An analytical model of subhalo population: mass function and stellar encounters

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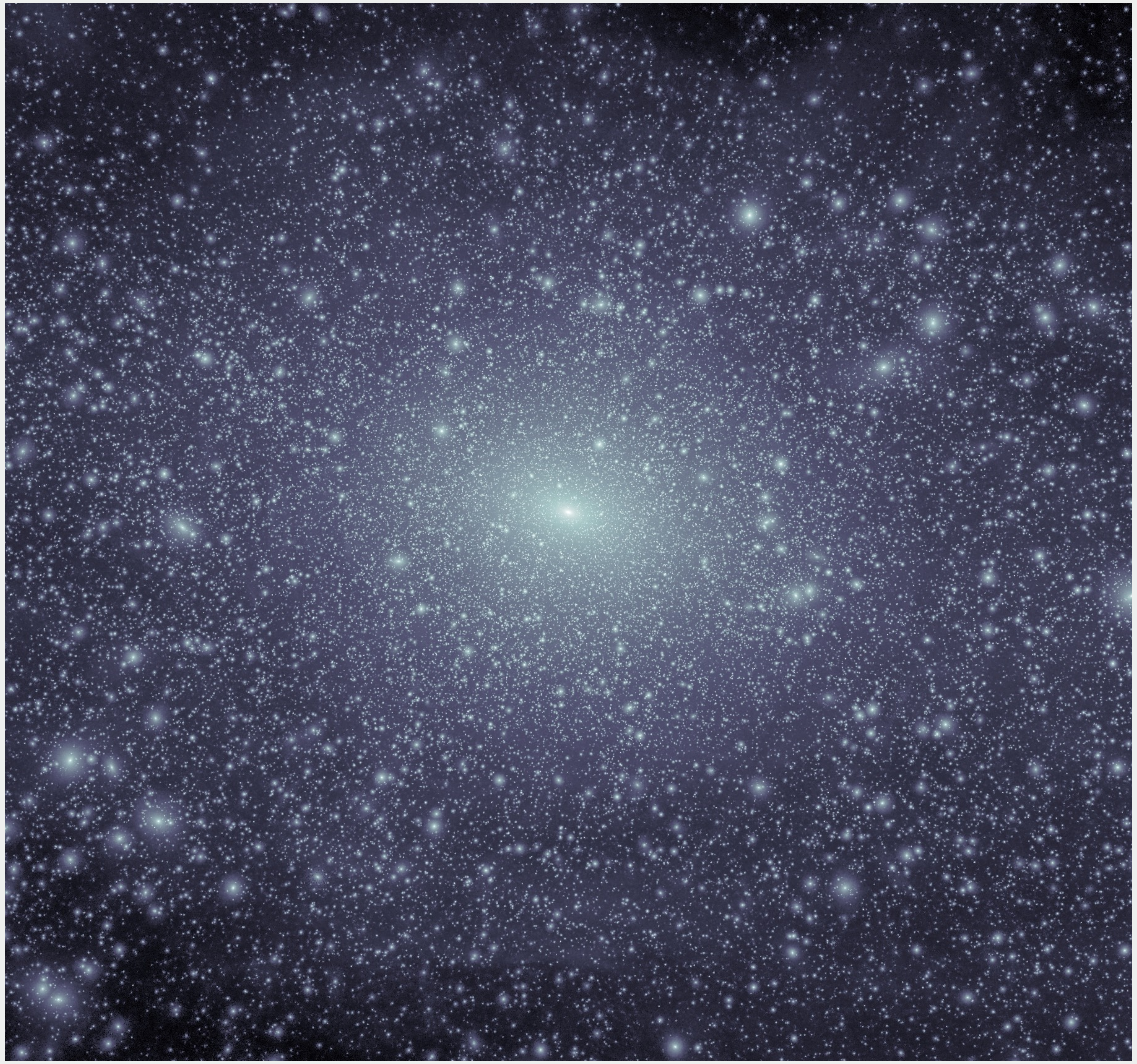
with Martin Stref and Julien Lavalle

Analytical model of subhalo population

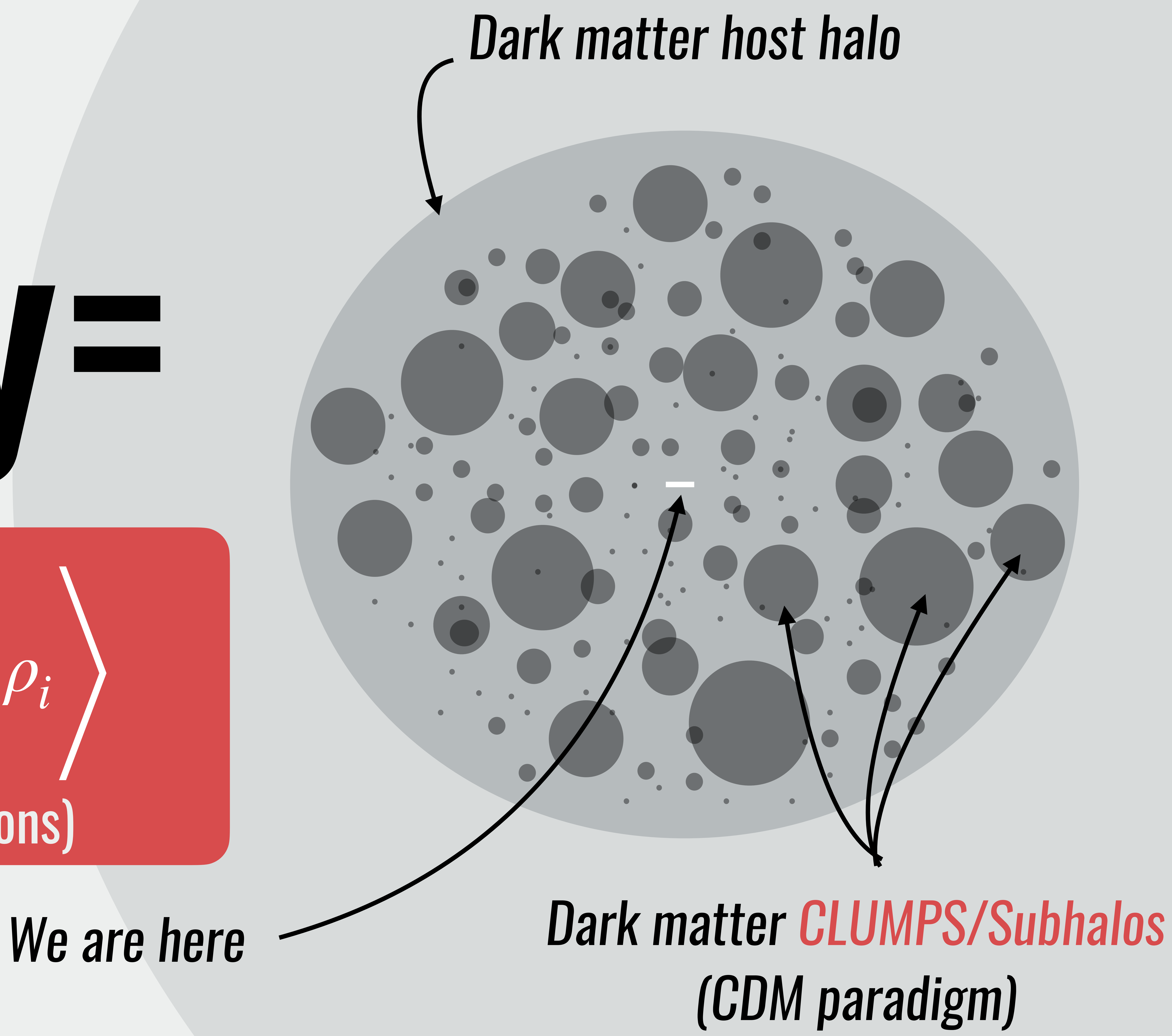


[The Via Lactea project - Diemand et al. 2008]

Galaxy=

$$\left\langle \rho_\chi = \rho_{\rm smooth} + \sum_{i=1}^{N_{\rm sub}} \rho_i \right\rangle$$

(Constrained by observations)



Why is looking for subhalos interesting?

Nature of DM: Cold DM? Warm DM? Self Interacting DM? ...

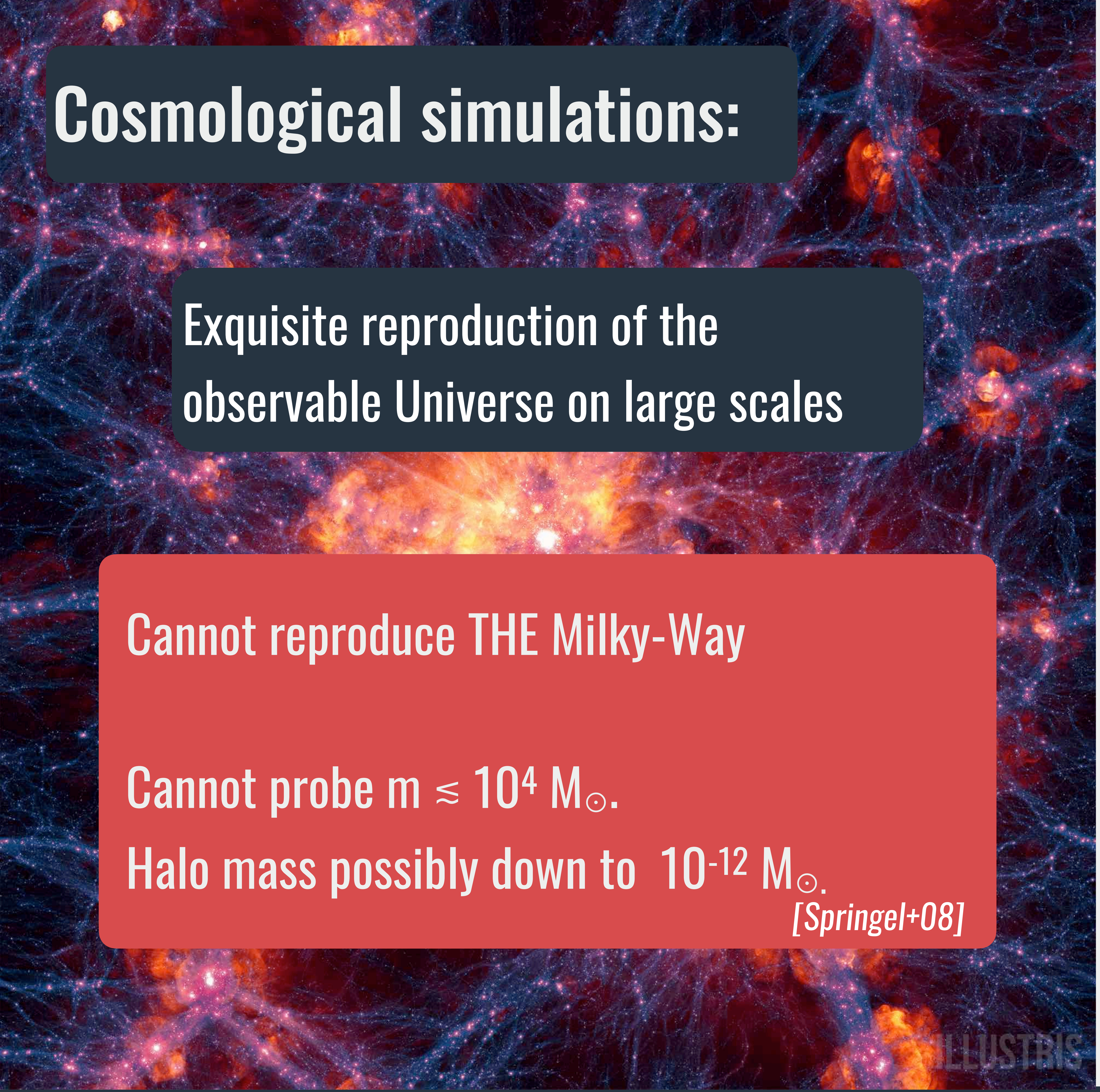
Can be looked for with several strategies (DM annihilation, lensing, ...)

Need a reliable population model for Galactic searches

[Ibarra+19, Hütten+19, Calore+19, Hütten+16, Ando+19, ...]

Important to understand,

how dark matter **(sub)halos** are distributed



Cosmological simulations:

Exquisite reproduction of the observable Universe on large scales

Cannot reproduce THE Milky-Way

Cannot probe m $\lesssim 10^4\ M_\odot$.

Halo mass possibly down to $10^{-12}\ M_\odot$.

[Springel+08]

Analytical models:

Number of CDM subhalos in a MW-like halo: $N_{\rm sub} \gtrsim 10^6$

Evaluate the statistical distribution of halos

[Stref+17, Hiroshima+18, Bartels+15, Zavala+14, Benson+12, Van den Bosch+05, Peñarrubia+05, ...]

Two main ideas to describe the subhalo population

A dynamically constrained semi-analytical model for the subhalo population in the Milky Way (MW)

From [Stref & Lavalle 2017]

Initial distribution:
(without dynamics)

$(\rho_s, r_s) \leftrightarrow (m, c)$

Initial mass distribution
(cosmological mass function)

$$p_{\rm sub}^{\rm init}(m, c, R) = p_{\mathbf{R}}(R) \frac{1}{N_{\rm sub}} \frac{\mathrm{d}N_{\rm sub}}{\mathrm{d}m} p_c(c\,|\,m)$$

Spatial distribution
(follows potential of the host)

[McMillan+17]

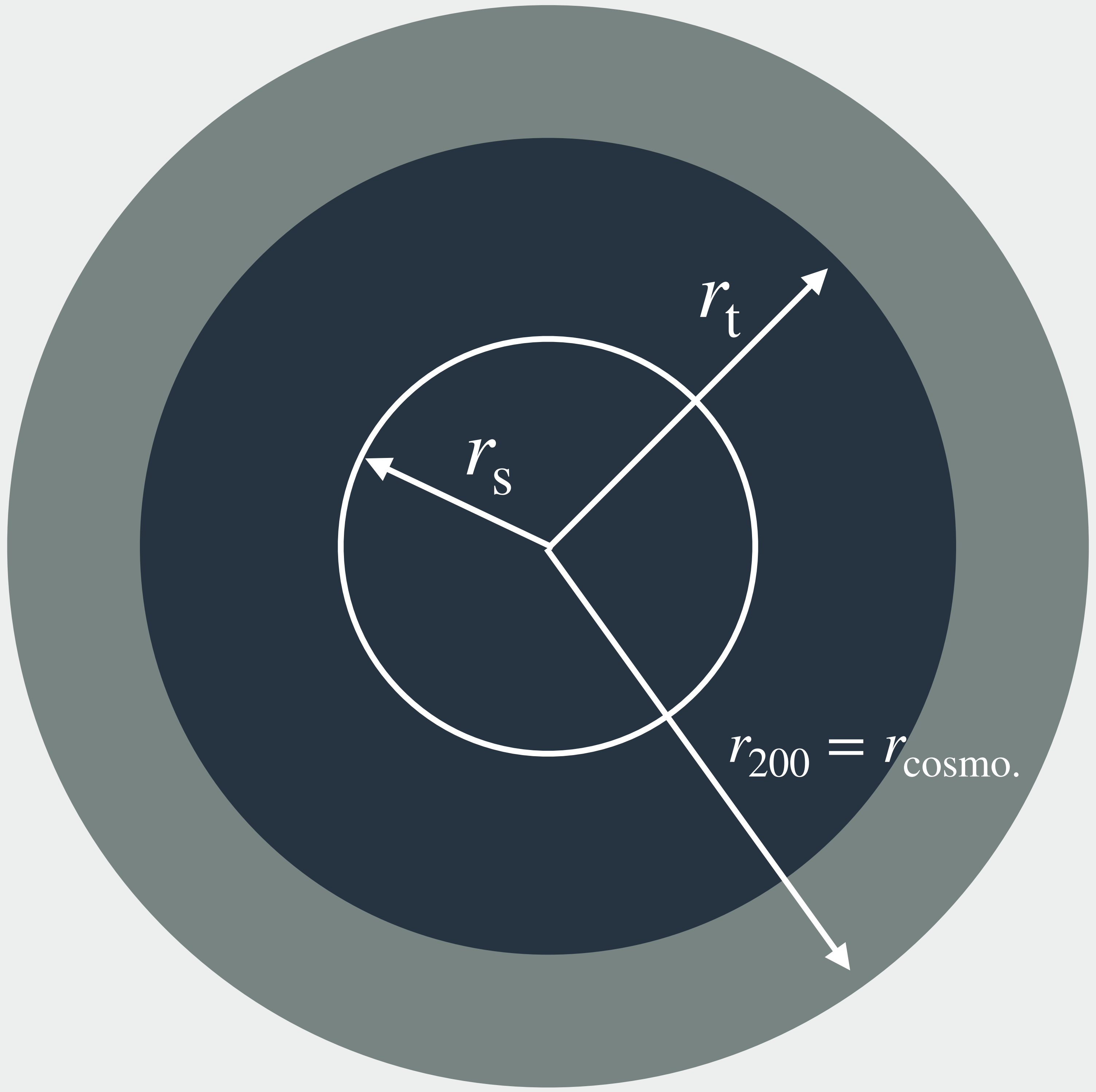
Distribution in concentration

[Bullock+01,Sánchez-Conde+14]

The model is constrained from dynamical effects

Tidal stripping effects

Subhalos lose mass and shrink



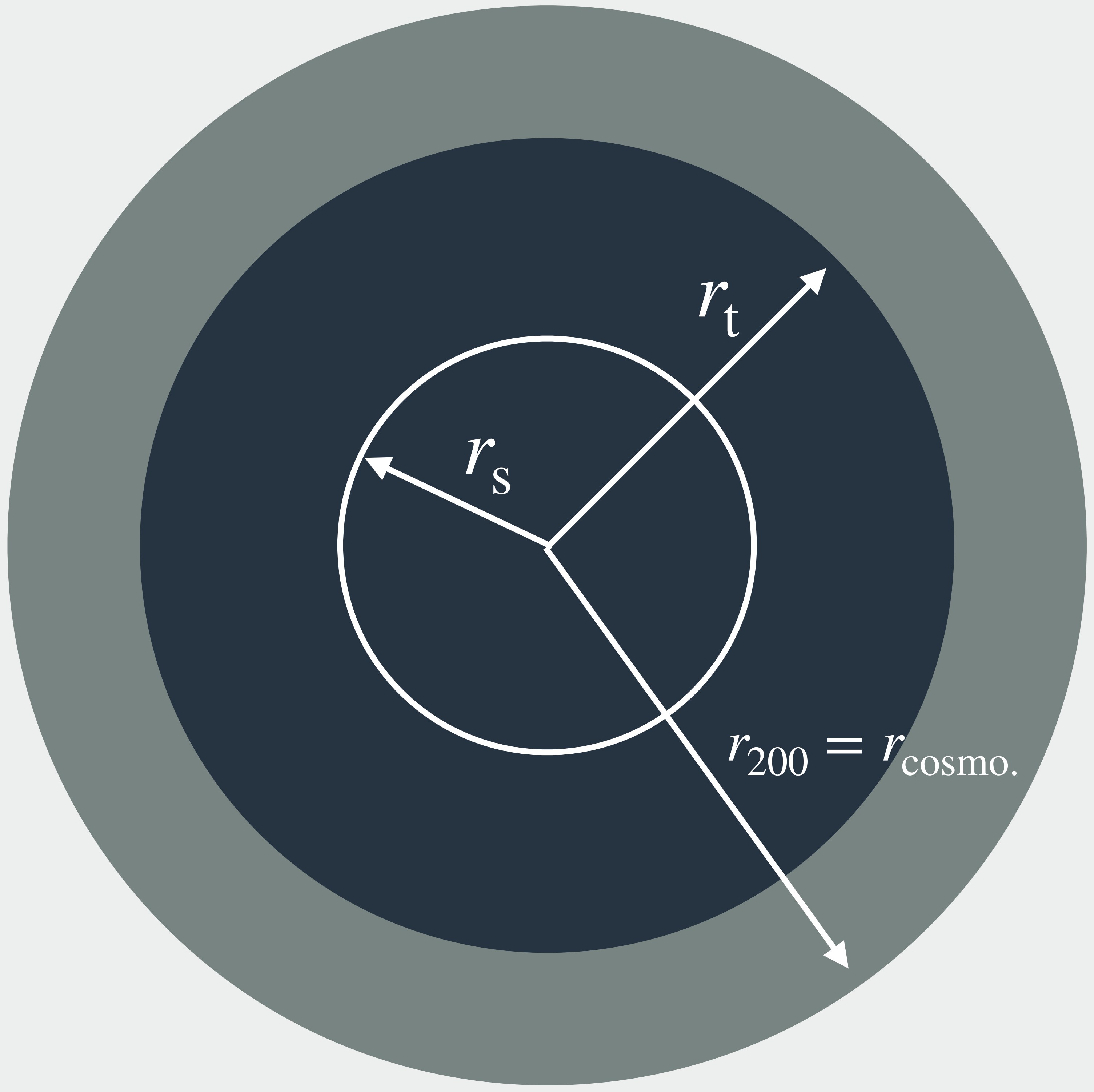
According to simulations if stripped too much subhalos are destroyed

[Tormen+98, Hayashi+03, Diemand+08, ...]

(But are they really?)

[Van den Bosch+18, Errani+20]

$$\begin{cases} \dfrac{r_t}{r_s} \geq \epsilon_t & \textit{the subhalo survives} \\ \dfrac{r_t}{r_s} < \epsilon_t & \textit{the subhalo is destroyed} \end{cases}$$

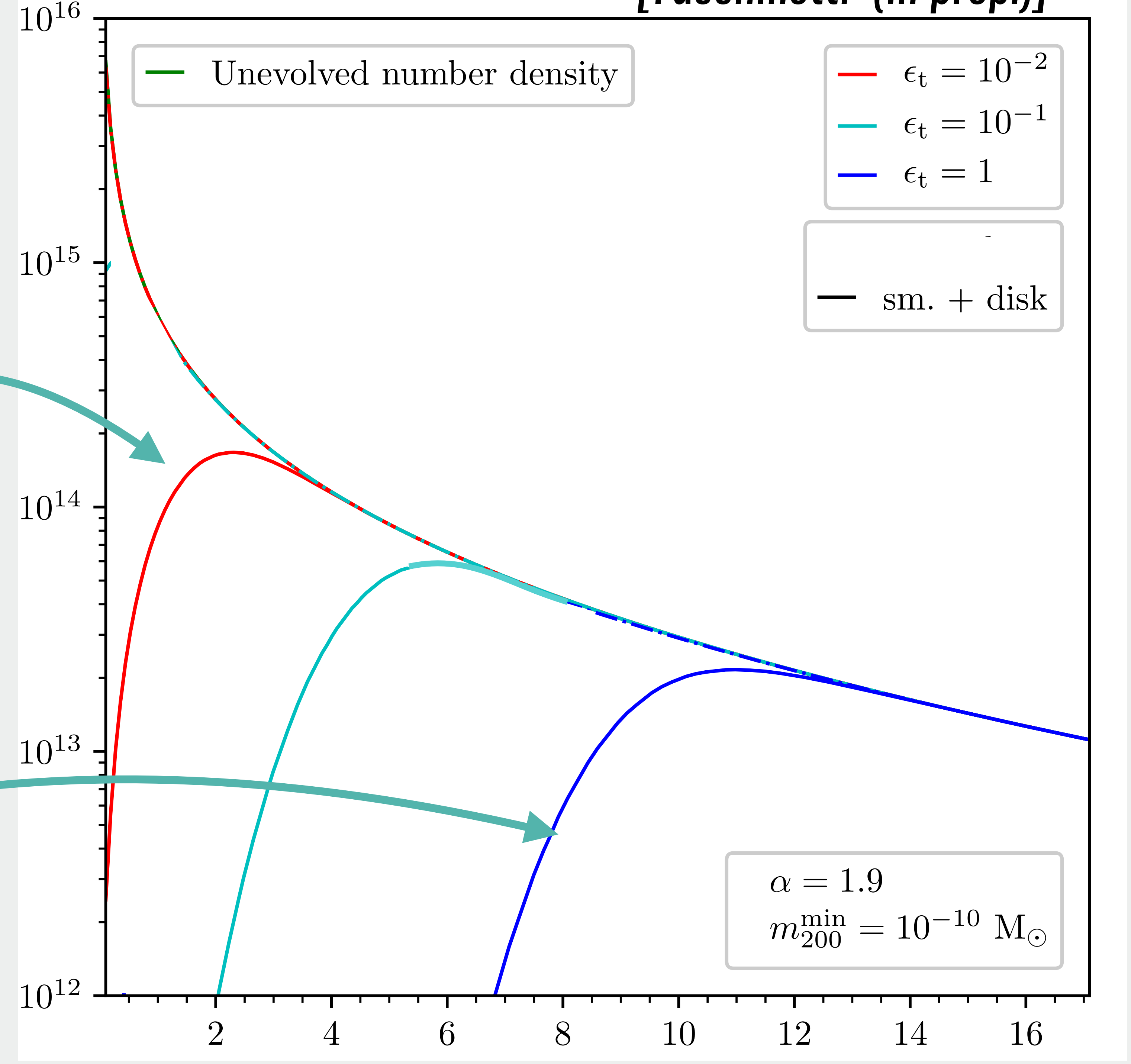


Number density of subhalos [kpc^{-3}]

Resilient subhalos (more physical)

Fragile subhalos (following simulations)

[Facchinetti+(in prep.)]

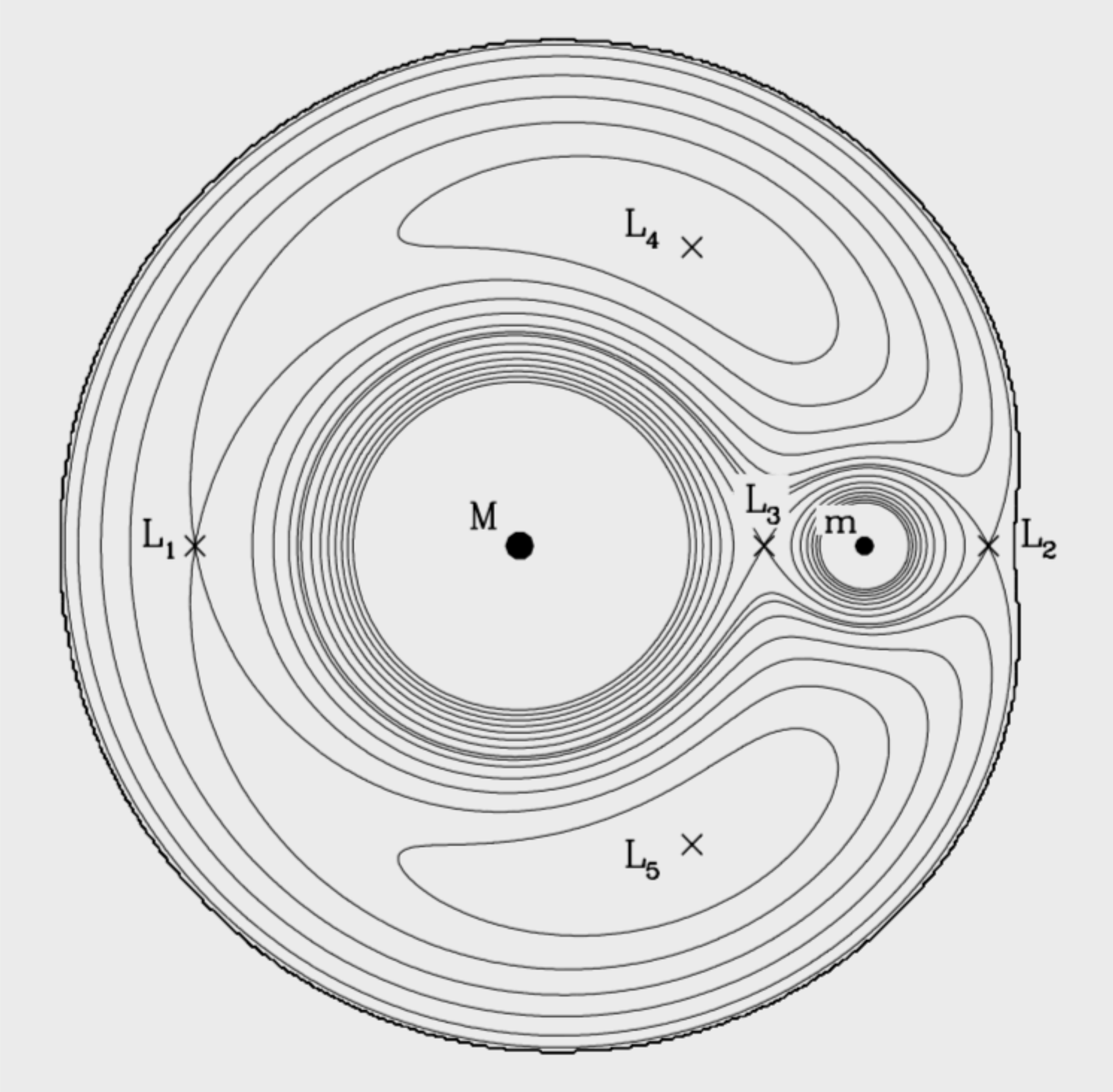


Distance from GC (MW) [kpc]

We set a disruption criterion

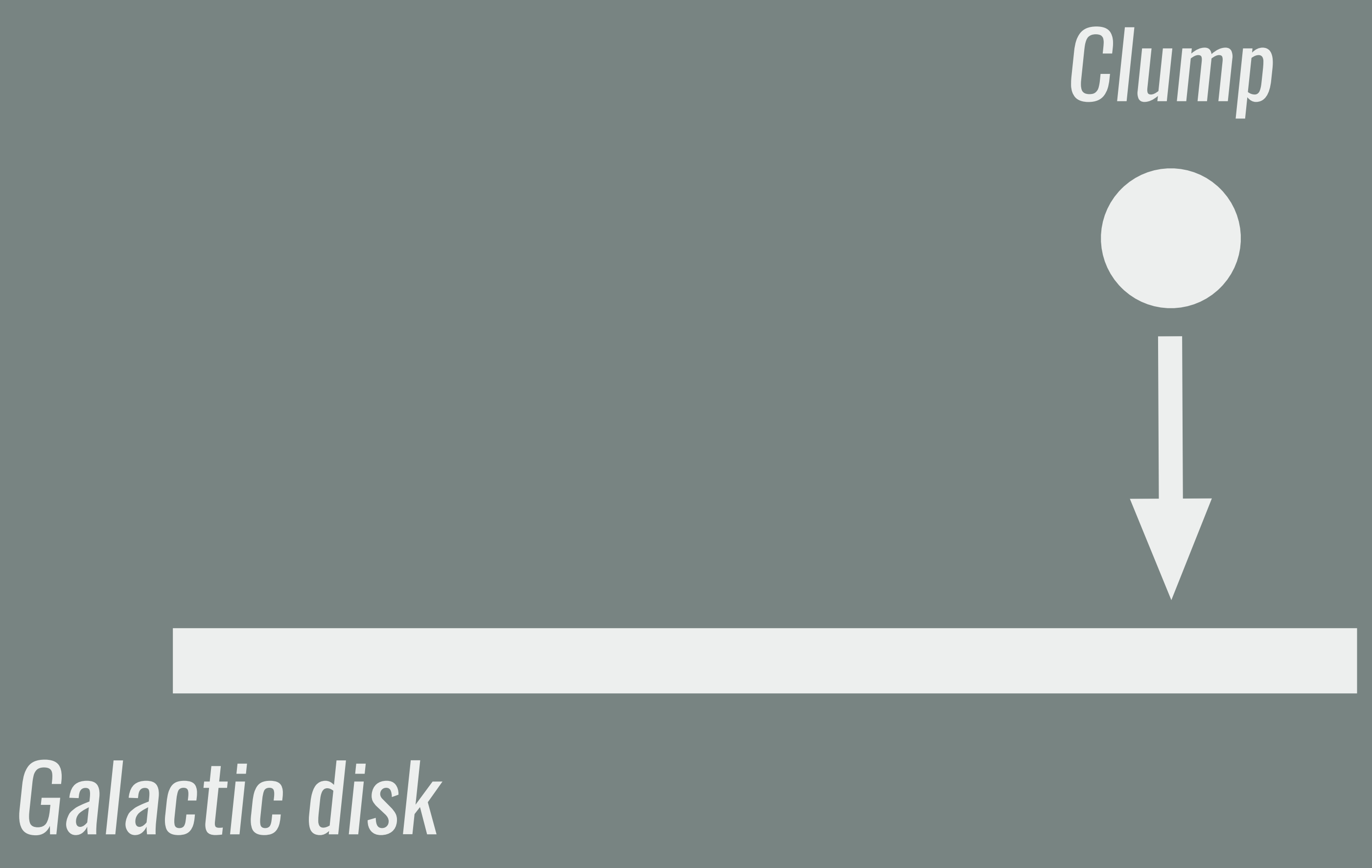
[Binney+08, Weinberg94, Gnedin+99, Stref+17]

$$r_t = R \left\{ \frac{M_{\rm int}(R)}{3M(R)f[M(R)]} \right\}^{1/3}$$



Global tides

$$\left\langle \frac{\delta E}{m_\chi} \right\rangle = \frac{2}{3} \frac{g_{\rm d}^2}{V_z^2} A(\eta) r^2$$



Disk shocking

Two sources of tidal stripping are considered and impact on the probability distribution

Initial distribution:
(without dynamics)

$(\rho_s, r_s) \leftrightarrow (m, c)$

Initial mass distribution
(cosmological mass function)

$$p_{\rm sub}^{\rm init}(m, c, R) = p_{\mathbf{R}}(R) \frac{1}{N_{\rm sub}} \frac{dN_{\rm sub}}{dm} p_c(c \mid m)$$

Spatial distribution
(follows potential of the host)

[McMillan+17]

Distribution in concentration

[Bullock+01,Sánchez-Conde+14]

+ Constraints from dynamical effects

$$p_{\rm sub}^{\rm init}(m, c, R) \rightarrow p_{\rm sub}^{\rm late}(m, c, R)$$

Initial/cosmological mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m} \propto m^{-\alpha}\Theta(m - m_{\mathrm{min}})$$

↓

Dynamical/tidal effects

↓

Evolved mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}(R)}{\mathrm{d}m_{\mathrm{t}}} = N_1 \iiint p_{\mathrm{sub}}^{\mathrm{late}}(m, c, R)\delta(m_t - m_t^{\star}(m, c, R))\mathrm{d}m\mathrm{d}c\mathrm{d}R$$

Tidal effect impact on the mass function

Initial/cosmological mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}}{\mathrm{d}m} \propto m^{-\alpha}\Theta(m - m_{\mathrm{min}})$$

Part I

Imply the calibration of mass fraction in subhalos on DM only simulations.
How to avoid that?

Dynamical/tidal effects

Part II

Impact of single star encounters

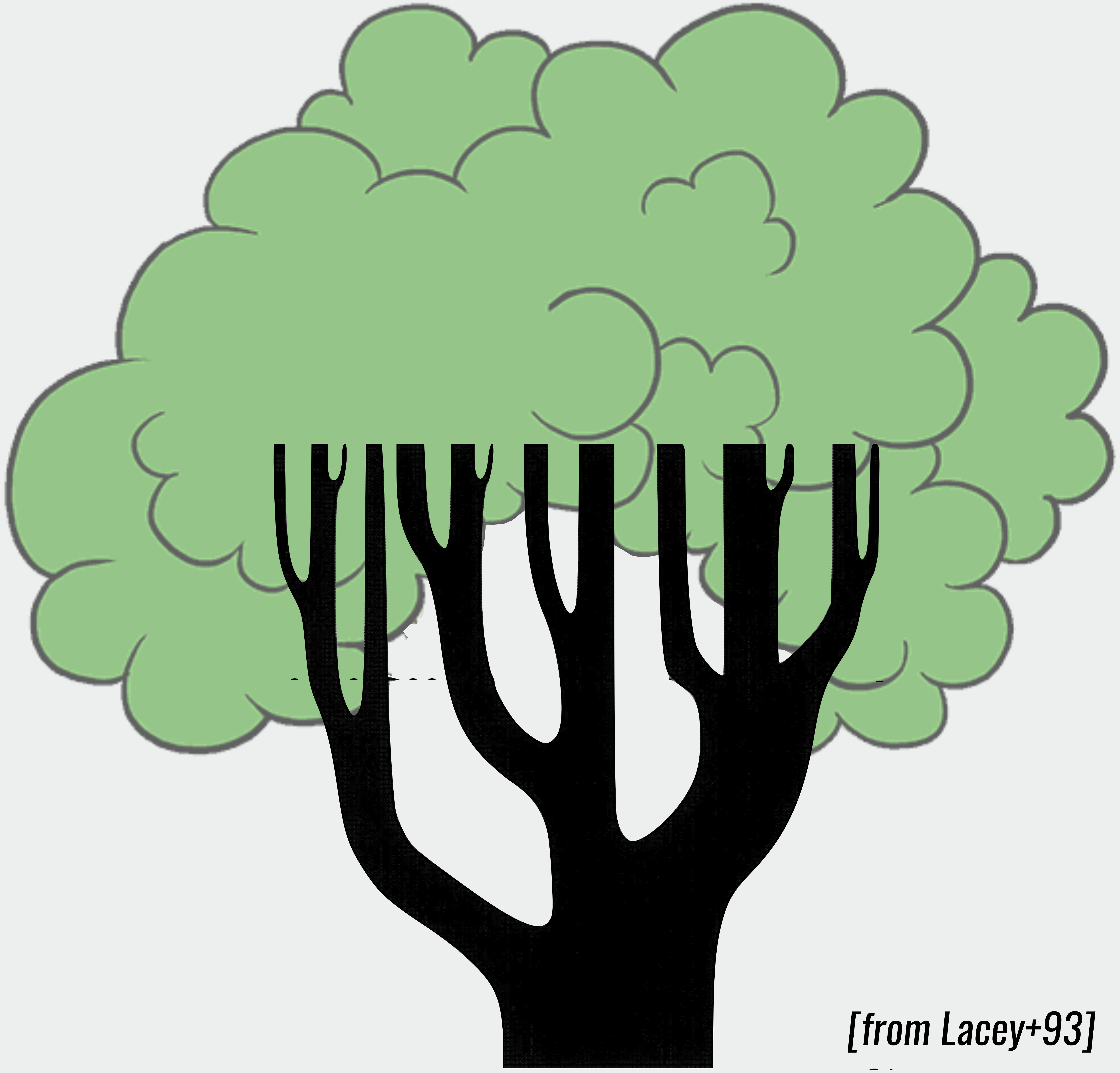
Evolved mass function

$$\frac{\mathrm{d}N_{\mathrm{sub}}(R)}{\mathrm{d}m_{\mathrm{t}}} = N_1 \iiint p_{\mathrm{sub}}^{\mathrm{late}}(m, c, R)\delta(m_t - m_t^{\star}(m, c, R))\mathrm{d}m\mathrm{d}c\mathrm{d}R$$

Tidal effect impact on the mass function

The cosmological mass function from merger trees

Formalism used in [Lacroix, GF+(in prep.)]



[from Lacey+93]

Two barrier-crossing problem

« Probability for a halo of mass M_2 formed at time t_2 to be in a halo of mass M_1 formed at time $t_1 > t_2$ »

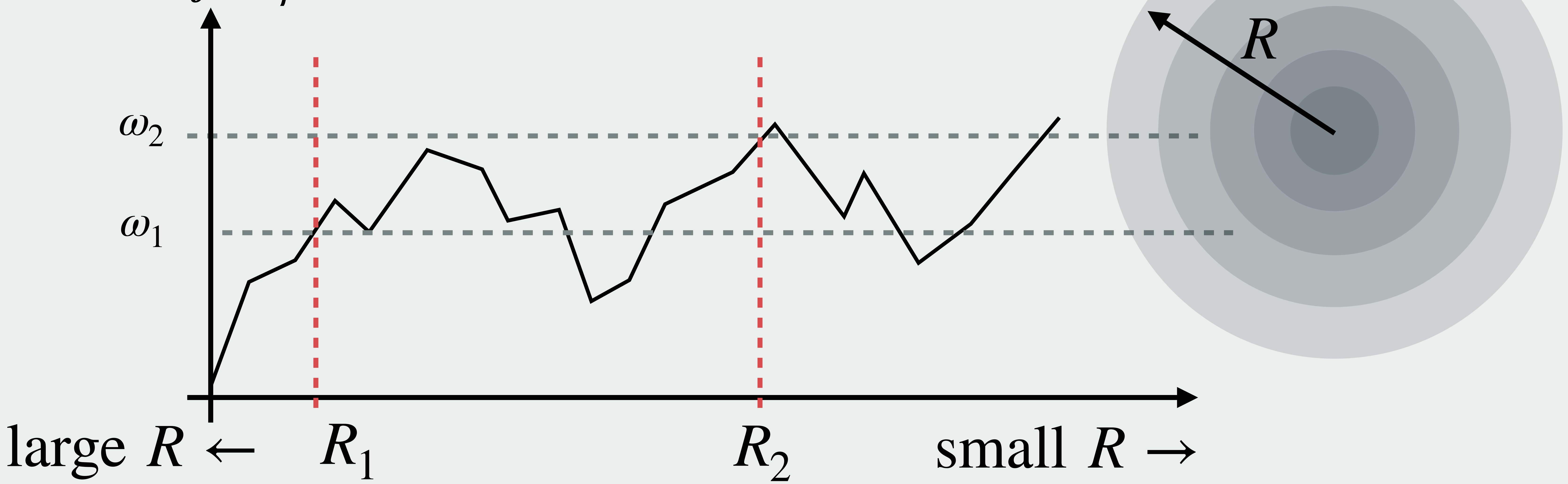
$$f(\omega_2, S_2 \,|\, \omega_1, S_1)\mathrm{d}S_2 = \frac{\omega_2 - \omega_1}{\sqrt{2\pi}(S_2 - S_1)^{3/2}} \exp\left(-\frac{(\Delta\omega)^2}{2(S_2 - S_1)}\right)\mathrm{d}S_2$$

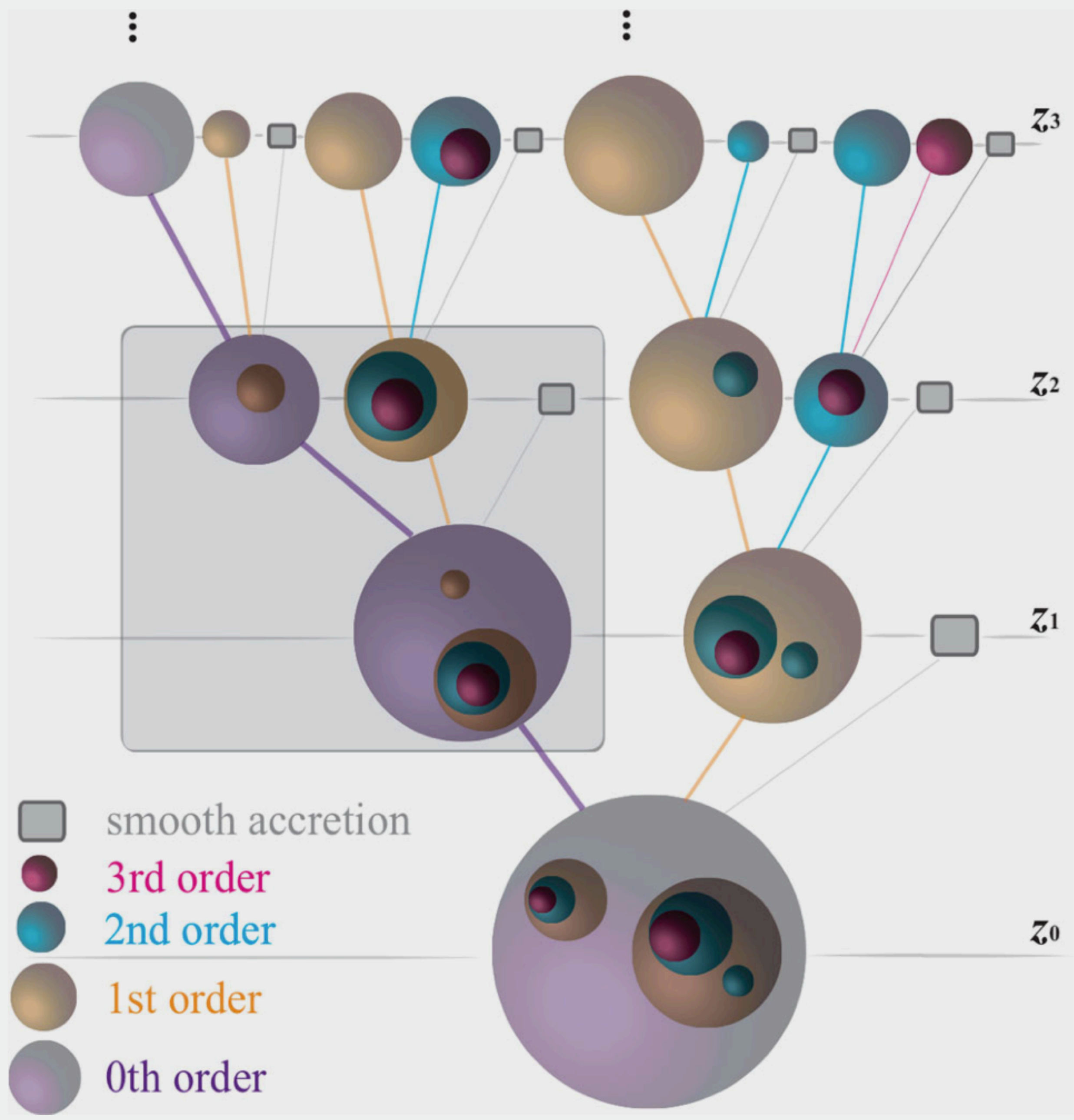
$$P_\mathrm{m}(k, z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{\mathrm{m},0}H_0^2}T(k)\right]^2 \mathscr{A}_S\left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_\mathrm{m}(k, z = 0)k^2\mathrm{d}k$$

[Bond+91]

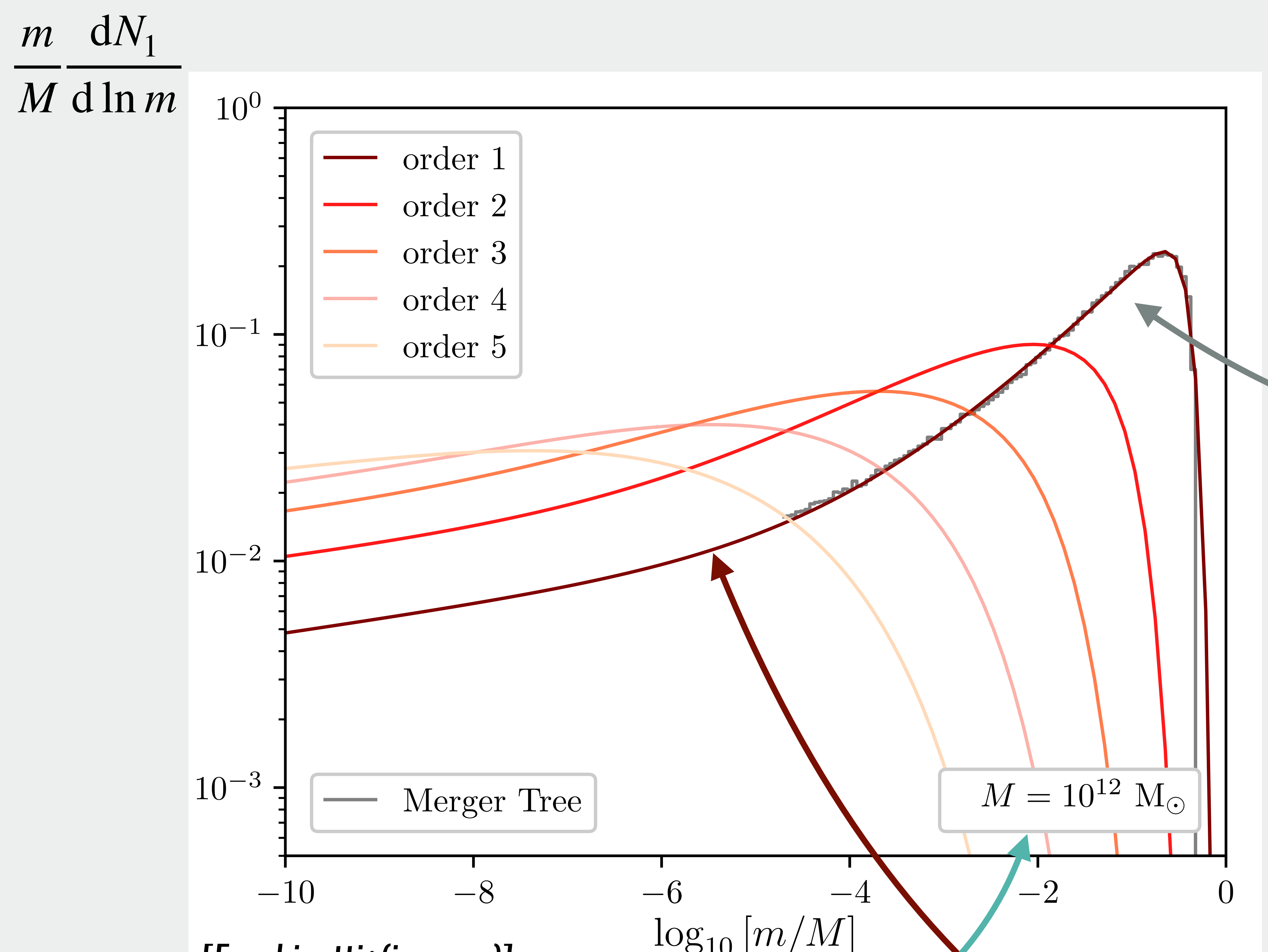
$$\delta_R(\mathbf{x}) = \int \mathrm{d}\mathbf{y}\frac{\delta\rho}{\bar{\rho}}W_R(|\mathbf{x} - \mathbf{y}|)$$ *(smoothed density contrast)*





[Jiang+14]

From the excursion set theory to merger trees



[Facchinetti+(in prep.)]

Host halo mass

Merger tree algorithm [Cole+00]

Mass function (on large masses)

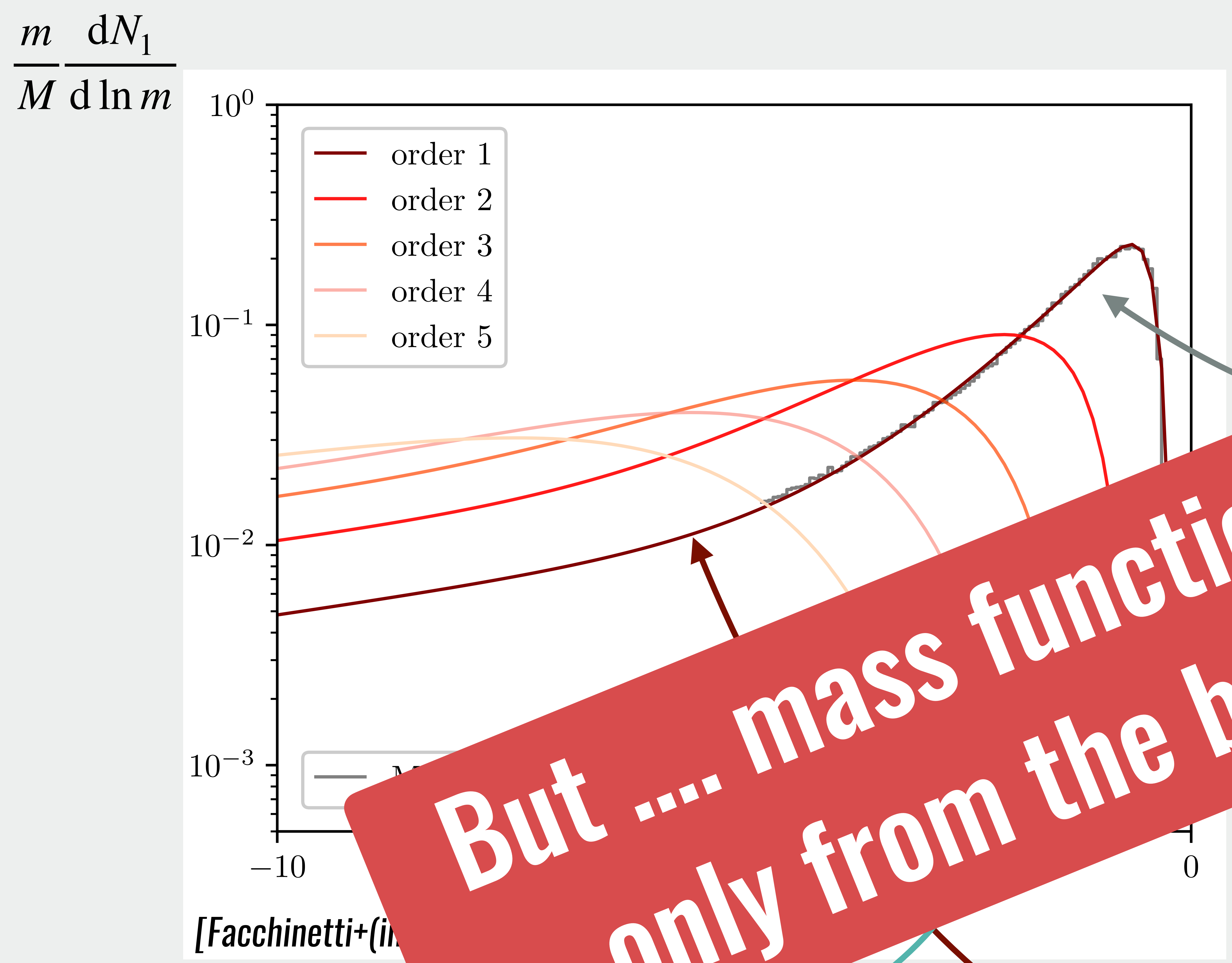
Fitting function (6 parameters)

$$f(m, M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

[Giocoli+08, Li+09, Jiang:+14]

Mass function (for all masses) $\frac{\mathrm{d}N_1}{\mathrm{d}m} = f(m, M)$

… it can be obtained from fits on the output of merger tree algorithms



[Facchinetti+(i

Host halo mass

Merger tree algorithm

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function (6 parameters)

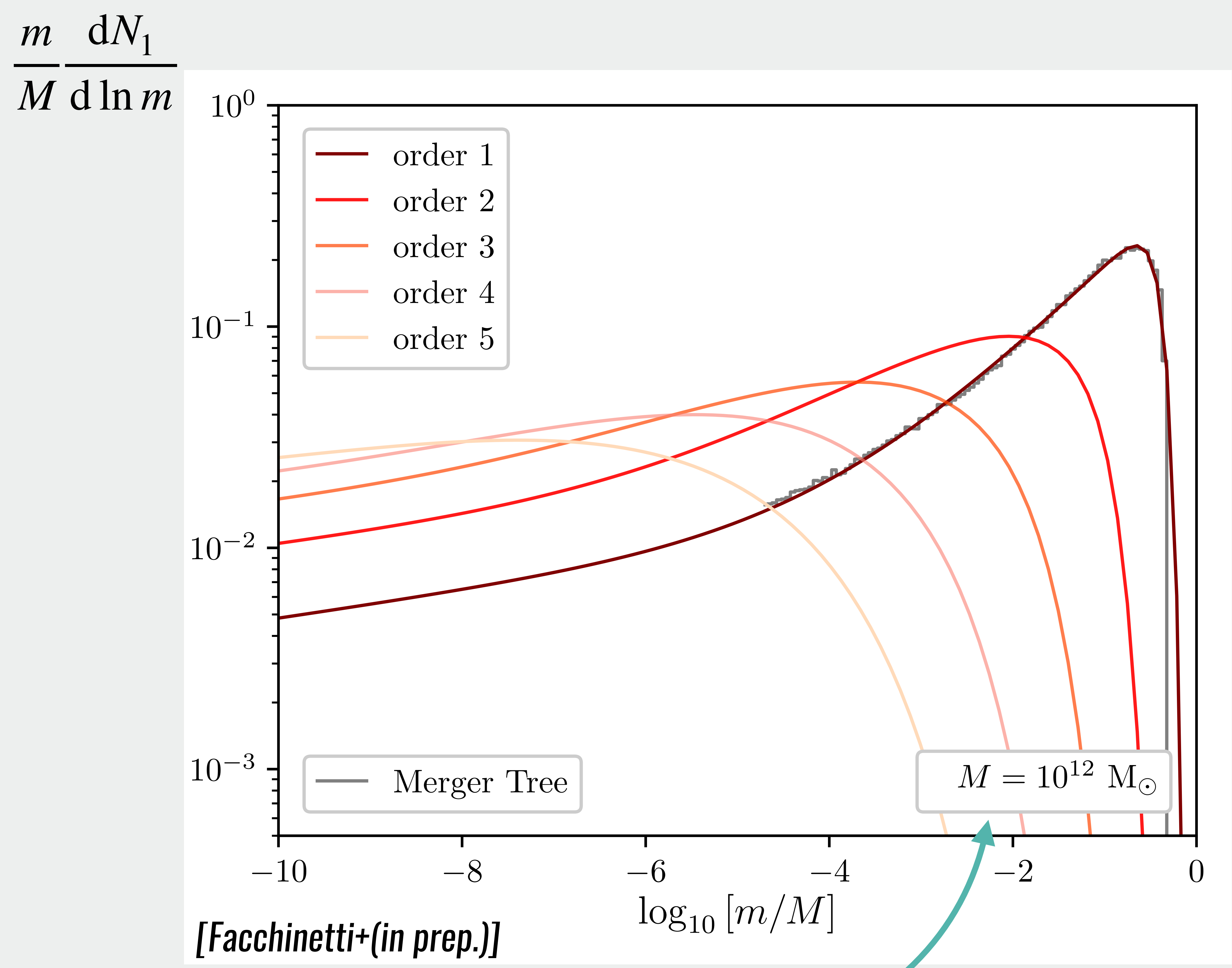
$$f(m, M) = \frac{1}{m}\left[\sum_{i=1,2}\gamma_i\left(\frac{m}{M}\right)^{-\alpha_i}\right]\exp\left\{-\beta\left(\frac{m}{M}\right)^{\zeta}\right\}$$

[Giocoli+08, Li+09, Jiang:+14]

Mass function (for all masses) $\frac{\mathrm{d}N_1}{\mathrm{d}m} = f(m, M)$

But .... mass function at small mass inferred only from the behaviour at large masse

... it can be obtained from fits on the output of merger tree algorithms



[Facchinetti+(in prep.)]

Host halo mass

New fitting procedure

Constraint on the shape by imposing the constraint

$$\frac{1}{M}\int_0^M m\frac{dN_1}{dm}dm = 1$$

The host halo is entirely made of subhalos
Consistent with the fractal picture

Fixes the slope at small mass

$$\frac{dN_1}{dm} \sim \gamma m^{-\alpha} \quad \text{with} \quad \alpha \sim 1.95$$

… it can be obtained from fits on the output of merger tree algorithms

Then truncate it from below to account for the minimal mass of halos

$$\frac{\mathrm{d}N_1}{\mathrm{d}m}(m, M) = f(m, M) \quad \longrightarrow \quad \frac{\mathrm{d}N_1}{\mathrm{d}m}(m, M) = f(m, M)\Theta(m - m_{\mathrm{min}})$$

Total number of subhalos (before tidal disruption)

$$N_1(M) = \int_0^M f(m, M)\Theta(m - m_{\mathrm{min}})\mathrm{d}m$$

Let us finish part I with a small computation (preliminary)

Assume self-similarity

$$\frac{\partial N_p(m,M)}{\partial m} = \int_0^M \frac{\partial N_1(m,m')}{\partial m} \frac{\partial N_{p-1}(m',M)}{\partial m'} \mathrm{d}m' \qquad \frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m \mathrm{d}m = 1$$

Define the total mass function

$$\frac{\partial N_{\mathrm{tot}}(m,M)}{\partial m} = \sum_{p=0}^{\infty} \frac{\partial N_p(m,M)}{\partial m}$$

$$\frac{\partial N_{\mathrm{tot}}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m} \frac{\partial N_{\mathrm{tot}}(m',M)}{\partial m'} \mathrm{d}m'$$

Start with

$$\frac{\partial N_{\rm tot}(m,M)}{\partial m} = \frac{\partial N_1(m,M)}{\partial m} + \int_0^M \frac{\partial N_1(m,m')}{\partial m}\frac{\partial N_{\rm tot}(m',M)}{\partial m'}\mathrm{d}m' \qquad \frac{1}{M}\int_0^M \frac{\partial N_p(m,M)}{\partial m} m\mathrm{d}m = 1$$

Change of variables
Assuming universality

$$\frac{\partial N_p(m,M)}{\partial m} = \frac{1}{m} g_p\left(-\ln\left(\frac{m}{M}\right)\right)$$

$$g_{\rm tot}(x) = g_1(x) + \int_0^x g_1(y) g_{\rm tot}(y-x)\mathrm{d}y \qquad \int_0^\infty g_p(x) e^{-x}\mathrm{d}x = 1$$

Laplace transform

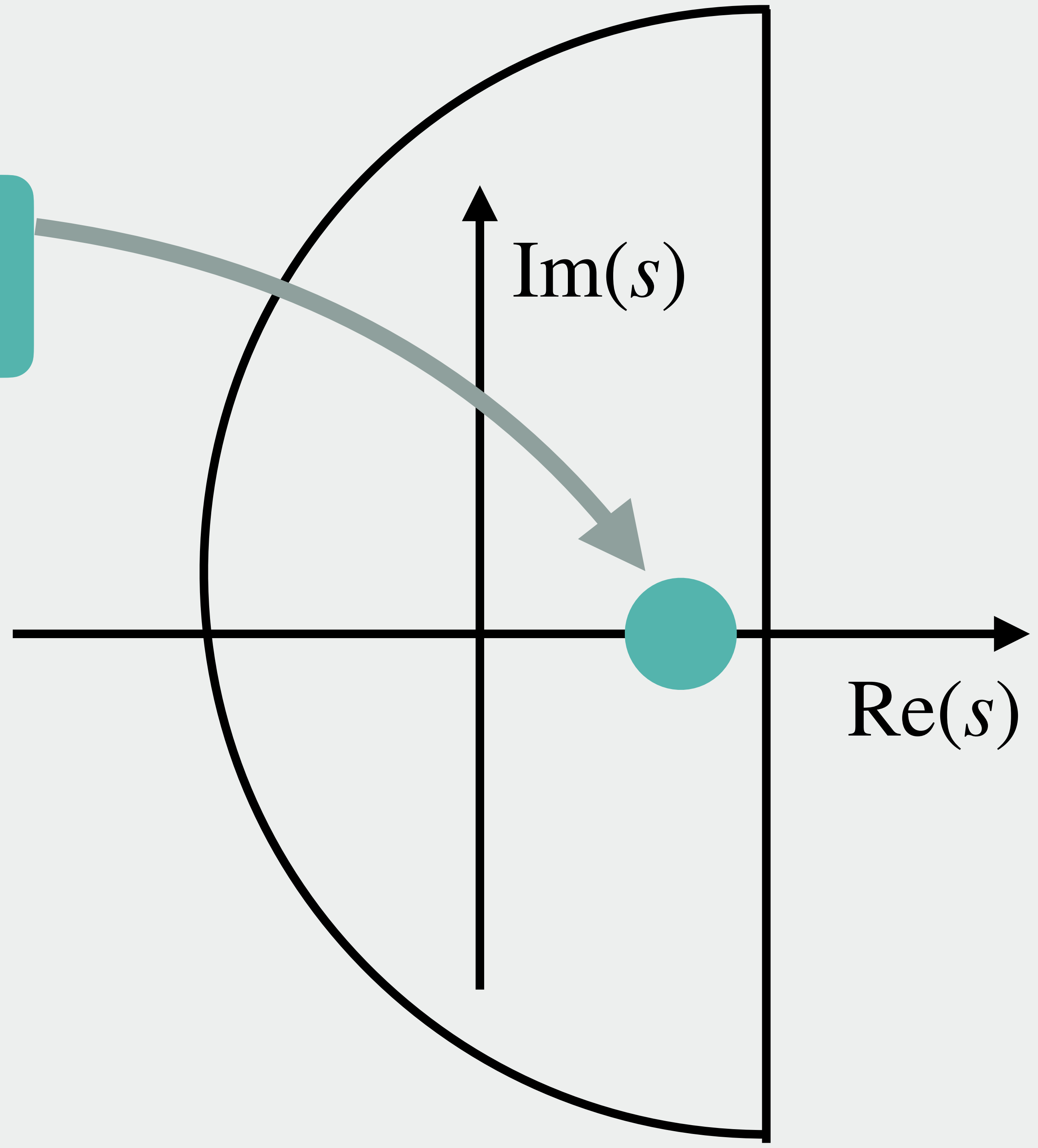
$$\hat{g}_p(s) \equiv \int_{[0,\infty[} g_p(x) e^{-sx}\mathrm{d}x$$

$$\hat{g}_{\rm tot}(s) = \frac{\hat{g}_1(s)}{1-\hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Start with

$$\hat{g}_{\text{tot}}(s) = \frac{\hat{g}_1(s)}{1 - \hat{g}_1(s)} \qquad \hat{g}_1(1) = 1$$

Pole in s=1



Use residue theorem (assuming we can)

$$g_{\text{tot}}(x) = \sum_{i=0}^{n_{\text{res}}} c_i e^{s_i x} \qquad c_i \equiv \text{Res}(\hat{g}_{\text{tot}}, s_i)$$

With the residue in s=1

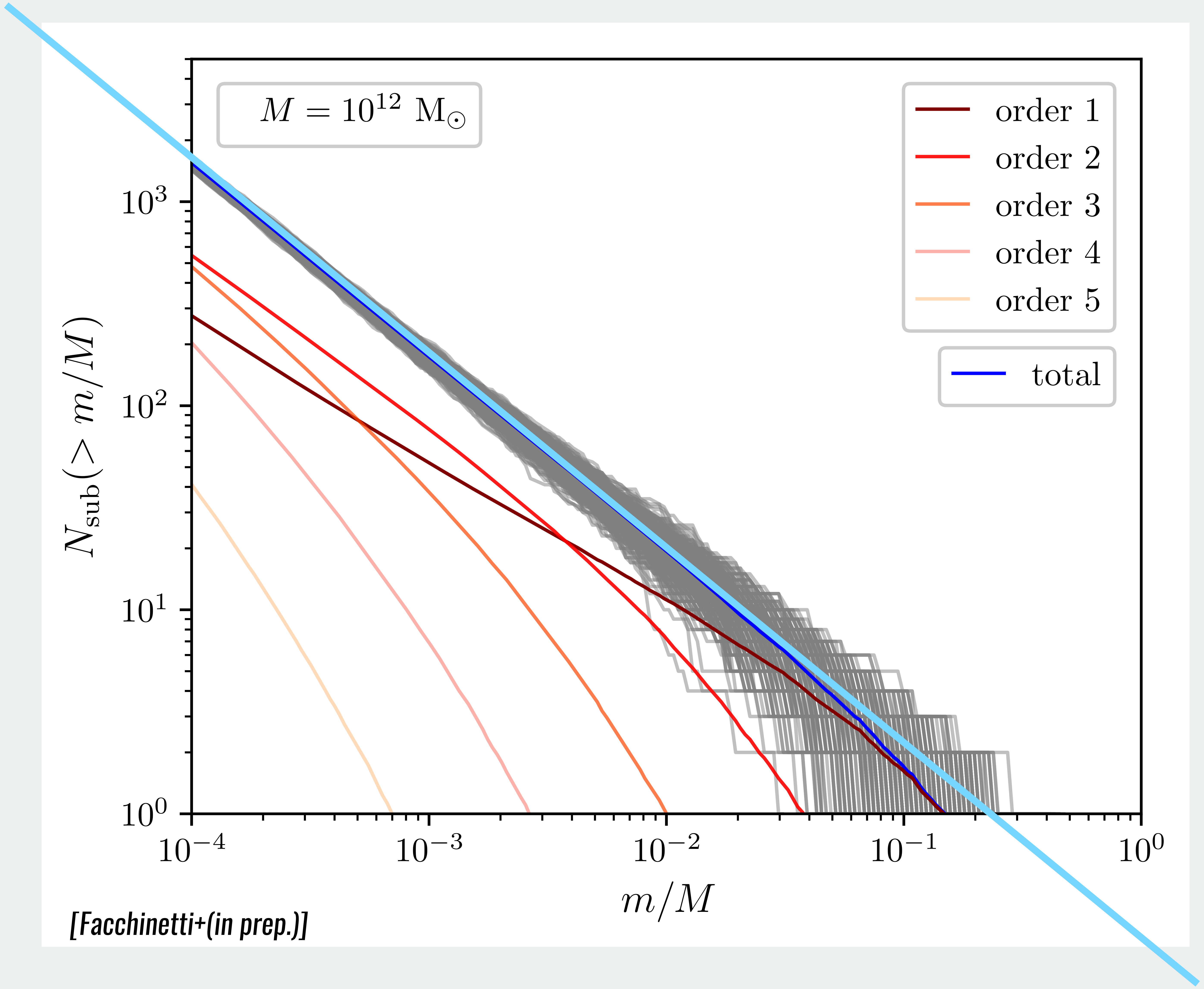
$$c_0 = \frac{1}{\hat{g}_1'(1)} \qquad s_0 = 1$$

$$g_{\text{tot}}(x) = \frac{1}{\hat{g}_1'(1)} e^x + \sum_{i>0} c_i e^{s_i x}$$

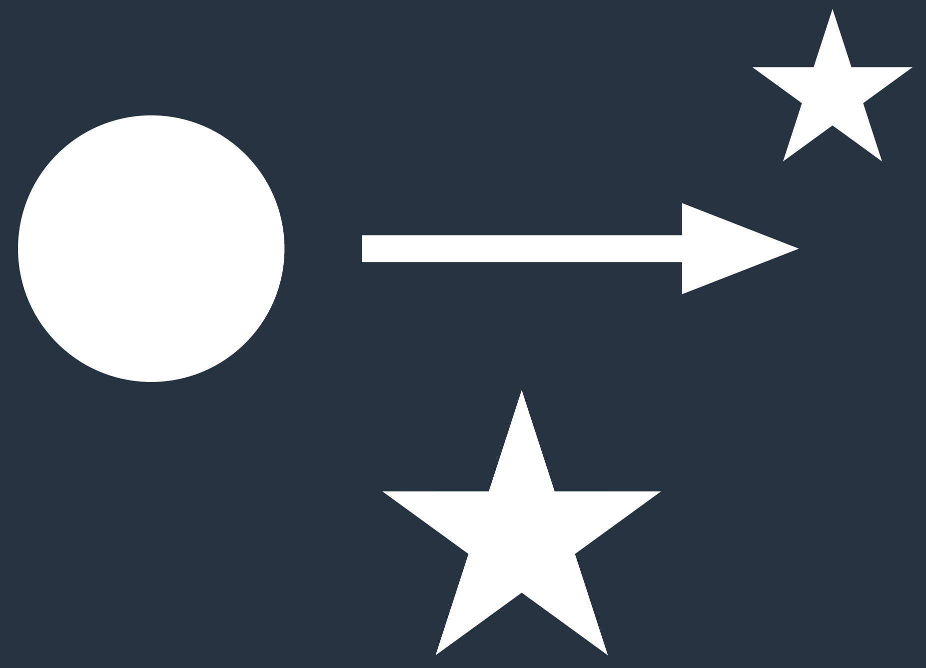
$$\frac{\partial N_{tot}(m, M)}{\partial m} = \frac{M}{\hat{g}_1'(1)} m^{-2} + \sum_{i>0} \frac{c_i}{m} \left(\frac{m}{M}\right)^{-s_i}$$

-2 is a critical exponent

$$\frac{\partial N_{tot}(m, M)}{\partial m} \underset{\sim}{\propto} m^{-2} \quad \textit{if} \quad \text{Re}(s_i) \ll 1 \ \forall i > 0$$

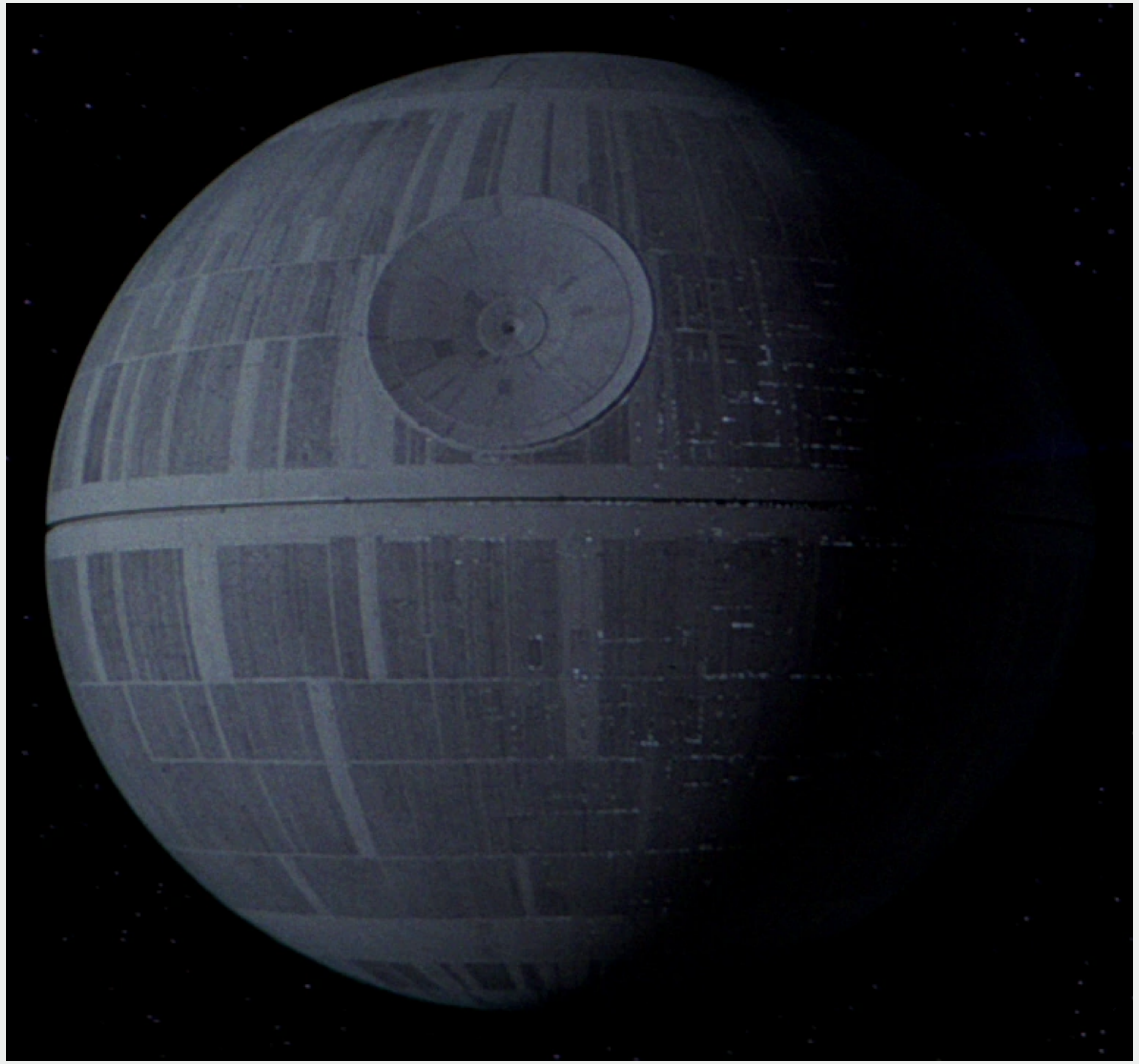


Merger Trees Monte Carlo results



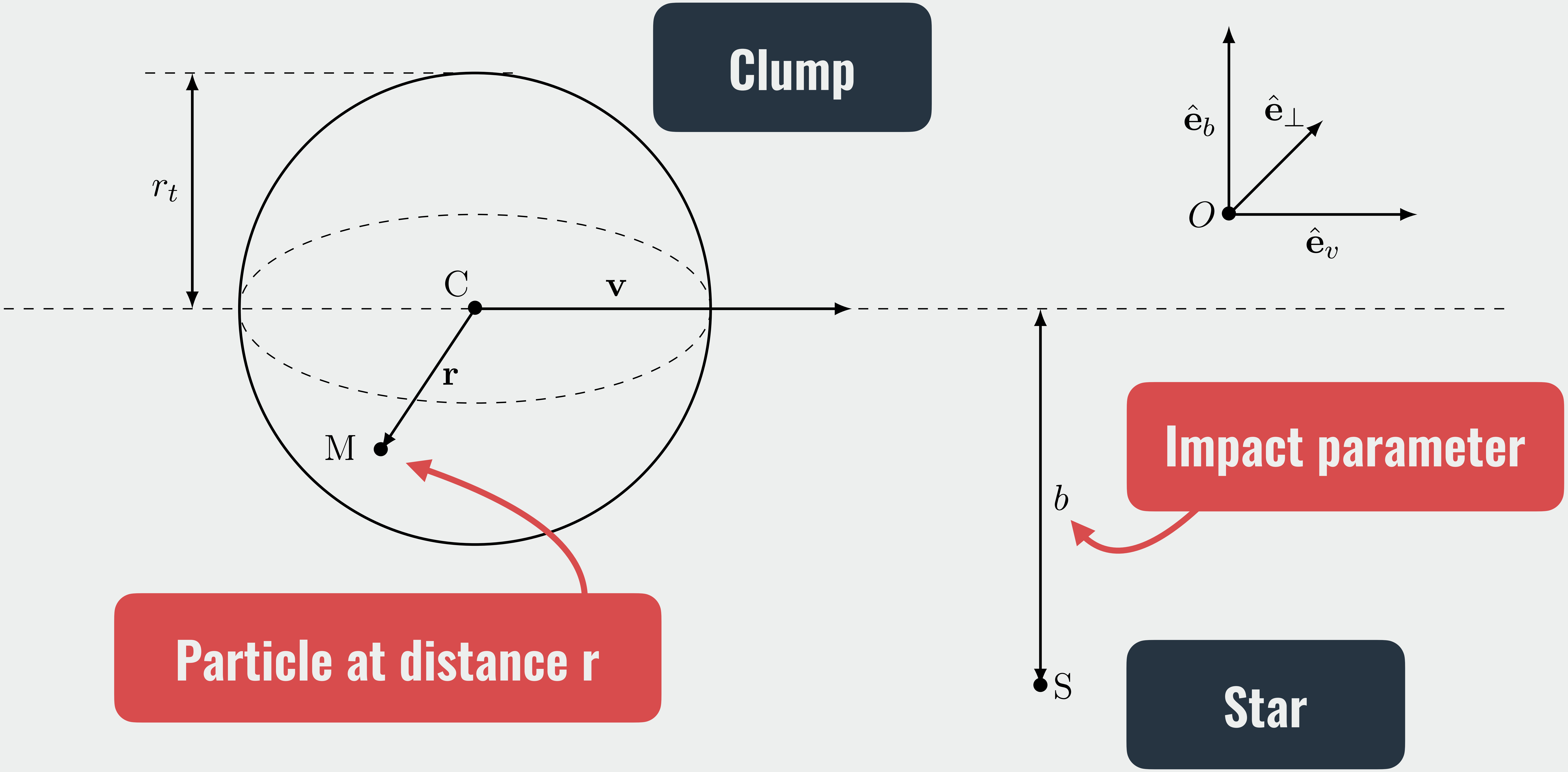
Stellar encounters

[arXiv:2201.xxxx]



[Darth Vador+(a long time ago)]

Encounter of one star and one subhalo



Modelisation of the encounter

We want to evaluate

The kinetic energy kick of particles

$$\delta E = E_{\mathrm{after}} - E_{\mathrm{before}}$$

Compare $\delta E(r)$

to the gravitational potential $|\Phi(r)|$

to evaluate the effects on the subhalo

We want to evaluate

The kinetic energy kick of particles

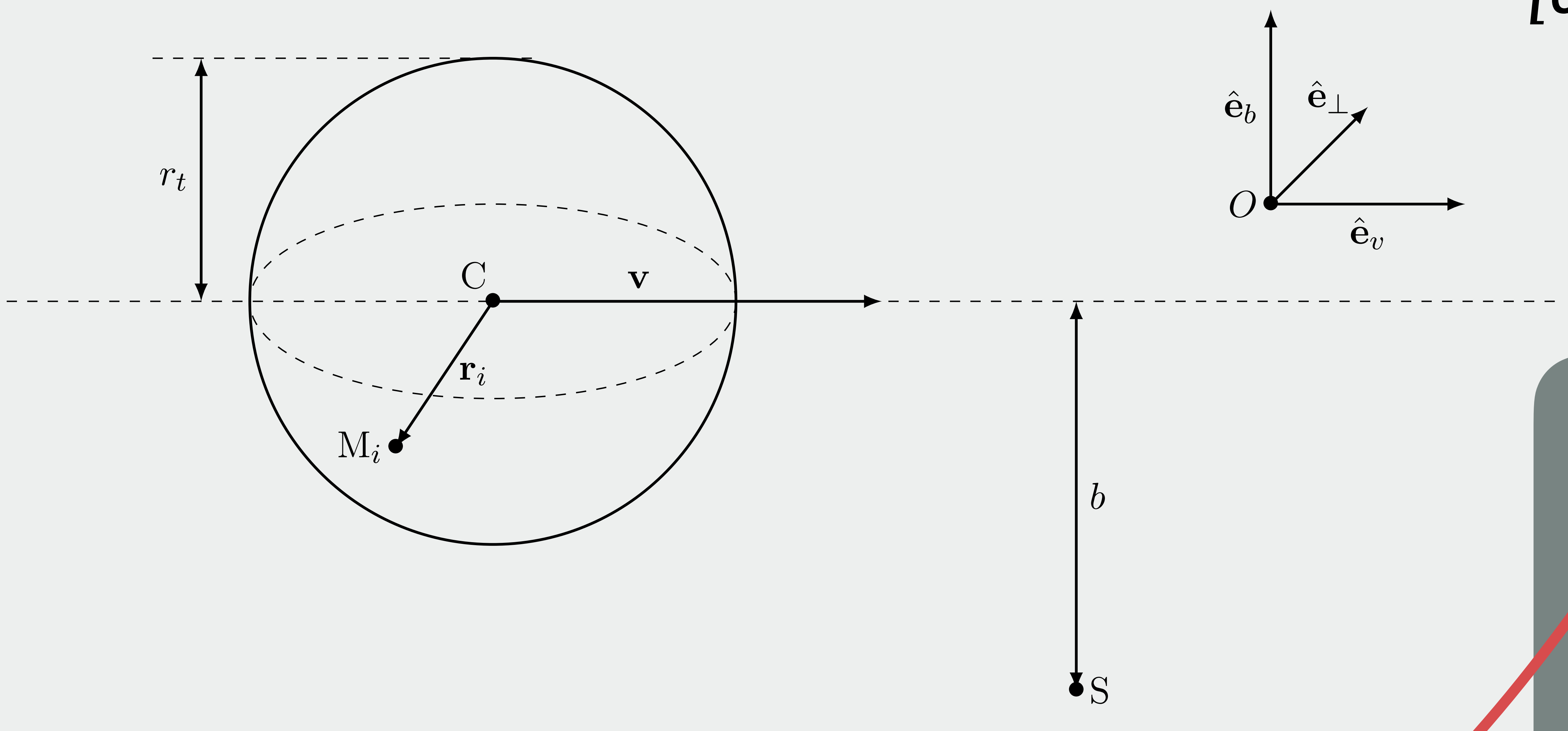
$$\delta E = \frac{1}{2}(\delta \mathbf{v})^2 + \mathbf{v} \cdot \delta \mathbf{v}$$

Velocity kick of particle w.r.t. the CM

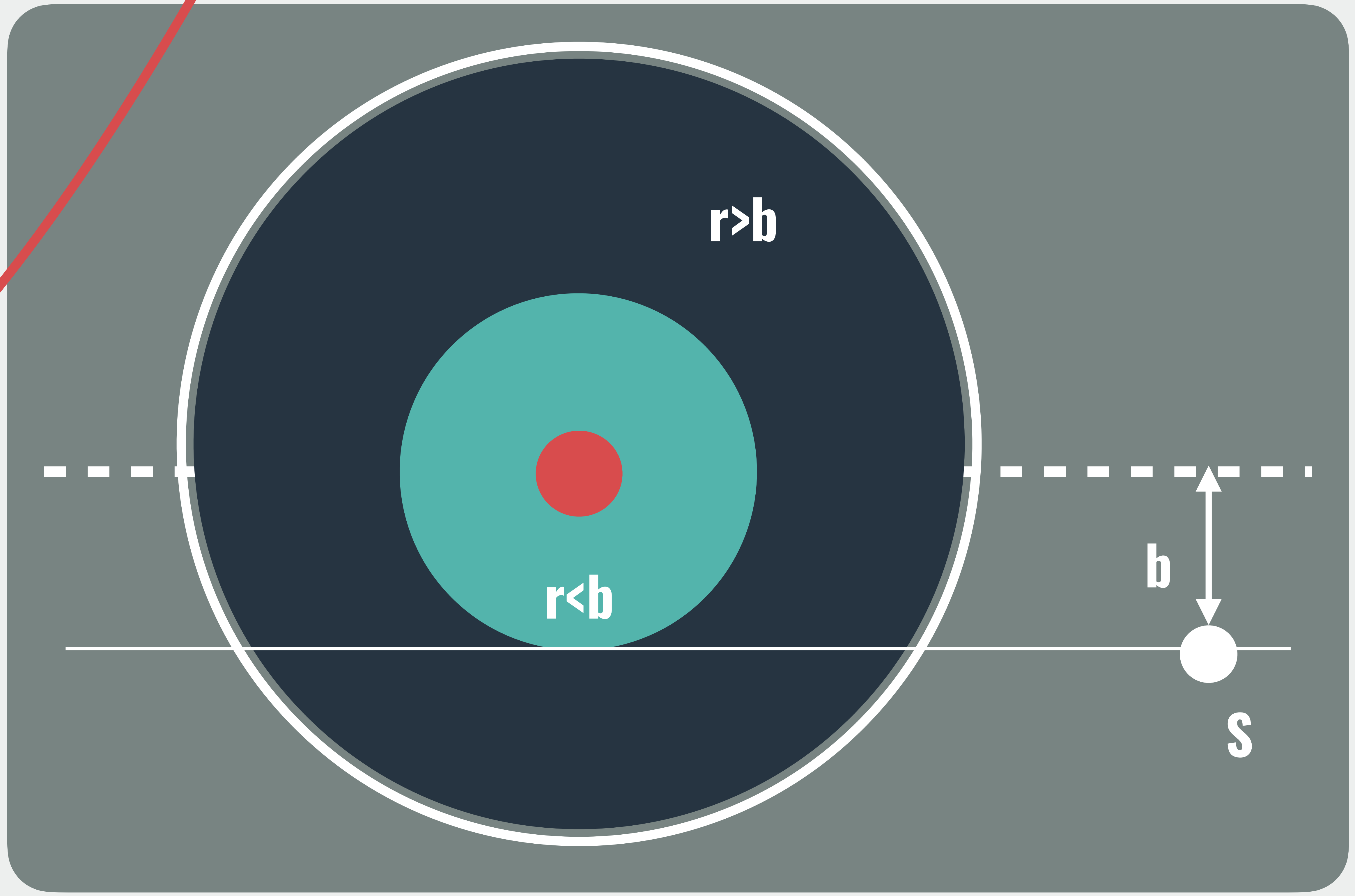
Initial velocity of particle w.r.t. the CM

[Spitzer58, Gerhard+83, Carr+99, Green+07, Schneider+10]

Complementary numerical simulation: [Delos19]



Encounter of two galaxies (Extended objects)



We improve the usual computation of $(\delta \mathbf{v})^2$

$$(\delta \mathbf{v})^2(\mathbf{r}) = \left(\frac{2G_\mathrm{N} m_\star}{v_\mathrm{r} b}\right)^2 \left[I^2 + \frac{b^2(1-2I) - 2I\mathbf{r}\cdot\mathbf{b}}{(\mathbf{r}+\mathbf{b})^2 - (\mathbf{r}\cdot\hat{\mathbf{e}}_{v_\mathrm{r}})^2}\right]$$

$$I(b, r_\mathrm{t}) = \frac{b^2 v_\mathrm{r}}{m_\mathrm{t}} \int_0^\infty \frac{m\left(<\sqrt{b^2 + v_\mathrm{r}^2 t^2}\right)}{\left(b^2 + v_\mathrm{r}^2 t^2\right)^{3/2}} \mathrm{d}t$$

It is convenient to average over angles

(because of the spherical symmetry)

$(\delta \mathbf{v})^2(\mathbf{r} = (r, \theta, \varphi)) \to \left\langle (\delta \mathbf{v})^2 \right\rangle (r)$ on every shell

However, infinities appear due to the diverging potential of the star

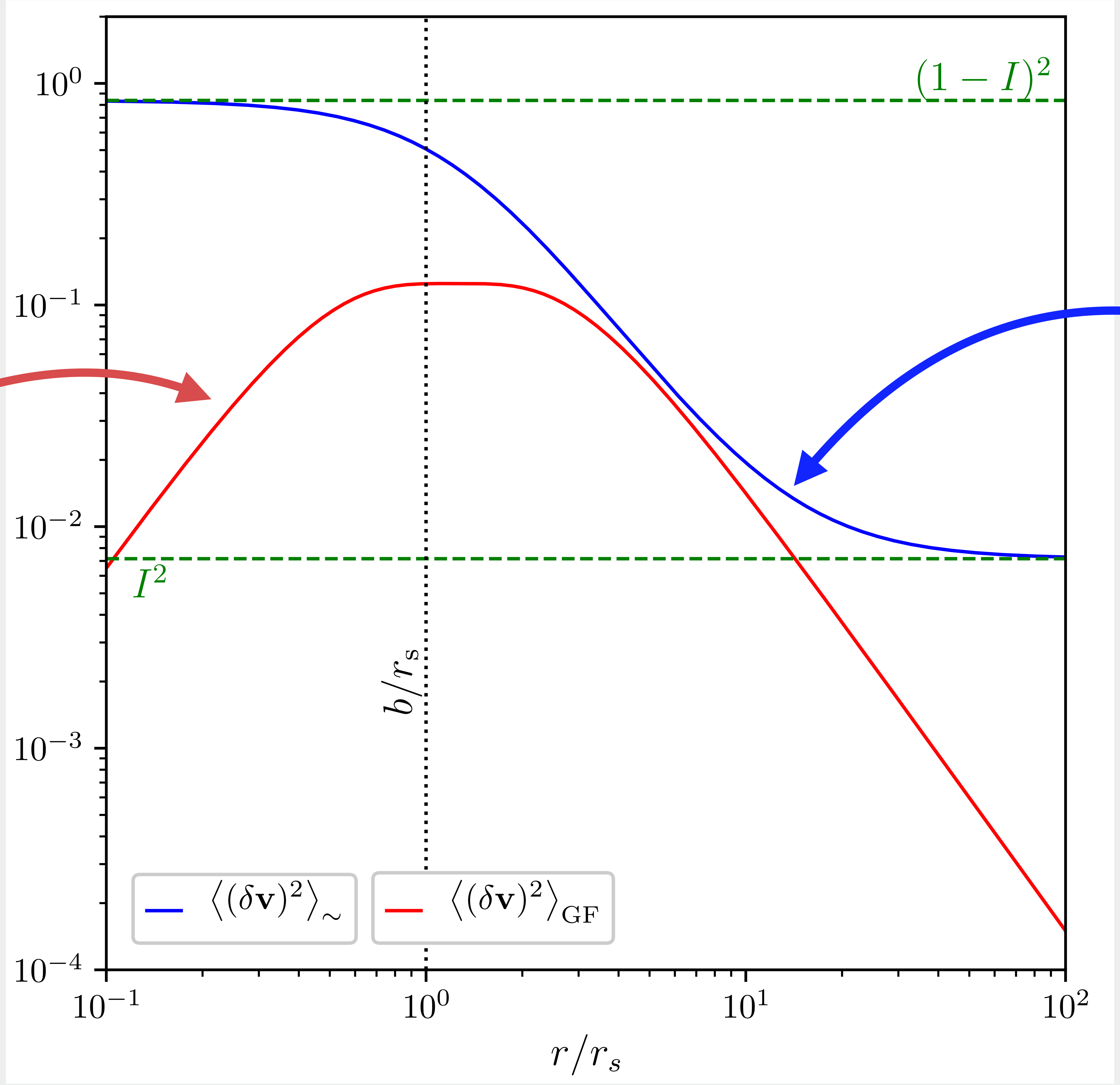
Solution: find a good ansatz

(our new proposal)

$$\left\langle (\delta \mathbf{v})^2 \right\rangle_{\sim}(r) = \left(\frac{2G_{\rm N} m_\star}{b v_{\rm r}} \right)^2 \left[I^2(b, r_t) + 3 \frac{1 - 2I(b, r_t)}{3 + 2(r/b)^2} \right]$$

Penetrative encounter

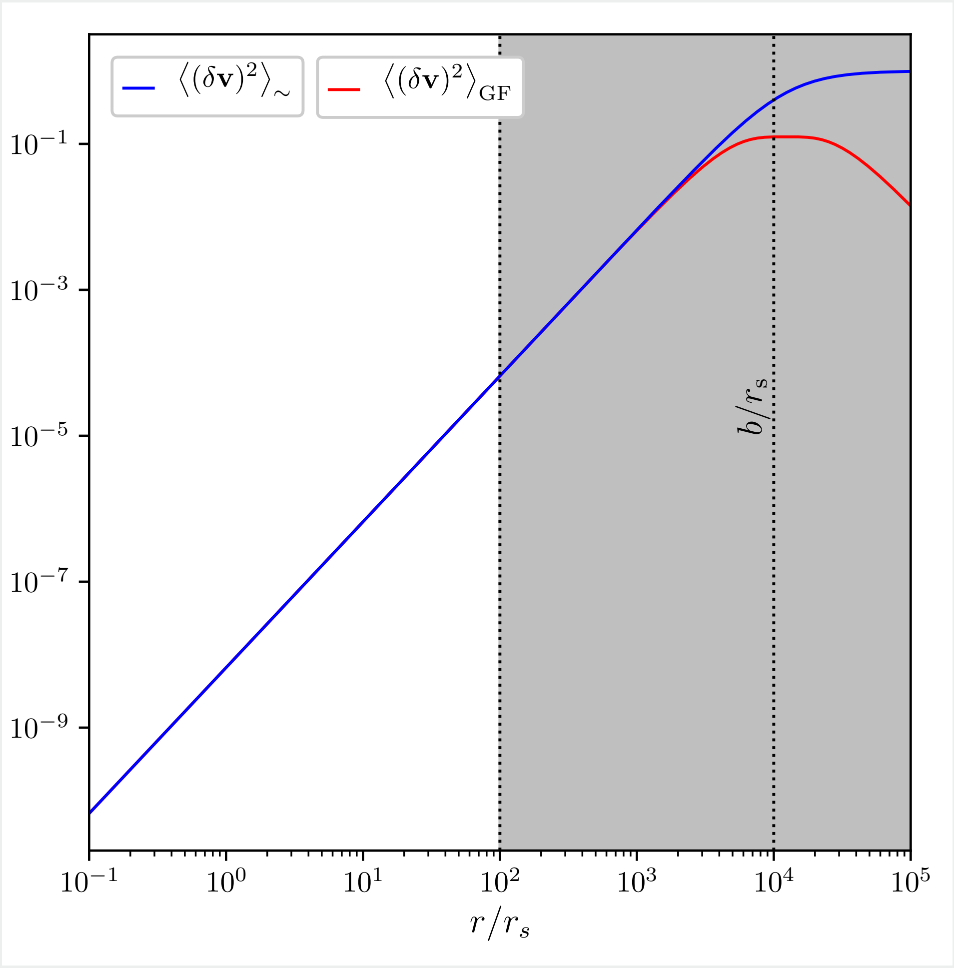
$(\delta \mathbf{v})^2$
(dimensionless)



Original extrapolation from [Gerhard+83]

Our ansatz

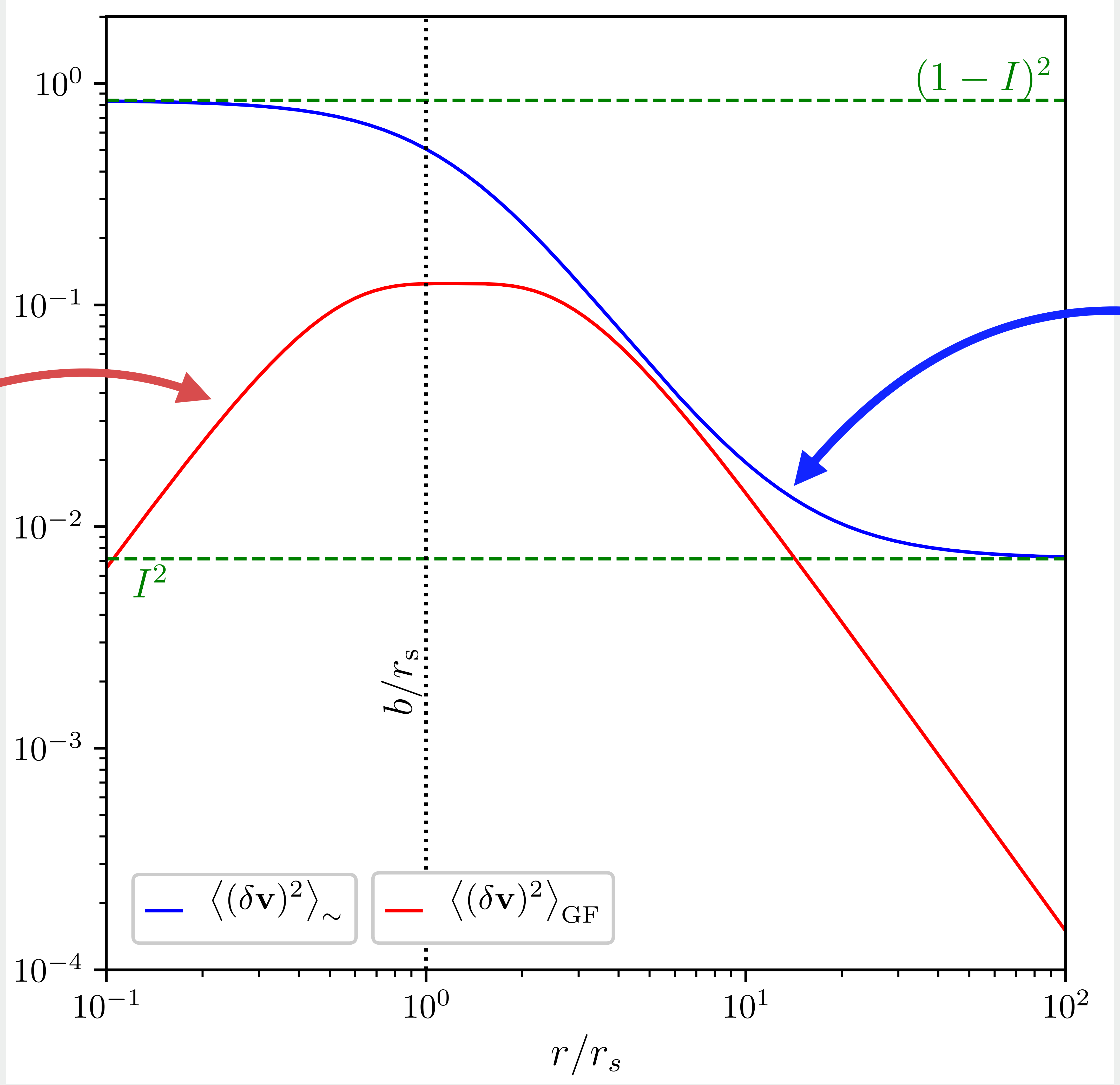
non-penetrative encounter



Our ansatz perform better for penetrative encounters

Penetrative encounter

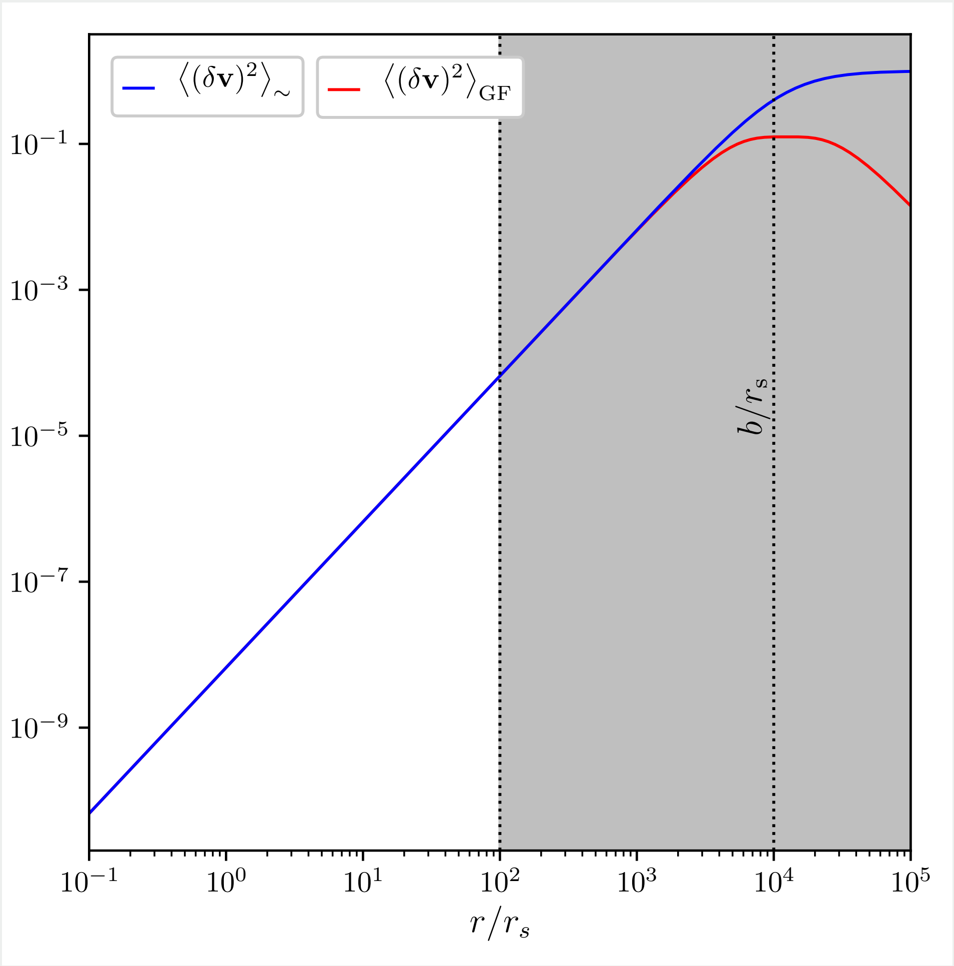
$(\delta \mathbf{v})^2$
(dimensionless)



Original extrapolation from [Gerhard+83]

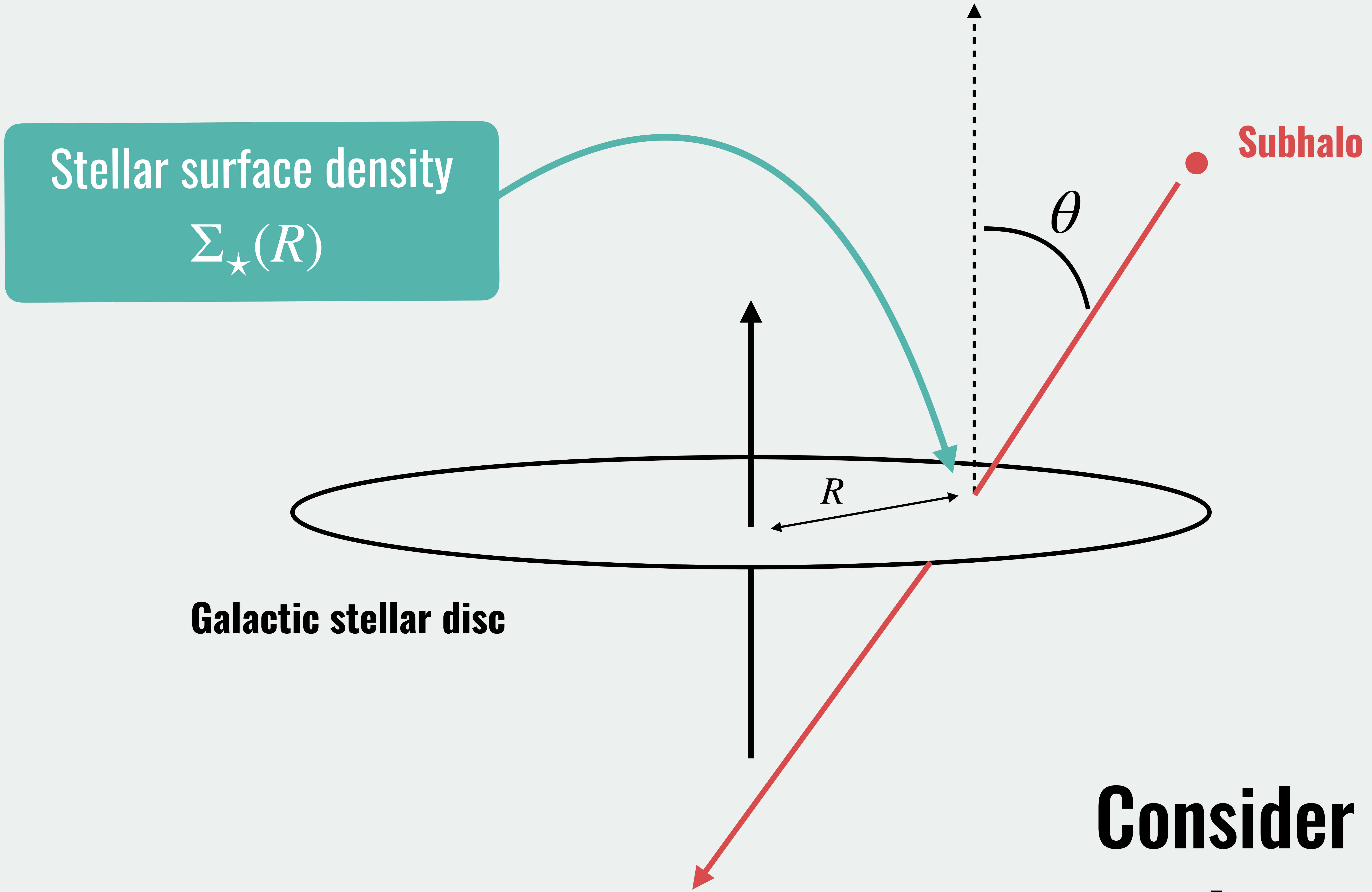
Our ansatz

non-penetrative encounter



Our ansatz perform better for penetrative encounters

Encounter of one subhalo with the stellar disc



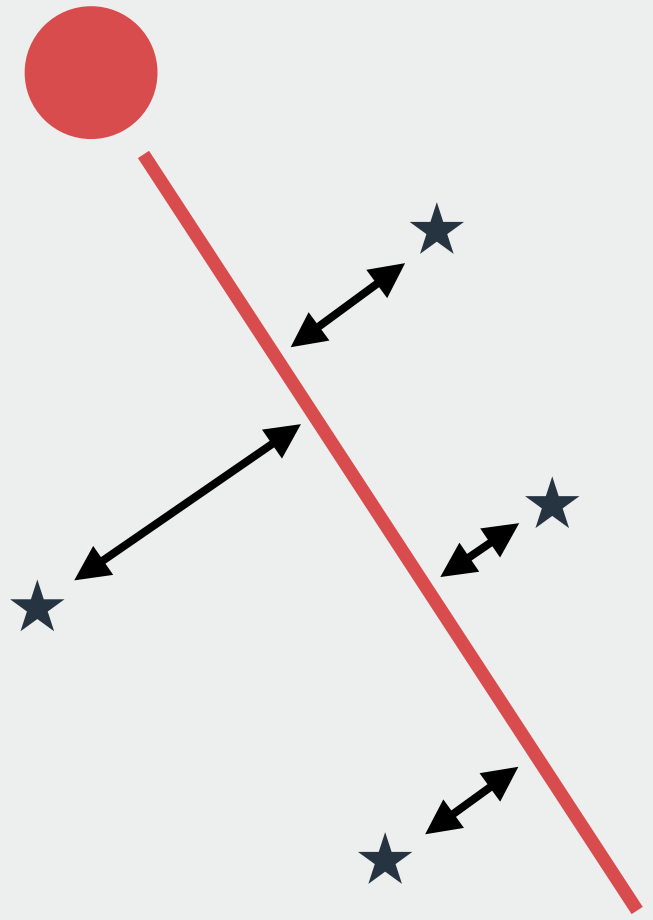
Consider a given subhalo and a given shell inside

Total energy/velocity kick:

Number of encountered stars

$$\Delta \mathbf{v} = \sum_{i=1}^{\mathcal{N}} \delta \mathbf{v}_i \qquad \Delta E = \frac{1}{2}(\Delta \mathbf{v})^2 + \mathbf{v} \cdot \Delta \mathbf{v}$$

Random walk in velocity space



Large N-limit velocity kick pdf:

$$p_{\Delta \mathbf{v}}(\Delta \mathbf{v}) = \frac{1}{\pi \mathcal{N} \overline{(\delta \mathbf{v})^2}} \exp\left(- \frac{(\Delta \mathbf{v})^2}{\mathcal{N} \overline{(\delta \mathbf{v})^2}} \right)$$

Central-Limit theorem

Average velocity squared per encounter

Large N-limit total energy kick pdf:

$$p_{\Delta E}(\Delta E) = \frac{1}{4 s \sigma_{\rm sub}^2 \sqrt{1+s^2}} \exp\left(\frac{\Delta E}{2\sigma_{\rm sub}^2} - \frac{|\Delta E|}{2\sigma_{\rm sub}^2} \frac{\sqrt{1+s^2}}{s} \right)$$

with initial velocity pdf: Maxwell-Boltzmann

The total velocity kick is the result of a random walk

Distribution of stars/encounters

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b\mathrm{d}m_\star} = \mathcal{N} p_b(b) p_{m_\star}(m_\star)$$

Result of one subhalo-one star encounter

Average velocity kick squared per encounter

$$\overline{(\delta \mathbf{v})^2} = \int_{b_{\min}\sim 0}^{b_{\max}} \mathrm{d}b \int \mathrm{d}m_\star p_b(b) p_{m_\star}(m_\star) \left\langle (\delta \mathbf{v})^2 \right\rangle$$

Stellar mass distribution [Chabrier03]

$p_b(b) \propto b$

We evaluate the average energy kick per encounter

The end of the story?

Problem! No convergence for innermost particles

$p_b(b) \propto b$

(steep and asymmetric)

and $\left\langle (\delta \mathbf{v})^2 \right\rangle \propto b^{-4}$

Small impact parameters: almost **never happen**

but contribute a lot to the integral of $\overline{(\delta \mathbf{v})^2}$

We need the analytical formulas to perform computations on the full subhalo population

$\overline{(\delta \mathbf{v})^2}$ **too large**

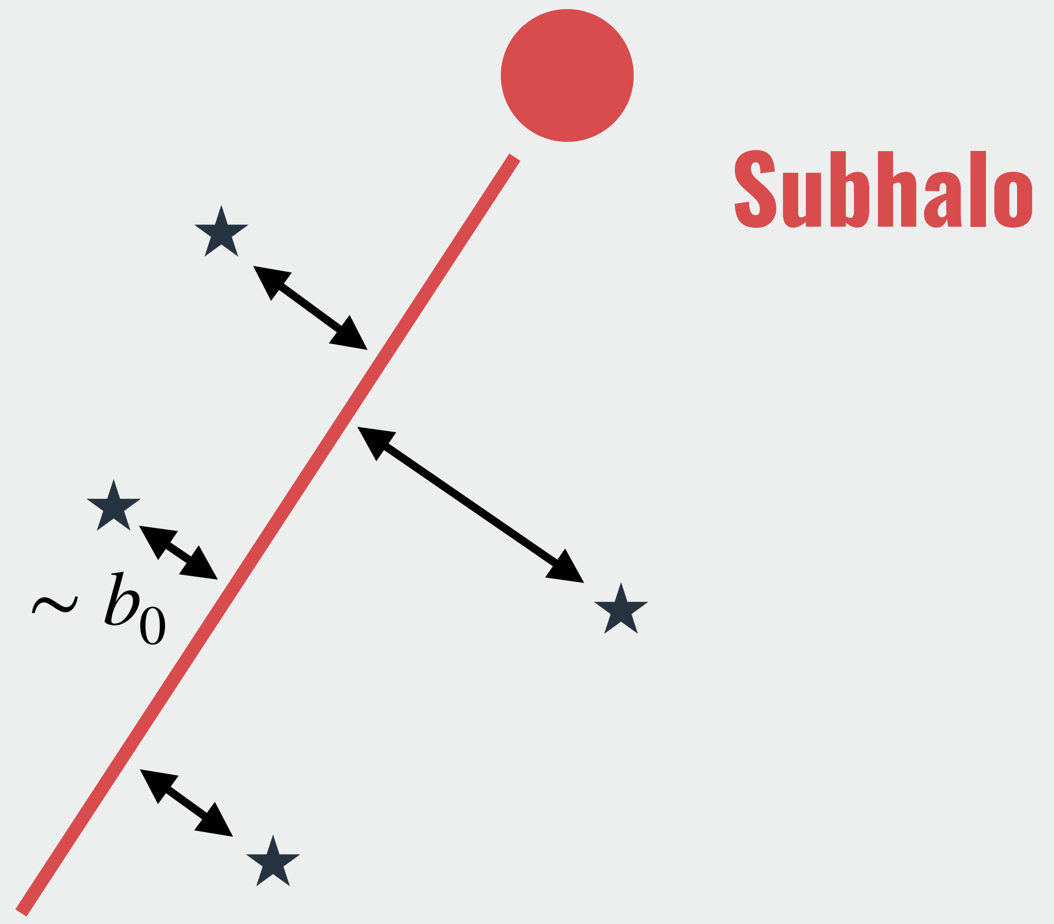
if $\mathcal{N} \neq \infty$

Solution! Find the typical minimal impact parameter (for each crossing of the disc)

$$b_0 \sim \frac{b_{\max}}{\mathcal{N}}$$

$$b_0(8\ \mathrm{kpc}) \sim 0.5 \times 10^{-4}\ \mathrm{pc}$$

$$b_0(1\ \mathrm{kpc}) \sim 0.8 \times 10^{-5}\ \mathrm{pc}$$

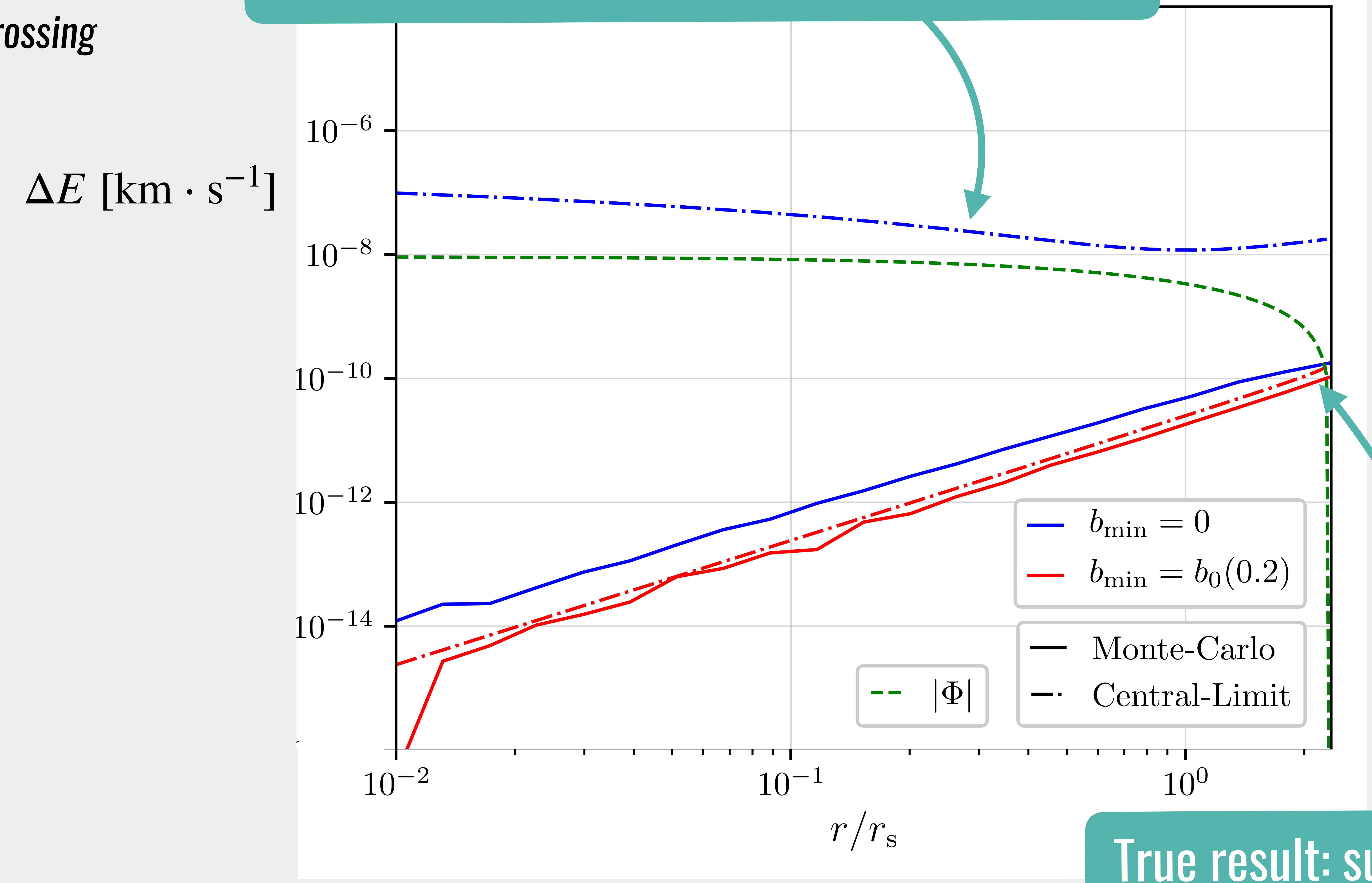


Cut-off on the impact parameter at b_0

Recover convergence

$$\overline{(\delta \mathbf{v})^2} = \int_{b_{\min} \sim 0}^{b_{\max}} \mathrm{d}b \int \mathrm{d}m_\star p_b(b) p_{m_\star}(m_\star) \left\langle (\delta \mathbf{v})^2 \right\rangle \qquad b_{\min} \to b_0$$

After one crossing of the disc



Naive Central-Limit result: subhalo disrupted

$v_{rel} = 334$ km . s^{-1}
$m_{200} = 1.6 \times 10^{-9}\, M_\odot$
$R = 8$ kpc
$\cos\theta = 1/2$

True result: subhalo slightly shrinks

[Facchinetti+(in prep.)]

The uncorrected CL-result overestimate the energy kick

Impact of stars with on the subhalo density profile

$$p_{\Delta E}(\Delta E) = \int \mathrm{d}^3\mathbf{v}\, \frac{f(v,r)}{\rho(r)} p_{\Delta E}(\Delta E \,|\, \mathbf{v})$$

$$\frac{1}{2} \leq \frac{\mathrm{Med}(\Delta E)}{\overline{\Delta E}} \leq \ln(2)$$

Median can be easily approximated by the average (« fast » to compute)

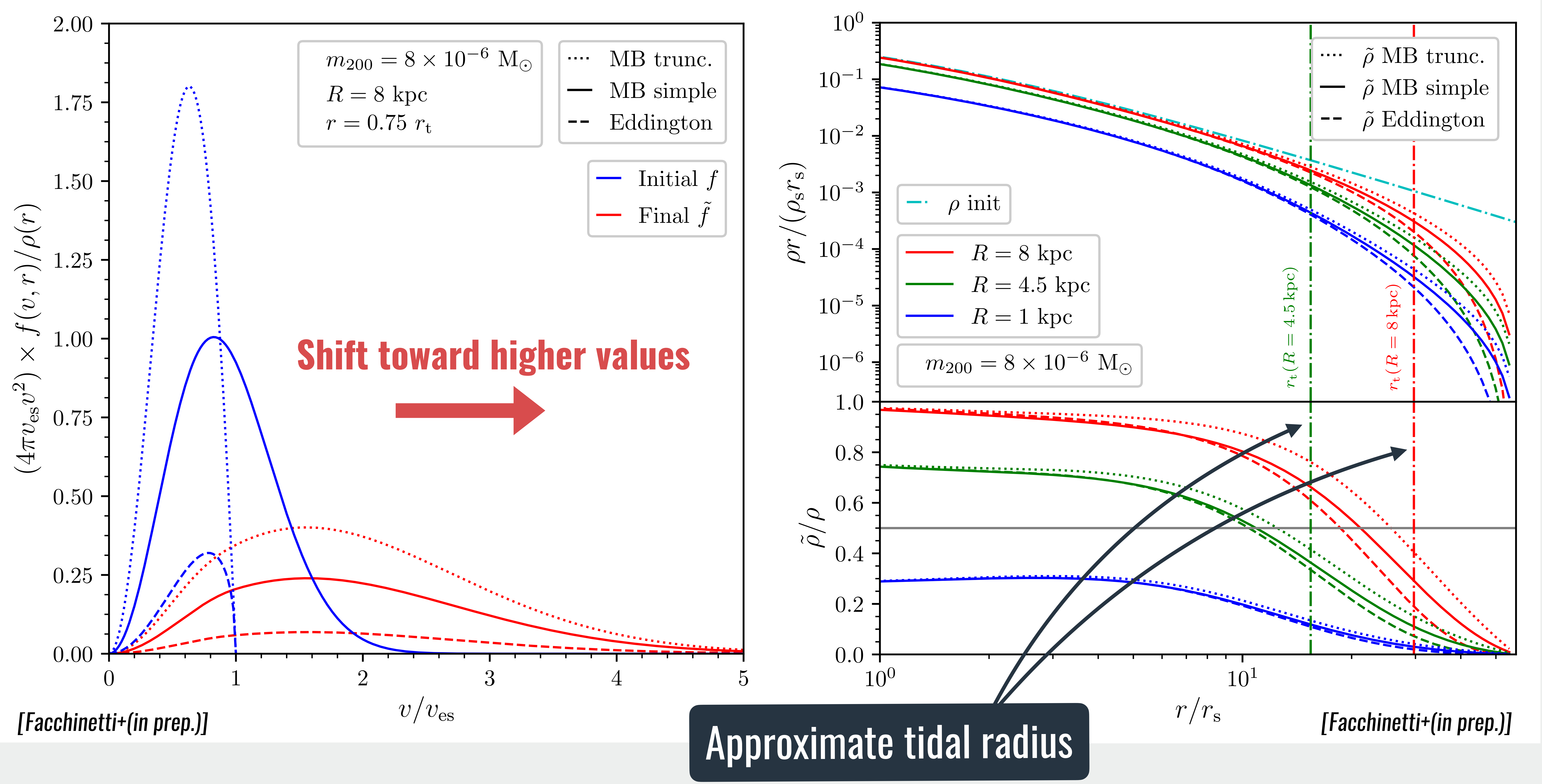
$$v_f = \sqrt{\mathbf{v}^2 + 2\Delta E}$$

$$p_{v_f}(v_f) = \int \mathrm{d}^3\mathbf{v}\, \frac{f(v,r)}{\rho(r)} p_{v_f}(v_f \,|\, \mathbf{v})$$

New phase space DF and profile:

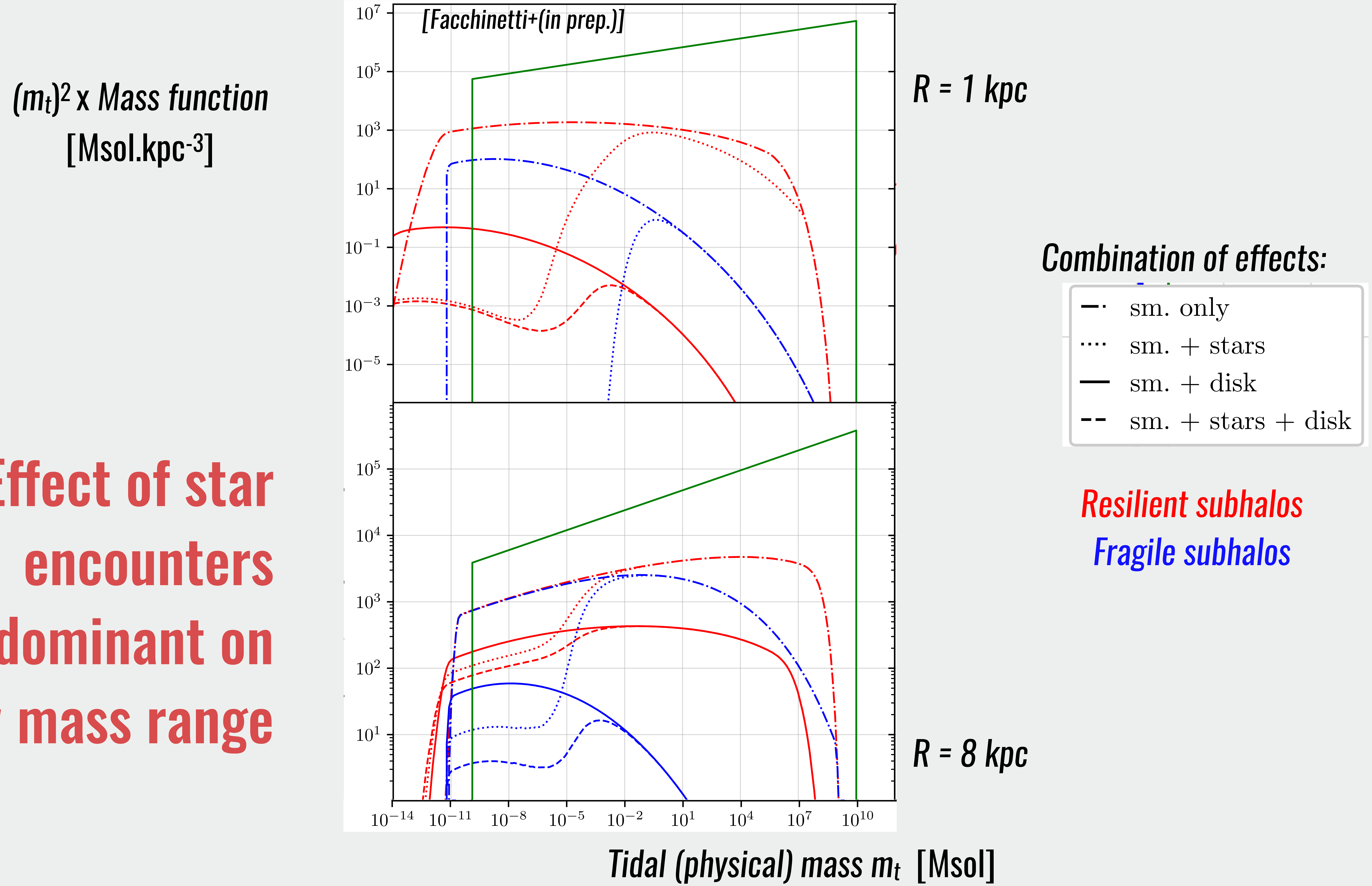
$$\tilde{f}(v_f, r) = \int_0^{v_{\mathrm{esc}}} \mathrm{d}^3\mathbf{v}\, f(v,r) p_{v_f}(v_f \,|\, \mathbf{v})$$

$$\tilde{\rho}(r) \simeq \int_0^{v_{\mathrm{esc}}} \mathrm{d}^3\mathbf{v}_f\, \tilde{f}(v_f, r) = \int_0^{v_{\mathrm{esc}}} \mathrm{d}^3\mathbf{v} f(v,r) F(<v_{\mathrm{esc}}, \mathbf{v})$$



$m_{200} = 8 \times 10^{-6}$ M$_\odot$
$R = 8$ kpc
$r = 0.75\ r_t$
MB trunc.
MB simple
Eddington
Initial f
Final $\tilde{f}$
Shift toward higher values
$(4\pi v_{es} v^2) \times f(v,r)/\rho(r)$
v/v_{es}
[Facchinetti+(in prep.)]
$\tilde{\rho}$ MB trunc.
$\tilde{\rho}$ MB simple
$\tilde{\rho}$ Eddington
ρ init
$R = 8$ kpc
$R = 4.5$ kpc
$R = 1$ kpc
$m_{200} = 8 \times 10^{-6}$ M$_\odot$
$r_t(R = 4.5\,\mathrm{kpc})$
$r_t(R = 8\,\mathrm{kpc})$
$\rho r/(\rho_s r_s)$
$\tilde{\rho}/\rho$
r/r_s
[Facchinetti+(in prep.)]
Approximate tidal radius

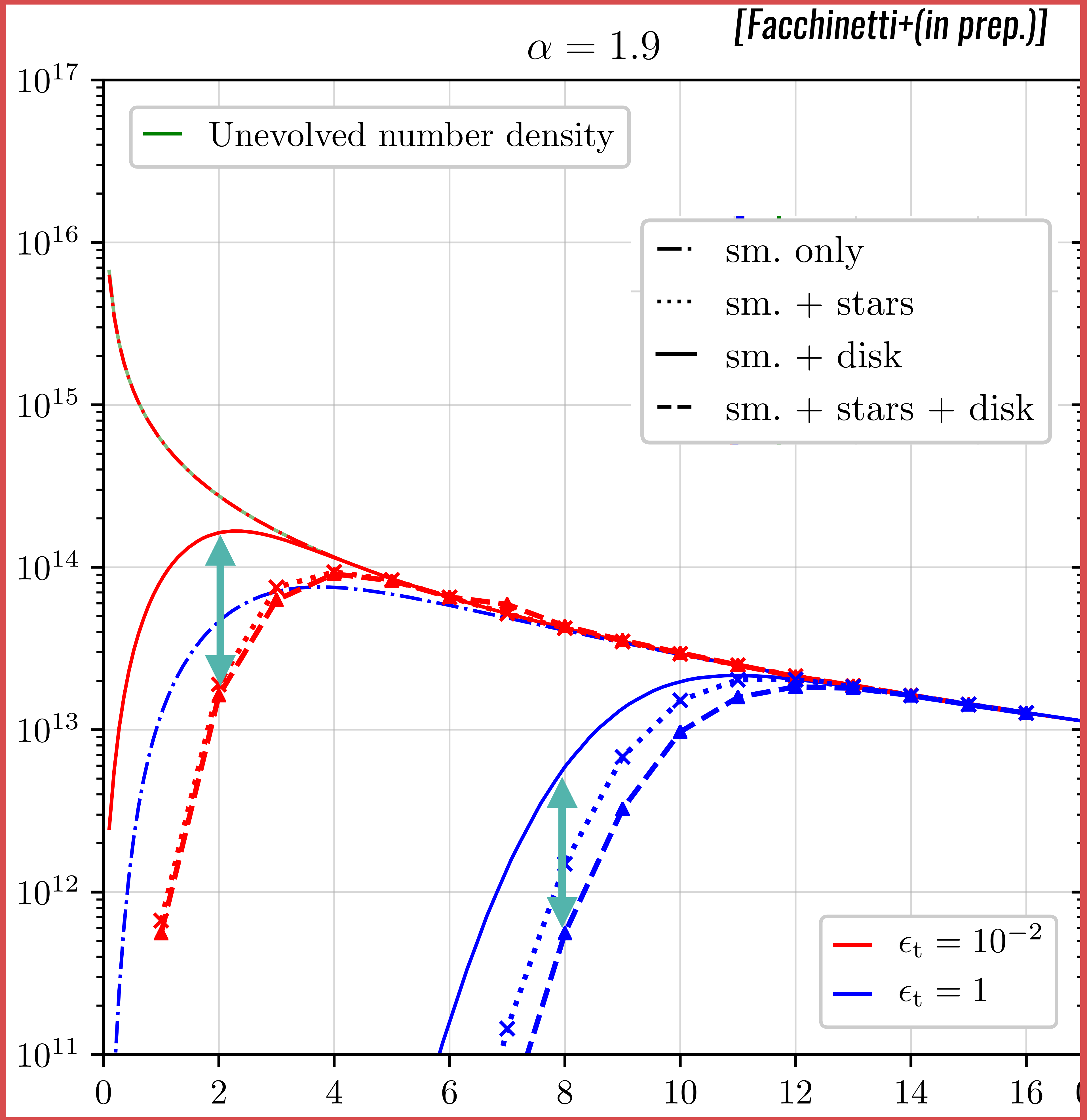
Impact of the stellar disc on the total subhalo population



Effect of star encounters dominant on low mass range

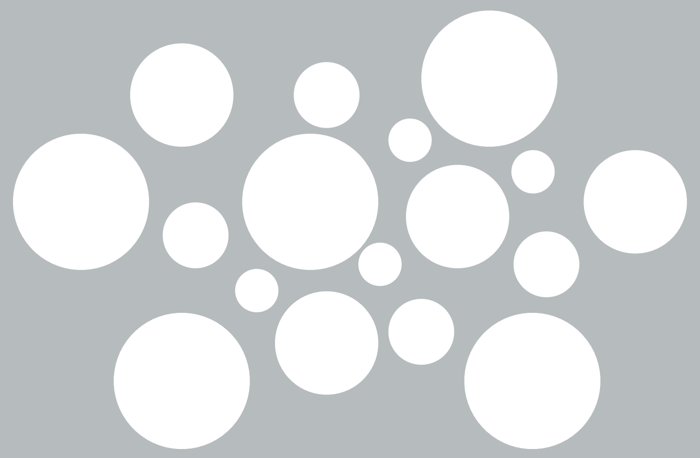
Stars impact the small mass range

Number density
[kpc^{-3}]



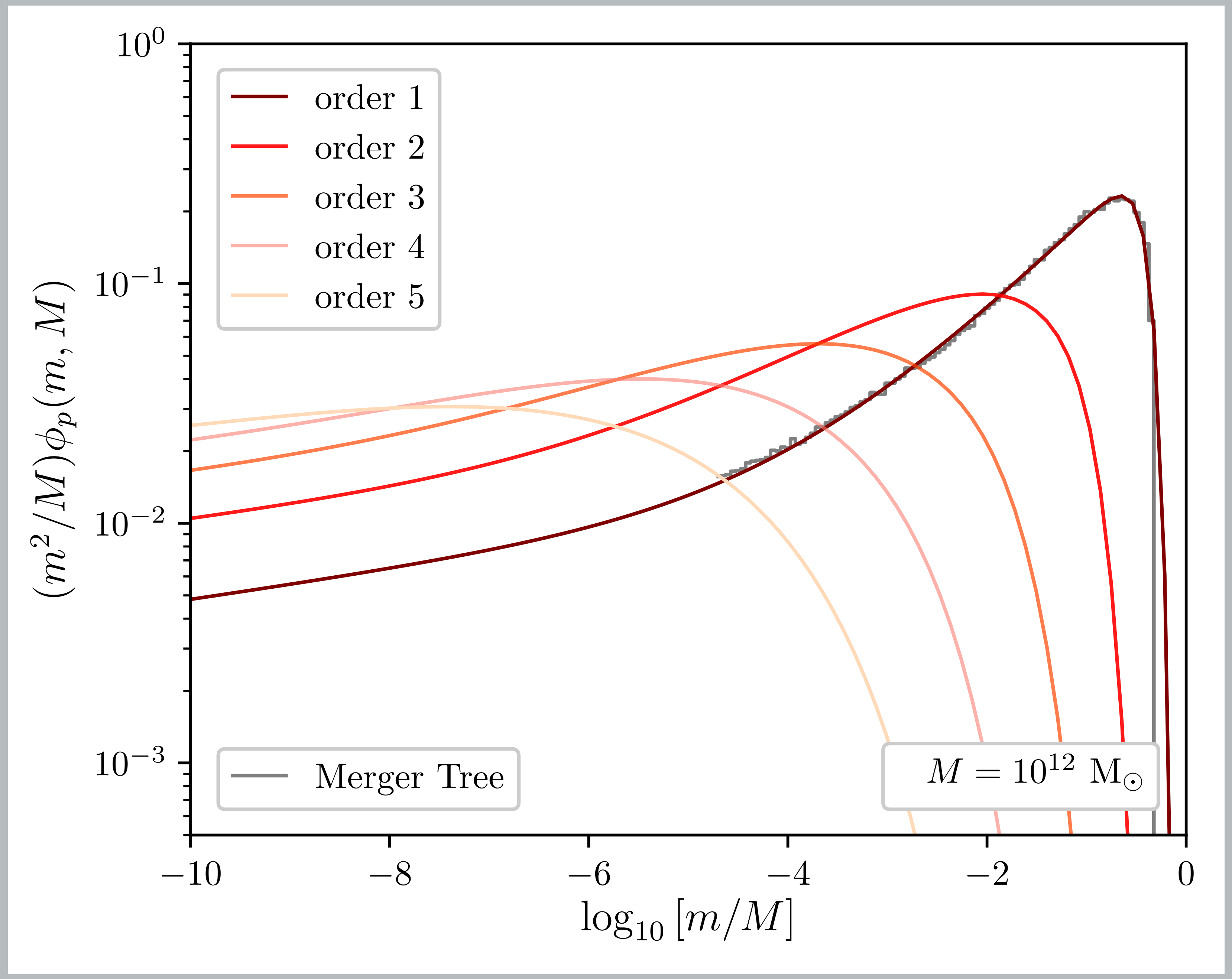
Distance from the Galactic center [kpc]

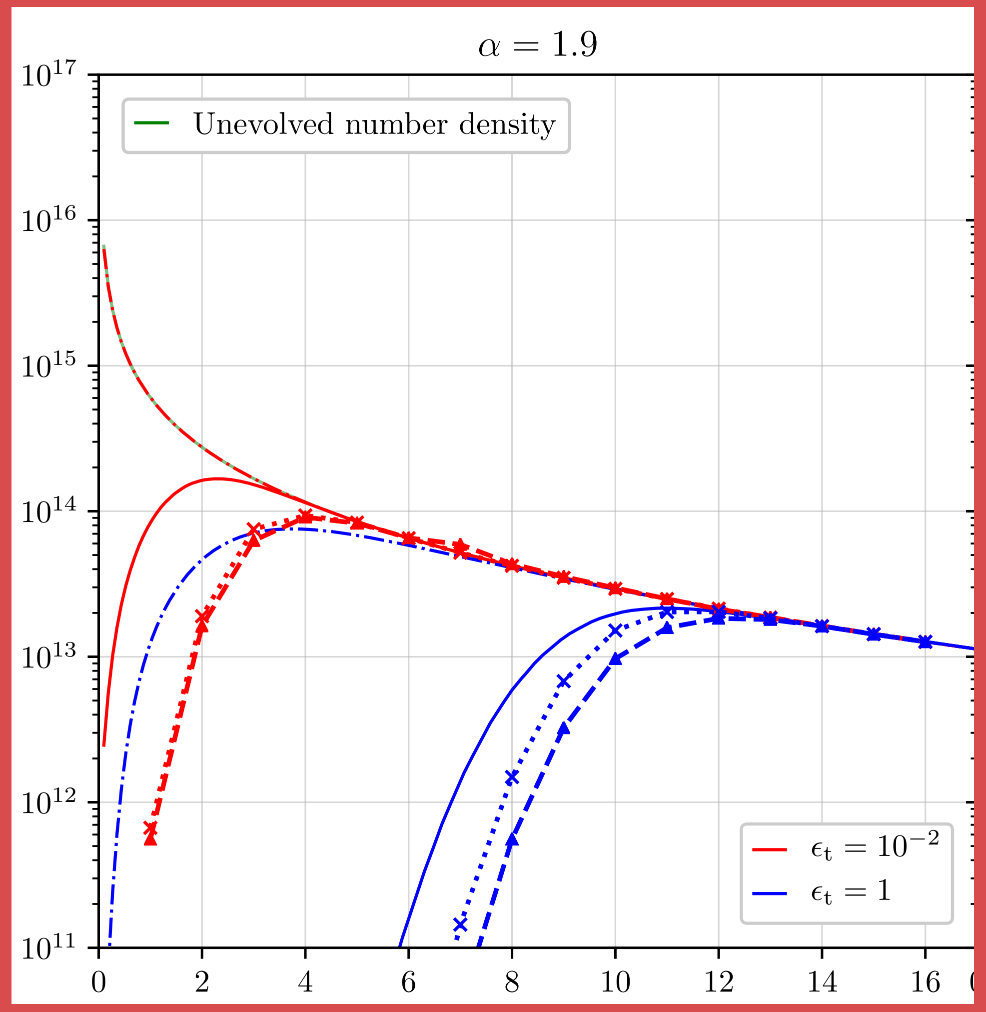
Star encounters have an important effect on the number density



New calibration of the model using merger tree algorithms.

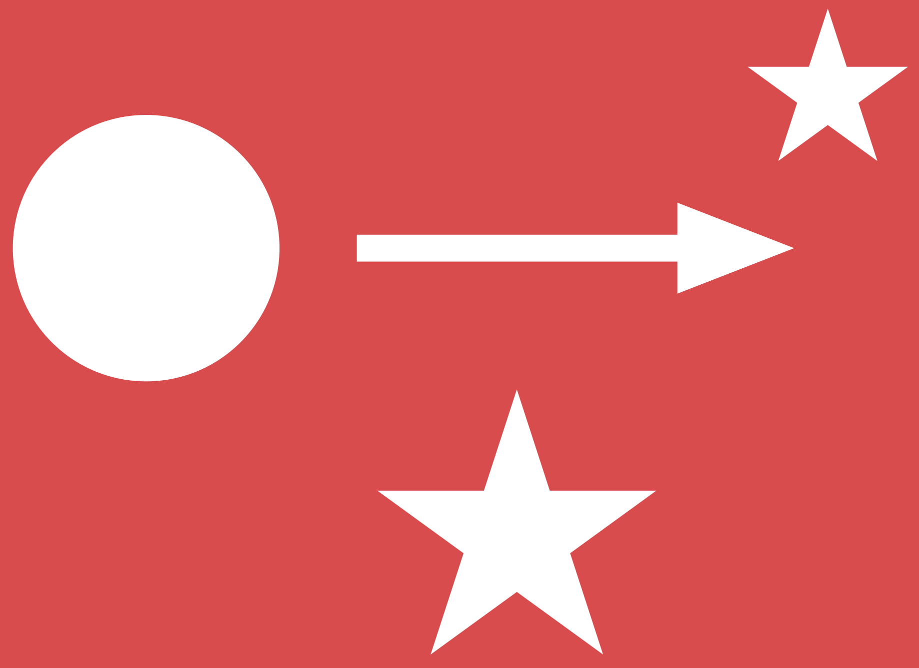
Can be used at any redshift and for any cosmology.





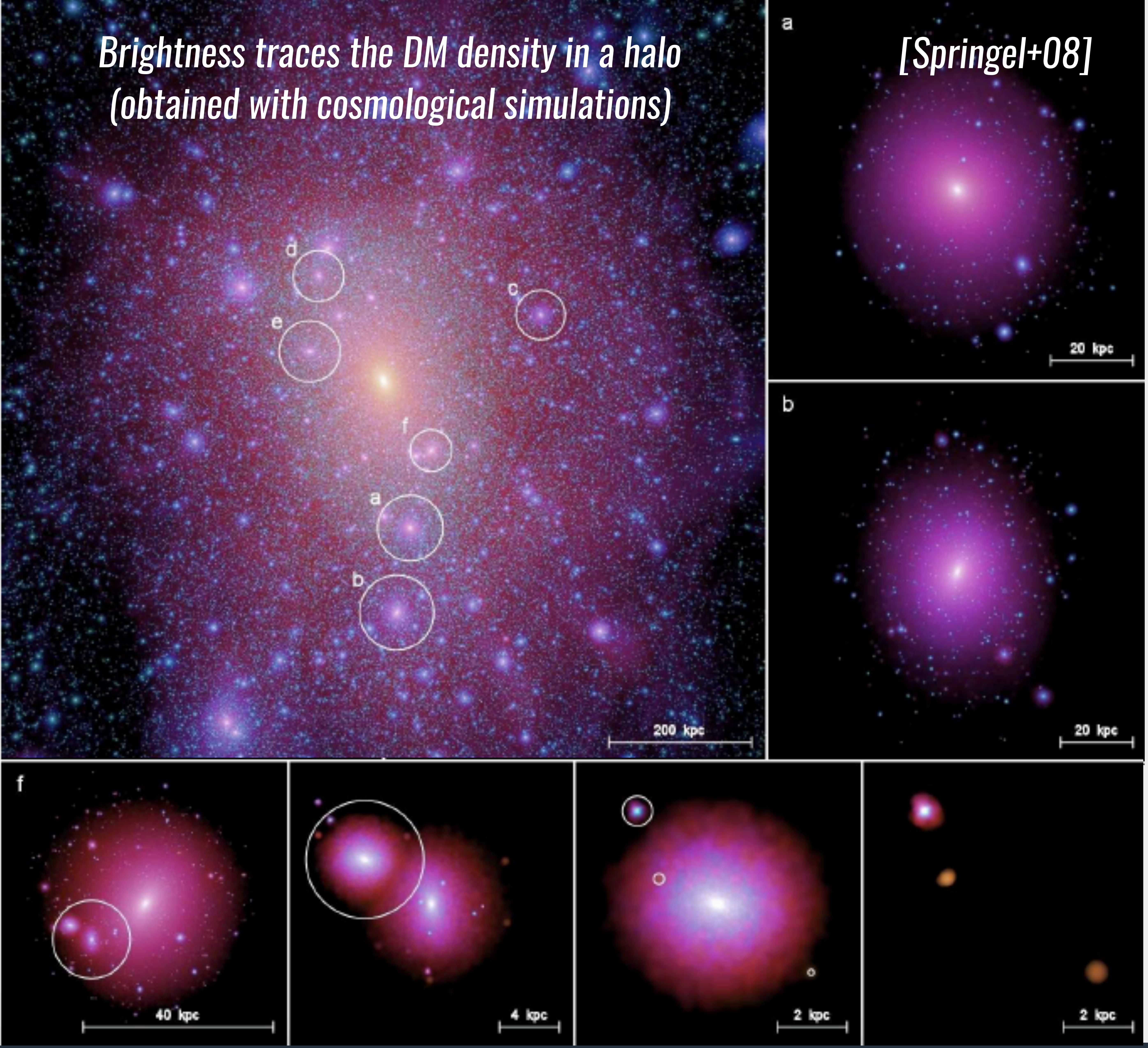
New computation for the encounter of one subhalo with one star.

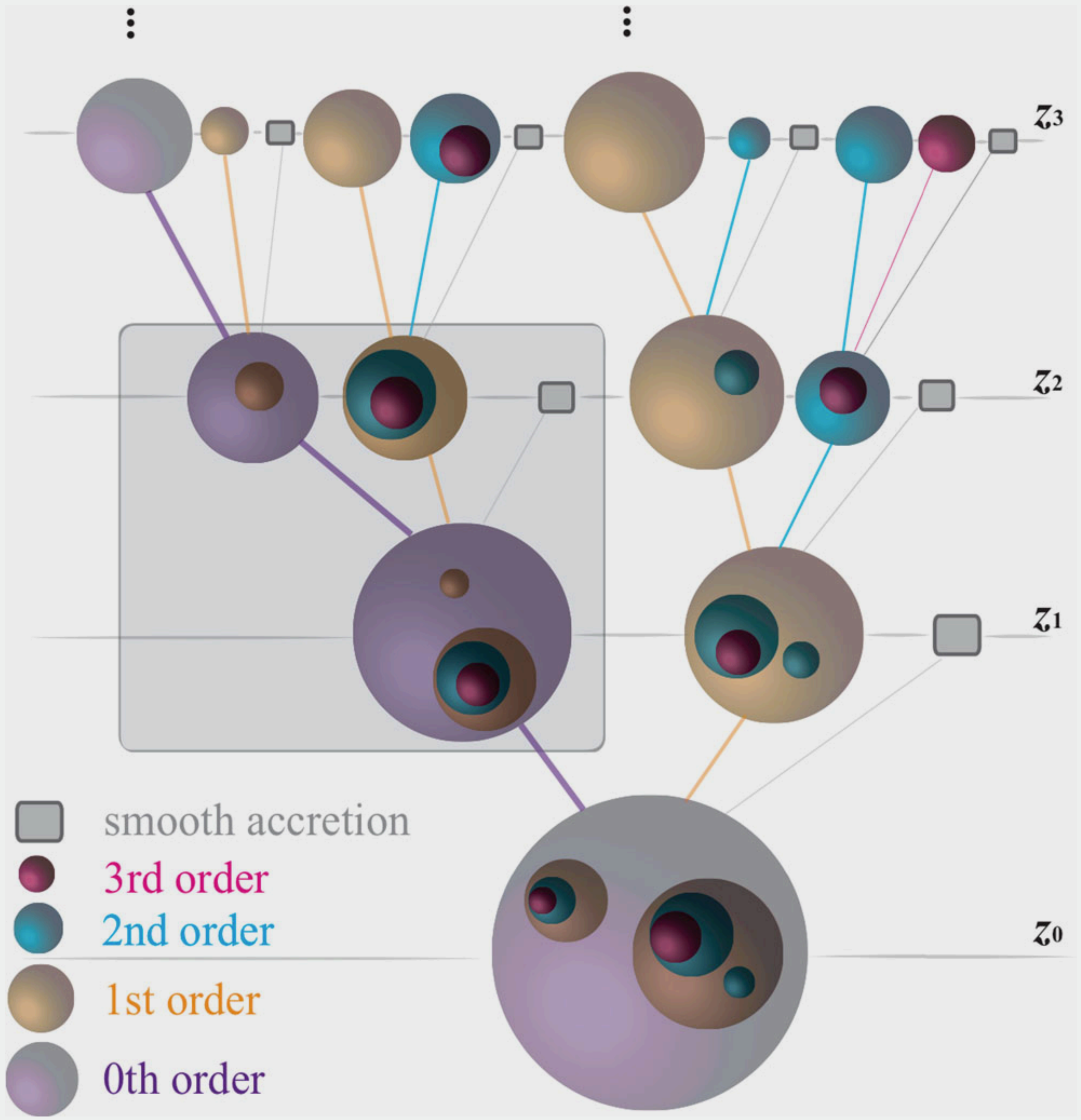
Proper definition of a global energy kick for one crossing of the disk.



Stars impact small subhalos towards the Galactic center. Shift the mass function to the small masses, desrupt fragile halos.

Back-up slides

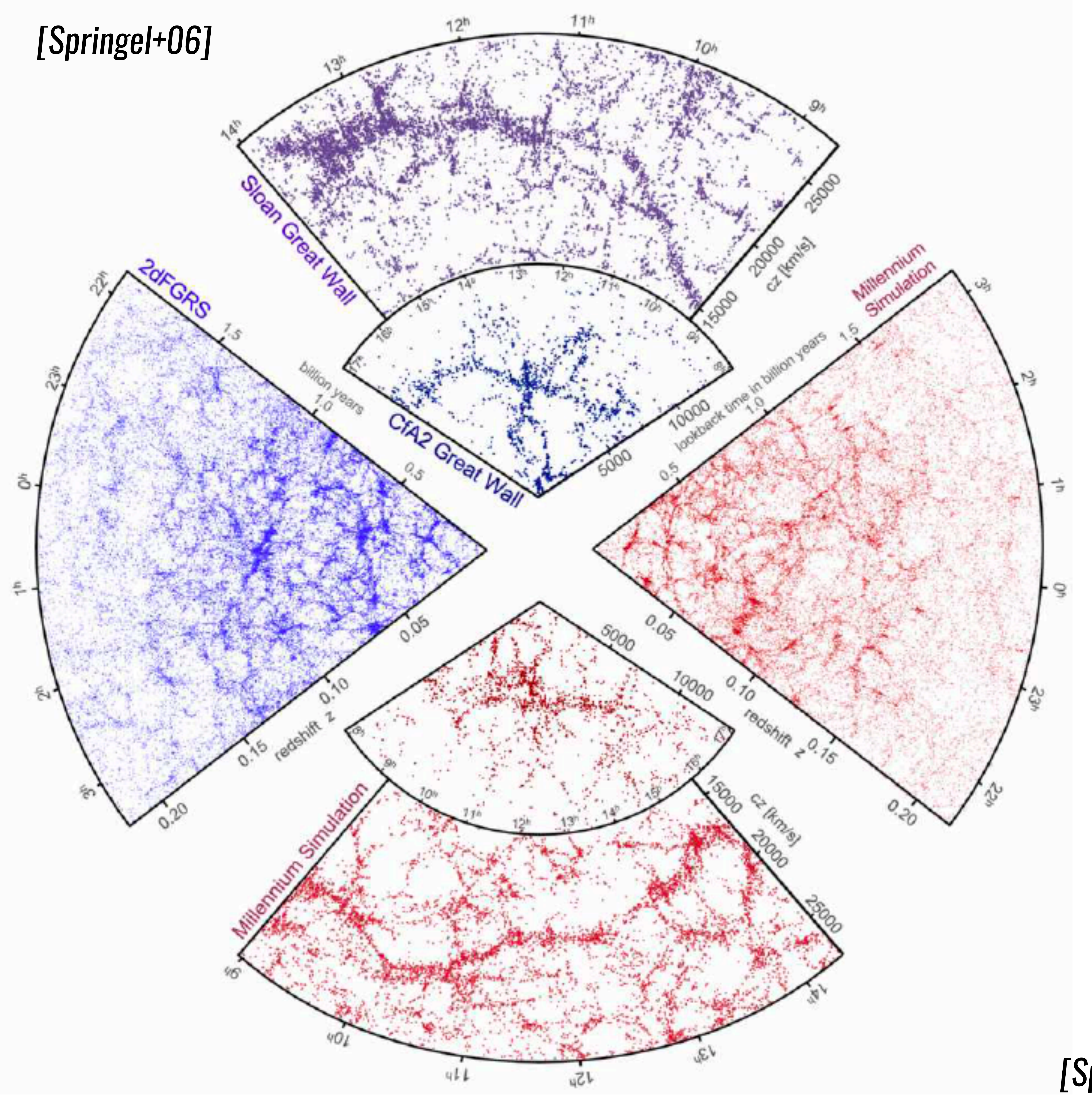




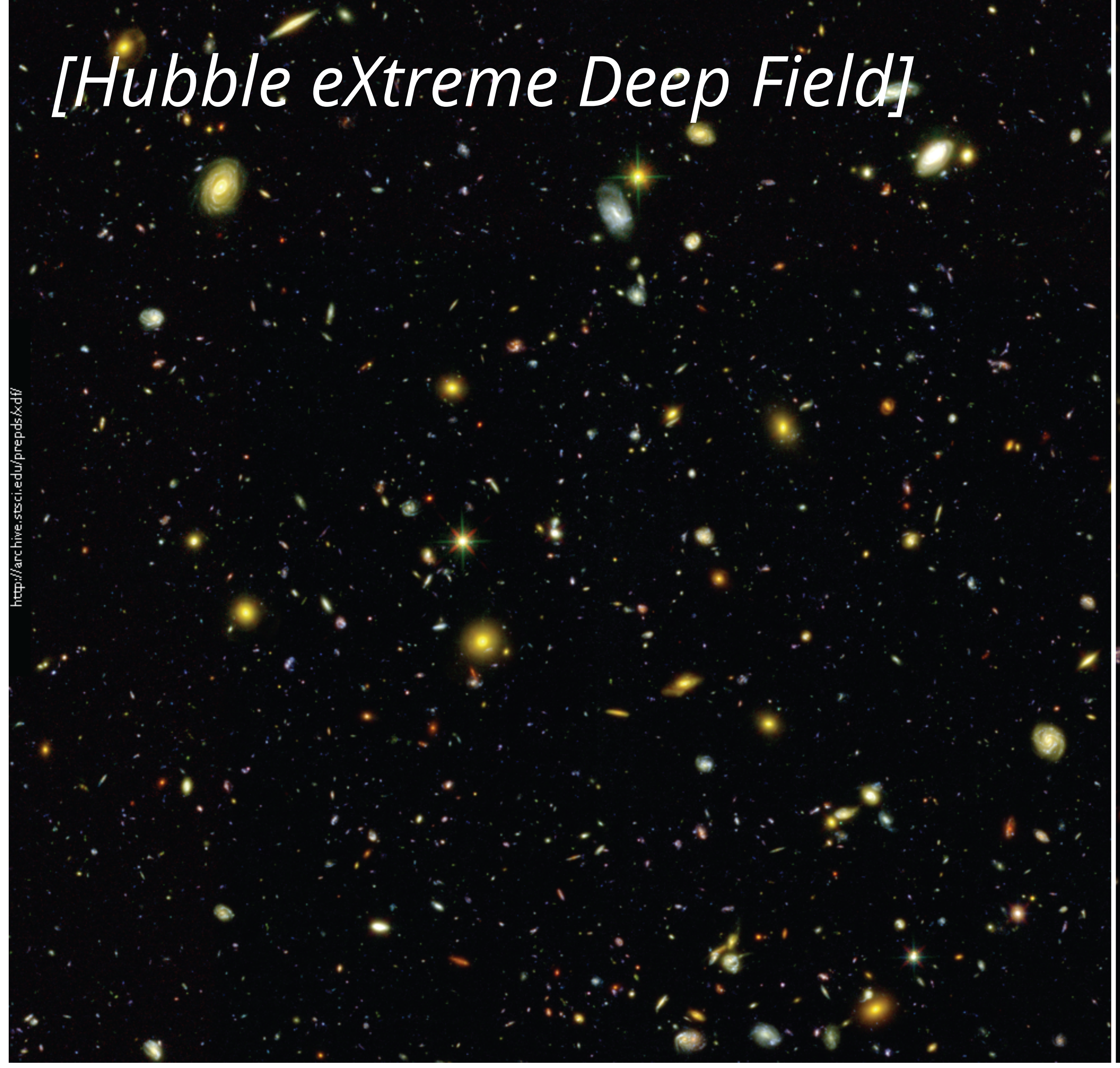
[Jiang+14]

Hierarchical formation leads to a fractal distribution

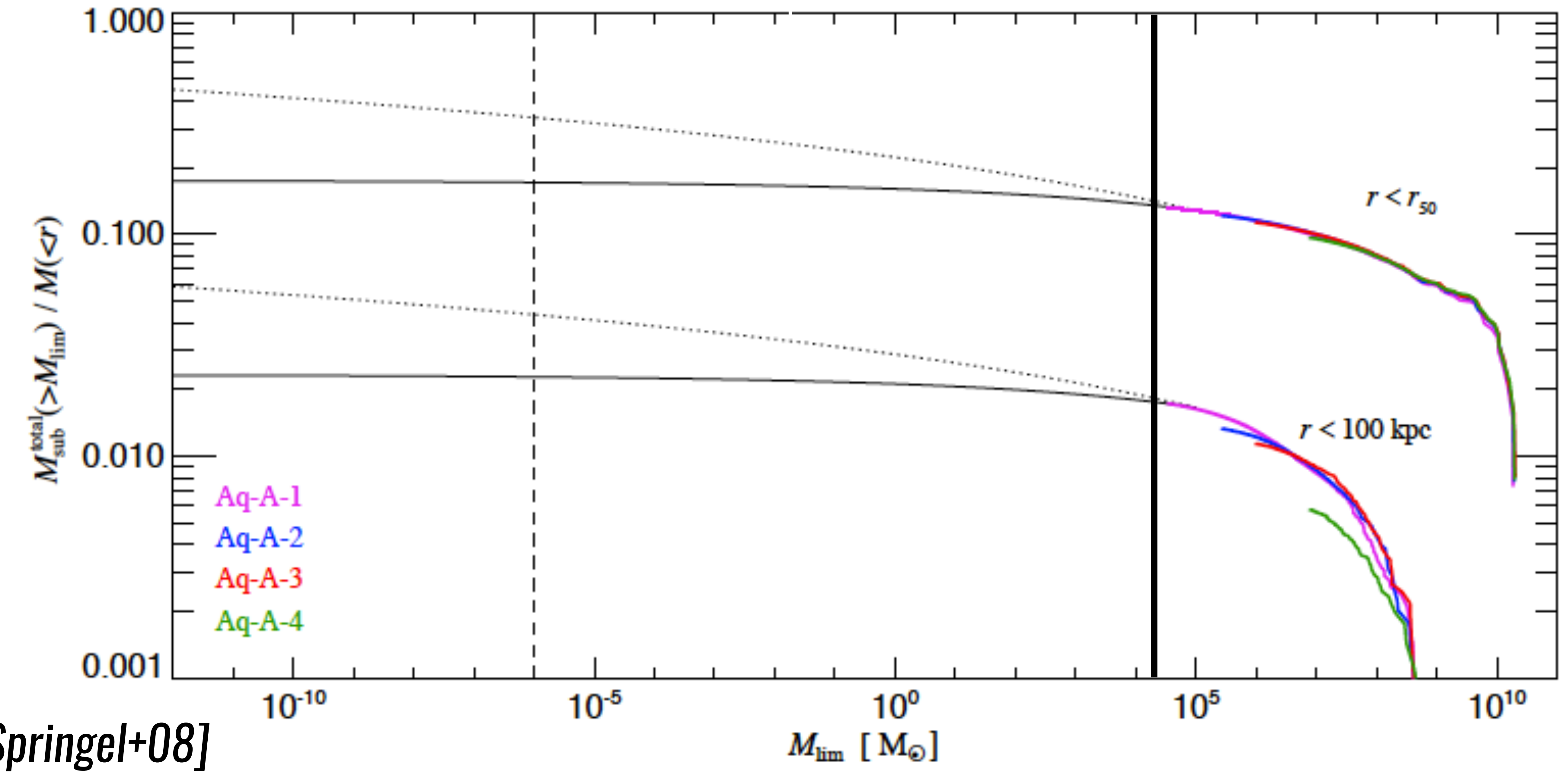
[Springel+06]



[Hubble eXtreme Deep Field]



[Illustris collaboration]



[Springel+08]

Cosmological simulations cannot probe very small scales

$$p_{\rm sub}^{\rm init}(\{m_i\}_i, \{c_i\}_i, \{\mathbf{R}_i\}_i) \simeq \left[p_{\rm sub}^{\rm init}(m, c, R)\right]^{N_{\rm sub}}$$

$$\Downarrow$$

$$p_{\rm sub}^{\rm init}(m, c, R) = \frac{1}{N_{\rm sub}} \frac{{\rm d}N_{\rm sub}}{{\rm d}m} p_c(c\,|\,m) p_{\mathbf{R}}(R)$$

$$\Downarrow$$

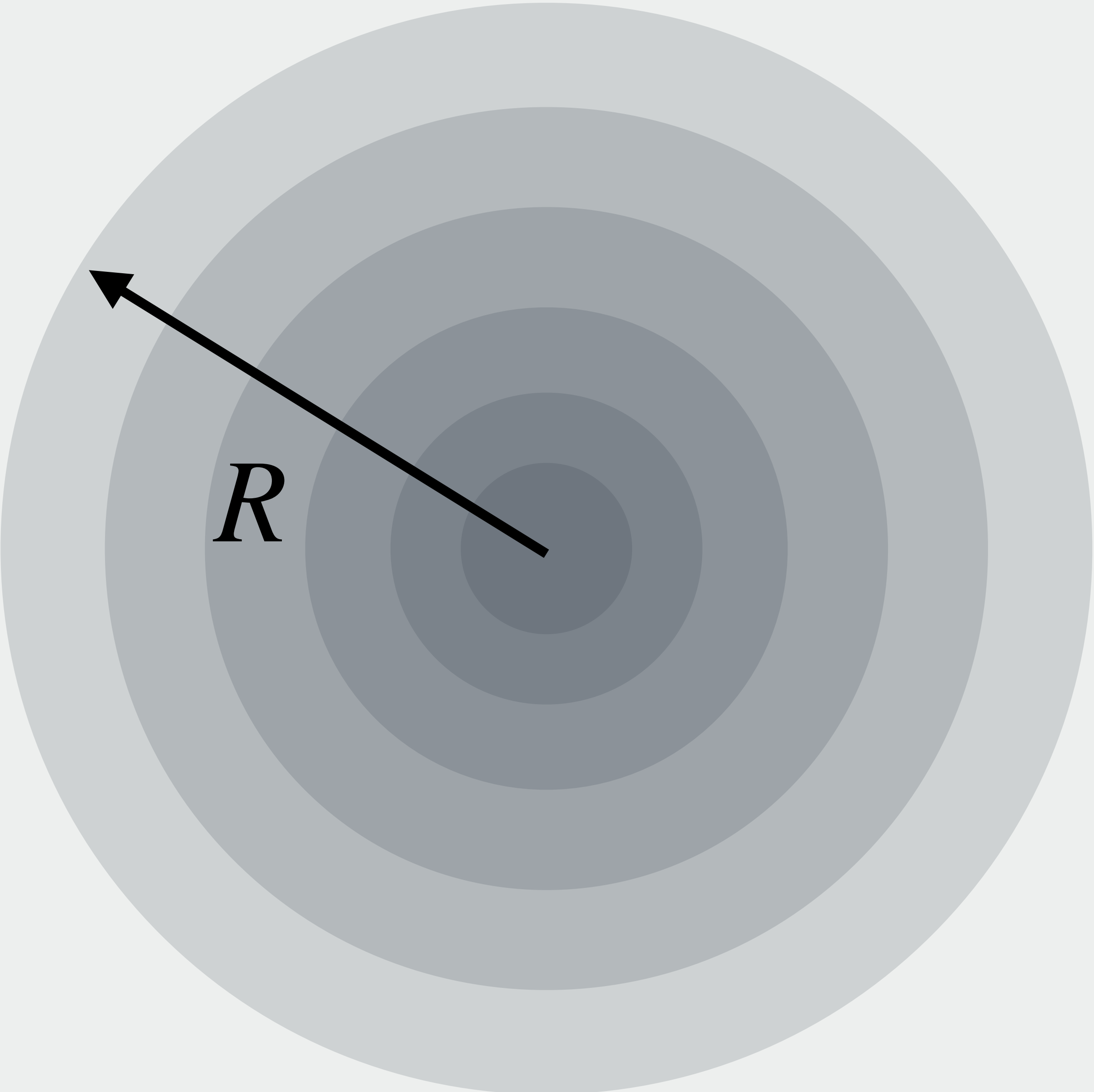
$$p_{\rm sub}^{\rm late}(m, c, R) = \frac{1}{K_{\rm t}} \frac{1}{N_{\rm sub}} \frac{{\rm d}N_{\rm sub}}{{\rm d}m} p_c(c\,|\,m) p_{\mathbf{R}}(R) \Theta[r_{\rm t}/r_{\rm s} - \epsilon_{\rm t}]$$

New number of subhalos $N_{\rm sub} \to K_{\rm t} N_{\rm sub}$

[Bond+91]

$$P_{\rm m}(k,z) = \frac{8\pi^2 k}{25}\left[\frac{D_1(z)}{\Omega_{\rm m,0}H_0^2}T(k)\right]^2 \mathscr{A}_S\left(\frac{k}{k_0}\right)^{n_s-1}$$ *(power spectrum of density fluctuations)*

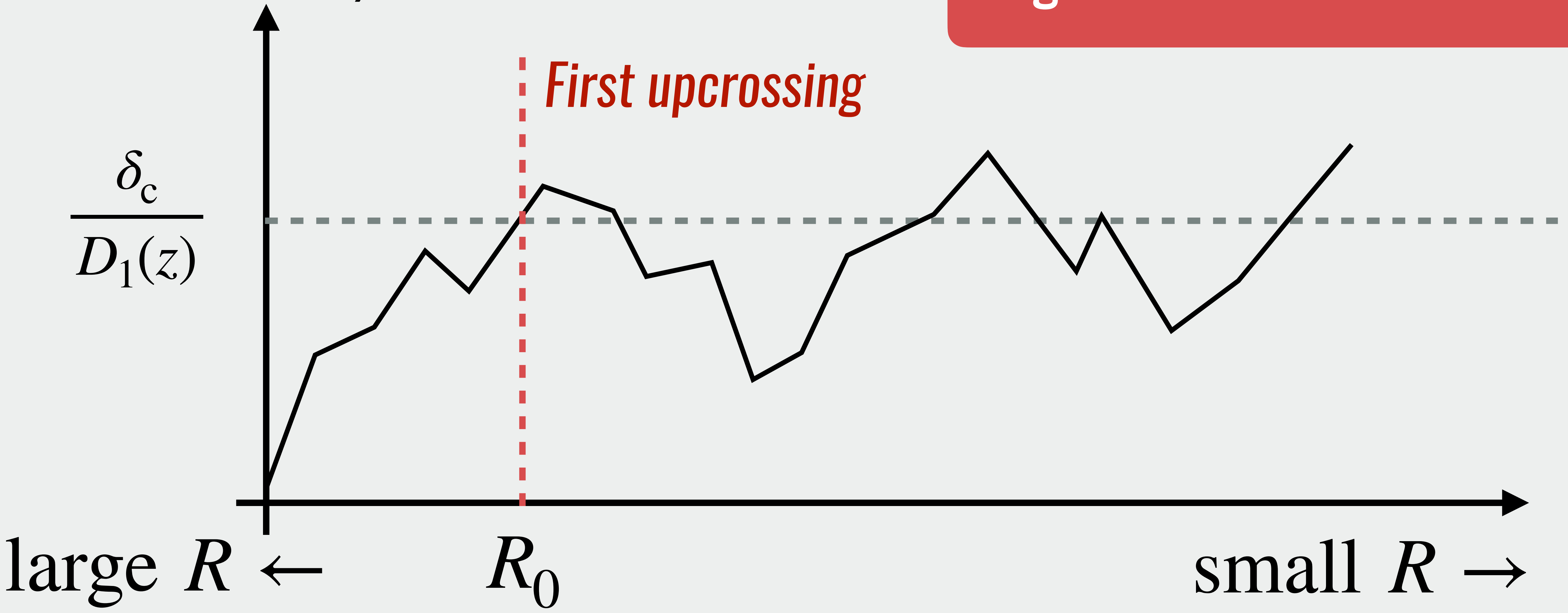
$$S(R) = \sigma_R^2 = \frac{1}{2\pi^2}\int_0^{1/R} P_{\rm m}(k, z=0)k^2 {\rm dk}$$ *(smoothed variance)*



(smoothed density contrast)

$$\delta_R(\mathbf{x}) = \int {\rm d}\mathbf{y}\frac{\delta\rho}{\bar\rho}W_R(|\mathbf{x}-\mathbf{y}|)$$

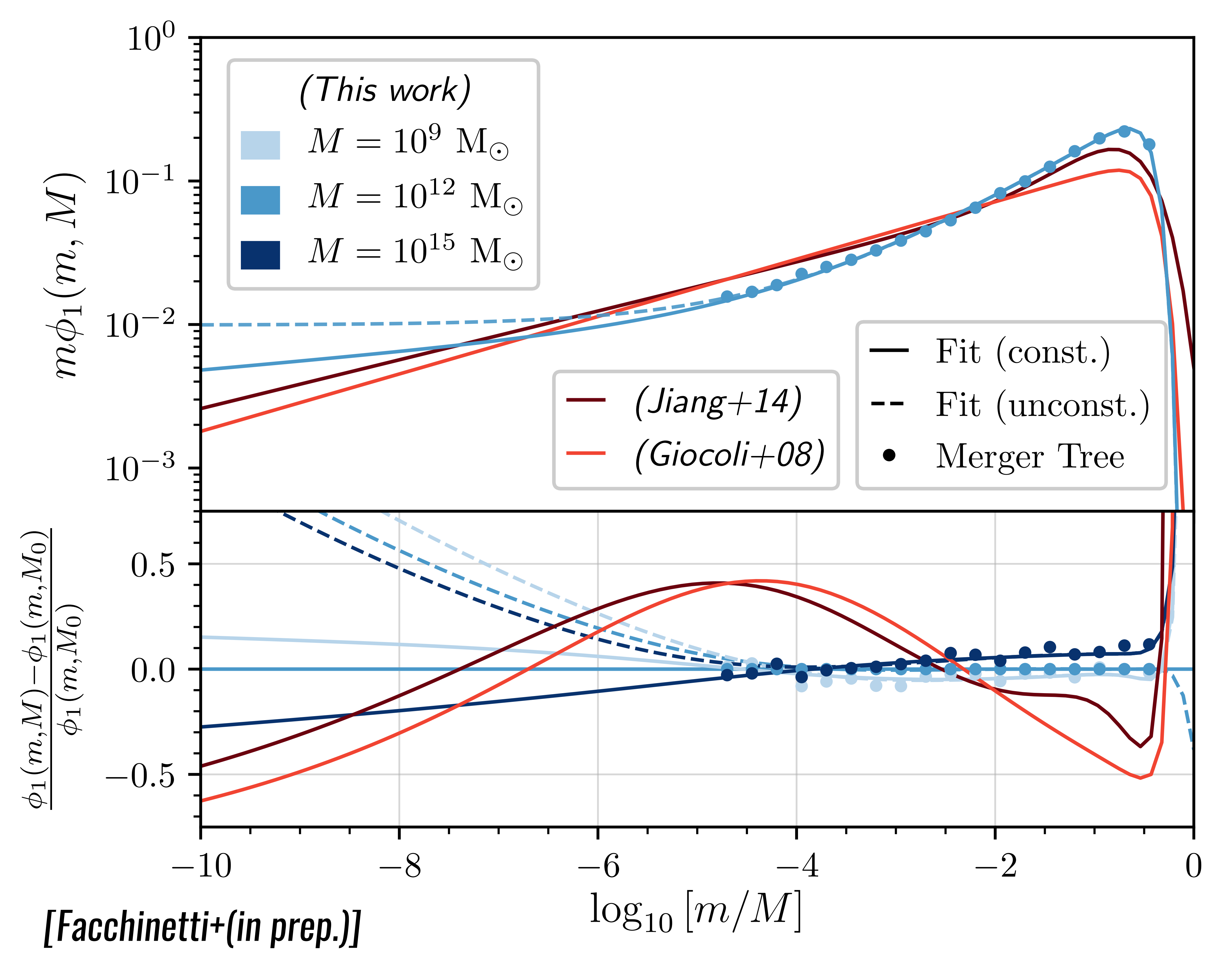
Region enclosed in a halo of size R_0

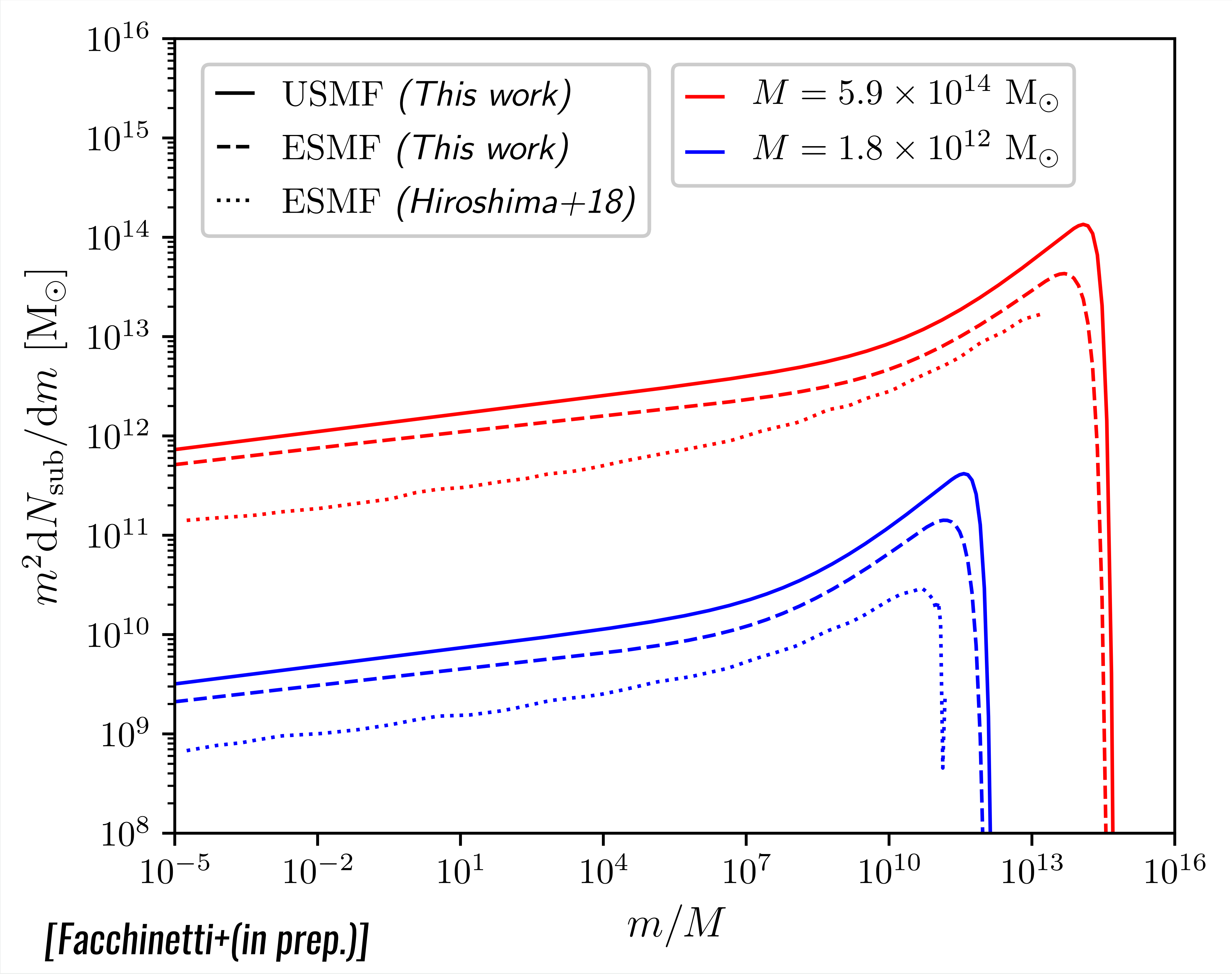


Fraction of mass in halos between M and M+dM

$$f(M)\left|\frac{{\rm d}S}{{\rm d}M}\right|{\rm d}M = \frac{\delta_c}{\sqrt{2\pi}S^{3/2}}\exp\left(-\frac{\delta_c}{2S}\right)\left|\frac{{\rm d}S}{{\rm d}M}\right|{\rm d}M$$

From the excursion set theory to merger trees

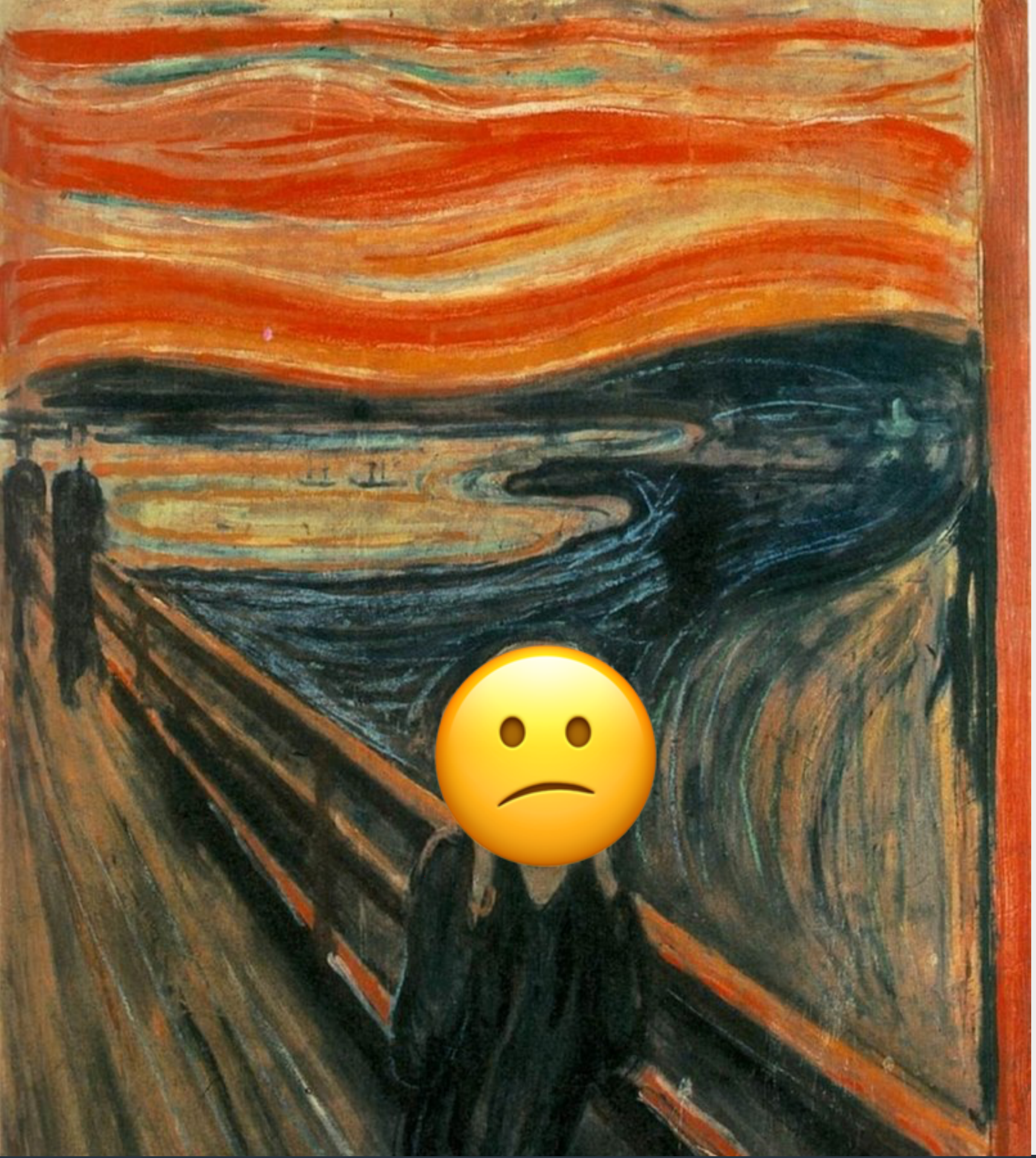




New calibration method

New velocity kick computation

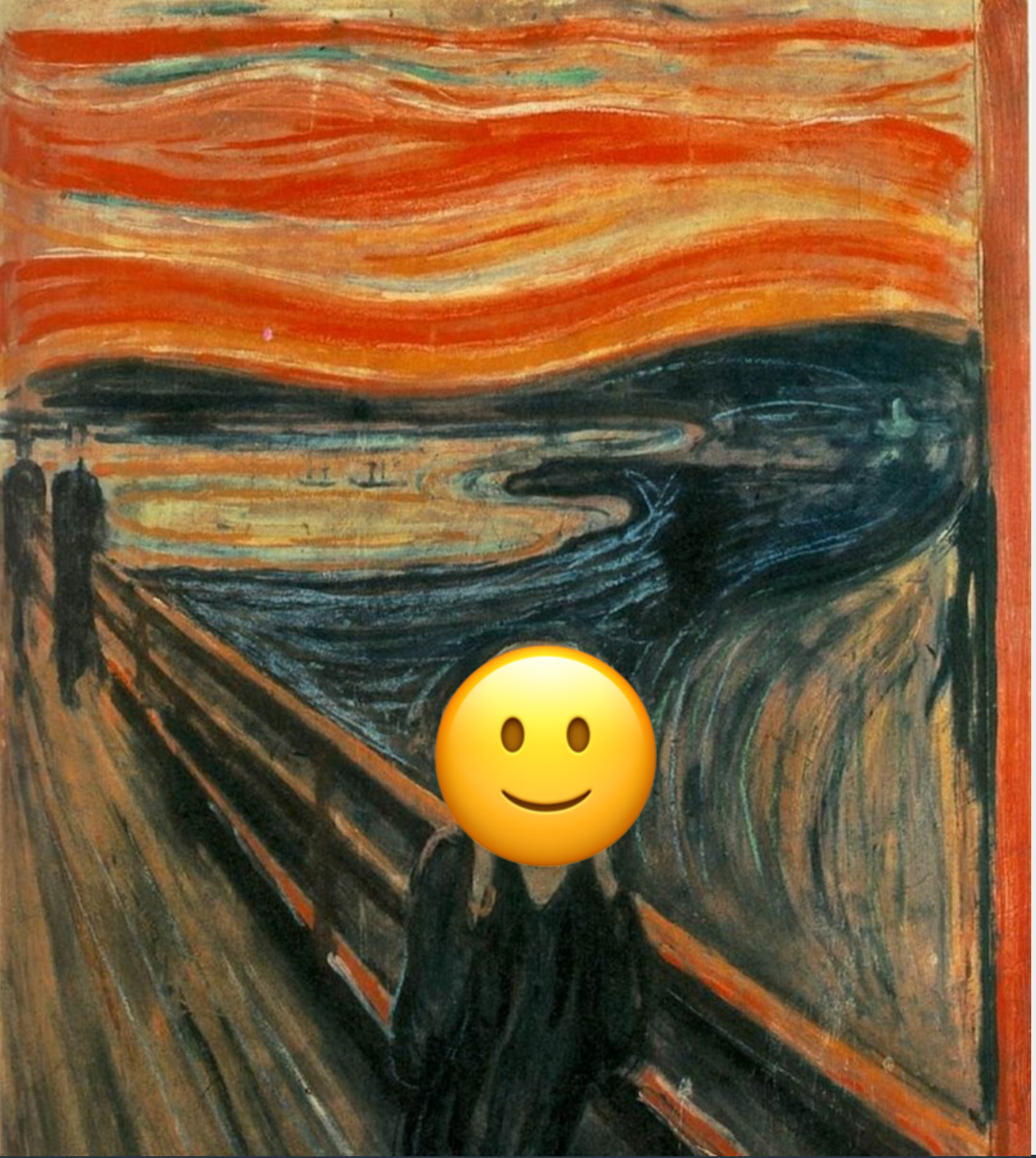
A N-body problem ...



1 High speed encounter:

The trajectory of the clump is a straight line

With two approximation the problem becomes analytical



1 **High speed encounter:**

The trajectory of the clump is a straight line

2 **Impulsive approximation:**

Particles inside the clump do not move during the encounter

With two approximation the problem becomes analytical

Spherical symmetry, average over angles

$$\langle \dots \rangle \equiv \frac{1}{4\pi} \oint \mathrm{d}\Omega \; \dots$$

r < b :

$$\left\langle |\delta \mathbf{v}|^2 \right\rangle \propto \left[I^2(b, r_t) - 2I(b, r_t) + \frac{b}{r} \frac{\arcsin(r/b)}{\sqrt{1-(r/b)^2}} \right]$$

r > b :

$$\left\langle |\delta \mathbf{v}|^2 \right\rangle \propto I(b, r_t) \left[I(b, r_t) - 1 + \frac{\sqrt{(r/b)^2 - 1}}{r/b} \right] + \int_0^{r/b} \mathrm{d}x \, \frac{x}{\sqrt{(r/b)^2 - x^2}} \frac{1}{|1 - x^2|}$$

We analytically evaluate the average - new approach

Spherical symmetry, average over angles

$$\langle \dots \rangle \equiv \frac{1}{4\pi} \oint \mathrm{d}\Omega \, \dots$$

r < b :

$$\left\langle |\delta \mathbf{v}|^2 \right\rangle \propto \left[I^2(b, r_t) - 2I(b, r_t) + \frac{b}{r} \frac{\arcsin(r/b)}{\sqrt{1-(r/b)^2}} \right]$$

r > b :

$$\left\langle |\delta \mathbf{v}|^2 \right\rangle \propto I(b, r_t) \left[I(b, r_t) - 1 + \frac{\sqrt{(r/b)^2 - 1}}{r/b} \right] + \fint_0^{r/b} \mathrm{d}x \, \frac{x}{\sqrt{(r/b)^2 - x^2}} \frac{1}{|1-x^2|}$$

$$+\infty$$

We analytically evaluate the average - new approach

Pdf for the energy kick (isotropic initial velocity distribution)

$$p_{\delta E}(\delta E \,|\, \delta \mathbf{v}) = \frac{1}{2\,|\,\delta \mathbf{v}\,|} \int_{\chi(|\delta \mathbf{v}|,\Delta E)}^{+\infty} \mathrm{d}^3\mathbf{v}\, \frac{f(v,r)}{\rho(r)} \quad \text{with} \quad \chi(\,|\,\delta \mathbf{v}\,|\,,\Delta E) \equiv \frac{|\,\delta E - (\delta \mathbf{v})^2/2\,|}{|\,\delta \mathbf{v}\,|}$$

Example: simple model

$$\frac{f(v,r)}{\rho(r)} \simeq \frac{1}{(2\pi\sigma_{\mathrm{sub}}^2)^{3/2}} \exp\left(-\frac{v^2}{2\sigma_{\mathrm{sub}}^2}\right)$$

Given by Jean's equation

What we computed

$$p_{\delta E}(\delta E \,|\, \delta \mathbf{v}) \simeq \frac{1}{\sqrt{2\pi\sigma_{\mathrm{sub}}^2(\delta \mathbf{v})^2}} \exp\left(-\frac{\left(\delta E - \frac{(\delta \mathbf{v})^2}{2}\right)^2}{2\sigma_{\mathrm{sub}}^2(\delta \mathbf{v})^2}\right)$$

Density of impact parameters:

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b}(b\,|\,R,\cos\theta) = \frac{b}{4\pi}\int_1^{\frac{1}{\cos^2\theta}} \frac{\mathrm{d}y}{\sqrt{1-y\cos^2\theta}\sqrt{y-1}} \sum_{i=1}^{4} \int \mathrm{d}\psi\, \frac{\Sigma_\star(R_i)}{m_\star}$$

$$R_i = b^2 y + R^2 + \frac{2Rb}{\sin\theta}\left[\pm\cos\psi\sqrt{y-1} \pm \sin\psi\sqrt{1-y\cos^2\theta}\right]$$

Number of encountered stars

$$\mathcal{N} \equiv \int_{b_{\min}}^{b_{\max}} \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b}\mathrm{d}b$$

Interstellar distance/2

Straight line approx. condition

If $\cos\theta \gg \frac{b}{3\,\mathrm{kpc}}$ (orbits not too close to the disc)

Then $p_b(b) \equiv \frac{1}{\mathcal{N}}\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b} \simeq \frac{2b}{b_{\max}^2 - b_{\min}^2}$ $\quad \mathcal{N} \simeq \frac{\pi}{\cos\theta}\frac{\Sigma_\star(R)}{m_\star}(b_{\max}^2 - b_{\min}^2)$

Average velocity kick squared per encounter

$$\overline{(\delta\mathbf{v})^2} = \int_{b_{\min}}^{b_{\max}} \mathrm{d}b \int \mathrm{d}m_\star p_b(b) p_{m_\star}(m_\star) \left\langle (\delta\mathbf{v})^2 \right\rangle$$

Result of part I

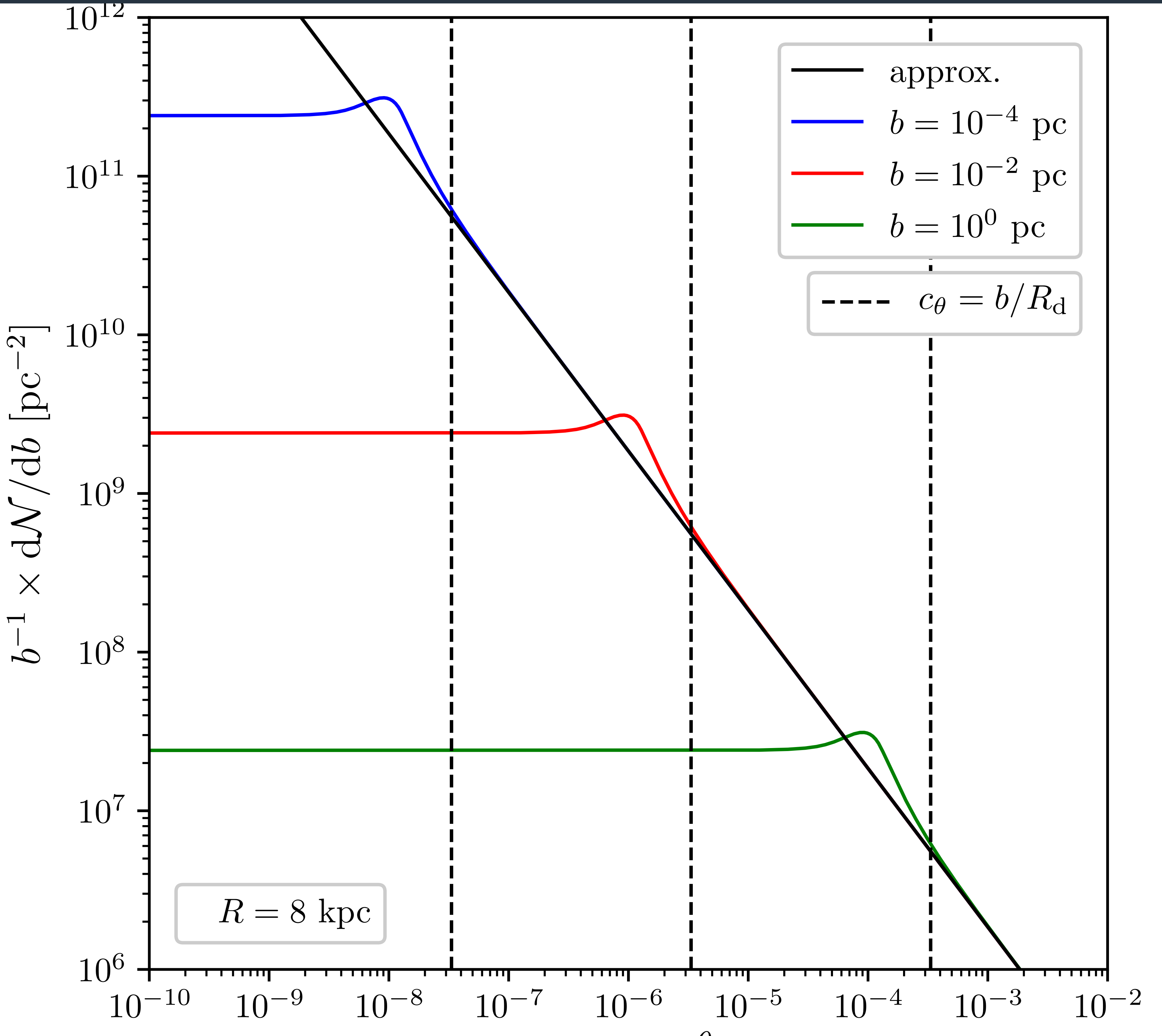
Stellar mass distribution

[Chabrier03]

The different ingredients

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}b}(b \mid R, \cos\theta) = \frac{b}{4\pi}\int_1^{\frac{1}{\cos^2\theta}} \frac{\mathrm{d}y}{\sqrt{1 - y\cos^2\theta}\sqrt{y-1}} \sum_{i=1}^{4} \int \mathrm{d}\psi \, \frac{\Sigma_\star(R_i)}{\overline{m_\star}}$$

$$R_i = b^2 y + R^2 + \frac{2Rb}{\sin\theta}\left[\pm\cos\psi\sqrt{y-1} \pm \sin\psi\sqrt{1 - y\cos^2\theta}\right]$$



[Facchinetti+(in prep.)]

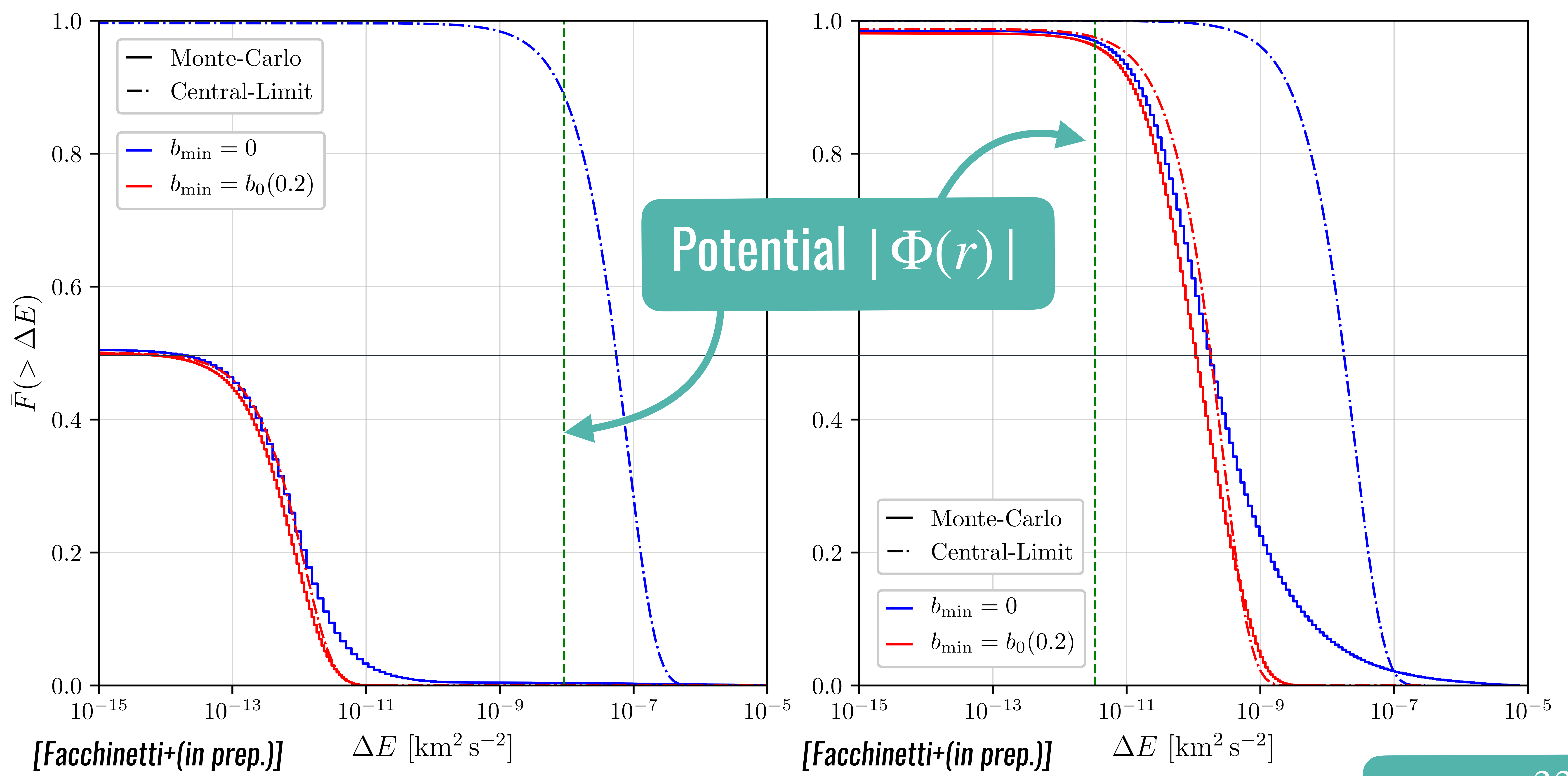
Comparison of impact parameter distributions

Mismatch with Central-Limit theorem

Compl. Cumulative Distribution Function

Centre of the subhalo $r/r_s = 0.01$

Outskirts of the subhalo $r = 0.999r_t$



[Facchinetti+(in prep.)]

[Facchinetti+(in prep.)]

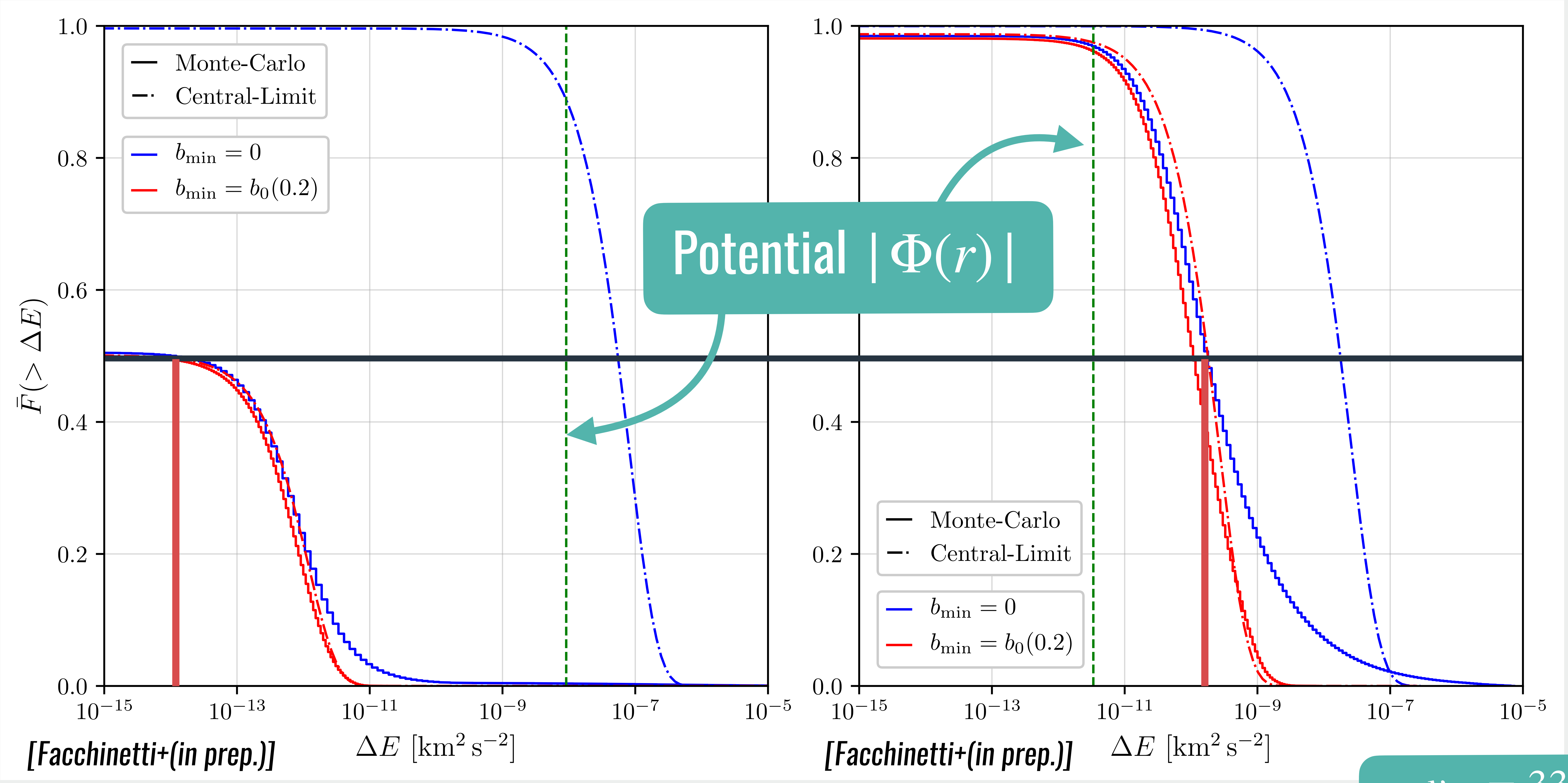
$v_{\rm rel} = 334\,{\rm km\,.\,s^{-1}}$
$m_{200} = 1.6 \times 10^{-9}\,{\rm M_\odot}$
$R = 8\,{\rm kpc}$
$\cos\theta = 1/2$

A Monte-Carlo simulation shows the issue and how to solve it

Centre of the subhalo r/r_s = 0.01

Outskirts of the subhalo $r = 0.999 r_t$

Compl. Cumulative Distribution Function

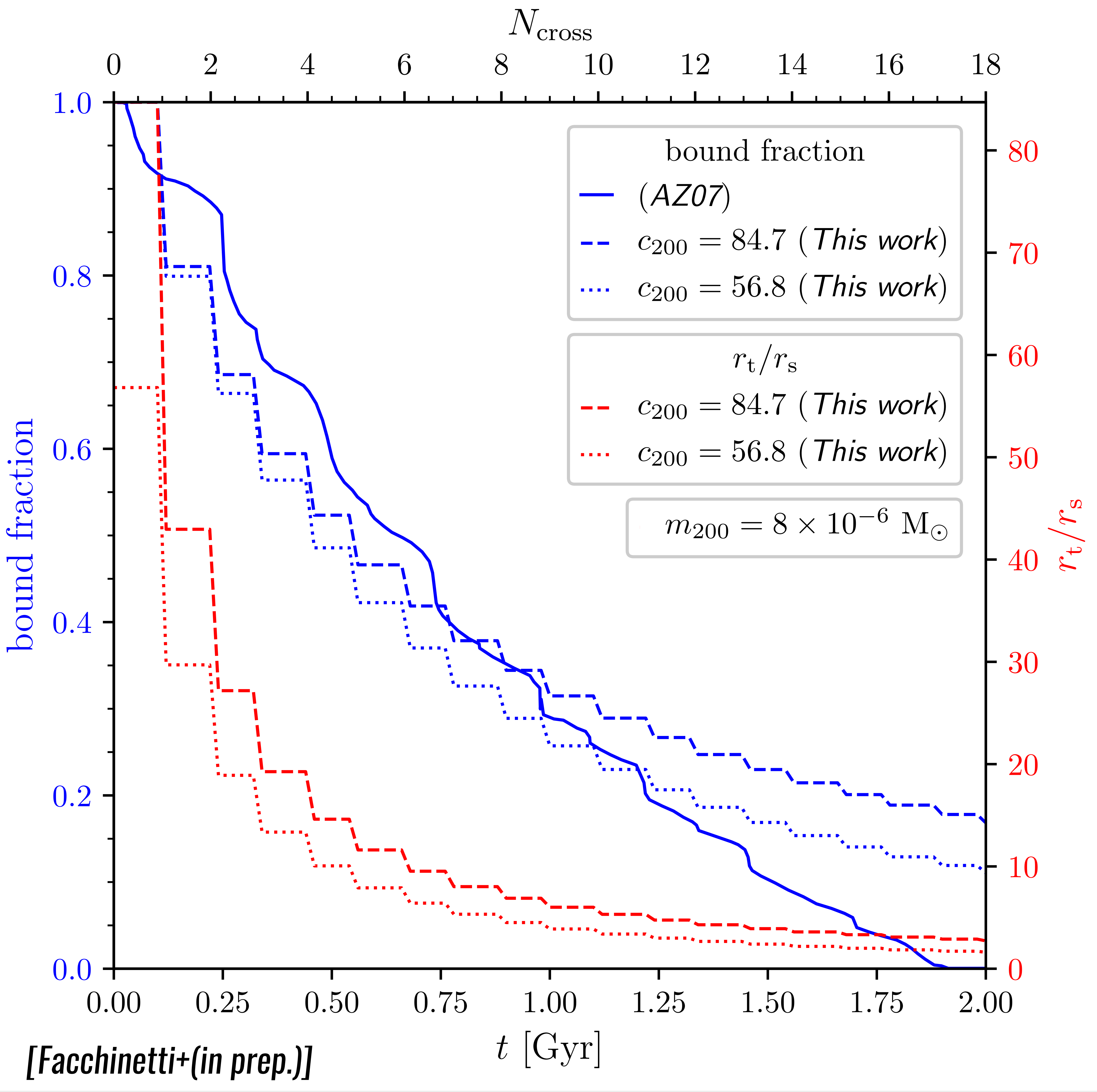


$v_{\rm rel} = 334\,{\rm km\,.\,s^{-1}}$
$m_{200} = 1.6 \times 10^{-9}\,{\rm M_\odot}$
$R = 8\,{\rm kpc}$
$\cos\theta = 1/2$

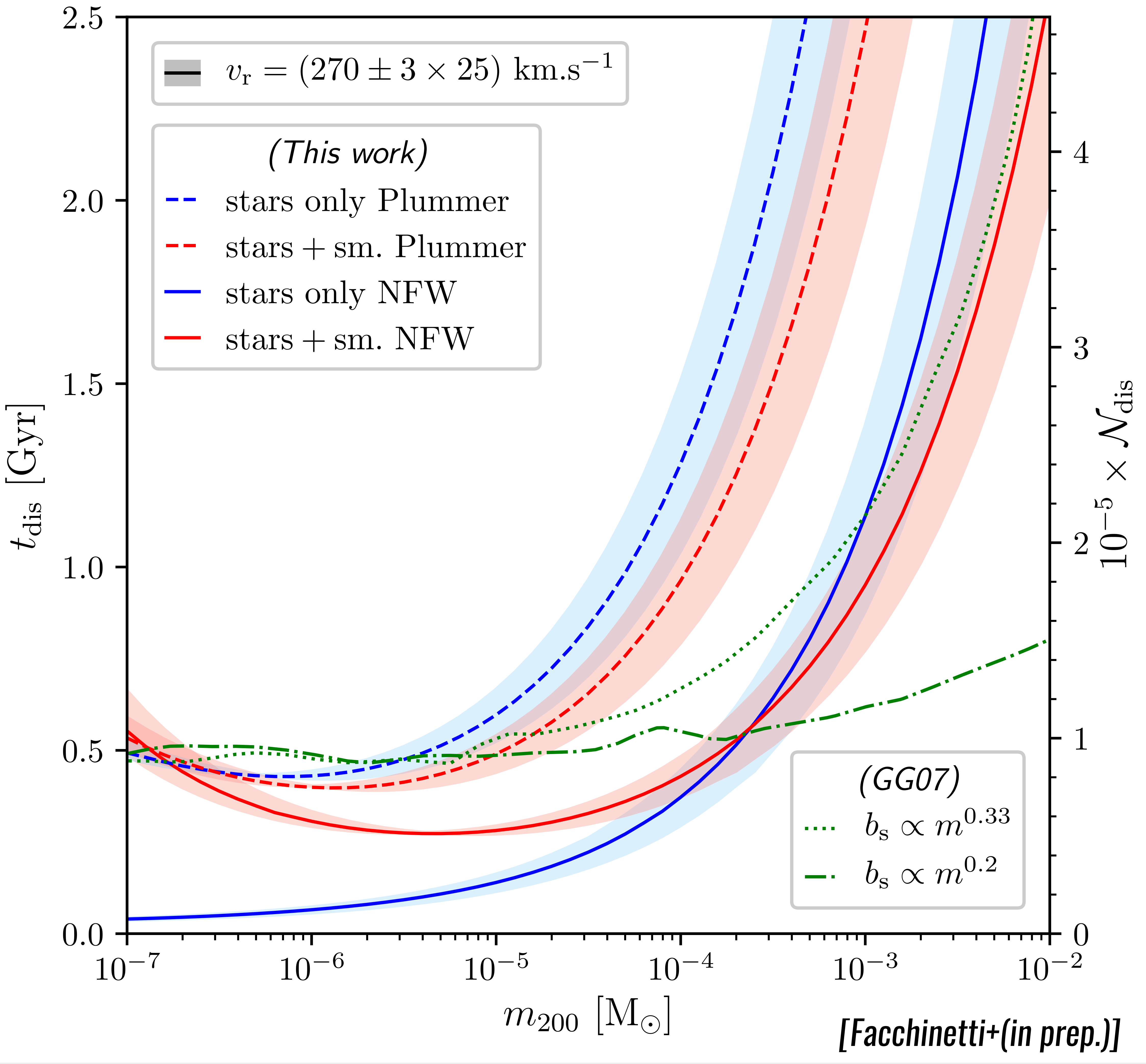
Definition of AN energy kick : $\Delta E \simeq {\rm Med}(\Delta E)$

A Monte-Carlo simulation shows the issue and how to solve it

Star encounters impact:
Comparison to the literature



Comparison with [Angus+07]



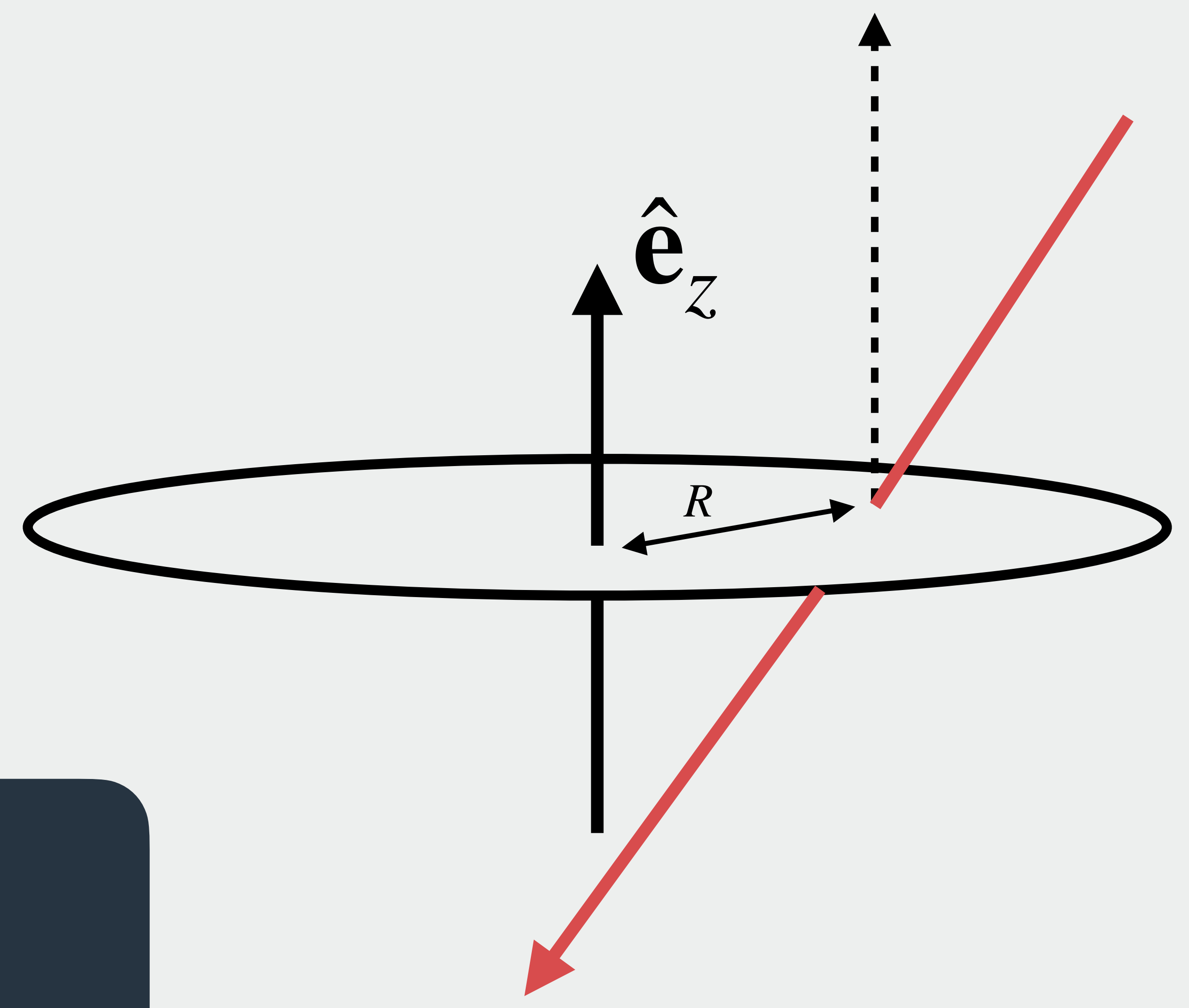
Comparison with [Green+07]

Star encounter impact: comparison to the literature

Total energy kick (disk shocking + stellar encounters)

$$\Delta E_{\mathrm{tot}} = \Delta E + \Delta E_{\mathrm{d}} + \Delta \mathbf{v} \,.\, \Delta \mathbf{v}_{\mathrm{d}} \sim \mathrm{Med}(\Delta E + \Delta E_{\mathrm{d}})$$

$$\sim 0.7\overline{\Delta E} + \overline{\Delta E_{\mathrm{d}}} = \frac{0.7}{2}\mathcal{N}(\delta \mathbf{v})^2 + \frac{4 g_{\mathrm{d}}^2(R)}{3(v_c \,.\, \hat{e}_z)^2} r^2 A_1(\eta)$$



Combination of energy kick

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