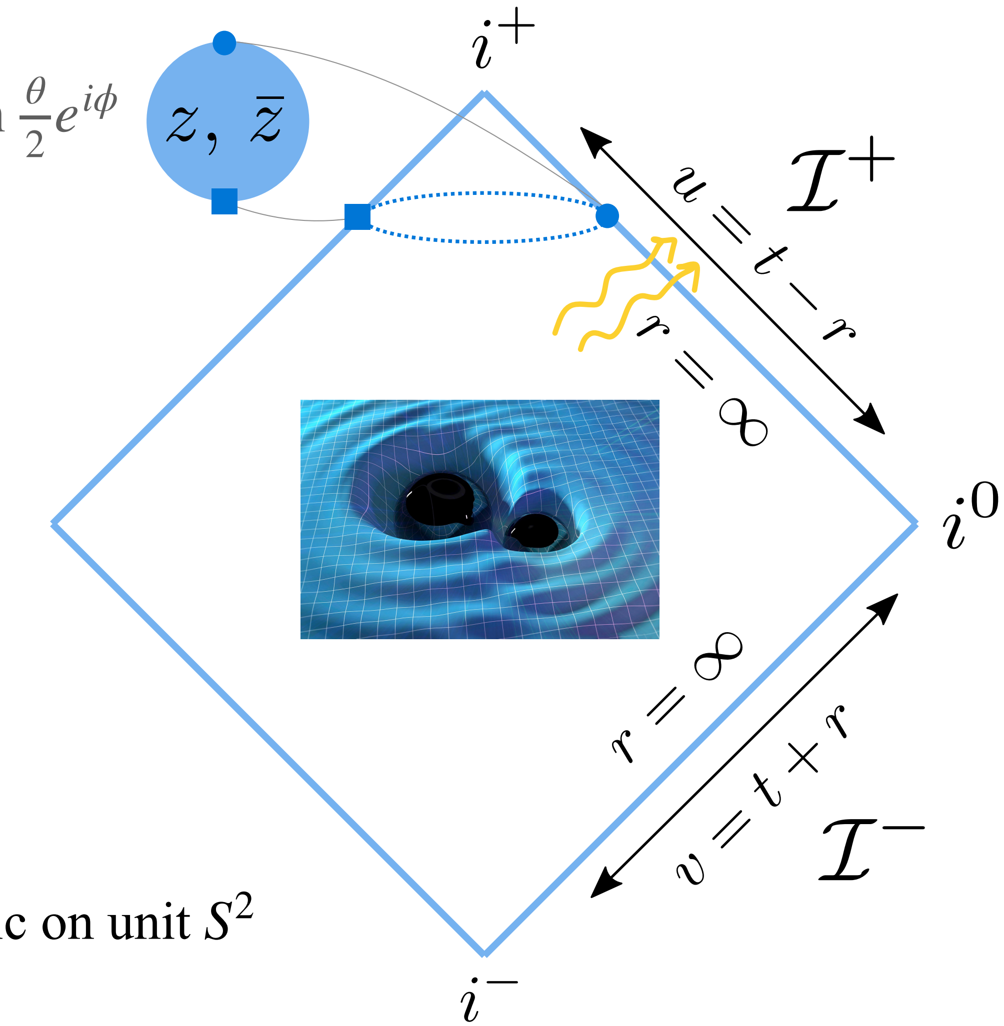


Asymptotically flat spacetime

Quantum gravity: metric fluctuates

$\Rightarrow$ flat + $\frac{1}{r}$ corrections

$z = \tan\frac{\theta}{2} e^{i\phi}$



Coordinates in which gravitational waves propagate radially outwards $r \to \infty$ at fixed u:

Bondi gauge: $\quad g_{rr} = 0\,,\quad g_{rA} = 0\,,\quad \partial_r \det\left(\frac{g_{AB}}{r^2}\right) = 0$

+ specify boundary conditions:

$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$ ... metric on unit S^2

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

$$+\frac{2m_B}{r}du^2 + rC_{zz}dz^2 + \left[D^zC_{zz} + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}D_z(C_{zz}C^{zz})\right)\right]dudz + c.c.$$

Radiative data at future null infinity $\mathcal{I}^+$: $\{m_B, N_z, C_{zz}\}$ [Bondi,van der Burg,Metzner;Sachs'62]

Bondi mass aspect

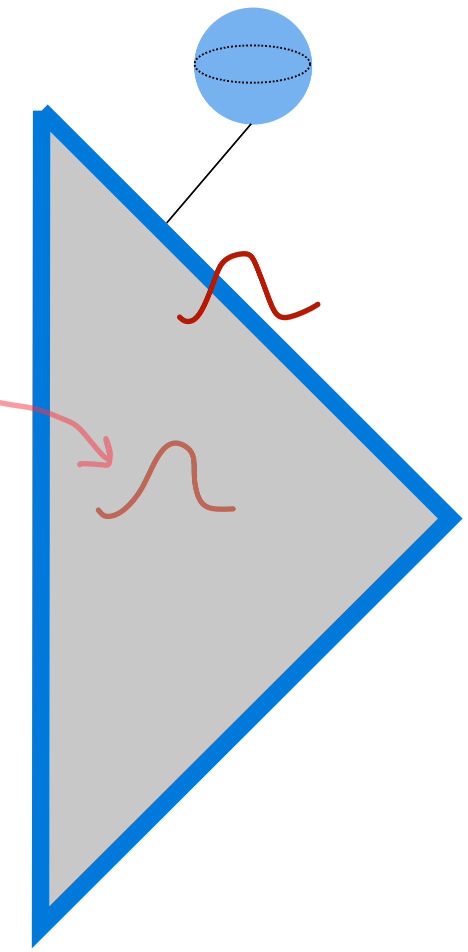
angular momentum aspect

gravitational data $\Rightarrow$ Bondi news

$$N_{zz} = \partial_u C_{zz}$$

gravitational waves

Asymptotic symmetries



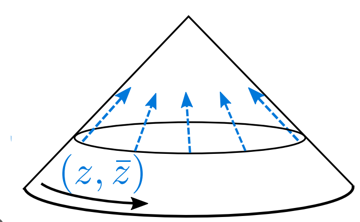
Asymptotic symmetry group = $\dfrac{\text{allowed diffeos}}{\text{trivial diffeos}}$

$\mathcal{L}_\xi g_{\mu\nu} \approx 0 \quad$ as $r \to \infty$

Diffeos that preserve Bondi gauge and fall-offs:

$$\xi = (1 + \tfrac{u}{2r})Y^z\partial_z - \tfrac{u}{2r}D^{\bar{z}}D_zY^z\partial_{\bar{z}} - \tfrac{1}{2}(u+r)D_zY^z\partial_r + \tfrac{u}{2}D_zY^z\partial_u + c.c.$$

$$+ f\partial_u - \tfrac{1}{r}(D^z f\partial_z + D^{\bar{z}}f\partial_{\bar{z}}) + D^zD_zf\partial_r$$



Translations: $\quad f \in \{1, z, \bar{z}, z\bar{z}\}$ $\quad$ e.g. $\xi_{f=1}|_{\mathscr{I}^+} = \partial_u$

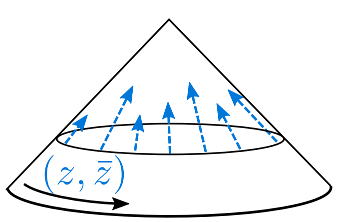
Lorentz: $\quad Y^z \in \{1, z, z^2, i, iz, iz^2\}$ $\quad$ global CKVs $\partial_{\bar{z}}Y^z = 0$

4+6=10 Poincare generators

$\downarrow$

∞ generators!

[Bondi,van der Burg,Metzner,Sachs'62]



BMS = Supertranslations $\ltimes$ Lorentz

$f(z, \bar{z})$

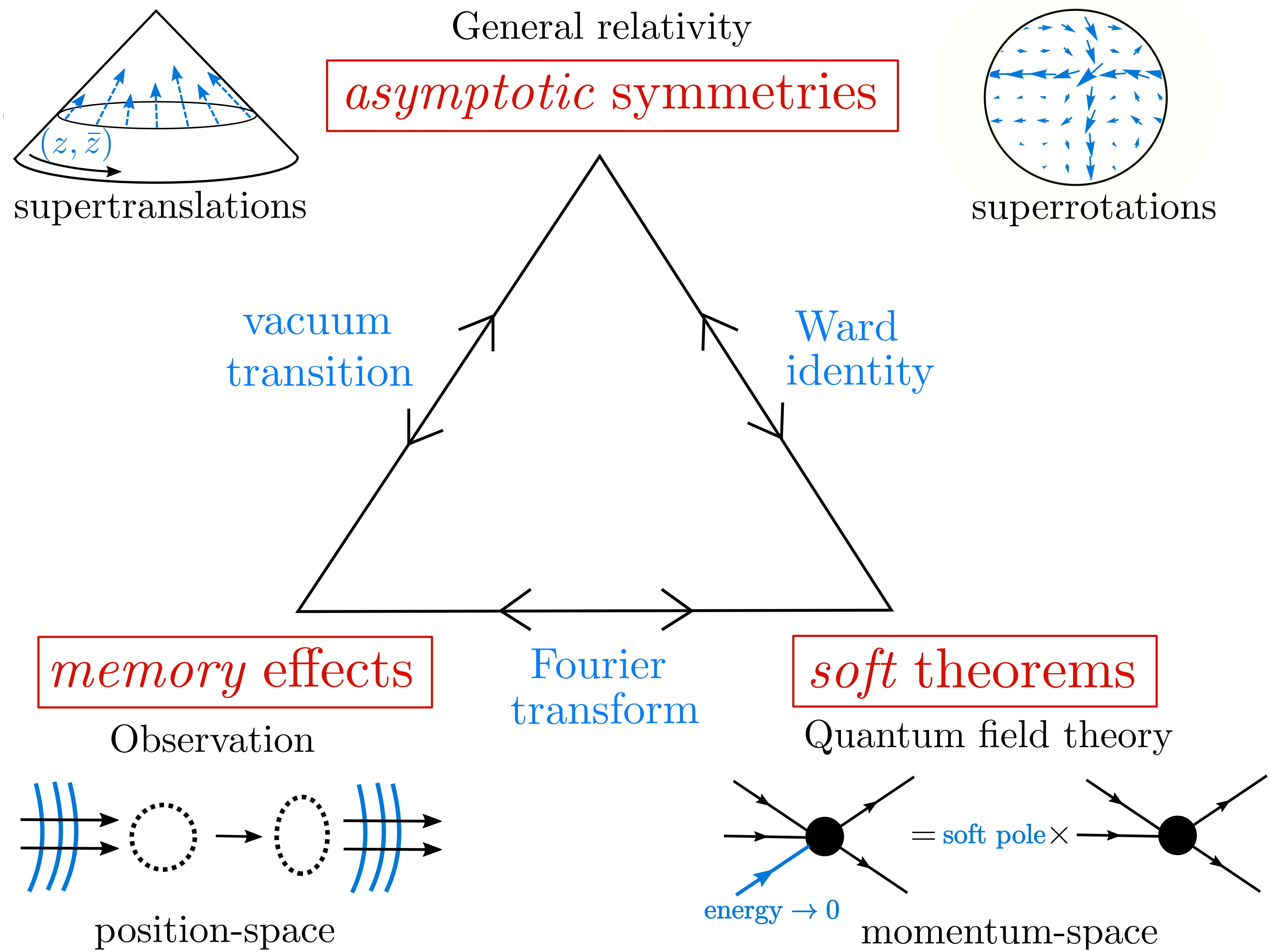
extended / generalized BMS = Supertranslations $\ltimes$ Superrotations

[deBoer,Solodukhin'03]
[Barnich,Troessaert'11-12']
[Campiglia,Laddha'15]

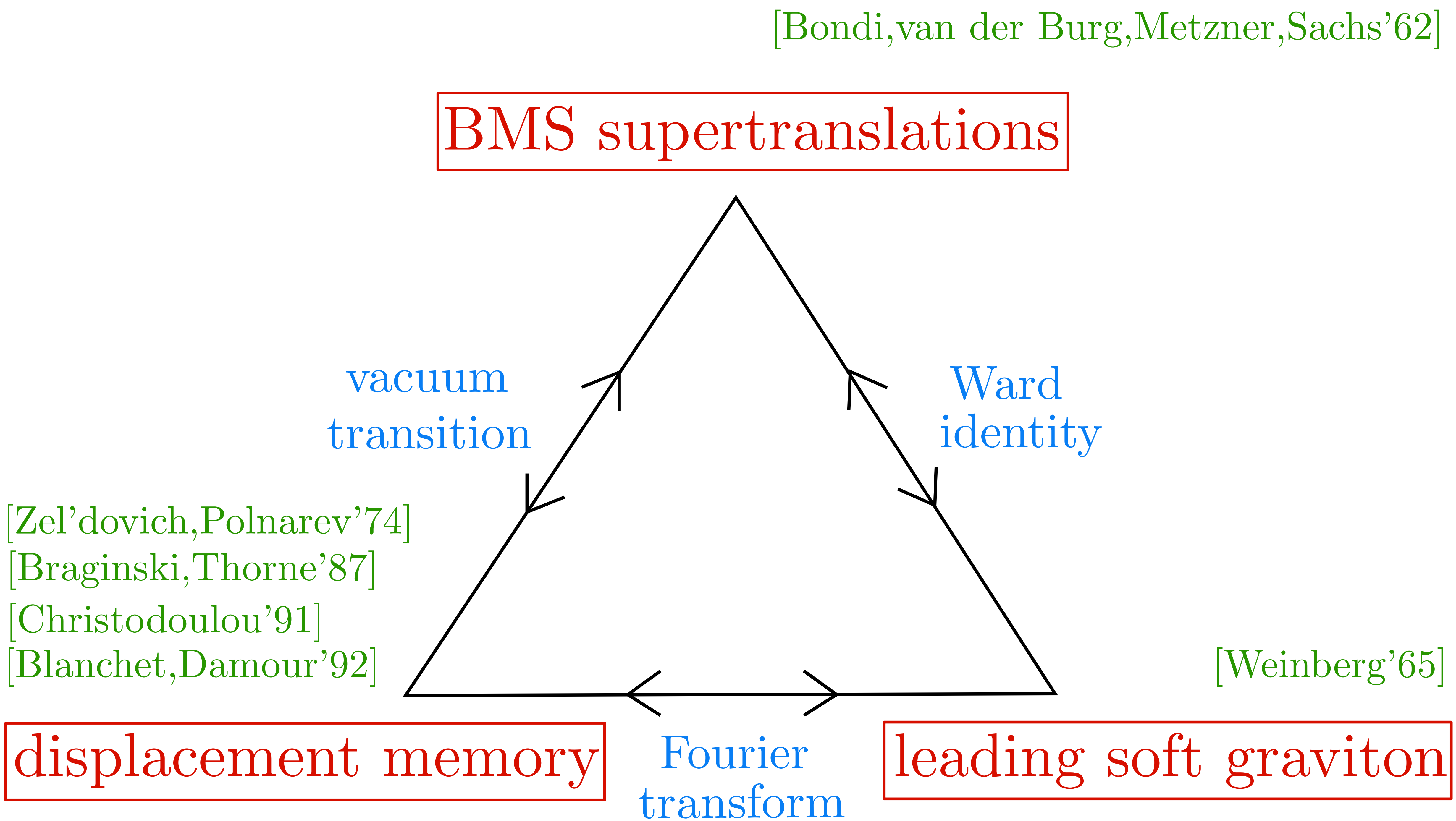
$Y^z(z)$ $\quad$ local CKVs $\longrightarrow$ Virasoro

$Y^z(z, \bar{z})$ $\quad\longrightarrow$ $\mathrm{Diff}(S^2)$

Gravity in the infrared



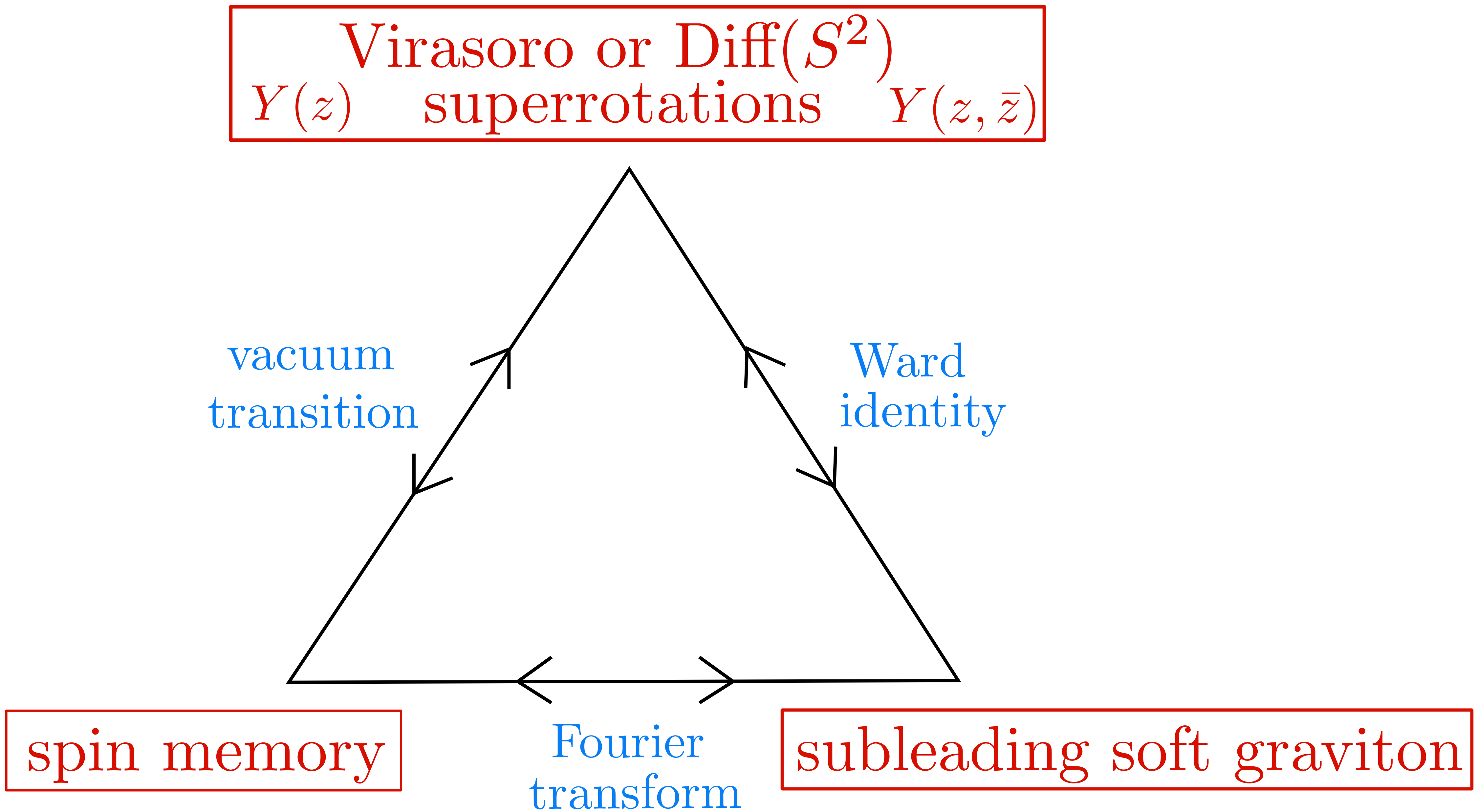
IR triangle for gravity - "leading"



IR triangle for gravity - "subleading"

[Banks'03]
[Barnich,Troessaert'09-'11]

[Campiglia,Laddha'15]
[Compère,Fiorucci,Ruzziconi'18]
[Campiglia,Peraza'20]



[Pasterski,Strominger,Zhiboedov'15]

[Cachazo,Strominger'14]

BMS symmetry

Supertranslations and superrotations change the physical state.

$\mapsto$ ***memory effect*** [Strominger,Zhiboedov'14] [Pasterski,Strominger,Zhiboedov'15]

BMS charge:

$$Q^{+}[f,Y] = \frac{1}{8\pi G}\int_{\mathcal{I}^{+}_{-}} d^2z\sqrt{\gamma}[2fm_{\rm B} + Y^A N_A]$$

$$\parallel$$

$$Q^{-}[f,Y] = \frac{1}{8\pi G}\int_{\mathcal{I}^{-}_{+}} d^2z\sqrt{\gamma}[2fm_{\rm B} + Y^A N_A]$$

antipodal matching

[Strominger'13]

Conserved charges commute with the $\mathcal{S}$-matrix:

$$\langle out\,|\,Q^{+}\mathcal{S} - \mathcal{S}Q^{-}\,|\,in\rangle \overset{!}{=} 0$$

$$Q^{\pm} = \underbrace{Q_S^{\pm}}_{\text{soft}} + \underbrace{Q_H^{\pm}}_{\text{hard}}$$

$$\langle out\,|\,Q_S^{+}\mathcal{S} - \mathcal{S}Q_S^{-}\,|\,in\rangle = -\,\langle out\,|\,Q_H^{+}\mathcal{S} - \mathcal{S}Q_H^{-}\,|\,in\rangle$$

soft graviton insertion

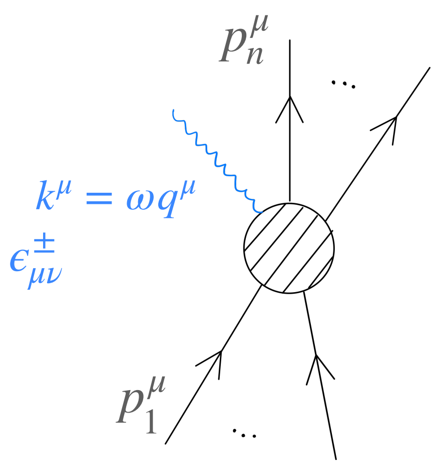
matter

Soft theorems

Universal behavior of gauge theory amplitudes when energy of gauge boson is taken *soft*, i.e. to zero. Let's look at gravity:

$$\langle out \,|\, a_\pm \mathscr{S} \,|\, in \rangle = \mathscr{A}^{\pm}_{n+1} \overset{\omega \to 0}{=} \left[\omega^{-1} S^{(0)\pm}_n + \omega^0 S^{(1)\pm}_n + \mathcal{O}(\omega)\right] \mathscr{A}_n, \qquad \mathscr{A}_n = \langle out \,|\, \mathscr{S} \,|\, in \rangle$$

[Weinberg'65] [Cachazo,Strominger'14]

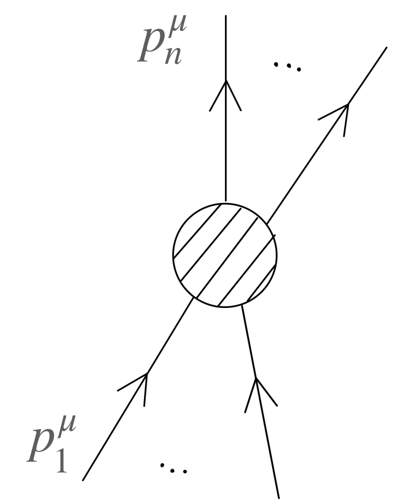


j-th momentum

$$S^{(0)\pm}_n = \frac{\kappa}{2} \sum_{j=1}^{n} \frac{(p_j \cdot \epsilon^{\pm})^2}{p_j \cdot q} \qquad \epsilon^{\pm}_{\mu\nu} = \epsilon^{\pm}_{\mu} \epsilon^{\pm}_{\nu} \qquad \epsilon^{\pm} \cdot q = 0$$

$$\kappa = \sqrt{32\pi G} \qquad S^{(1)\pm}_n = -i \frac{\kappa}{2} \sum_{j=1}^{n} \frac{(p_j \cdot \epsilon^{\pm})(q \cdot J_j \cdot \epsilon^{\pm})}{p_j \cdot q}$$

j-th angular momentum



Soft graviton theorem implies BMS symmetry:

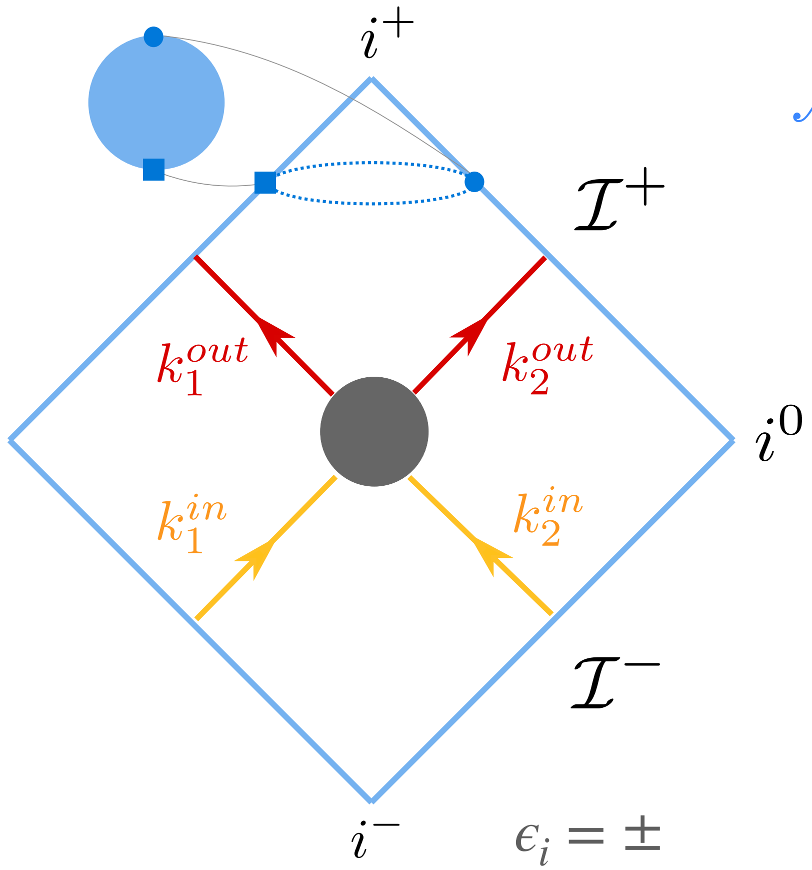
$$Q^+[f, Y] = Q^-[f, Y]$$

[Strominger'13] [He,Lysov,Mitra,Strominger'14]

4D $\mathcal{S}$-matrix ⇒ 2D correlator

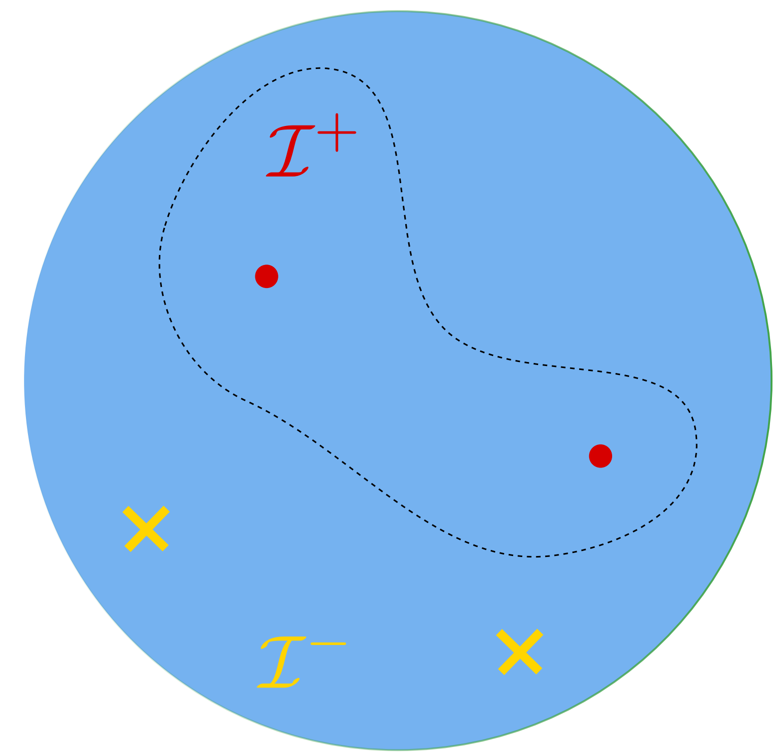
[de Boer,Solodhukin'03] [Cheung,de la Fuente,Sundrum'16][Pasterski,Shao,Strominger'16]

For massless scattering the map is a Mellin transform:



$$\mathcal{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta}(\,.\,)$$

$$=$$



$\epsilon_i = \pm$

$$k_i^\mu = \epsilon_i \omega_i q_i^\mu(w_i, \bar{w}_i)$$

$$\mathcal{A}_n(\omega_i, \ell_i, w_i, \bar{w}_i) \equiv \langle out \,|\, \mathcal{S} \,|\, in \rangle$$

momentum-space amplitude

$$\widetilde{\mathcal{A}}(\Delta_i, J_i, w_i, \bar{w}_i) \equiv \langle \prod_{i=1}^{n} \mathcal{O}^{\epsilon_i}_{\Delta_i, J_i}(w_i, \bar{w}_i) \rangle$$

celestial amplitude

Conformal primary wavefunctions

[Pasterski,Shao'17]

massless: $$\Phi^{s=|J|}_{\Delta,J}(X^\mu; w, \bar{w}) \simeq \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta \epsilon_{\mu_1\ldots\mu_s} e^{\pm i\omega q\cdot X}$$

on-shell plane waves

4D spin-s field under Lorentz transformations

2D conformal primary with conformal dimension Δ and spin J

Lorentz transformation

bulk point $X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu$

conformal transformation

boundary point $w \mapsto \dfrac{aw+b}{cw+d}$ $\quad \bar{w} \mapsto \dfrac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}$

$ad - bc = 1 = \bar{a}\bar{d} - \bar{b}\bar{c}$

$$\Phi^s_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J}\underbrace{D_s(\Lambda)}\Phi^s_{\Delta,J}(X^\mu; w, \bar{w})$$

3+1D spin-s representation of the Lorentz algebra

2D conformal primary operators: $\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) \equiv i(\hat{O}^s(X), \Phi^{s,\pm}_{\Delta^*,-J}(X; w, \bar{w}))_\Sigma$

Radiative: $J = \pm s$ & solve the linearized eom for massless spin-s particles

Generalized: $|J| \leq s$ & allow sources and distributions [Pasterski,AP'20]

Conformal primary backgrounds

Conformal primary wavefunctions satisfy Kerr-Schild double copy:

[Pasterski,AP'20]

$$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{m_\mu m_\nu \varphi^\Delta}_{h_{\Delta,J=\pm 2;\mu\nu}}$$

Kerr-Schild vector m^μ null and geodesic wrt $\eta_{\mu\nu}$ and $g_{\mu\nu}$. [Monteiro,O'Connell,White'14]

$\Rightarrow h_{\Delta,J=\pm 2;\mu\nu}$ gives *exact* radiative *solution to Einstein's equations!*

Exact *generalized* conformal primary solution to Einstein's equations:

- Aichelburg-Sexl shockwave or ultraboosted Schwarzschild [Aichelburg,Sexl'71]

$$g_{\mu\nu} = \eta_{\mu\nu} - \underbrace{4G_N \alpha q_\mu q_\nu \log(X^2)\delta(q\cdot X)}_{h^{gen}_{\Delta=-1,J=0;\mu\nu}} \qquad E = \alpha q^0$$

generalized conformal primary metric

- Kerr gyraton or ultraboosted Kerr see [Cristofoli'20] [Arkani-Hamed,Huang,O'Connell'20]...

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N \alpha q_\mu q_\nu \log(|X^2 - a^2|)\delta(q\cdot X) \qquad a^\mu = s^\mu/m$$

Spectrum vs Symmetries

[Pasterski,Shao'17]

Conformal basis of finite energy wavefunctions for $\Delta \in 1 + i\mathbb{R}$.

After summing over all energies what are soft particles?

Notion of *soft* particle $\mapsto$ ***conformally soft*** particle $\Delta = 1$. [Donnay,AP,Strominger'18]

only tip of ∞ tower!

Translations shift the conformal dimension:

[Donnay,AP,Strominger'18]
[Stieberger,Taylor'18]

$$\mathscr{P}^\mu = q^\mu e^{\partial_\Delta} \Leftrightarrow \Delta \mapsto \Delta + 1$$

$$\int_0^\infty \frac{d\omega}{\omega}\omega^\Delta \omega = \int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta+1}$$

Takes finite energy $\Delta \in 1 + i\mathbb{R}$ off the principal series.

Celestial amplitudes make conformal symmetry manifest,
at the expense of obscuring translation symmetry.

Conformally soft limits

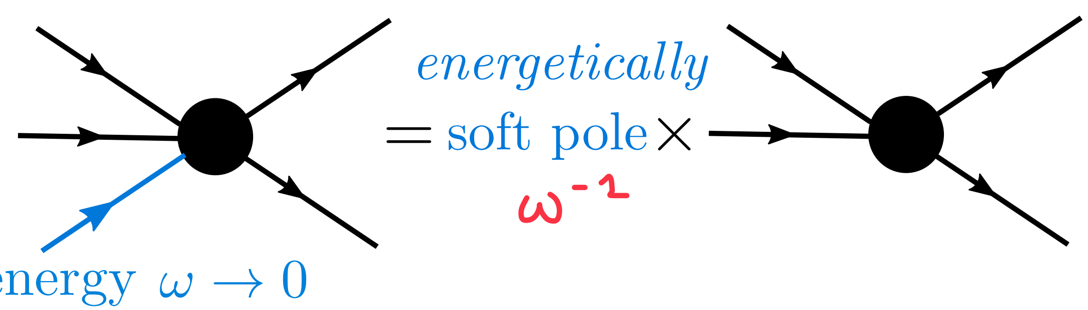
Soft graviton theorem:

$$\lim_{\omega\to 0} \mathscr{A}^{\pm}_{n+1} = \left[\omega^{\overset{\#}{-1}} S_n^{(0)\pm} + \omega^{\overset{\#}{0}} S_n^{(1)\pm} + \mathcal{O}(\omega)\right] \mathscr{A}_n$$

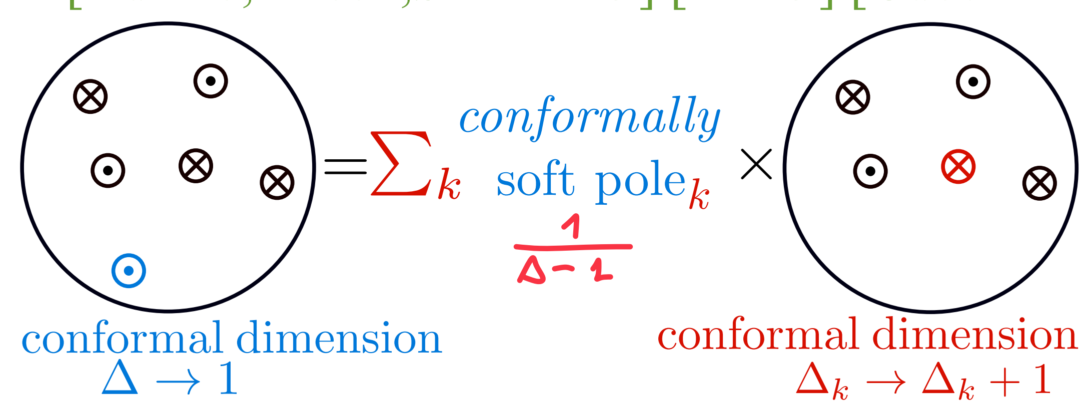
$$\int_0^{\omega_*} \frac{d\omega}{\omega}\omega^{\Delta+\#} = \frac{\omega_*^{\Delta+\#}}{\Delta+\#}$$

Soft limits at different orders in $\omega \to 0 \Leftrightarrow$ "conformally soft" Δ poles.

Leading soft graviton: [Weinberg'65]



[Adamo,Mason,Sharma'19] [AP'19] [Guevara'19]



$$\mathscr{P}_w \mathcal{O}_\omega(z,\bar{z}) \sim \frac{\omega}{w-z}\mathcal{O}_\omega(z,\bar{z})$$

$$\mathscr{P}_w \mathcal{O}_\Delta(z,\bar{z}) \sim \frac{1}{w-z}\mathcal{O}_{\Delta+1}(z,\bar{z})$$

[Donnay,AP,Strominger'18]

supertranslation current

unusual looking OPE!

[Strominger'13]

Asymptotic symmetries

Continue finite energy $\Delta = 1 + i\mathbb{R}$ to conformally soft $\Delta \in \frac{1}{2}\mathbb{Z}$.

Conformally soft limits of celestial amplitudes as ***asymptotic symmetries***:

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi_{\Delta^*,-J})_\Sigma$$

asymptotic symmetry generator

pure gauge

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP'21]

$s = \|J\|$	Δ	$\tilde{\Delta} = 2 - \Delta$	energetically soft pole	celestial current	asymptotic symmetry
1	1	1	ω^{-1}	$\mathcal{J}$	large U(1)
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\omega^{-\frac{1}{2}}$	$\mathcal{S}$ & $\tilde{\mathcal{S}}$	large SUSY
2	1	1	ω^{-1}	$\mathcal{P}$	supertranslation
2	0	2	ω^{0}	$\mathcal{T}$ & $\tilde{\mathcal{T}}$	superrotation

2D stress tensor!

An ∞ of new celestial symmetries

Continue finite energy $\Delta = 1 + i\mathbb{R}$ to conformally soft $\Delta \in \frac{1}{2}\mathbb{Z}$.

Conformally soft limits of celestial amplitudes as ***new celestial symmetries***:

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi_{\Delta^*,-J})_\Sigma$$

conformally soft

but ***not*** pure gauge

- $\exists$ more (conformally) soft theorems:

  e.g. subsubleading soft graviton $\Delta = -1$

  [Fotopoulos,Taylor,Stieberger,Zhu'20]
  [Adamo,Mason,Sharma'19][Guevara,19]

- $\exists$ ∞ many more conformally soft $\mathcal{O}_{\Delta=-n,J}$

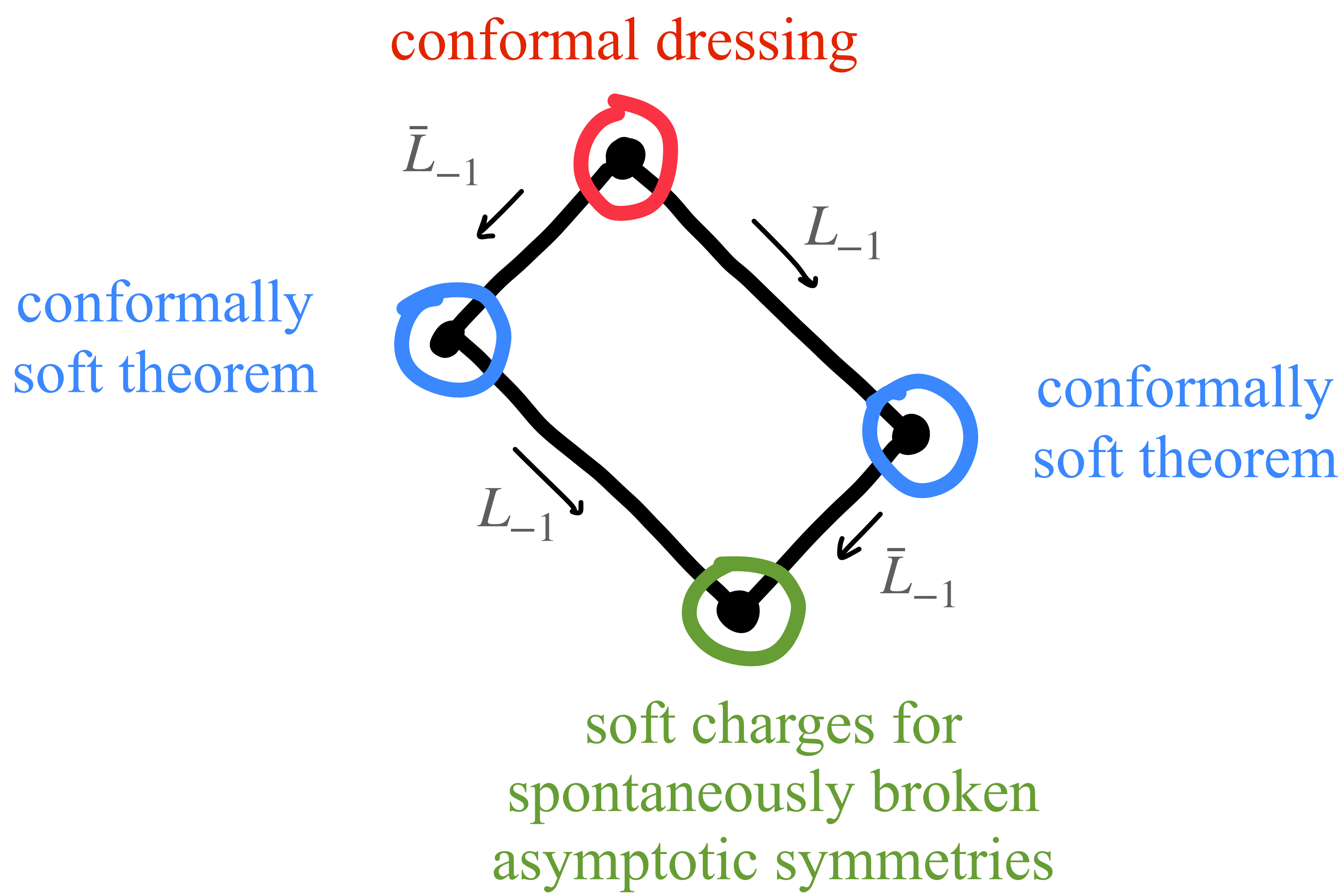
  e.g. $w_{1+\infty}$ symmetry

  [Strominger'21]
  [Guevara,Himwich,Pate,Strominger'21]
  [Himwich,Pate,Singh'21] [Jiang'21]

$\Rightarrow$ Conformally soft limits without clear asymptotic symmetry interpretation.

Celestial diamonds

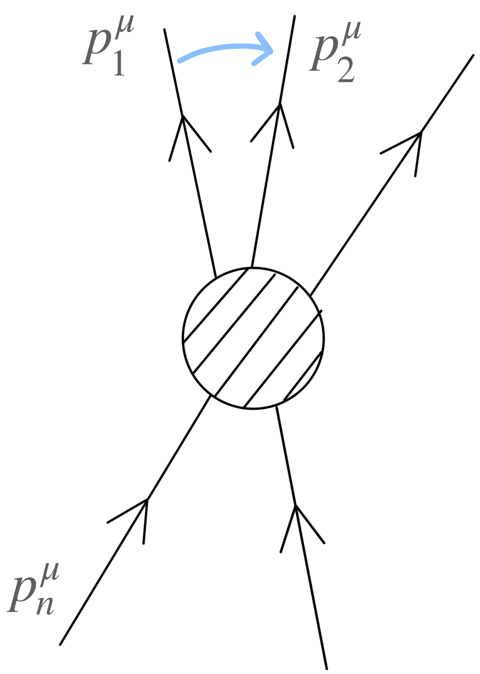
All celestial symmetries can be described in unified celestial CFT framework via $SL(2,\mathbb{C})$ conformal multiplets or "celestial diamonds". [Pasterski,AP,Trevisani'21] [Banerjee'18] [Banerjee,Pandey,Paul'19]



● $SL(2,\mathbb{C})$ primary

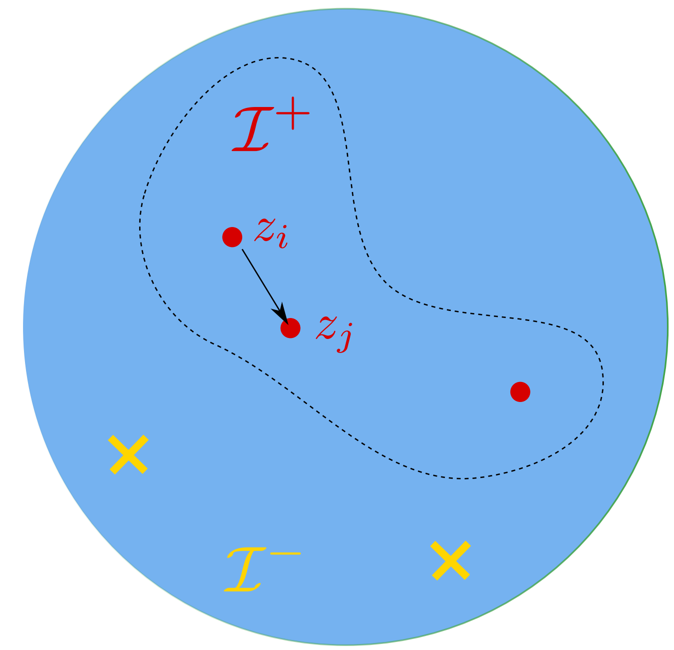
↙ ↘ $L_{-1}, \bar{L}_{-1}$ generate descendants

Collinear limits ⇒ celestial OPEs



The collinear limit of 4D amplitudes is captured by the 2D celestial OPE:

$$p_i^\mu \,||\, p_j^\mu \qquad \leftrightarrow \qquad z_{ij} \equiv z_i - z_j \to 0$$



Brute force from Mellin transform of collinear amplitudes limit:

$$\mathscr{A}_{\ell_1,\ldots,\ell_n}(p_1,\ldots,p_n) \overset{z_i \to z_j}{\to} \sum_{\ell} \mathrm{Split}^{\ell}_{\ell_i \ell_j}(p_i,p_j)\mathscr{A}_{\ell_1\ldots\ell\ldots\ell_n}(p_1,\ldots,P,\ldots,p_n)$$

$$P^\mu = p_i^\mu + p_j^\mu \qquad \omega_p = \omega_i + \omega_j$$

$$\lim_{z_i \to z_j} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\mathcal{O}_{\Delta_j,J_j}(z_j,\bar{z}_j) = \int_0^\infty \frac{d\omega_i}{\omega_i}\omega_i^{\Delta_i}\int_0^\infty \frac{d\omega_j}{\omega_j}\omega_j^{\Delta_j}\mathrm{Split}^{\ell}_{\ell_i\ell_j}(p_i,p_j)\,|\,P,\ell\rangle + \ldots$$

[Fan,Fotopoulos,Taylor'19]
[Fotopoulos,Stieberger,Taylor,Zhu'19]

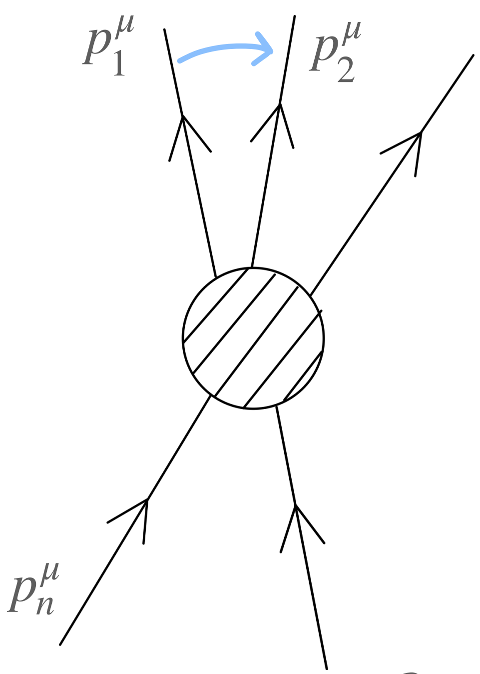
$$\mathrm{Split}^2_{22}(p_i,p_j) = -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}}\frac{\omega_P^2}{\omega_i\omega_j}$$

Euler beta fct $\quad B(x,y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

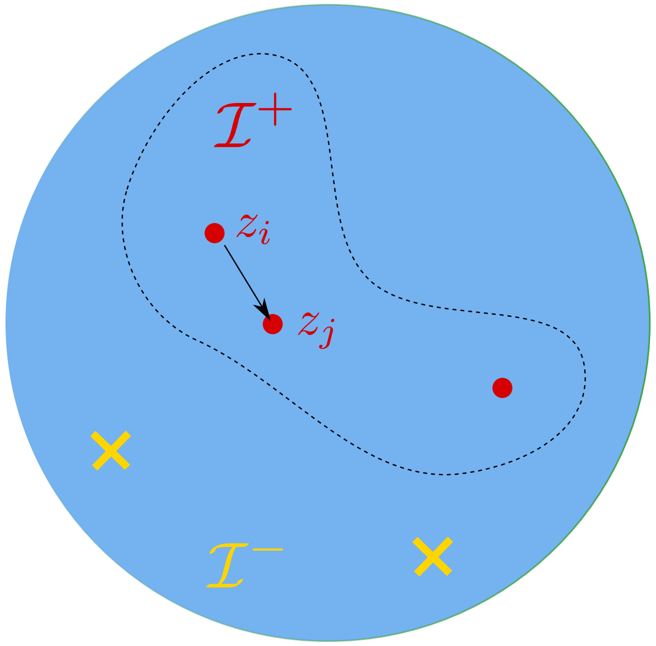
celestial OPE of gravitons:

$$\mathcal{O}_{\Delta_i,+2}(z_i)\mathcal{O}_{\Delta_j,+2}(z_j) \sim -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}}B(\Delta_i-1,\Delta_j-1)\mathcal{O}_{\Delta_i+\Delta_j;+2}(z_j)$$

Collinear limits ⇒ celestial OPEs



The collinear limit of 4D amplitudes is captured by the 2D celestial OPE:



$$p_i^\mu || p_j^\mu \qquad \leftrightarrow \qquad z_{ij} \equiv z_i - z_j \to 0$$

From symmetry considerations:
[Pate,Raclariu,Strominger,Yuan'19]

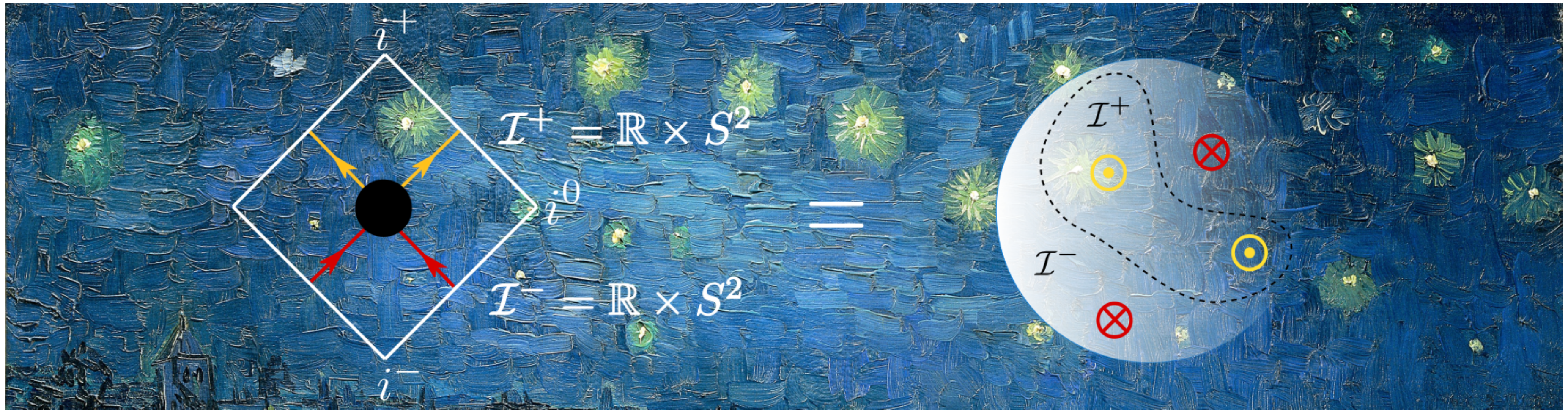
$$\mathcal{O}_{\Delta_i,+2}(z_i,\bar{z}_i)\mathcal{O}_{\Delta_j,+2}(z_j,\bar{z}_j) \sim \frac{\bar{z}_{ij}}{z_{ij}} C(\Delta_i,\Delta_j)\mathcal{O}_{\Delta_i+\Delta_j,+2}(z_j,\bar{z}_j)$$

- Translation invariance: $C(\Delta_i,\Delta_j) = C(\Delta_i+1,\Delta_j) + C(\Delta_i,\Delta_j+1)$
- Residue of pole at $\Delta_i \to 1$ fixed by leading soft graviton theorem: $\lim_{\Delta_i \to 1} C(\Delta_i,\Delta_j) \sim -\frac{\kappa}{2}\frac{1}{\Delta_i - 1}$
- Subsubleading soft graviton: extra recursion relation

celestial OPE of gravitons:

$$\mathcal{O}_{\Delta_i,+2}(z_i)\mathcal{O}_{\Delta_j,+2}(z_j) \sim -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}} B(\Delta_i - 1,\Delta_j - 1)\mathcal{O}_{\Delta_i+\Delta_j,+2}(z_j)$$

Outlook: celestial amplitudes



S-matrix as celestial correlator:

- makes asymptotic $\supset$ conformal symmetries manifest
- reorganizes (conformally) soft and collinear behavior:

Collinear limits $\Rightarrow$ OPE data for celestial CFT.

Conformally soft sector: ∞ (asymptotic) symmetries & conformal dressings.

- anti-Wilsonian paragidm: probes all energies, sensitive to UV

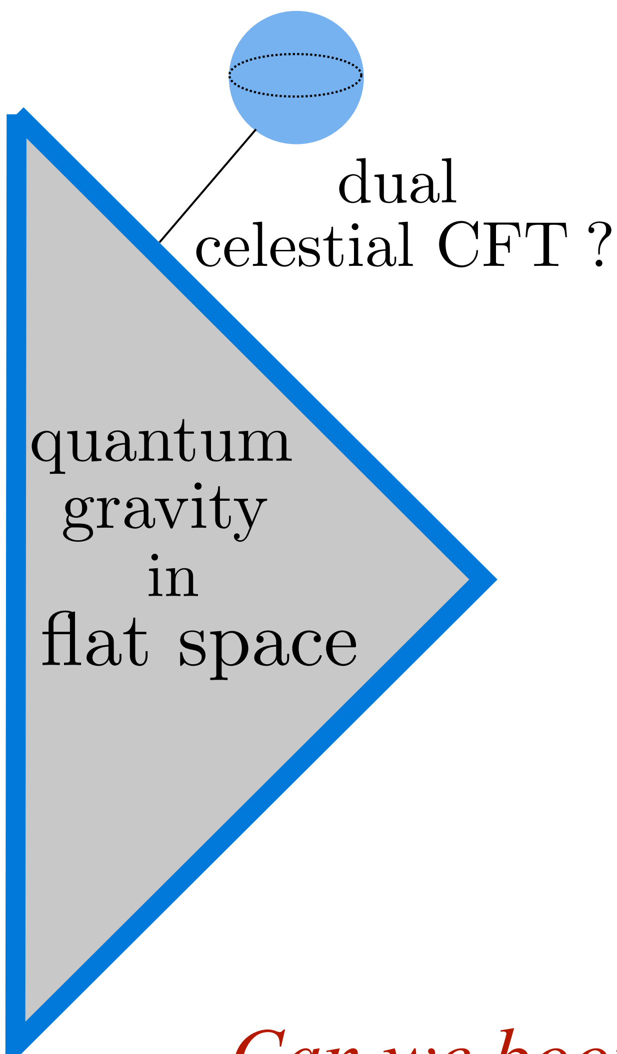
Novel framework for studying scattering amplitudes!

Outlook: celestial holography

Novel approach to bootstrapping quantum gravity in flat space.

$$ {}_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta_1,J_1}(w_1,\bar{w}_1)\dots\mathcal{O}^{\pm}_{\Delta_n,J_n}(w_n,\bar{w}_n)\rangle_{\text{celestial CFT}} $$

What is this CFT?



What is the spectrum?

What are all the symmetries?

What is the organizing principle?

Can we bootstrap the (conformally) soft sector?

…

Can we bootstrap quantum gravity in asymptotically flat space?

Exploration of celestial territory has only begun!

Thank you!