Testing the predictions of axisymmetric inversion method on hydrodynamical simulations

M. Petač, J. Lavalle, A. Núñez-Castiñeyra, E. Nezri, JCAP 08 (2021) 031 / arXiv: 2106.01314

Mihael Petač

Laboratoire Univers et Particules de Montpellier (LUPM), CNRS & University of Montpellier

Center for Astrophysics and Cosmology (CAC), University of Nova Gorica

News from The Dark 2021
23rd November 2021

Phase-space distribution functions (PSDFs) of DM halos

Interesting from several perspectives:

- Structure of collisionsless self-gravitating systems
- Galactic dynamics
- **DM searches** (direct & indirect detection, microlensing, ...)

Test the predictions of **axisymmetric inversion method** on high-resolution **hydrodynamical simulations**:

1. Extract $\rho(R,z)$, $\Psi(R,z)$ and $\bar{v}_\phi(R)$ from simulations
2. Compute PSDFs using the axisymmetric inversion method
3. Compare the predicted PSDFs with simulation

Follow-up study of: Lacroix et al., JCAP 10 (2020) 031

Axisymmetric inversion method

Assume that the phase-space distribution parameterized by **two integrals of motion**:

1. Relative energy: $\mathcal{E} = \Psi(r) - \frac{v^2}{2}$
2. Angular momentum around z-axis: $L_z = R \cdot v_\phi$

$$f(\mathcal{E}, L_z) = f_+(\mathcal{E}, |L_z|) + f_-(\mathcal{E}, L_z)$$

$$f_+(\mathcal{E}, L_z) = \frac{1}{4\pi^2 i\sqrt{2}} \oint_{C(\mathcal{E})} \frac{\mathrm{d}\xi}{\sqrt{\xi - \mathcal{E}}} \left.\frac{\mathrm{d}^2\rho(R^2, \Psi)}{\mathrm{d}\Psi^2}\right|_{\substack{\Psi=\xi \\ R^2=\frac{L_z^2}{2(\xi-\mathcal{E})}}}$$

$$f_-(\mathcal{E}, L_z) = \frac{\mathrm{sign}(L_z)}{8\pi^2 i} \oint_{C(\mathcal{E})} \frac{\mathrm{d}\xi}{\xi - \mathcal{E}} \left.\frac{\mathrm{d}^2\left(\rho\bar{v}_\phi\right)}{\mathrm{d}\Psi^2}\right|_{\substack{\Psi=\xi \\ R^2=\frac{L_z^2}{2(\xi-\mathcal{E})}}}$$

Hunter C. & Qian E.: MNRAS262, 401 (1993) ; https://github.com/mpetac/AIM

Hydrodynamical simulations

High-resolution hydro simulations: *Mochima, Halo B & Halo C*

Mollitor et al., MNRAS 447-2 (2015); Nuñez-Castiñeyra et al., MNRAS 501-1 (2021)

- Millions of particles ($m_{\rm DM} \sim 10^5\ M_\odot$, $m_\star \sim 10^4\ M_\odot$)
- Similar initial setups, but appreciably different final galaxies

Analysis of the simulations:

1. Center on the most gravitational bound particle
2. Align cylindrical coordinates according to inertia tensor of stellar particles
3. **Extract** $\rho(R, z)$, $\Psi(R, z)$ **and** $\bar{v}_\phi(R)$

Baryonic gravitational potential

Parameterize as: $\Psi_{\mathrm{B}}(R,z) = \Psi_{\mathrm{MN}}(R,z) + \Psi_{\mathrm{H}}(R,z)$

$$\Psi_{\mathrm{MN}}(R,z) = \frac{GM_{\mathrm{disc}}}{\sqrt{R^2 + (a_{\mathrm{d}} + \sqrt{z^2 + b_{\mathrm{d}}^2})^2}}$$

$$\Psi_{\mathrm{H}}(R,z) = \frac{GM_{\mathrm{bulge}}}{\sqrt{R^2 + z^2} + a_{\mathrm{b}}}$$

Azimuthally average the simulated baryonic potential:

$$\bar{\Psi}_{\mathrm{B,sim}}(R,z) = \langle\, \Psi_{\mathrm{B,sim}}(\vec{r}(R,z,\phi)\,)\rangle_\phi$$

$$\sigma^2_{\bar{\Psi}_{\mathrm{B}}}(R,z) = \langle\, \left(\Psi_{\mathrm{B,sim}}(\vec{r}(R,z,\phi) - \bar{\Psi}_{\mathrm{B,sim}}(R,z)\right)^2 \rangle_\phi$$

Perform a χ^2 fit for grid of 20^2 points in range $R, z \in [0.1, 20]$ kpc

Baryonic potential fit

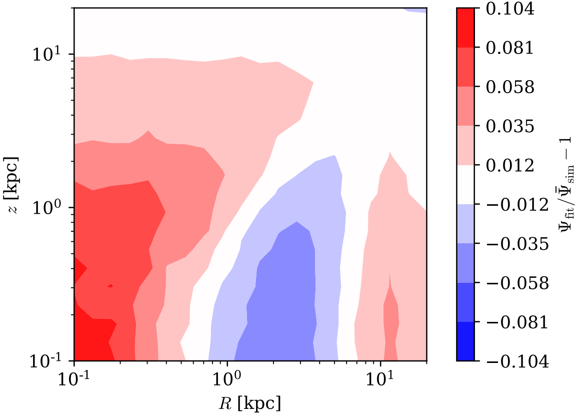


Figure: Relative difference between the fit and azimuthally averaged gravitational potential in Mochima simulation.

DM density profile

Use either **NFW or Burkert** DM density profile:

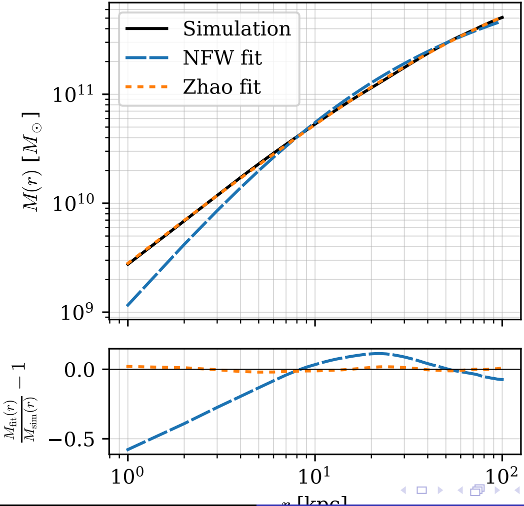
$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{r/r_s \cdot (1 + r/r_s)^2} \quad , \quad \rho_{\mathrm{Bur}}(r) = \frac{\rho_s}{(1 + r/r_s) \cdot (1 + r^2/r_s^2)}$$

Fit the enclosed DM mass (numerically more stable than density):

$$M_{\mathrm{sim}}(r) = \sum_{r_i < r} m_{\mathrm{DM,i}}$$

Minimize $f = \sum_{r_i} \left(\frac{M_{\mathrm{NFW/BUR}}(r_i \,|\, \rho_s, r_s)}{M_{\mathrm{sim}}(r_i)} - 1 \right)^2$ over hundred evenly-distributed radial points in the range $r \in [1, 100]$ kpc

DM mass profile



Halo rotation

Parameterize the halo rotation profile as:

$$\bar{v}_\phi(R,z) = \frac{\omega R}{1 + (R^2 + z^2)/r_a^2}$$

- **Small correction** in the velocity distribution ($\bar{v}_\phi \ll \sigma_\phi$)
- Can not be accessed by observations
- Irrelevant for indirect detection, but can be important for direct detection experiments

In the following **neglect the halo rotation**:

$$\bar{v}_\phi = 0 \quad \Rightarrow \quad f(\mathcal{E}, L_z) \equiv f_+(\mathcal{E}, |L_z|)$$

Results

Focus on the **comparison of DM velocity distribution** predicted by the inversion methods and the one extracted from simulations

- Speed distribution
- Meridional velocity distribution
- Azimuthal velocity distribution

Quantify the similarity in terms of **relative entropy**:

$$D_{\mathrm{KL}}(P|Q) = \sum_{x \in \mathcal{X}} P(x) \cdot \log\left(\frac{P(x)}{Q(x)}\right)$$

Derived quantities

- **Moments** of the velocity distribution
- **Velocity anisotropy**
- Astrophysical factors for direct and indirect **DM searches**

Speed distribution

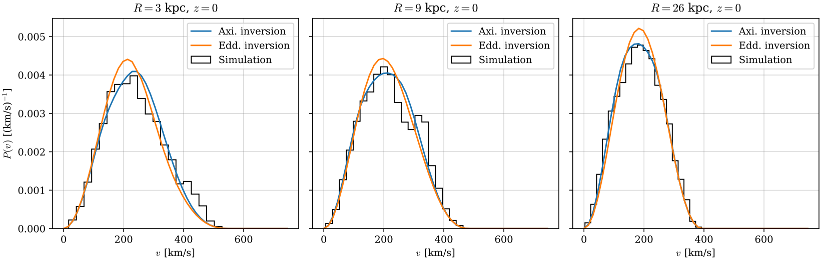


Figure: Comparison the predicted speed distributions with the ones extracted from simulation.

Components of the velocity distribution

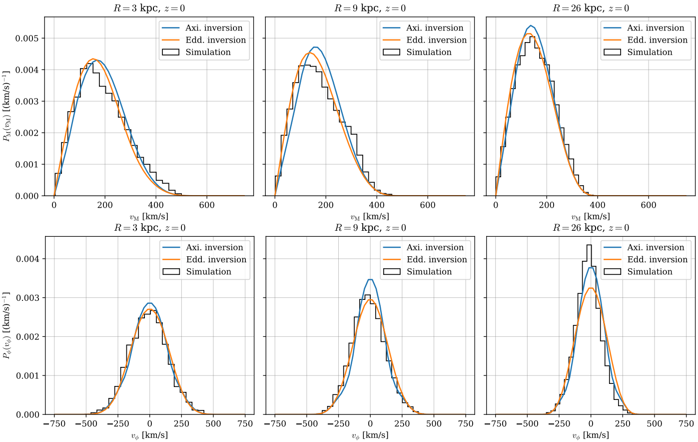


Figure: Comparison the meridional and azimuthal distributions.

Relative entropy (Kullback-Leibler divergence)

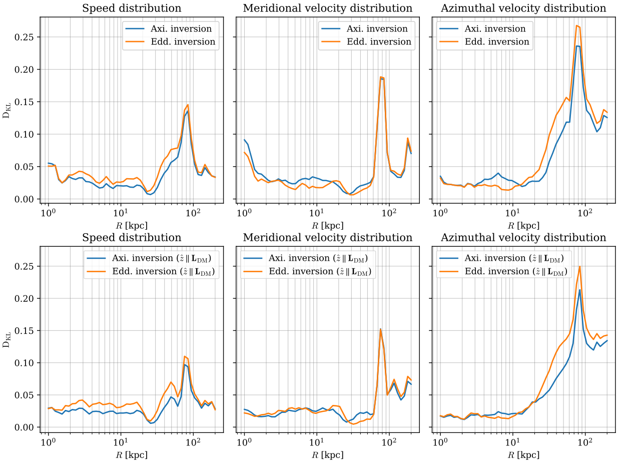


Figure: Relative entropy between true and predicted velocity distributions.

Moments of velocity distribution

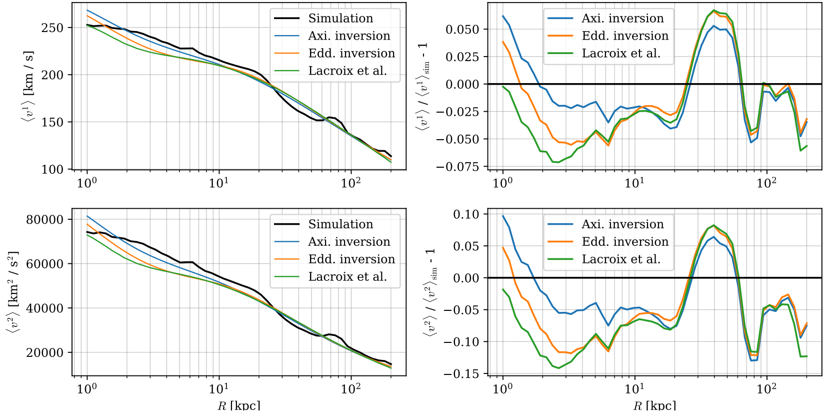


Figure: First two positive moments of the speed distribution.

Moments of velocity distribution

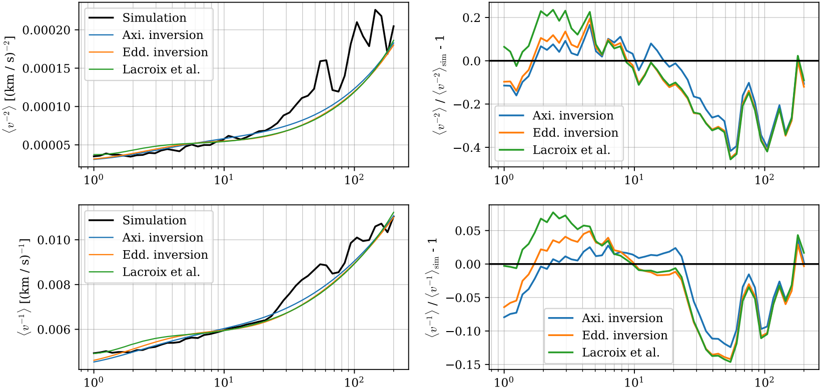


Figure: First two negative moments of the speed distribution.

Velocity anisotropy

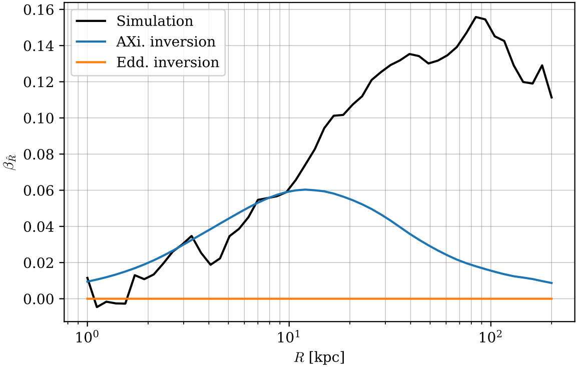


Figure: Velocity anisotropy in the disk plane; $\beta_{\hat{R}}(R) \equiv \frac{1}{2} - \frac{\sigma_\phi^2(R, z{=}0)}{\sigma_\mathrm{M}^2(R, z{=}0)}$.

Direct and indirect DM searches

DM-nucleon recoil rate in **direct searches** proportional to:

- $g(v_{\rm min}) \equiv \frac{1}{\rho_\odot} \int_{|\vec{v}|>v_{\rm min}} d^3v \;\; v^{-1} \cdot f_{\rm lab}(\mathcal{E}, L_z)$
- $h(v_{\rm min}) \equiv \frac{1}{\rho_\odot} \int_{|\vec{v}|>v_{\rm min}} d^3v \;\; v \cdot f_{\rm lab}(\mathcal{E}, L_z)$

Expected annihilation signal in **indirect searches** proportional to:

$$J(\Delta\Omega) = \int_{\Delta\Omega} d\Omega \int_0^\infty d\ell \; \rho^2_{\rm DM}(\vec{r}) \cdot \langle S(v_{\rm rel})\rangle(\vec{r})$$

- p-wave annihilations: $\langle S(v_{\rm rel})\rangle = \langle v^2_{\rm rel}\rangle$
- Sommerfeld enhancement: $\langle S(v_{\rm rel})\rangle = \langle v^{-\alpha}_{\rm rel}\rangle$ with $\alpha \in [0,2]$

Direct detection

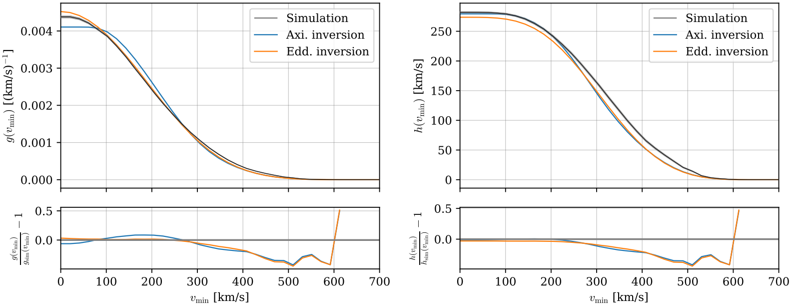


Figure: Astrophysical factors entering direct detection.

Moments of relative velocity distribution

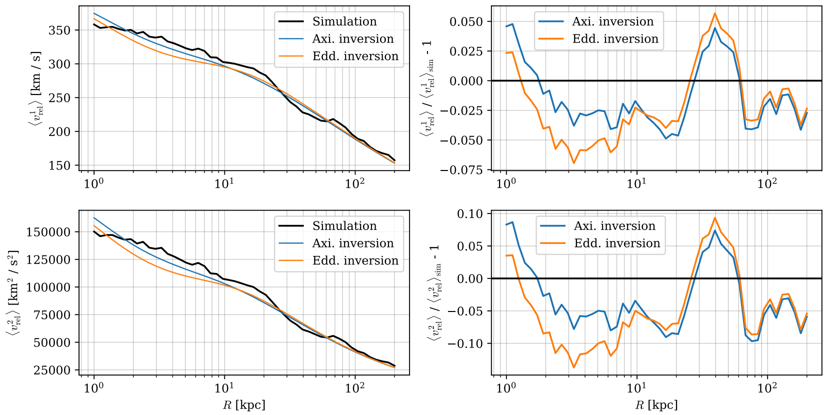


Figure: First two positive moments of the relative speed distribution.

Moments of relative velocity distribution

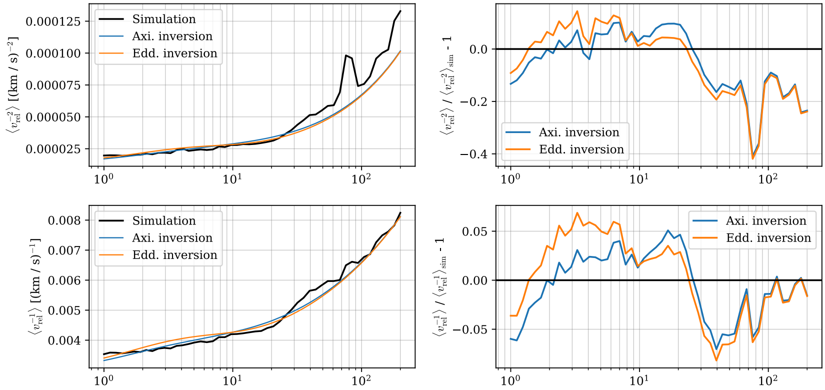


Figure: First two negative moments of the relative speed distribution.

Conclusions and outlook

Incremental improvement in modelling of the DM distribution

- Up to 50% smaller relative entropy and relative errors in the moments of velocity distribution
- Important for certain aspects of direct detection experiments
- Could serve as a starting point for more complex models

Caveats:

- Axisymmetric inversion can lead to **non-physical solutions**
- Accuracy depends on **unobservable "degrees of freedom"**

Some questions for the future:

- From (mock) observations to distribution function
- Models for the DM that is not in dynamical equilibrium