Self-consistent galaxy models

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Outline

- The value of multicomponent, self-consistent model galaxies
- Why DFs should be f(J)
- Application to the Fornax dSph
- Application to the MW
- Choosing f(J) avoid unphysical features

Need for f(x,v)

- V-distributions are always non-Gaussuan
  - $\langle v^2 \rangle^{1/2} = v_{esc}/2$ from VT
- V usually anisotropic & LOSVD known to be only hope of breaking mass-anisotropy degeneracy
- Data are increasingly discrete (Gaia…) but incomplete
  - Seriously uncertain distances, often lack $v_\parallel$ (Gaia..) or $v_\perp$ (MUSE..)
  - Discrete data best exploited by computing likelihood

$$\mathcal{L} = \prod_i \int \mathrm{d}x_\parallel \mathrm{d}v_\perp \; f_i(\mathbf{x}, \mathbf{v})$$

f(x,v) or N-body?

- N-body models deliver realisations of *f* not *f*
  - So can't compute likelihood of data
  - Model & data need to be binned
- Need > 10^6 particles to suppress 2-body relaxation so expensive
- Hard to control (via ICs) and hard to diagnose
  - *f(x,v)* can help with both tasks

f(E,..) or f(J)?

- Hard to solve for Φ if E appears in *f*
  - $\Phi(0)$ initially unknown, so range $[\Phi(0)\text{-}0]$ of E unknown & each iteration for E uses a different range
- Models need to be multicomponent (minimum is $f_* + f_{DM}$)
  - With E in f you can't specify M_* and M_{DM} up front
- What integrals to use alongside E?
  - In spherical case clearly L; axisymmetric case clearly L_z – both are actions
  - But generally?

Action integrals stand out

- Fixed range $(0,\infty)$ or $(-\infty,\infty)$
- Can be complemented to make up set (θ,J) of canonical coordinates
- Trivial Jacobian $d^3x d^3v = d^3\theta d^3J$
- M easily specified up front: $$M = (2\pi)^3 \int d^3\mathbf{J}\, f(\mathbf{J})$$
- (θ,J) the natural coordinates of perturbation theory

Example: Fornax (Pascale+ 2018)

- DFs for stars & DM
- logL computed as product chisq(density profile) and logL($x_\perp$,$v_\parallel$)
- Data from Battaglia (2006) and Walker (2009)

$$f_\star(\mathbf{J}) = \frac{M_{0,\star}}{J_{0,\star}^3} \exp\left[-\left(\frac{k(\mathbf{J})}{J_{0,\star}}\right)^\alpha\right]$$

with

$$k(\mathbf{J}) = J_r + \eta_\phi |J_\phi| + \eta_z J_z,$$

$$f_{\rm dm}(\mathbf{J}) = f(\mathbf{J}) g(\mathbf{J}) T(\mathbf{J}),$$

where

$$f(\mathbf{J}) = \frac{M_{0,\rm dm}}{J_{0,\rm dm}^3} \frac{[1 + J_{0,\rm dm}/h(\mathbf{J})]^{5/3}}{[1 + h(\mathbf{J})/J_{0,\rm dm}]^{2.9}},$$

$$g(\mathbf{J}) = \left[\left(\frac{J_{\rm c,dm}}{h(\mathbf{J})}\right)^2 - \mu \frac{J_{\rm c,dm}}{h(\mathbf{J})} + 1\right]^{-5/6}$$

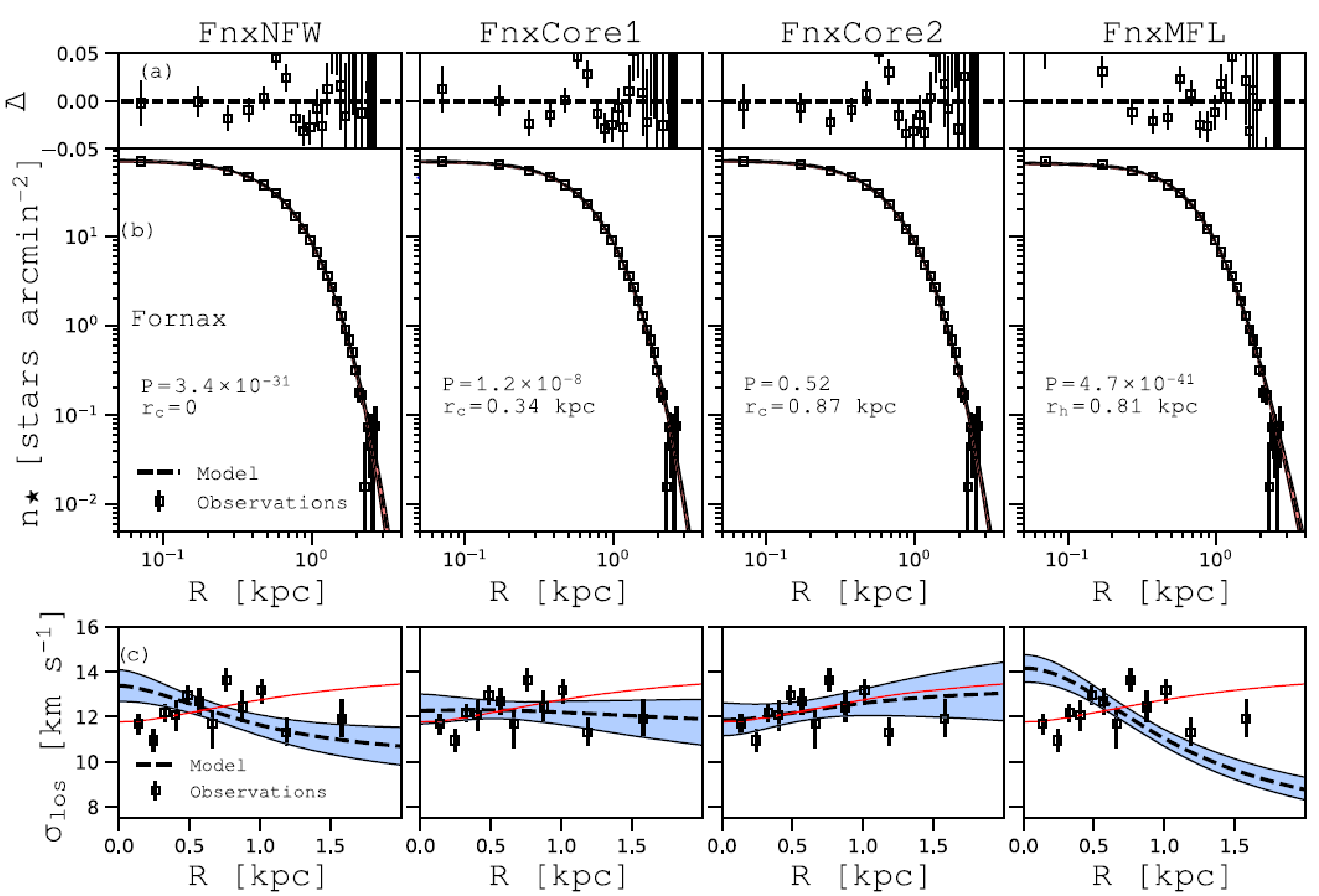
$$h(\mathbf{J}) = J_r + \delta_{h,\phi} |J_\phi| + \delta_{h,z} J_z$$

Fornax results

- Parameters set/coupled so just 5 free parameters

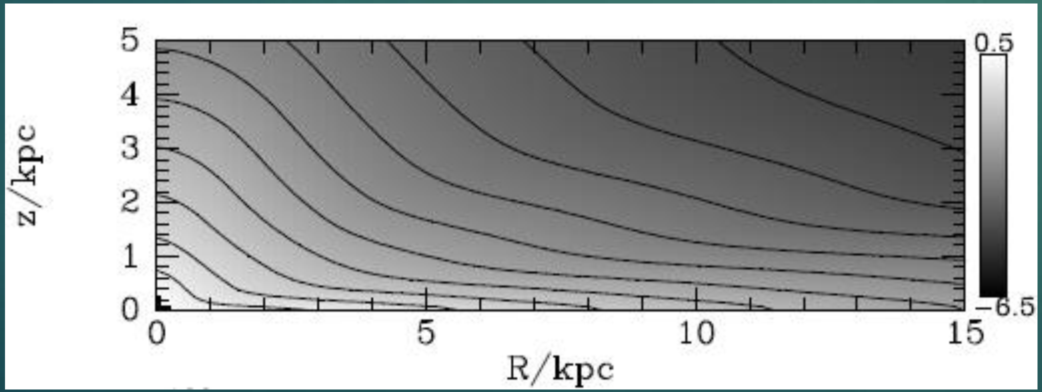
$(\alpha,\ \eta,\ \tilde{M}_{\mathrm{tot,dm}},\ J_{0,\star},\ M_{0,\star})$.

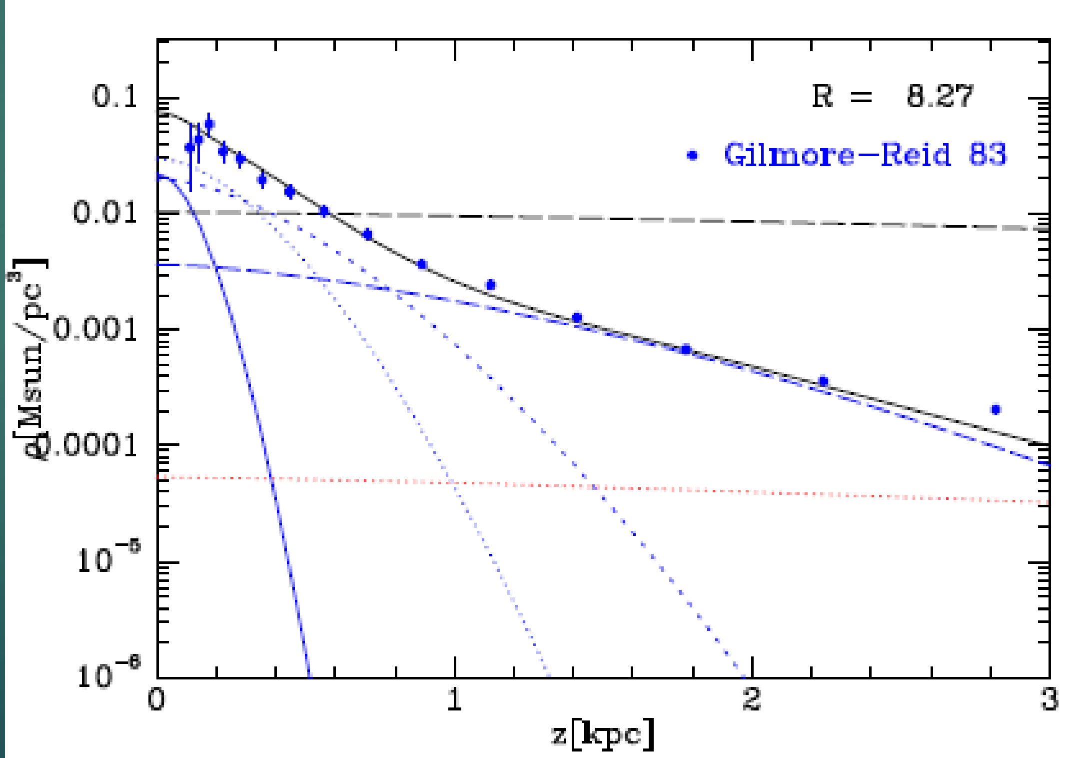
- Explored 4 values of J_c (0,…)
- Largest core (Fnx3)best



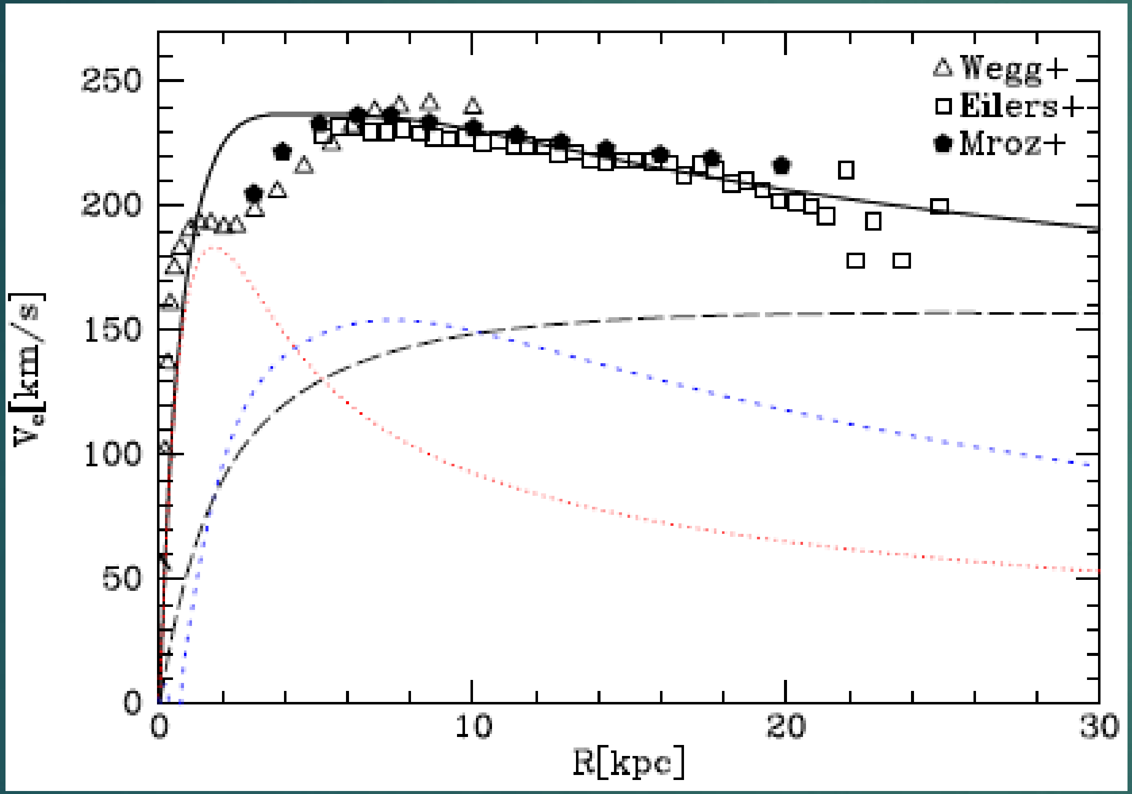
Modelling MW
(Binney & Vasiliev in prep)

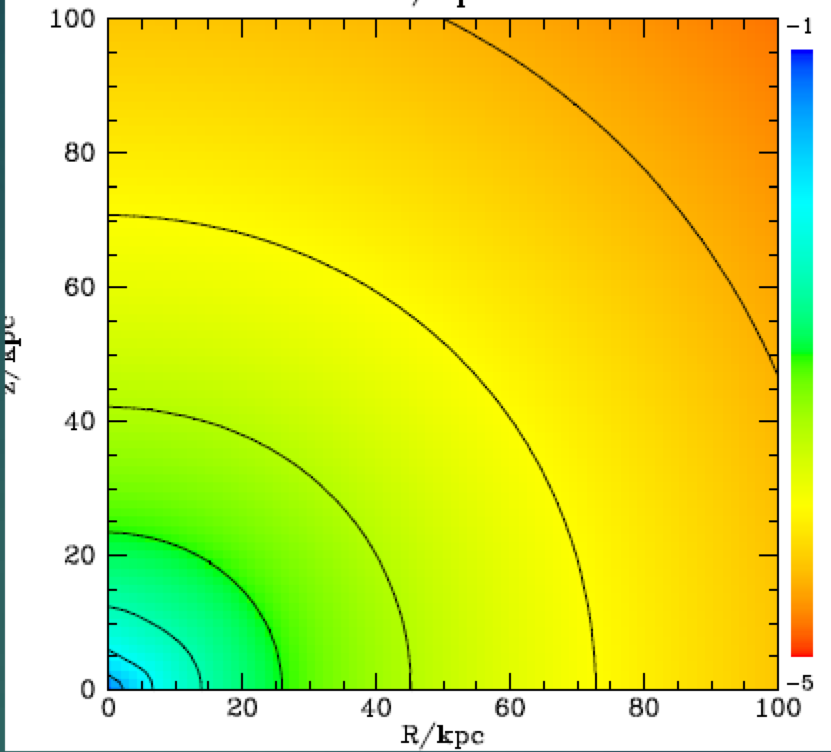
- New class of disc DF: 4 component disc
- Spheroids with old double power-law DFs
- Fitted by hand to
- RVS data from Gaia DR2
- vertical structure at R0 from Gilmore & Reid (1983)

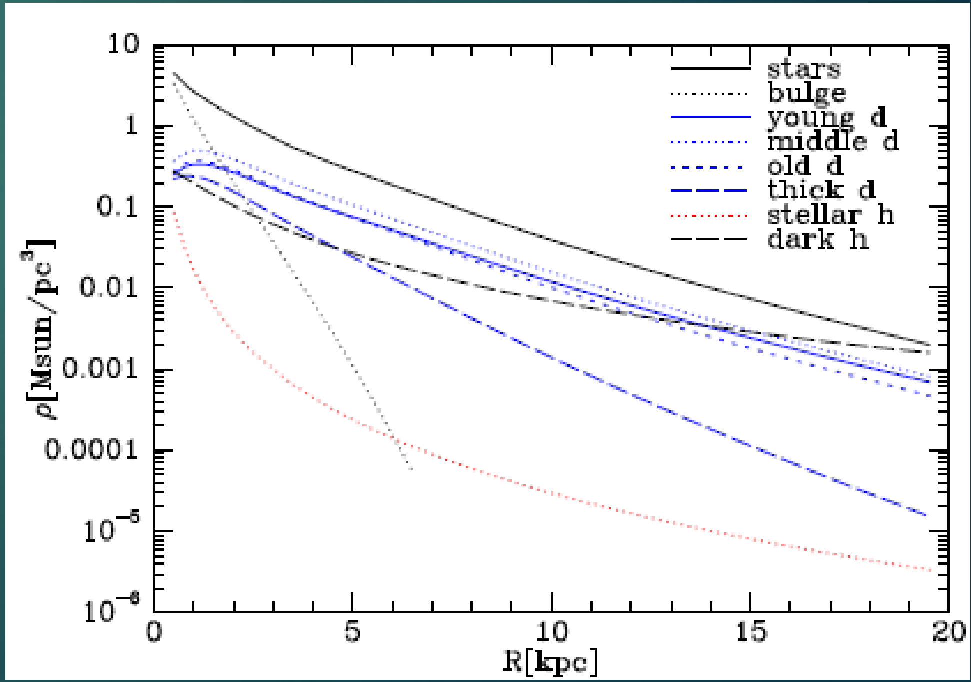




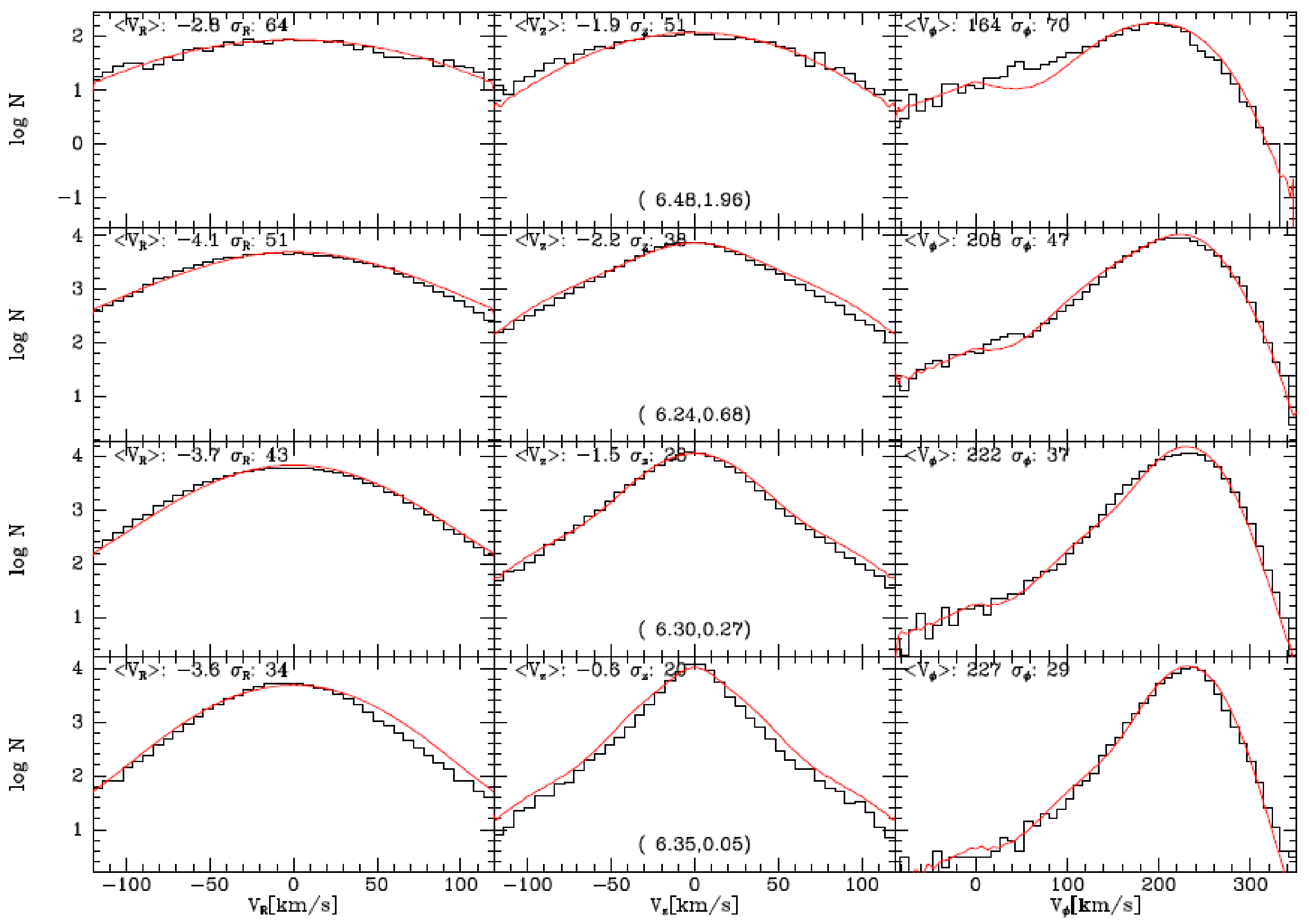
Self-consistent MW



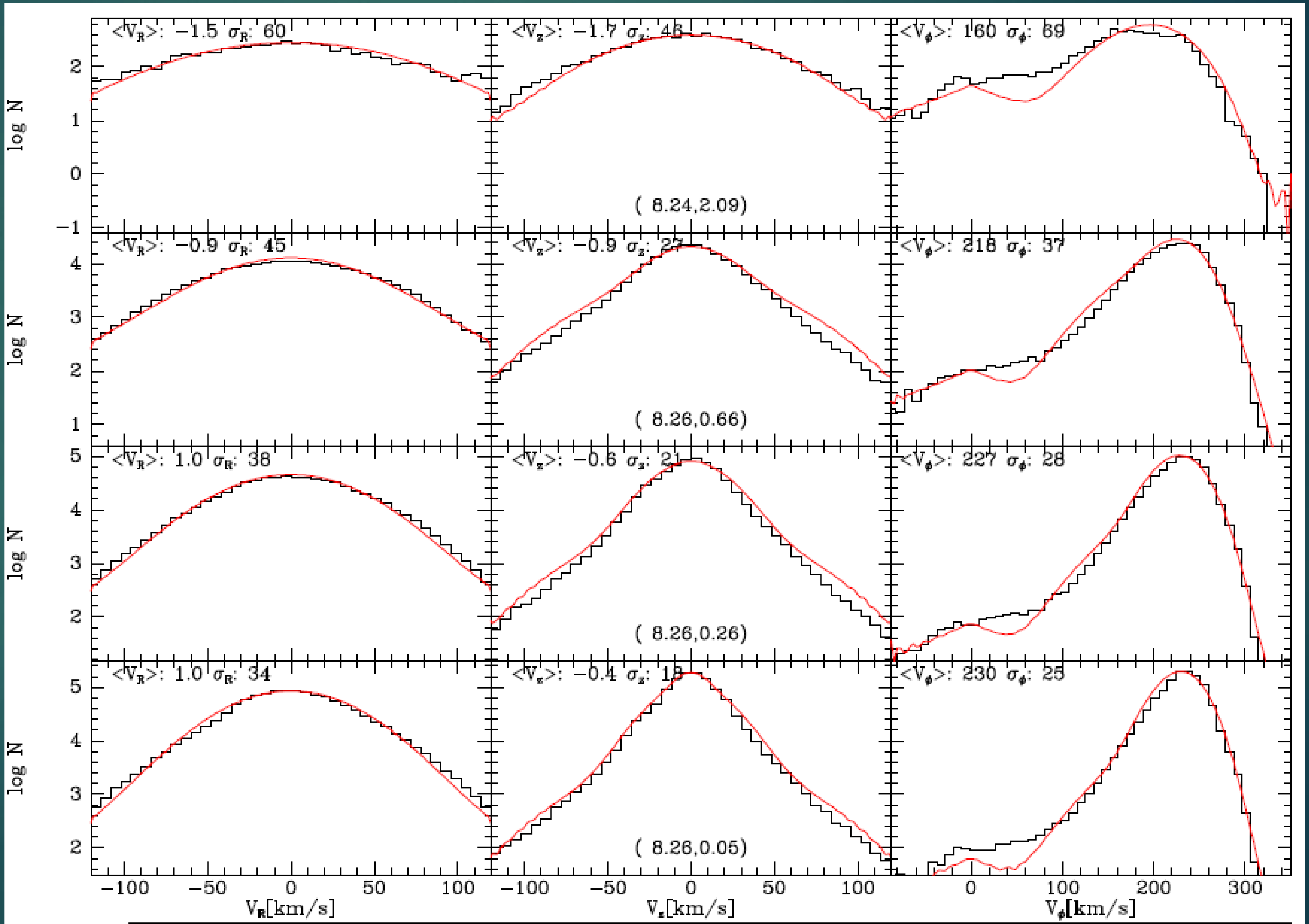




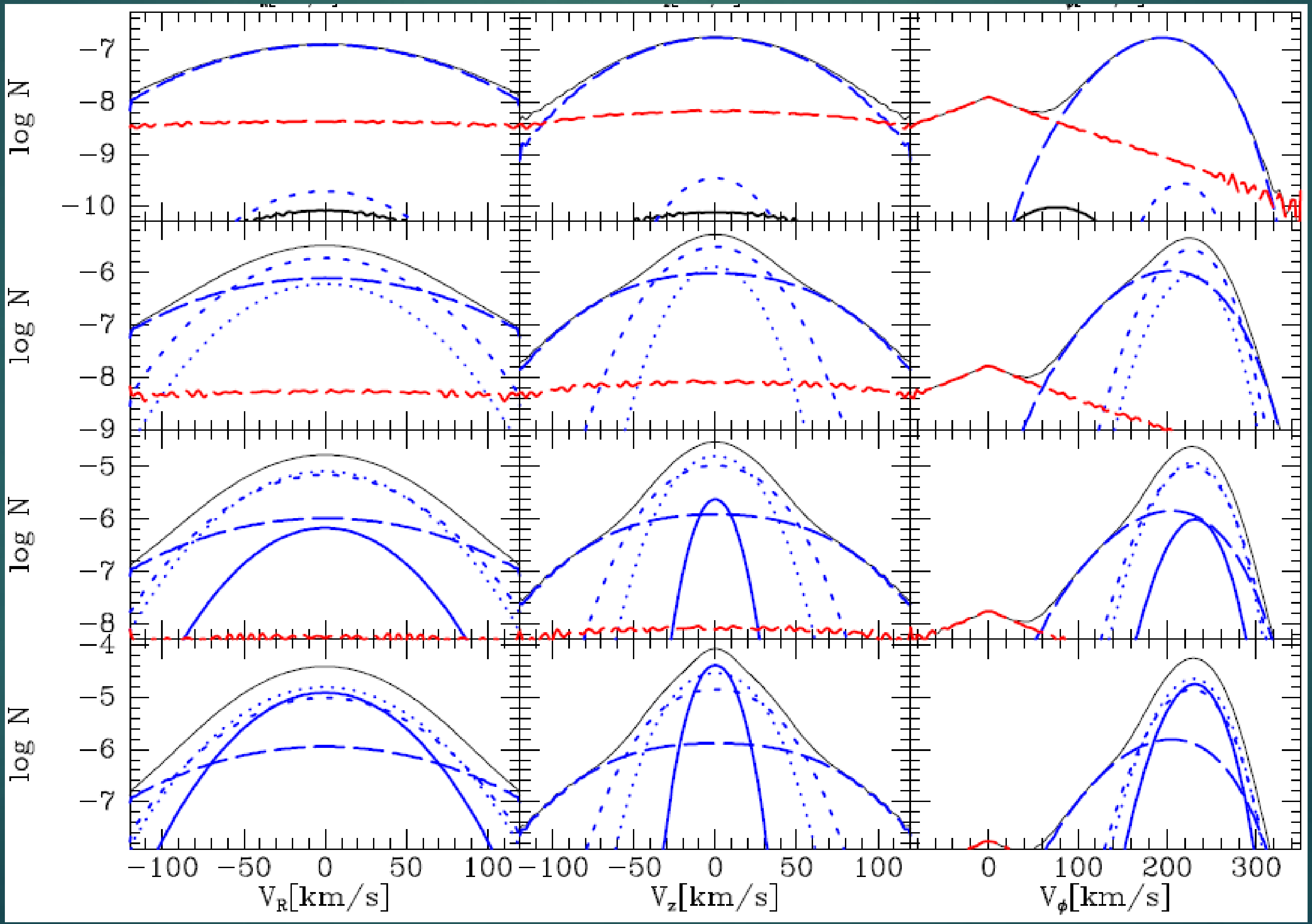
Kinematics interior to Sun



Kinematics of the solar cylinder

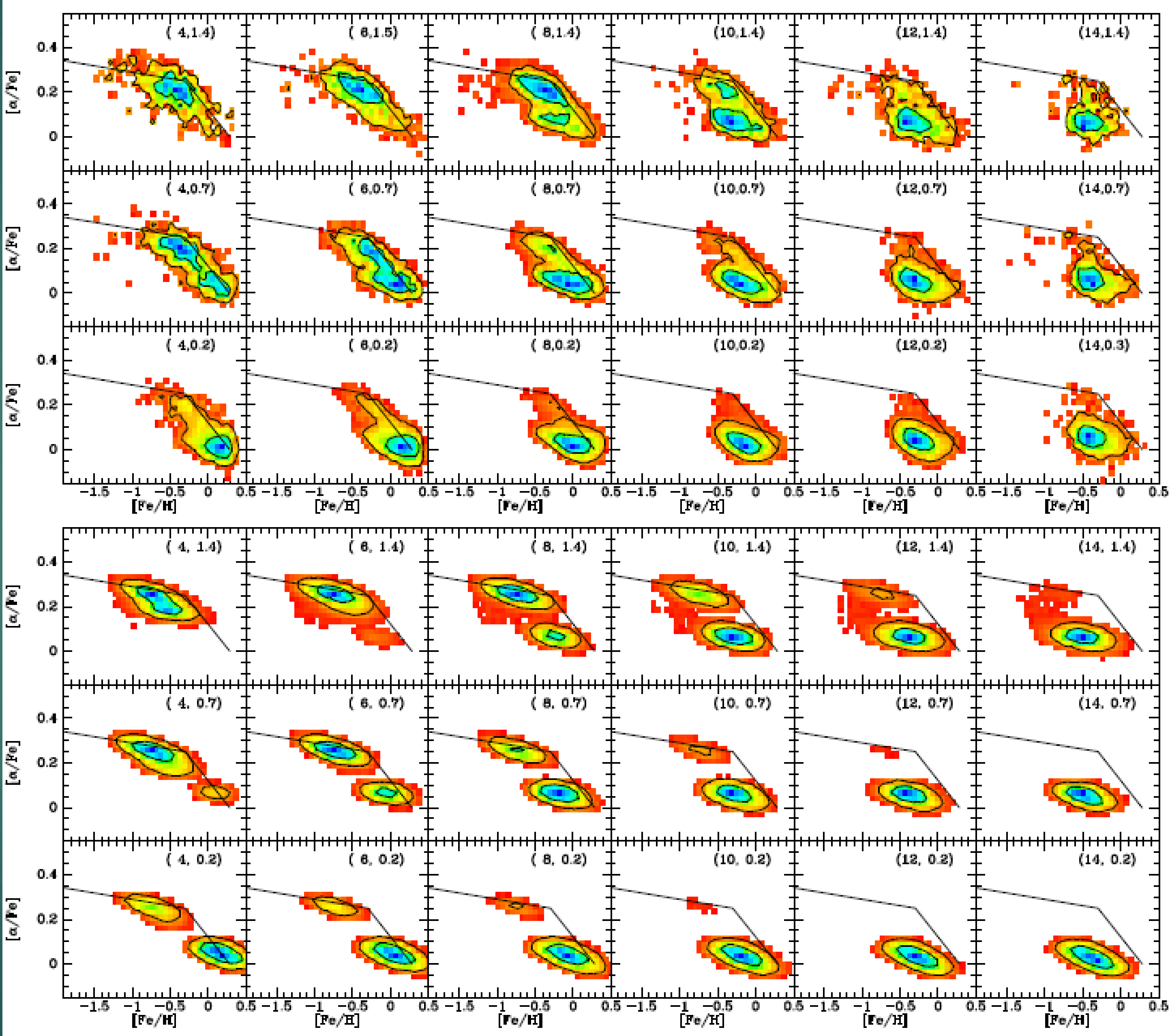


The MW Xrayed
Contributions of stellar components



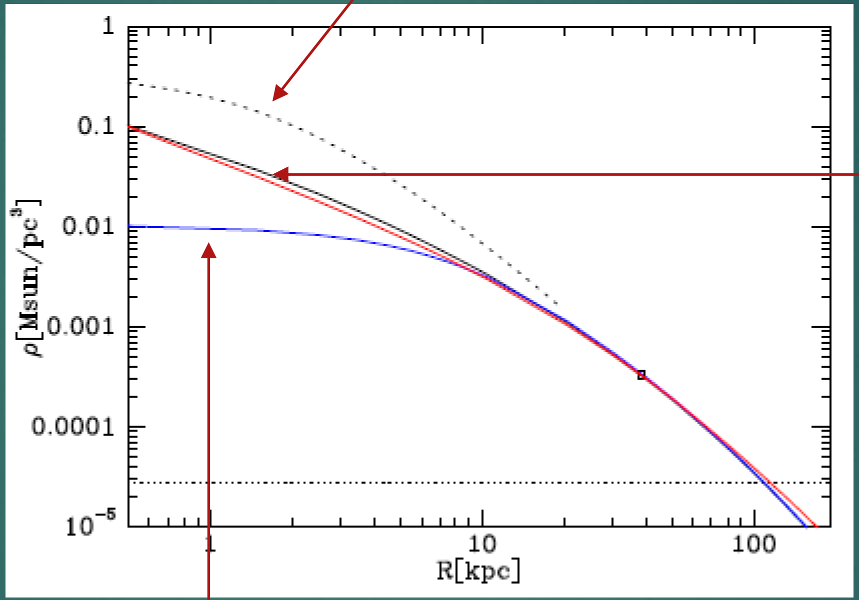
Adding chemistry

- Hayden+ 2015 data
- f(J) model



The dark halo unburdened

Current DM



Isolated f_{DM} minus core

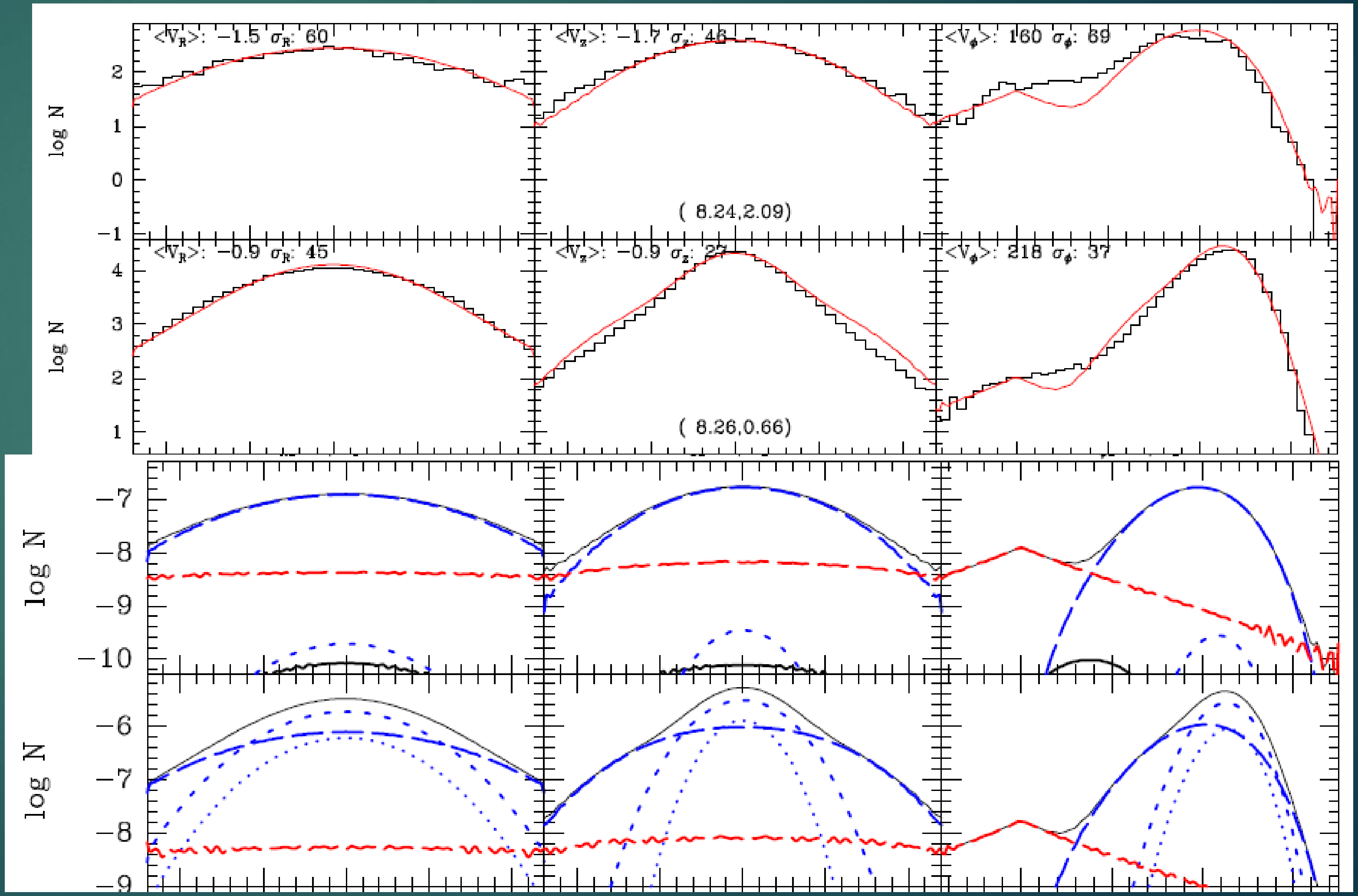
Isolated f_{DM}

Choice of DFs

- *f(J)* is a powerful approach to galaxy modelling
  - but it relies on a library of function forms for DFs
- Given any non-negative, integrable *f* of 3 variables you can construct a galaxy model
  - but the model will be unphysical unless *f* satisfies conditions near L_z=0
  - Consider ergodic model *f(H)* so $\frac{dn}{\partial v_\phi} = \frac{\mathrm{d}f}{\mathrm{d}H}\left(\Omega_r \frac{\partial J_r}{\partial v_\phi} + \Omega_z \frac{\partial J_z}{\partial v_\phi} + \Omega_\phi R\right)$
  - Since $H(v^2)$, $\partial n/\partial v_\varphi = 0$ at $v_\varphi = 0$ but this requires cancellations involving frequencies
- The scheme for introducing anisotropy proposed by Binney (2012) and implemented by Posti+(2015) destroys cancellation

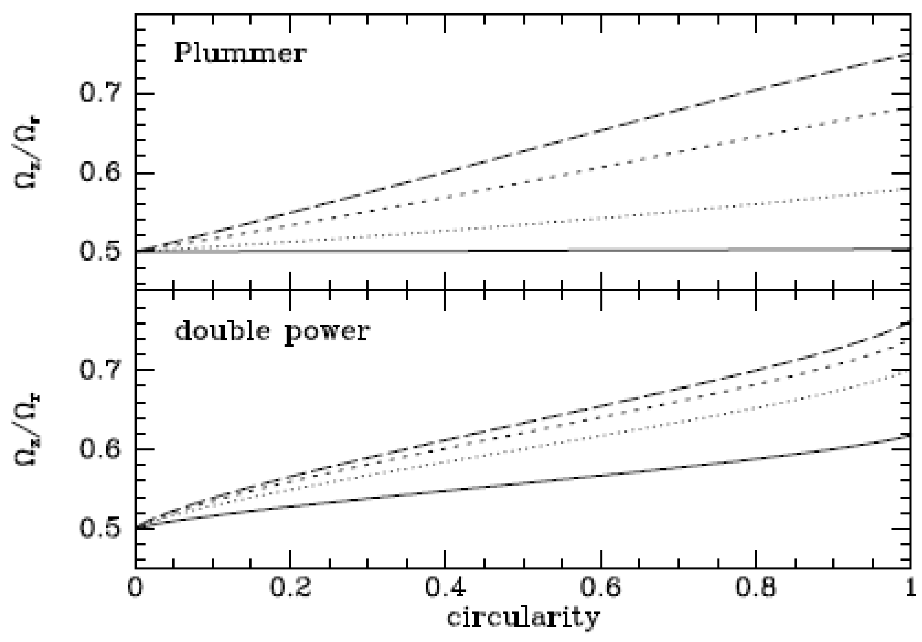
Not an academic problem

- In solar cylinder
- Stellar halo flattened & radially biased
- With Posti-style DF can't model low-L_z stars



We need frequency ratios

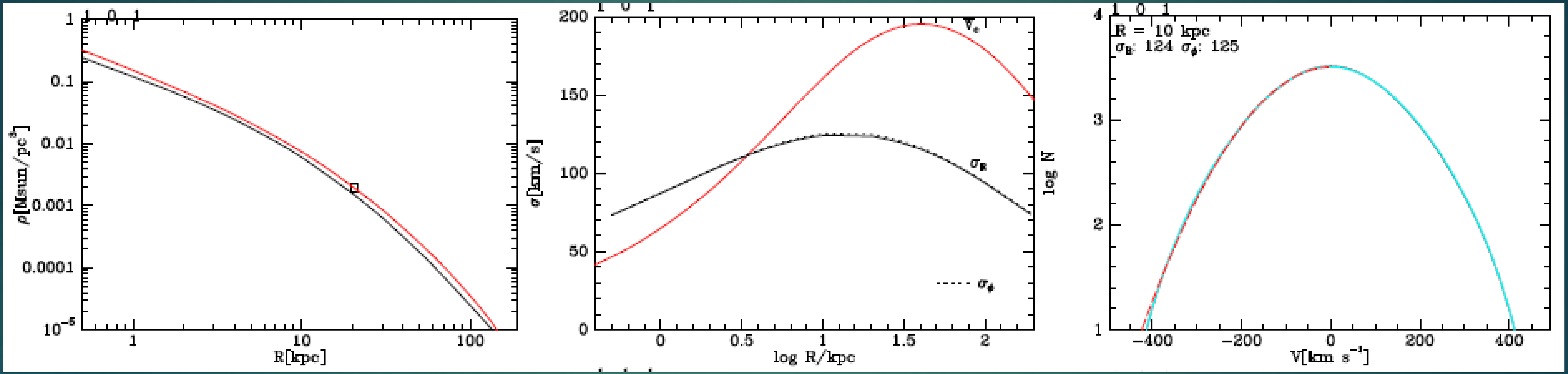
- Widely used ‘quasi-isothermal’ DF for discs (B2010, B&McMillan 2011) references frequencies
  - A bad idea because DF should come before Φ
- New disc DFs (B&Vasiliev in prep) free of freqs
  - But reference to freq *ratios* at low L_z unavoidable in spheroid DFs
- Fortunately these ratios are moderately generic
  - Simple analytic funcs fit them to sufficient precision



Ergodic f(J)

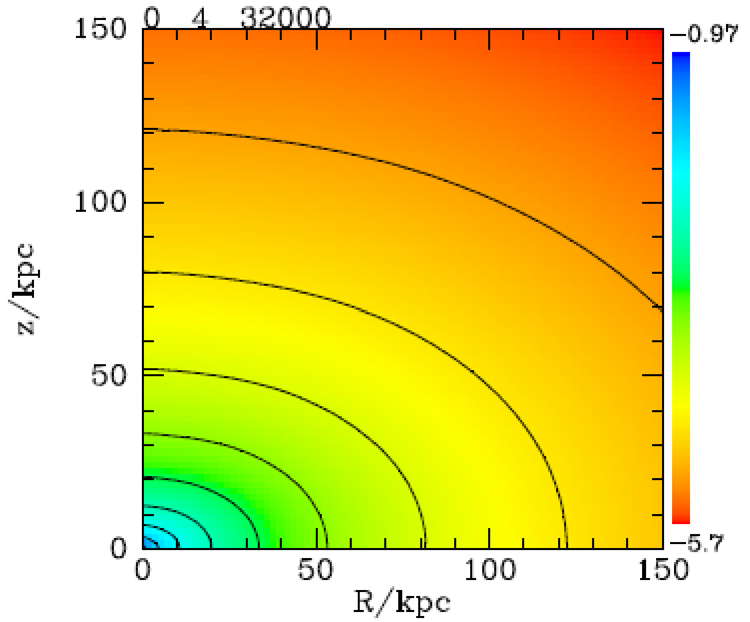
- Not so interesting per se but starting point for physical isotropic models
- In spherical case $$0 = \mathrm{d}H = \Omega_r \mathrm{d}J_r + \Omega_L \mathrm{d}L \;\Rightarrow\; \frac{\mathrm{d}L}{\mathrm{d}J_r} = -\frac{\Omega_r}{\Omega_L}$$
  - Integrate ode from to $J_r = 0$ (circular orbit) and set f to same value all along path.
  - Then f(H)
- In flattened case $$0 = \mathrm{d}H = \Omega_r \mathrm{d}J_r + \Omega_z \mathrm{d}J_z + \Omega_\phi \mathrm{d}J_\phi$$
  - Integrate $$\frac{\mathrm{d}J_z}{\mathrm{d}J_\phi} = -\frac{\Omega_\phi}{\Omega_z} \text{ at fixed } J_r$$
  - to $J_z = 0$ and then integrate $$\frac{\mathrm{d}J_r}{\mathrm{d}J_\phi} = -\frac{\Omega_\phi}{\Omega_r}$$
  - to $J_r = 0$ and note J_φ ; now $h(J)=L_{circ}(E)$ can be the argument of ergodic DF

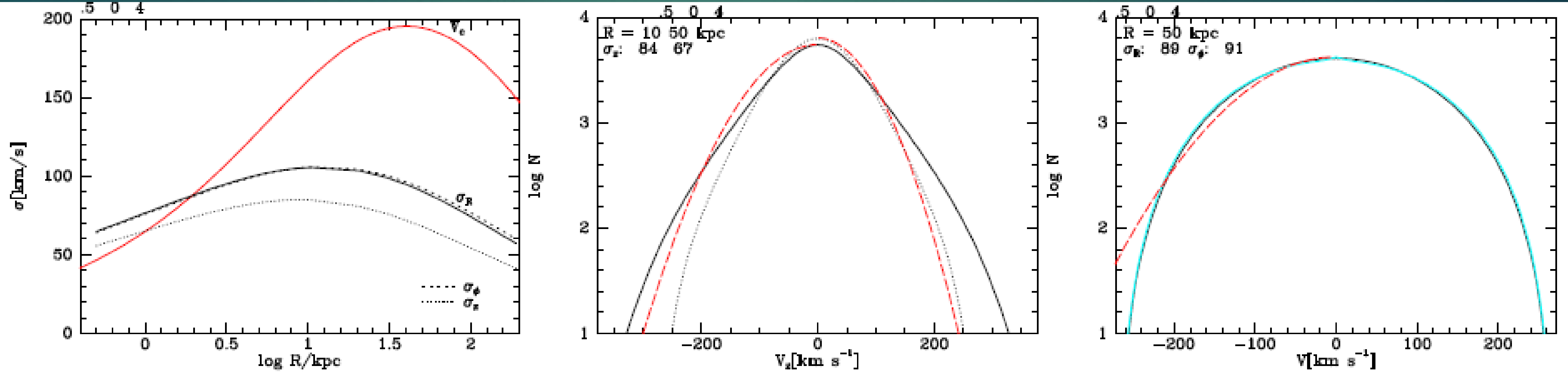
particles in spherical NFW Φ



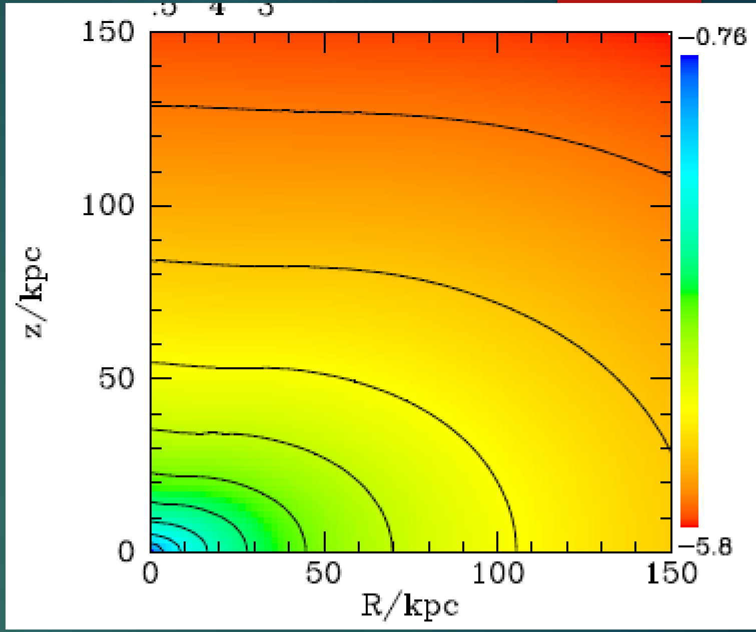
Introducing anisotropy

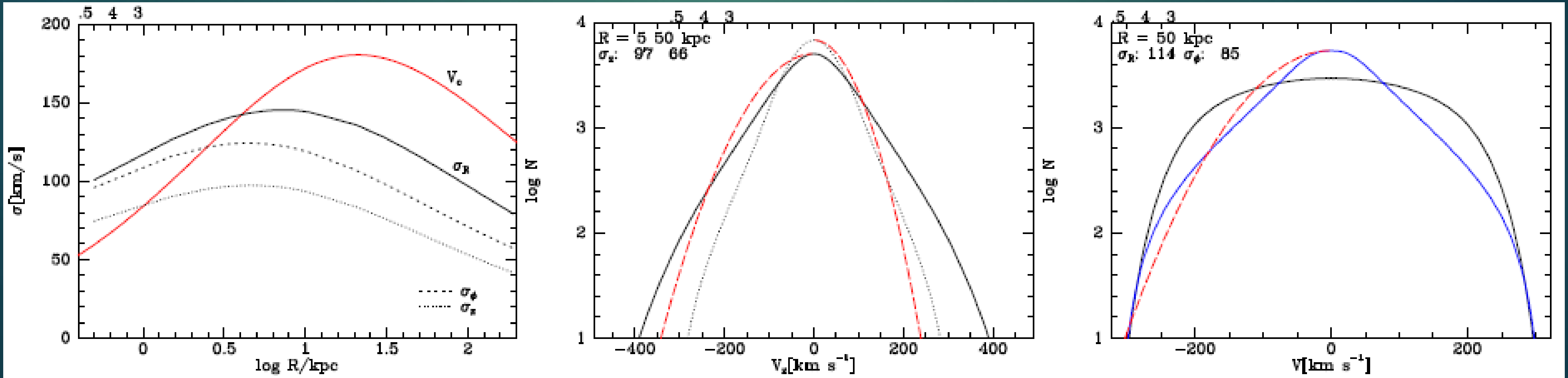
- (i) f(h(J))*g(L) introduces radial bias
- (ii) f(h(J))*g(J_φ) with dg/dJ_φ -> 0 as J_φ-> 0 to flatten model
- Self-consistent model flattened by (ii)





Fully anisotropic model





Conclusions

- Self-consistent, multi-component models are powerful tools for interpreting data for the MW, nearby galaxies and N-body simulations
- Models of oblate axisymmetric systems are easily generated by the AGAMA package using the *f(J)* technique
- Care is, however, required in the choice of *f* if unphysical behaviour at V_φ=0 is to be avoided
- Near the symmetry axis (J=0) *f* must resemble the DF of the ergodic model
- That DF can be recovered by integrating odes in action space
- DFs for plausible anisotropic models are easily constructed by multiplying by factors