Inflationary Correlators Beyond Weak Coupling: a Numerical Approach

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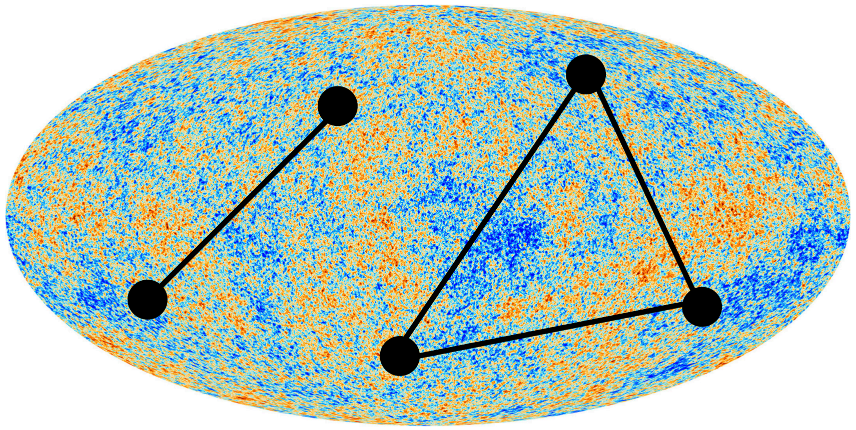
$$\mathcal{L}(\delta X^a) \longrightarrow$$



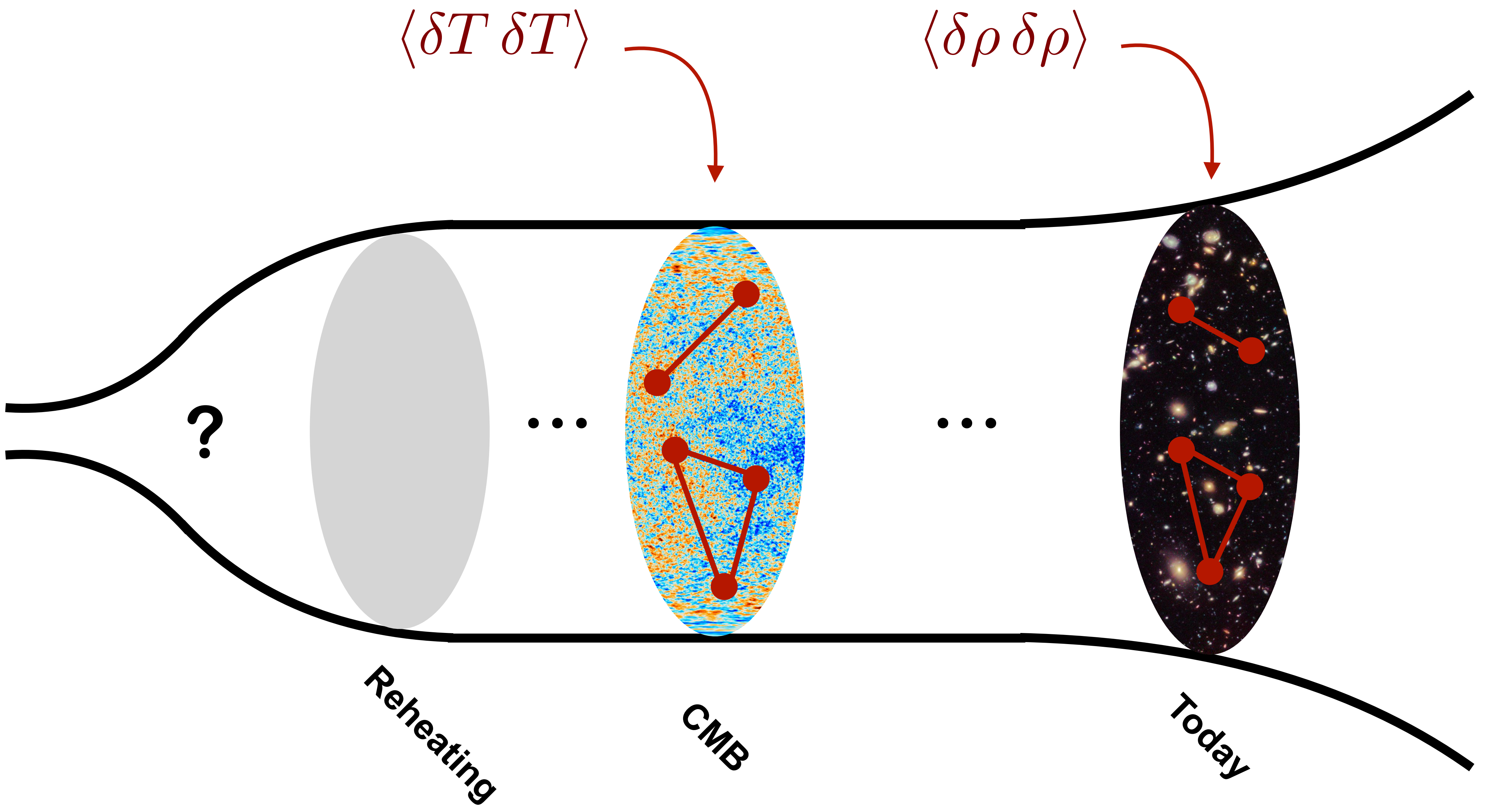
Denis Werth

Based on work with

Sébastien Renaux-Petel
Lucas Pinol

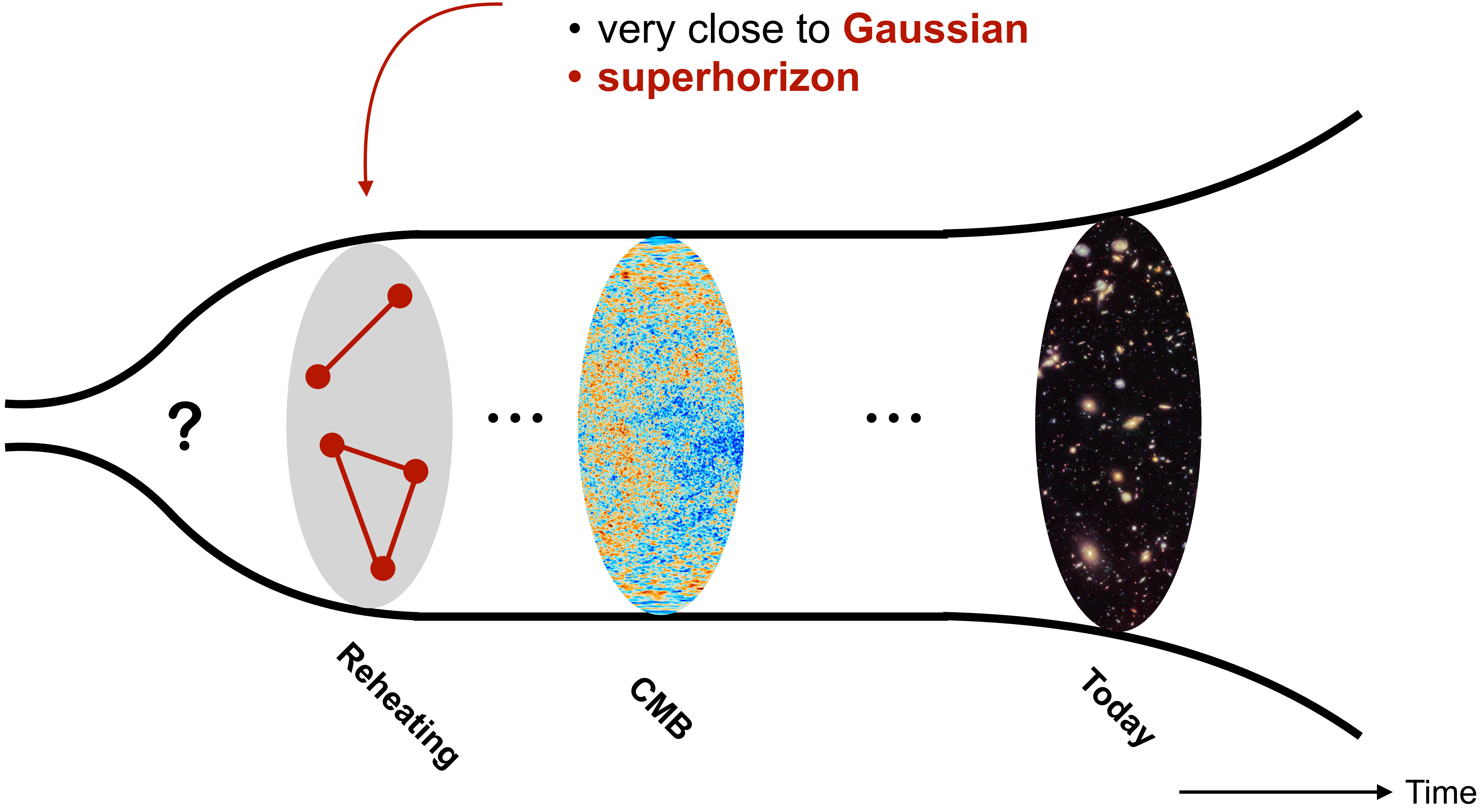
Cosmology: Observing Fluctuations

Cosmological structures are **correlated** on large scales

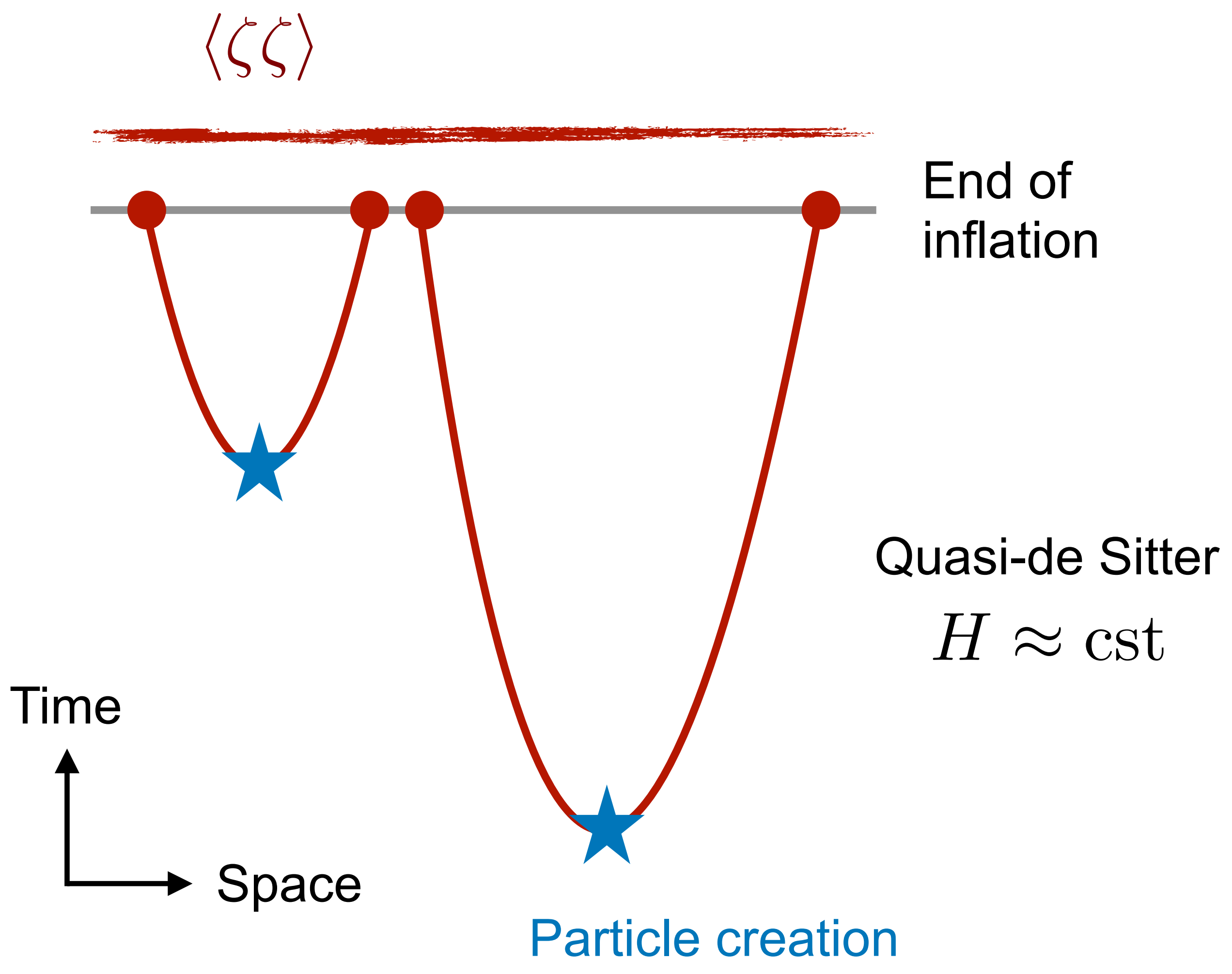


Cosmology: Observing Fluctuations

- **adiabatic**
- almost **scale-invariant**
- very close to **Gaussian**
- **superhorizon**



Inflationary Paradigm: the Big Picture

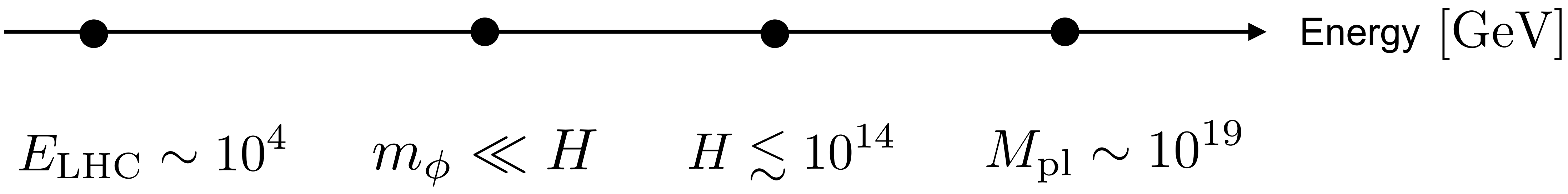


- At least one scalar mode in every inflationary model:

  ζ : scalar ($\sim \gamma_{ii}$)

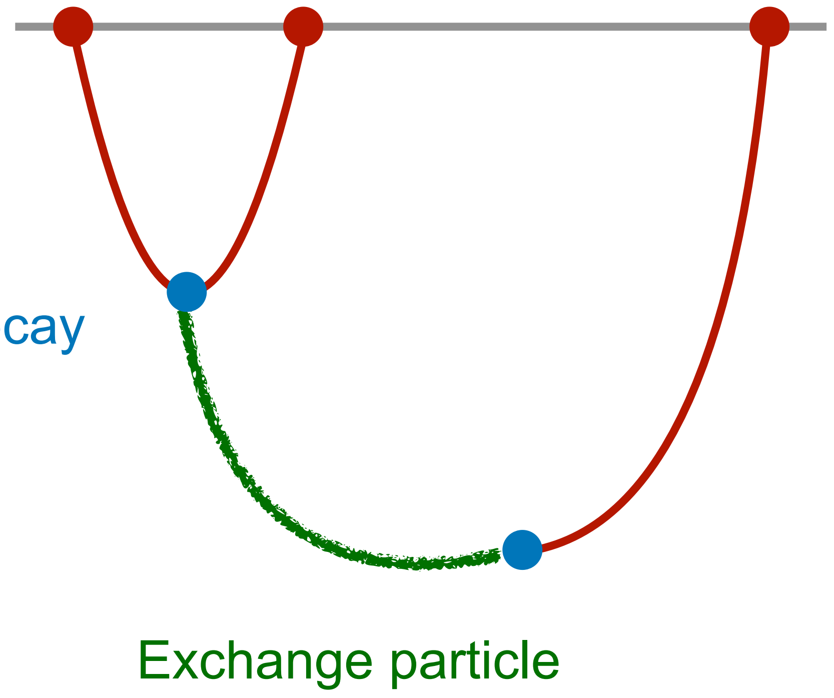
- Spontaneous particle production (vacuum fluctuation) leads to non-trivial correlations of ζ

- Observables of interest are correlation functions of fields at the end of inflation



Inflation as a Cosmological Collider

During inflation, very **massive particles** ($\sim 10^{14}$ GeV) can be produced whose decays lead to correlations of curvature perturbation



We want to know the physics of inflation:

- Particle content of inflation (number of fields, mass spectra, spins, etc)
- Interactions?
- Build a complete theory of the bulk (like SM)

Boundary correlators:

- Time evolution of correlators is not observable
- Observables should emerge from a consistent time evolution in the bulk

In-In Formula

Vacuum of the full interacting theory

Equal-time correlators

Interacting Hamiltonian

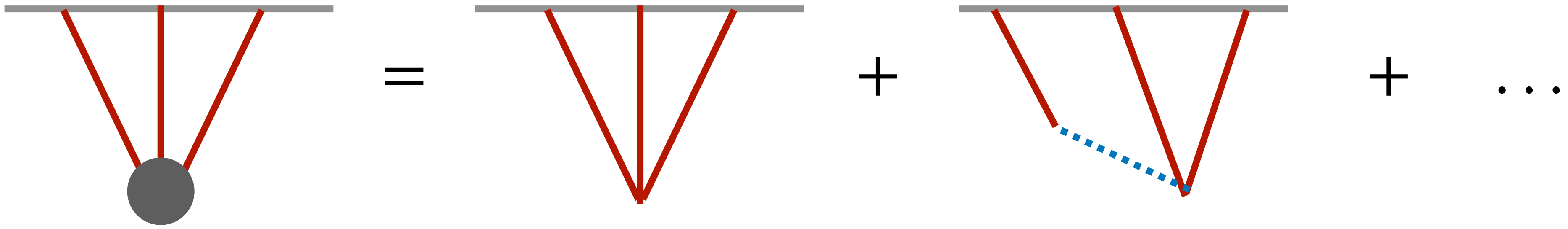
Vacuum of the free theory

$$\langle\Omega|\mathcal{Q}(\tau_0)|\Omega\rangle = \langle 0|\left[\bar{T}e^{i\int_{-\infty}^{\tau_0}\mathrm{d}\tau H_I(\tau)}\right]\mathcal{Q}(\tau_0)\left[Te^{-i\int_{-\infty}^{\tau_0}\mathrm{d}\tau H_I(\tau)}\right]|0\rangle$$

Simple analytical treatment:

- Weak coupling expansion
- Use of uncoupled mode functions
- Diagrammatic expansion

Weinberg [2005]



Difficulties

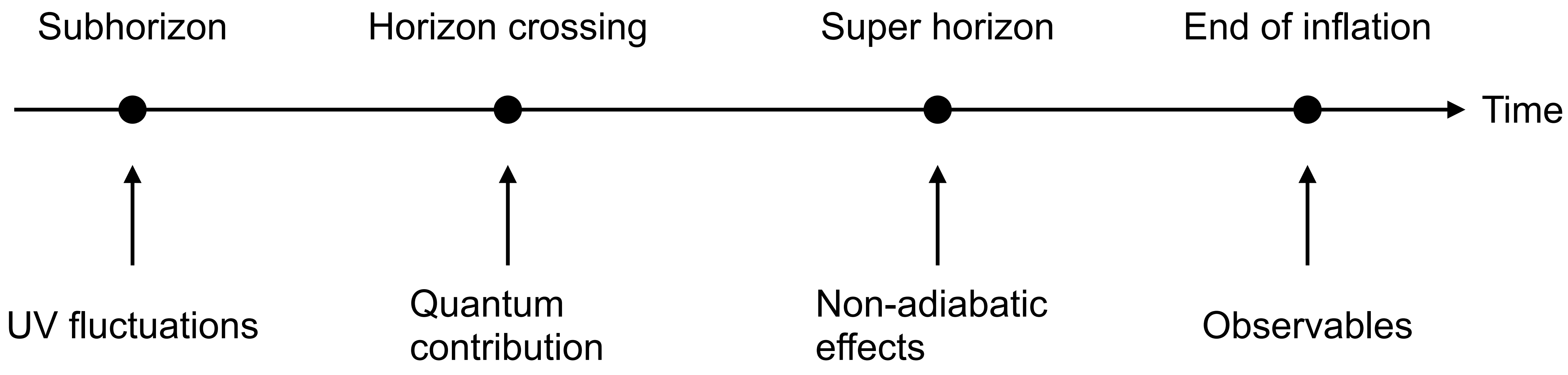
Conceptual difficulties:

- Background is time-dependent (Lorentz invariance is broken)

Practical difficulties:

- Algebraic complexity
- Hard to accurately include all necessary masses and rates

Correlators receive contributions from all times



Subhorizon

Horizon crossing

Super horizon

End of inflation

Time

UV fluctuations

Quantum contribution

Non-adiabatic effects

Observables

A Useful Method: Transport Approach

A useful method to tackle all these issues is to **switch to numerics**

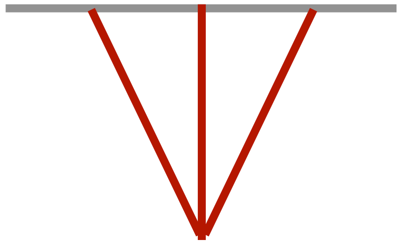
Collider Phenomenology

- Extract observational predictions from QFT
- Feynman diagram generators (LanHEP, FeynRules, …)
- Automatic computation of collision events (CompHEP, MadGraph, …)

We want to apply the same philosophy in cosmology

Numerically follow the **time evolution** of cosmological correlators including all effects

Feynman-type Integrals: a Simple Illustrative Example



$$\langle\zeta^3\rangle(\tau) \sim \int_{-\infty}^{\tau} \mathrm{d}\tau'\, \tau'^n g(\tau') e^{iK(\tau'-\tau)} \qquad \text{with} \quad K = k_1 + k_2 + k_3$$

$$\mathcal{L}^{(3)}/a^3 \supset -\frac{g(t)}{3!}\dot{\zeta}^3$$

Direct Calculation

$$\langle\zeta^3\rangle = \frac{g}{iK}$$

Indirect Calculation

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\langle\zeta^3\rangle = g - iK\langle\zeta^3\rangle$$

Change of perspective

- Translate the problem of computing Feynman-type integrals to **solving differential equations**
- Enables one to follow the time evolution of correlators in different regimes

Transport Approach Formalism

At the level of the fluctuations:

Mulryne, Seery et al. [2016]

- General theory

$$H = \frac{1}{2!} H_{\alpha\beta} \delta X^\alpha \delta X^\beta + \frac{1}{3!} H_{\alpha\beta\gamma} \delta X^\alpha \delta X^\beta \delta X^\gamma$$

- Write the transport equations

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \langle \delta X^\alpha \delta X^\beta \rangle = u^\alpha_\rho \langle \delta X^\rho \delta X^\beta \rangle + u^\beta_\rho \langle \delta X^\alpha \delta X^\rho \rangle$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \langle \delta X^\alpha \delta X^\beta \delta X^\gamma \rangle = u^\alpha_\rho \langle \delta X^\rho \delta X^\beta \delta X^\gamma \rangle + u^\alpha_{\rho\sigma} \langle \delta X^\rho \delta X^\beta \rangle \langle \delta X^\sigma \delta X^\gamma \rangle + \text{perms}$$

Model dependence:

- Various theories are encoded in u^α_β and $u^\alpha_{\beta\gamma}$
- Time-dependent coupling constants

Initial conditions:

- In the far past, modes do not feel the effect of spacetime curvature
- Set of **uncoupled dofs**
- Analytical approximations become both **tractable** and **accurate**

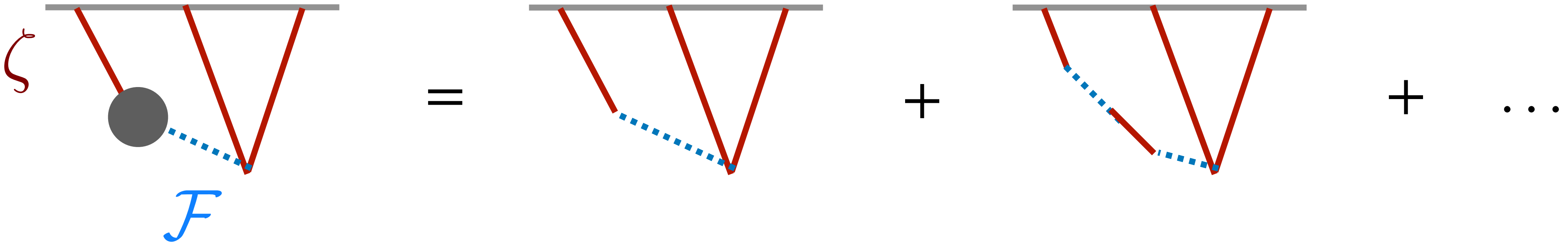
Resummed Diagrams: Beyond Weak Coupling

Numerical approach enables us to use the **full propagators**, effectively resumming an infinite number of diagrams

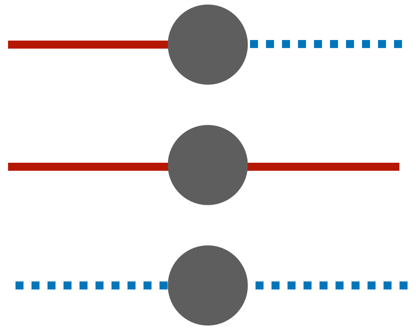
- Ever-present quadratic interaction

$$\mathcal{L}^{(2)} = \mathcal{L}^{(2),\mathrm{free}}(\zeta) + g(\tau)\dot{\zeta}\mathcal{F} + \mathcal{L}^{(2),\mathrm{free}}(\mathcal{F})$$

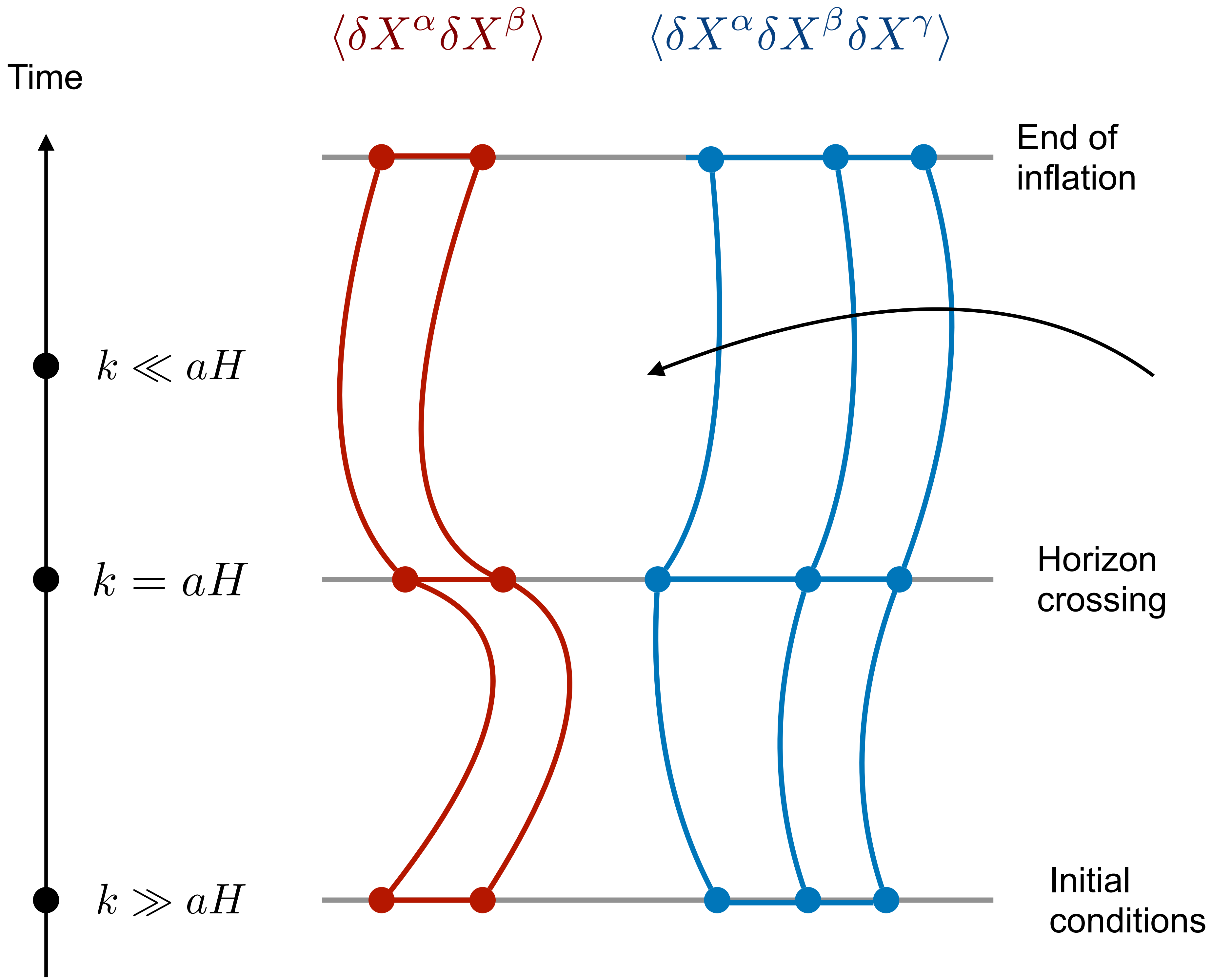
- Usually treated as a perturbation



Directly computed with the transport approach



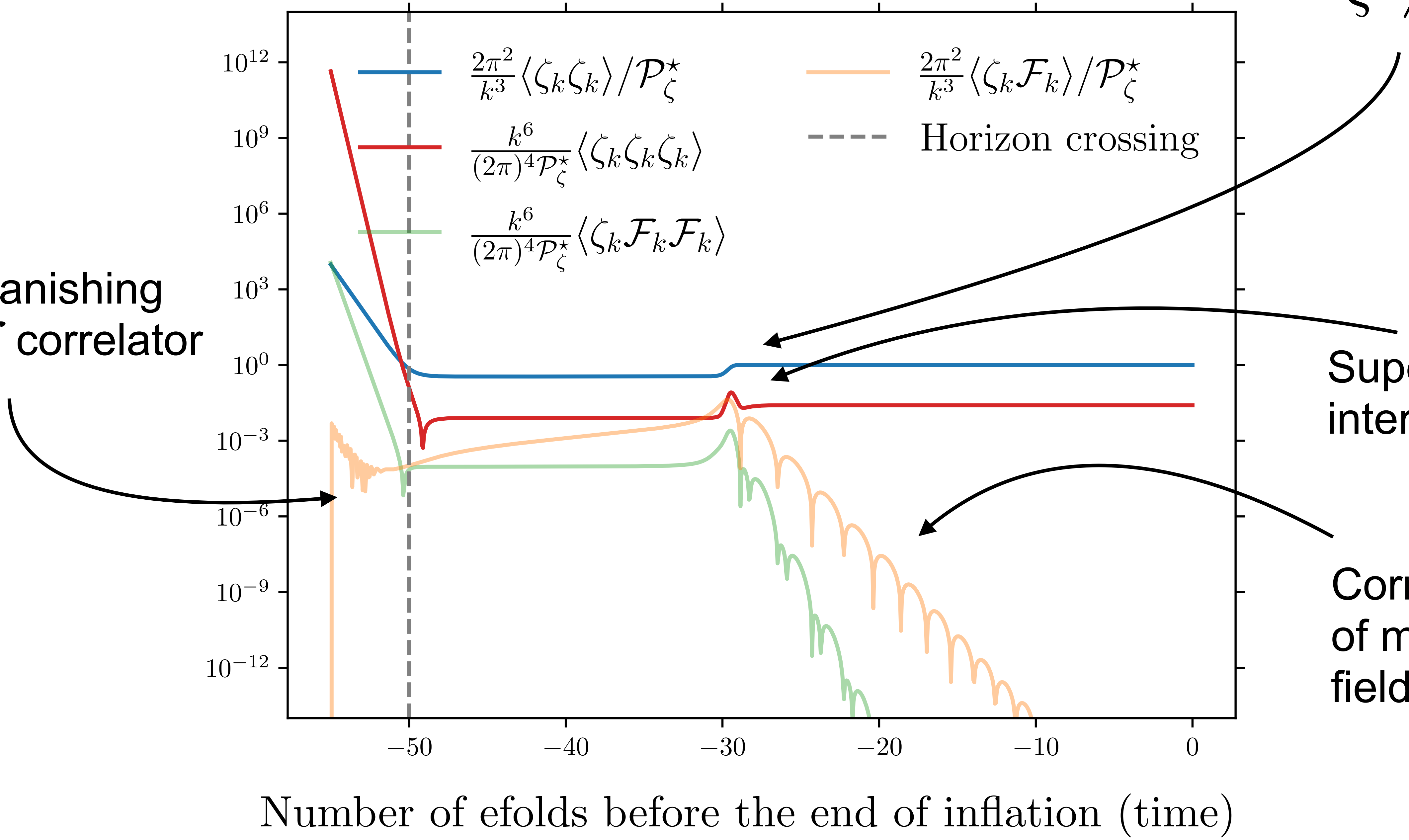
Transport Approach: Summary



Solve **non-trivial physics** during inflation by following the time evolution of all correlators using **full propagators**

Time Evolution of Various Correlators

We take as a benchmark example a **two-field model** with a **turn** in field space 30 folds before the end of inflation



$\dot{\zeta} \neq 0$

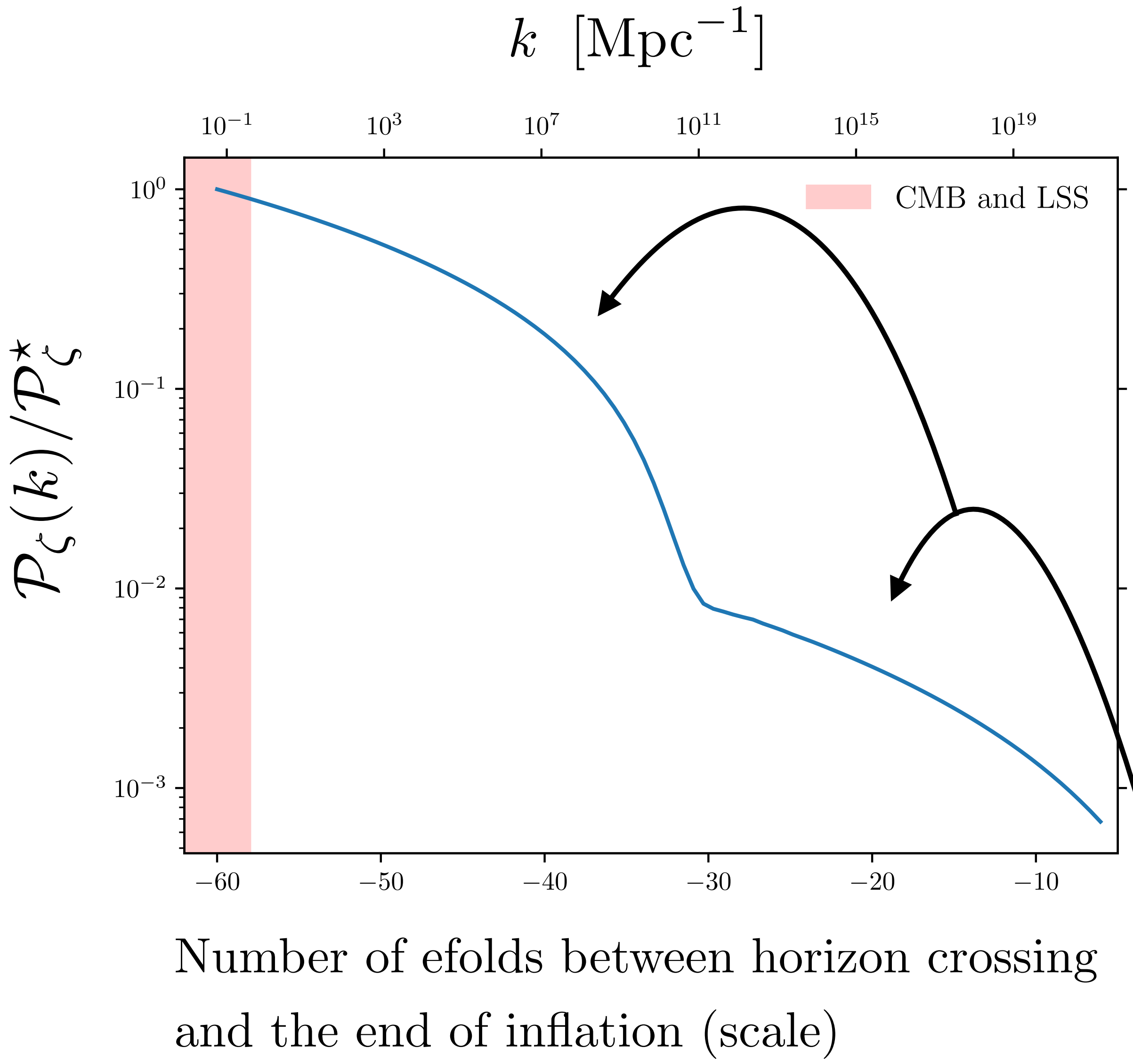
Non-vanishing $\zeta - \mathcal{F}$ correlator

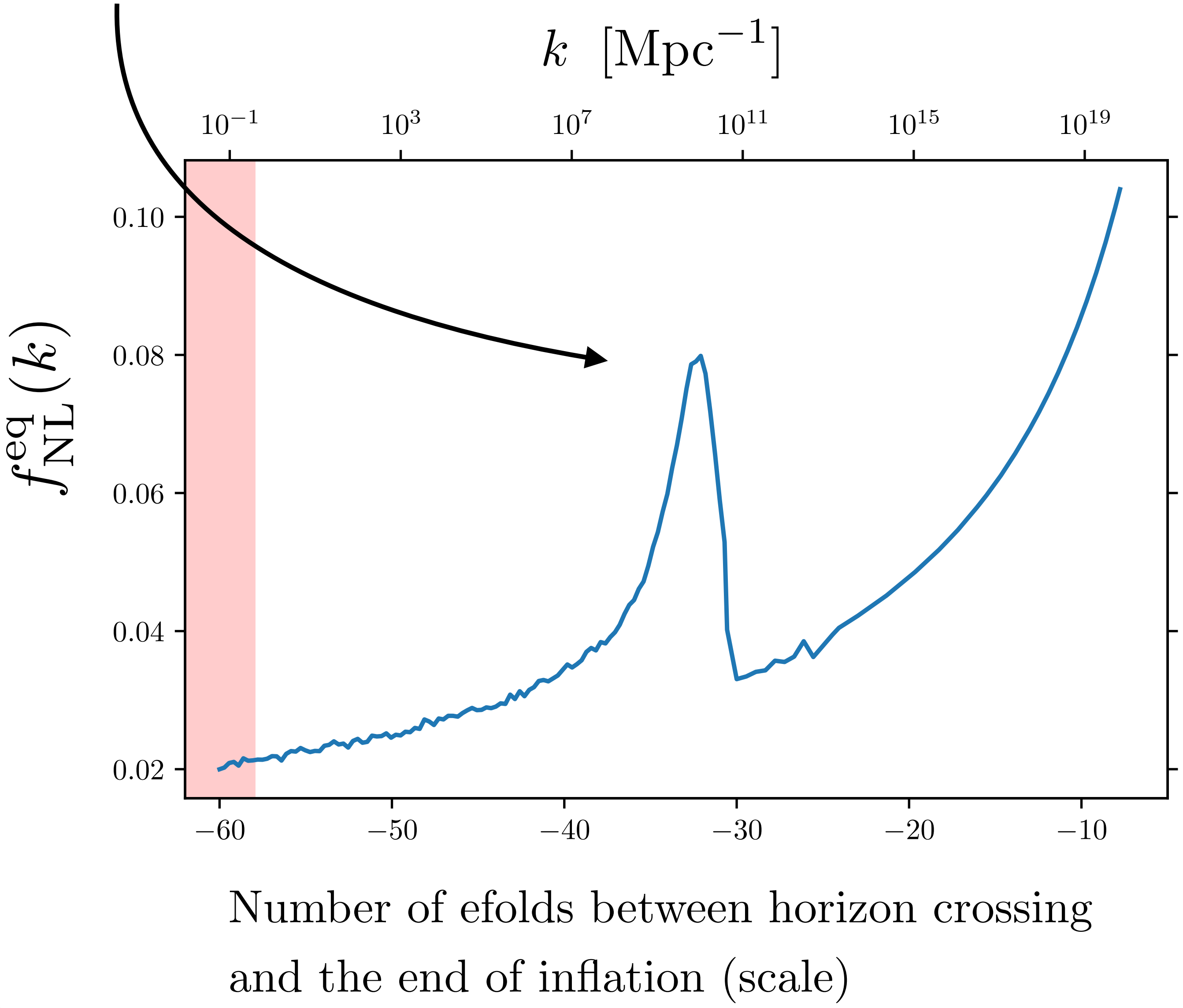
Super horizon interactions

Correlators of massive fields decay

Scale-dependence of Various Correlators

Features in the bispectrum



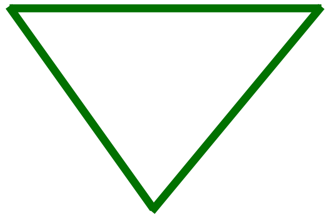


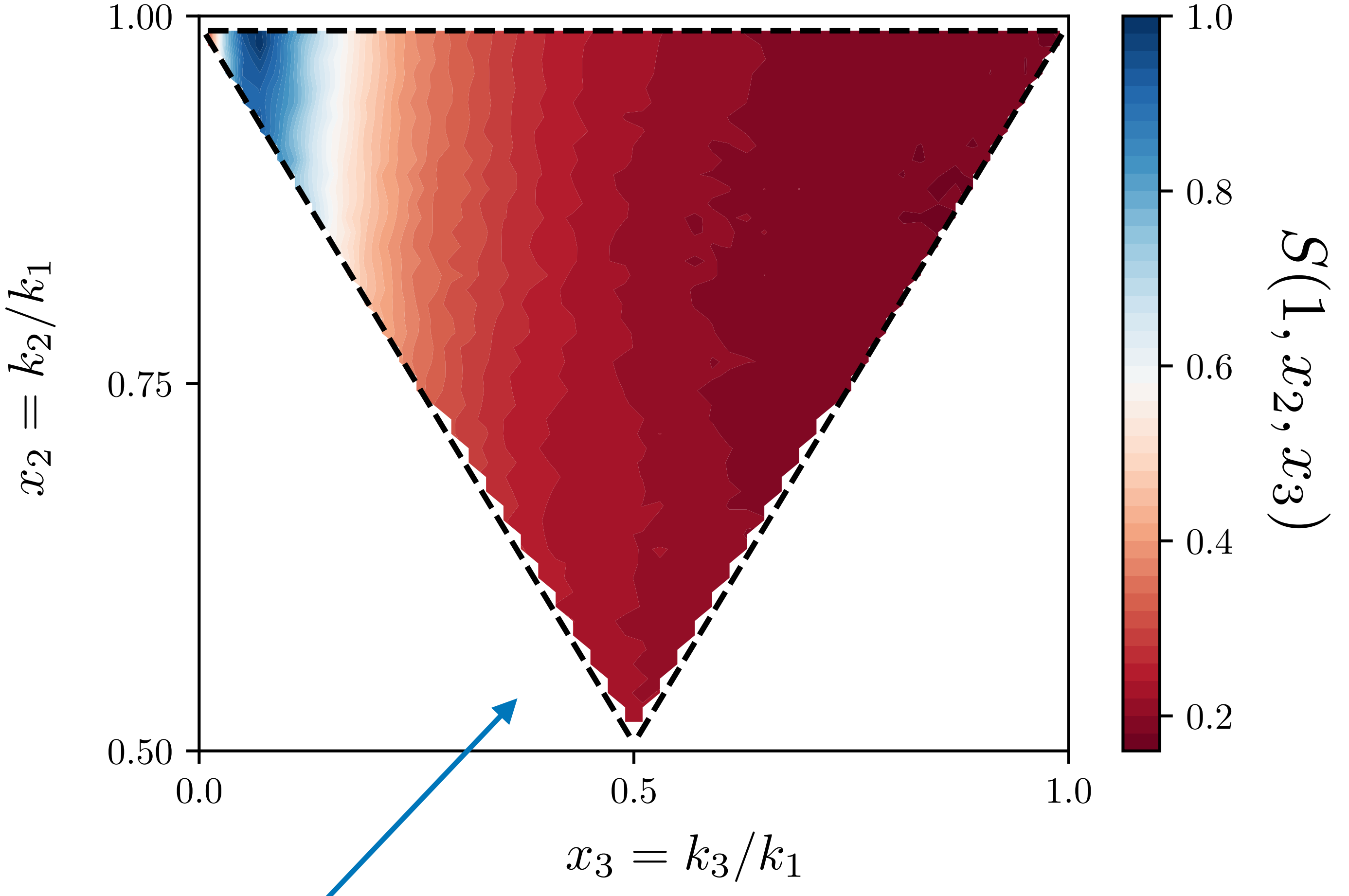
Two-stage inflation

Bispectrum Shape

Squeezed limit:
- Multi-field inflation
- Cosmological collider signal

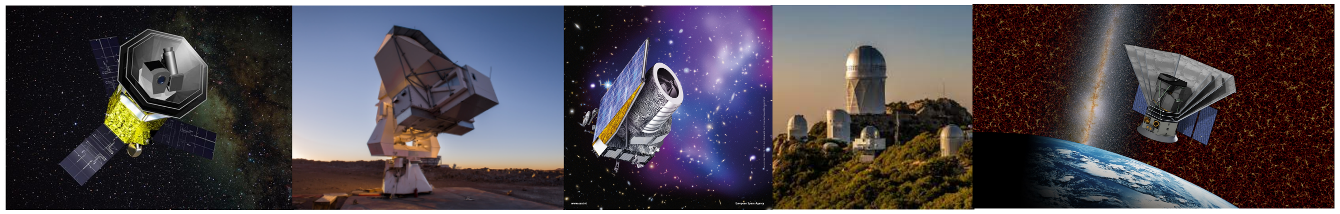
Equilateral configuration:
- Probe contact (self-) interactions





Folded configuration:
- Excited states

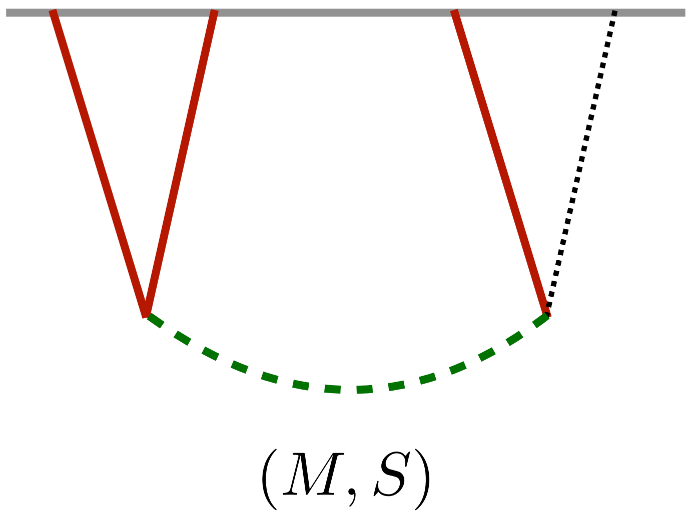
Prospects



Cosmological collider physics:

- Spectroscopy by probing the **squeezed** limit of the 3pt correlation function
- Extend the results to **non scale-invariant** theories
- Add **spinning fields**
- …

$$\langle \zeta_{k_L} \zeta_{k_S} \zeta_{k_S} \rangle \sim \left(\frac{k_L}{k_S}\right)^{3/2} \cos\left[\frac{M}{H}\log\left(\frac{k_L}{k_S}\right)\right] \mathbb{P}_S(\cos\theta)$$



Chen, Wang [2009]
Baumann, Green [2011]
Arkani-Hamed, Maldacena [2015]
Baumann, Lee, Pimentel [2016]
Cheung, Creminelli, Senatore [2007]
Senatore, Zaldarriaga [2010]
Renaux-Petel, Turzynski [2015]
Garcia-Saenz, Pinol, Renaux-Petel [2020]
Pinol [2020]
…

Study EFT-driven theories for fluctuations:

- **Speed of sound** breaking dS boosts
- Additional **Planck suppressed operators**
- Beyond two fields
- Study resonant/sharp features
- …

Conclusion and Take-home Message

Inflation is fascinating as it allows us to probe the **laws of physics** at the **highest reachable energies**

Present a **complete formalism** to numerically follow the time evolution evolution of **all 2- and 3-pf correlation functions**

Develop a code that automatically computes observables from an **EFT for fluctuations**

This method is **powerful** because

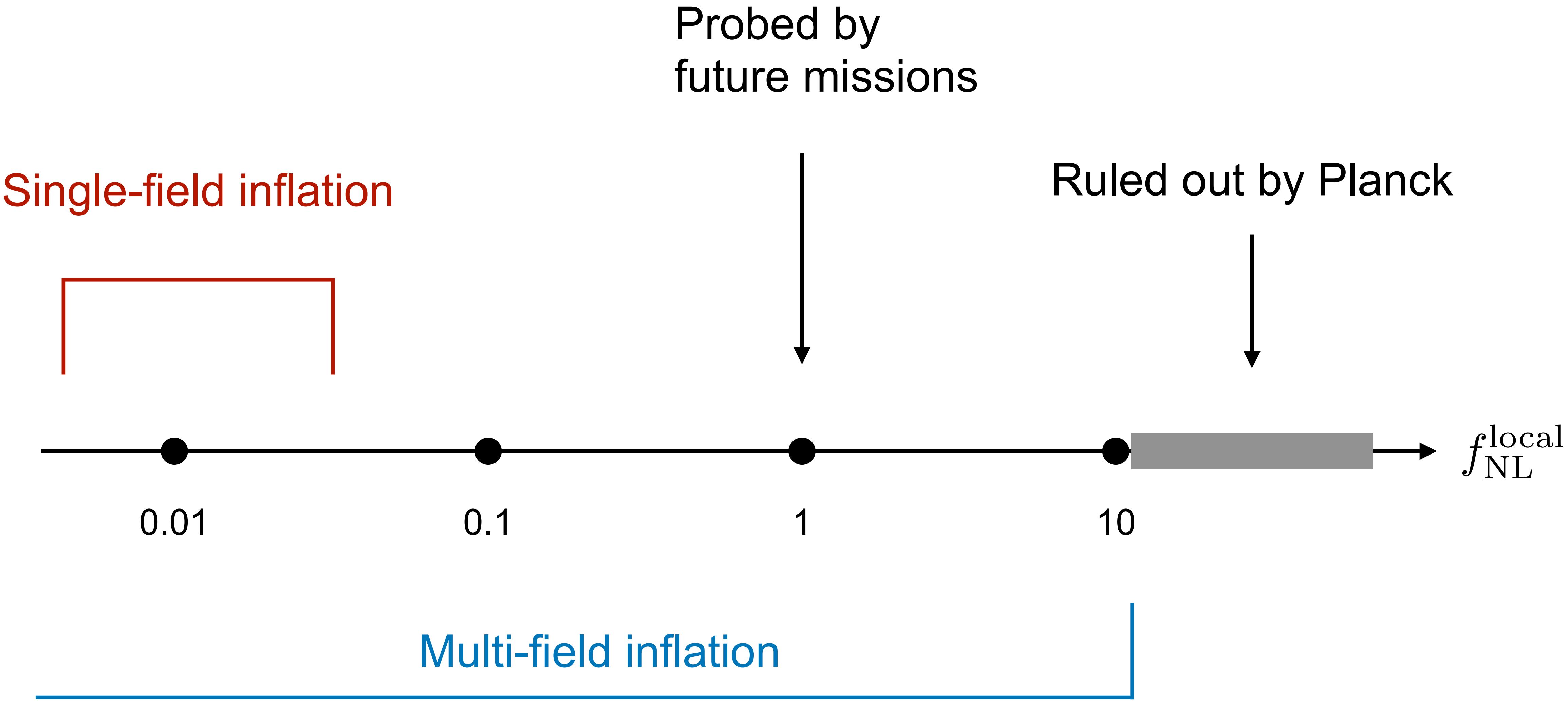
- Include all effects
- Full propagators (resummed diagrams)

Future work is exciting !

Some Missions

- **SphereX** : infrared space telescope (NASA), observe LLS and constrain NGs
- **Simons Observatory** : ground based (Chile), measure CMB polarisation, gravitational lensing of the CMB, primordial bispectrum, measure tensor-to-scalar ratio
- **Euclid** : near-infrared space telescope (ESA), dark energy, measure galaxy resifts (<2), 3D galaxy distribution
- **DESI** : ground based (Arizona), construct 3D map of galaxy distribution, test models of dark energy
- **LiteBIRD** : space satellite (JAXA), measure B-mode polarisation in the CMB

Measuring non-Gaussianities



Probed by
future missions

Single-field inflation

Ruled out by Planck

Multi-field inflation

Weakly broken dS boosts
(Slow-roll suppression)

Strongly broken dS boosts
(Non-trivial speed of sound)

Codes Available for Inflationary Calculations

Two-point function solvers:

- FieldInf
- ModeCode & MultiModeCode
- PyFlation

Three-point function solvers:

- BINGO (single-field inflation)

Transport approach:

- CppTransport
- PyTransport

Our code:

- Decouple from a specific background
- EFT at the level of the fluctuations

Ringeval, Brax, van de Bruck, Davis, Martin [2006]
Price, Frazer, Xu, Peiris, Easther [2015]
Huston, Malik [2009][2011]
Hazra, Sriramkumar, Martin [2013]
Dias, Fazer, Seery [2015]
Mulryne [2016]