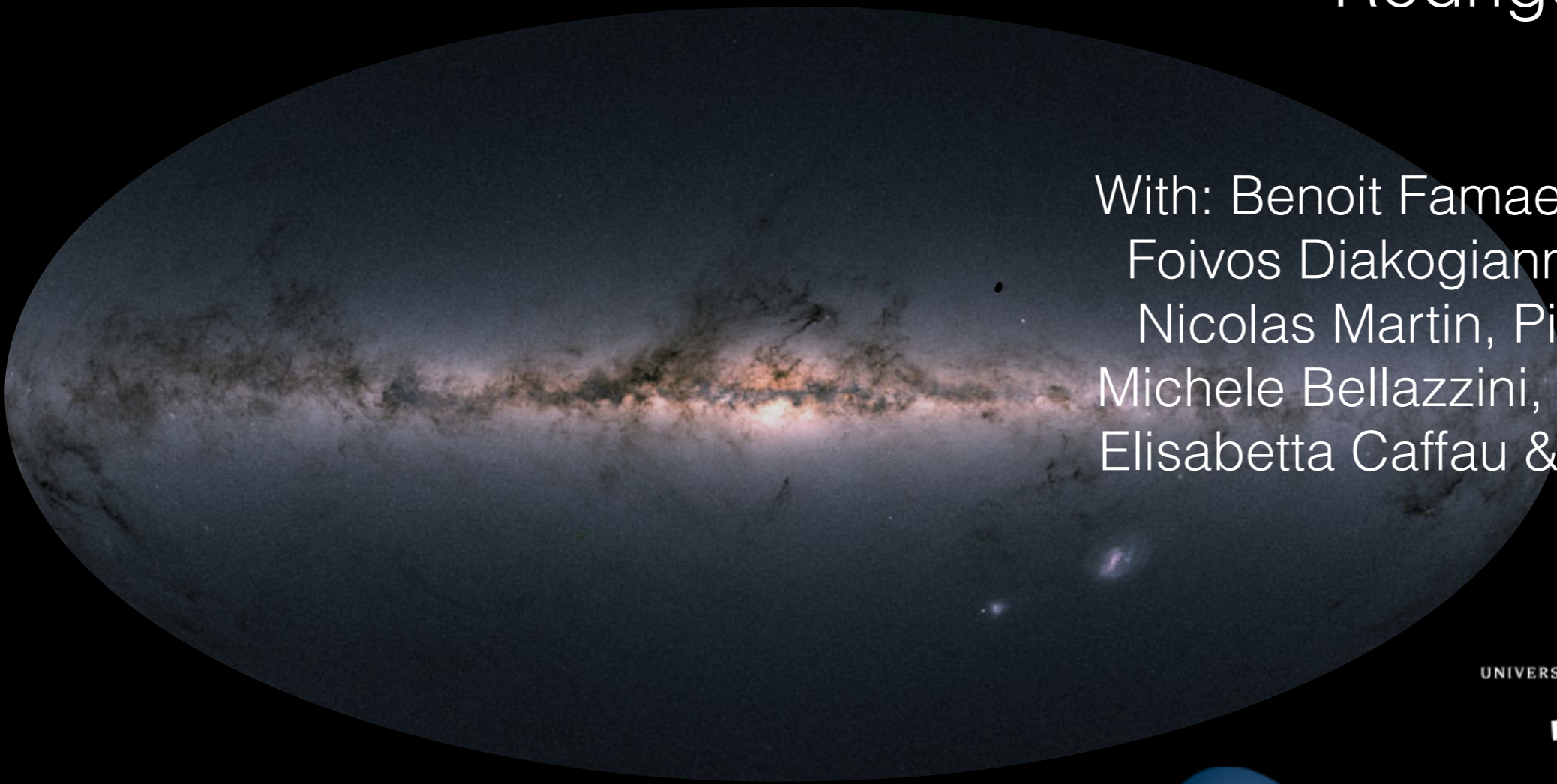


# Learning the properties of our Galaxy's Dark Matter with Stellar Streams

Rodrigo Ibata

With: Benoit Famaey, Giacomo Monari,  
Foivos Diakogiannis, Khyati Malhan,  
Nicolas Martin, Pierre-Antoine Oria,  
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Elisabetta Caffau & Guillaume Thomas



# Outline

- **Dynamics with Machine Learning (ActionFinder)**
- The STREAMFINDER search for stellar streams
- Perturbations of the GD-1 stream

# Canonical Transformations

$$\dot{q} = \frac{\partial H}{\partial p} \quad \dot{p} = -\frac{\partial H}{\partial q}$$

$H$  is the scalar field whose derivatives (rotated by  $\pi/2$ ) show how to advance the point through  $(q, p)$  phase space

consider a change of coordinates  $(q, p)$  to  $(Q, P)$ , that preserves this “canonical” form, i.e.

$$\dot{Q} = \frac{\partial K}{\partial P} \quad \dot{P} = -\frac{\partial K}{\partial Q}$$

so again:  
 $K$  is the scalar field whose derivatives (rotated by  $\pi/2$ ) show how to advance the point through  $(Q, P)$  phase space

# Action-angle variables

consider further a canonical transformation  $(q, p)$  to  $(\theta, J)$ ,  
in which the new Hamiltonian  $K$  is independent of  $\theta$

Then:

$$\dot{\theta} = \frac{\partial K(J, t)}{\partial J} \quad \dot{J} = -\frac{\partial K(J, t)}{\partial \theta} = 0$$

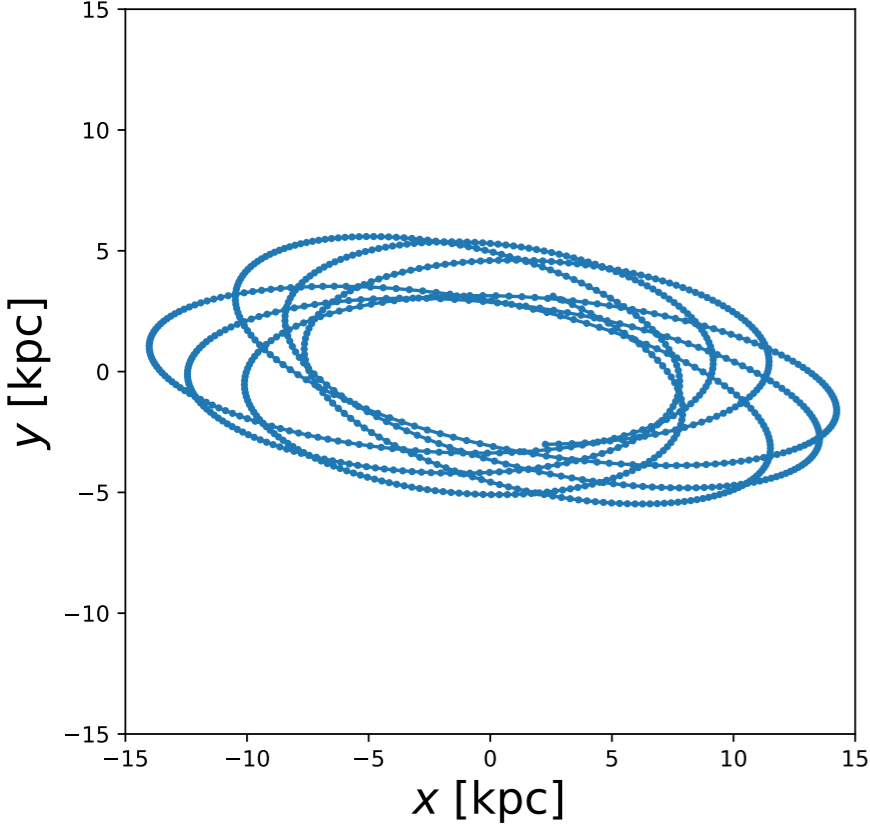
Hence:  $J = J_0$  (the constant “action” along the path)

$\theta$  is called the “angle” variable

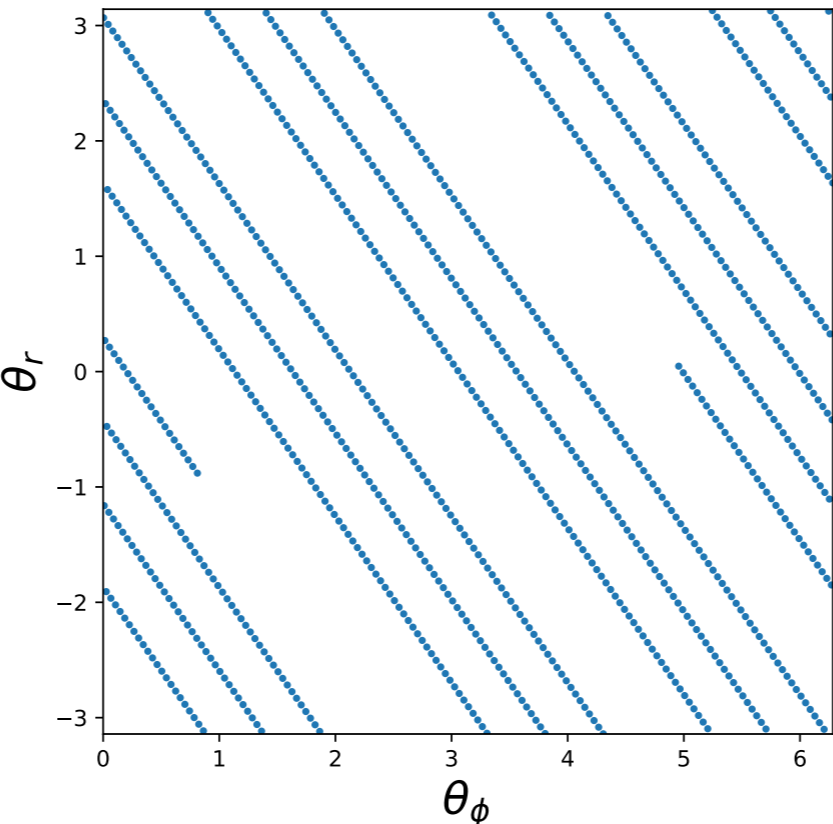
Dynamics in action-angle variables is very simple  
if  $K$  is independent of  $t$ :

$$\theta = \theta_0 + t \left. \frac{\partial K}{\partial J} \right|_{J_0} \quad (\text{i.e. } \theta \text{ advances linearly in time})$$

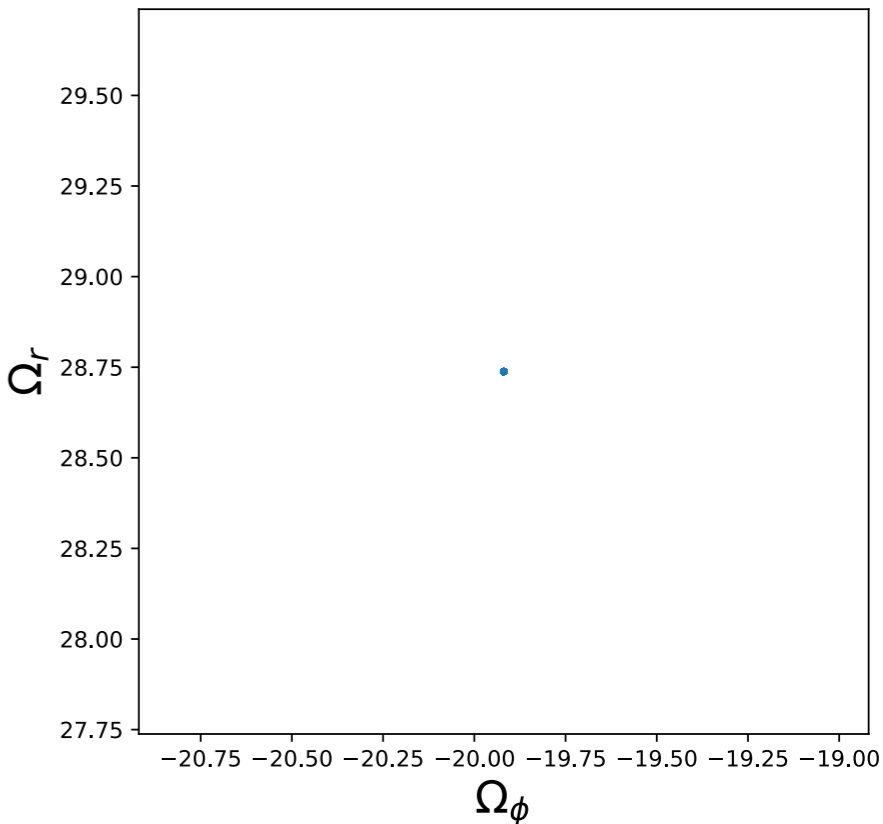
**orbit (like that of Palomar 5)**



**positions**



**angles**



**frequencies**

**2 Gyr integration in isochrone model with  $v_c(R = 8 \text{ kpc}) = 220 \text{ km s}^{-1}$**

# Action-angle variables

Avantages:

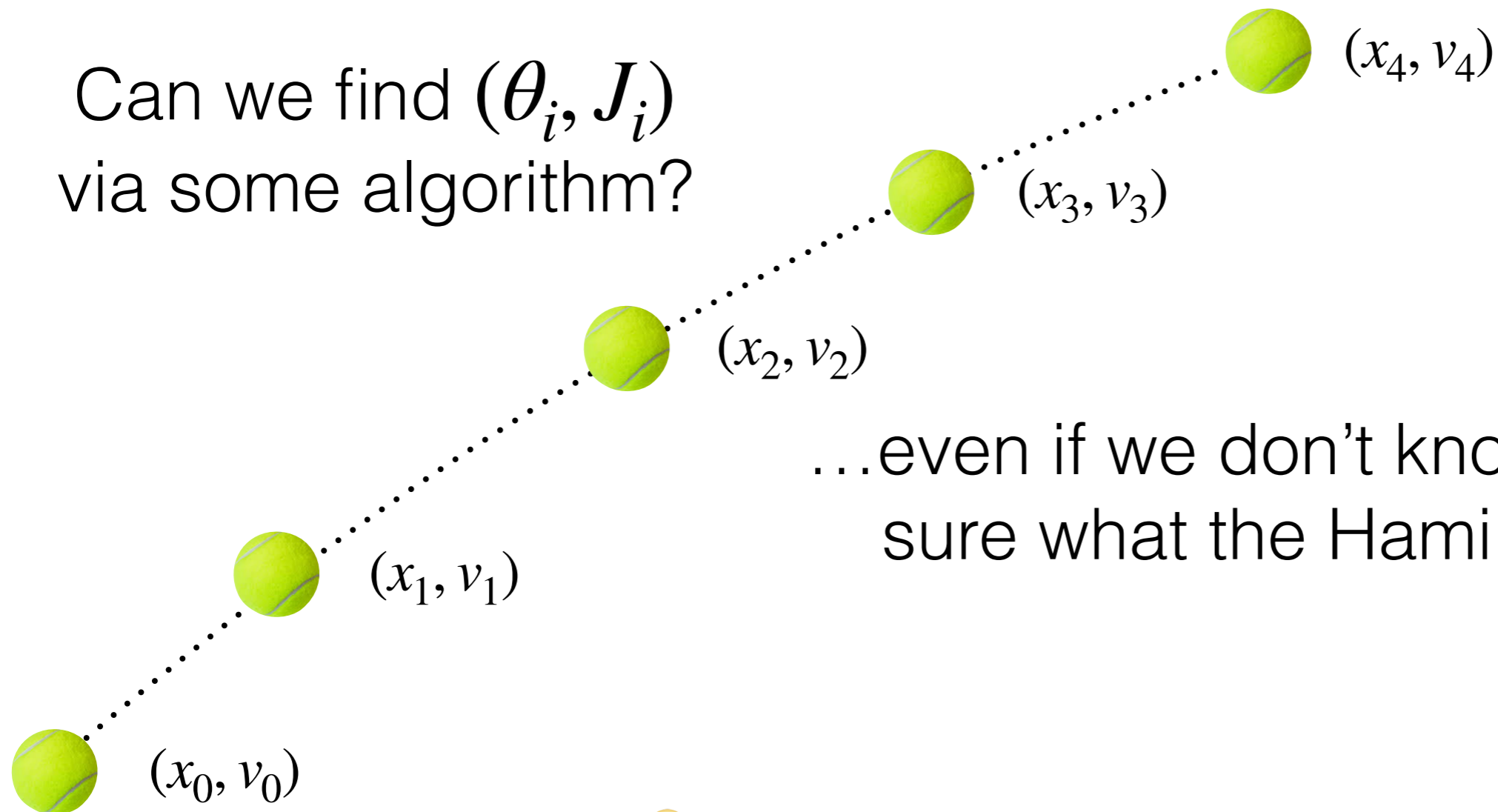
- $J$  are integrals of motion, giving a means to identify objects on the same phase-space trajectory
- $J$  are adiabatic invariants, hence are the best “archeological” coordinates
- $(\theta, J)$  are natural coordinates for perturbation theory
- If system is static:  $H(J)$
- Acceleration field can be derived from

$$\dot{J}_i = \frac{\partial J_i}{\partial x_j} \dot{x}_j + \frac{\partial J_i}{\partial \dot{x}_j} \ddot{x}_j = 0$$

But the coordinate transformation is challenging to compute.

Suppose we have measurements of phase space trajectories of a deterministic and reversible system

Can we find  $(\theta_i, J_i)$   
via some algorithm?



...even if we don't know or are not  
sure what the Hamiltonian is?

💡 **Lockdown idea 1:**

Can we implement this with a neural net

$$(\theta, J) = N_{\text{net}}(x, v) ?$$

## 💡 Lockdown idea 1:

Can we implement the transformation in 6D phase space with  
a neural net

$$\eta = N_{\text{net}}(\xi) ?$$

Objective function: minimize spread of  $J$  on same trajectory,  
& minimize symplectic constraint  $M^T \mathbb{J} M - \mathbb{J}$

where

$$M_{ij} = \partial \eta_i / \partial \xi_j \quad \mathbb{J} = \begin{pmatrix} 0 & \mathbb{I}_3 \\ -\mathbb{I}_3 & 0 \end{pmatrix}$$

Result: a huge waste of time!

reason:  $M_{ij}(\xi)$  has  $(2n)^2 = 36$  entries, whereas only  
 $2n^2 + n = 21$  should be independent if symplectic



## 💡 Lockdown idea 2:

Actually ~200 year old idea of Jacobi of using a generating function involving the old and new canonical coordinates

Consider the transformation generated by  $S(q, P, t)$

$$p = \frac{\partial S}{\partial q} \quad Q = \frac{\partial S}{\partial P}$$

This transformation is canonical as  $\{Q_i, P_j\}_{q,p} = \delta_{ij}$

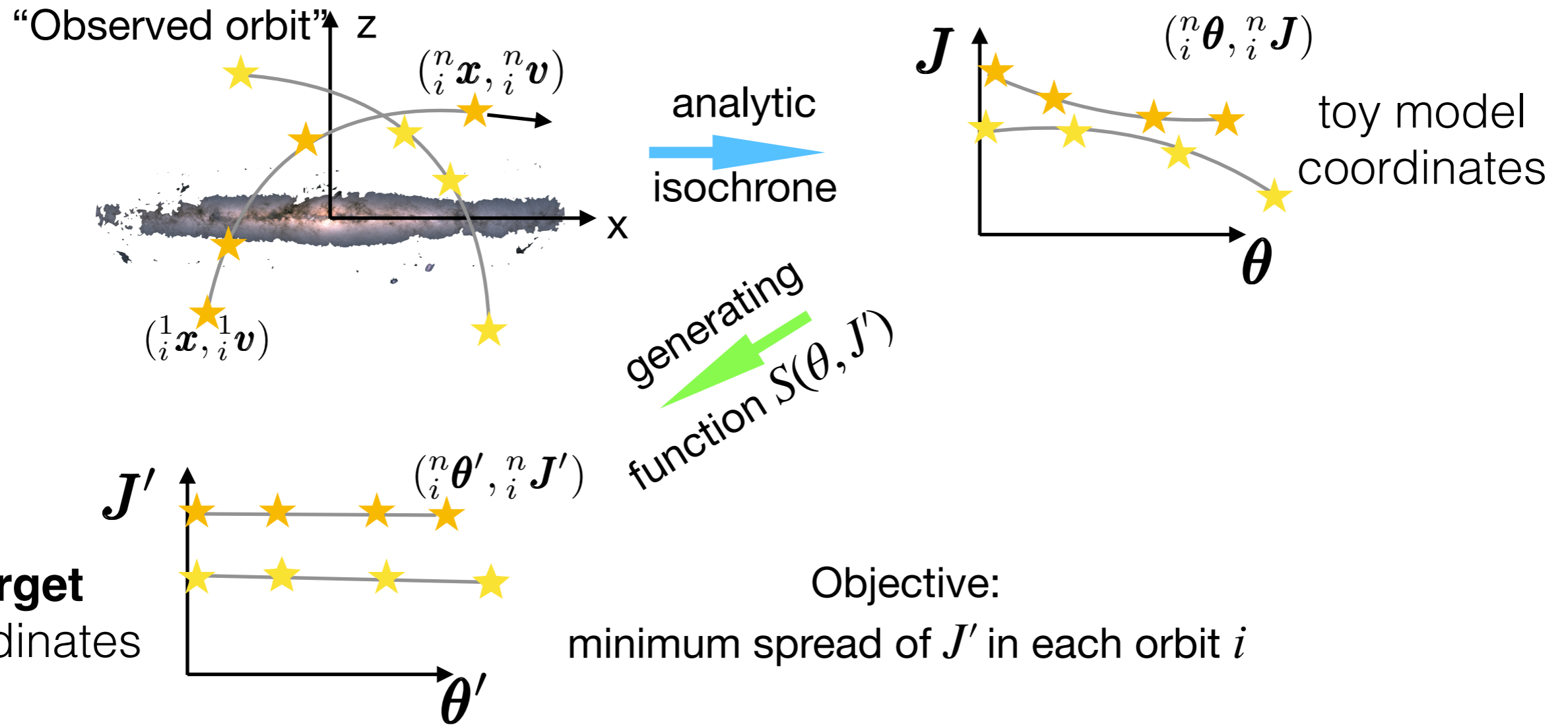
In general  $S$  is very challenging to find!

If  $H$  is known and system is axisymmetric  $S$  can be approximated with a Fourier series (McGill & Binney 1990)

Can we make this more powerful with a **neural network**, and avoid needing to know  $H$  in advance?

# Learn the transformation from orbits

We convert  $(\mathbf{x}, \mathbf{v}) \rightarrow (\theta', \mathbf{J}')$  in an **unsupervised** way,  
**learning** a generating function  $S(\theta, \mathbf{J})$  for the canonical transformation...



**ADVANTAGE:** We have not had to assume  $H(\mathbf{x}, \mathbf{v})$  or  $\Phi(\mathbf{x})$  or any symmetry

Network **derives** the acceleration field.

RI, Diakogiannis, Famaey, Monari (2021)

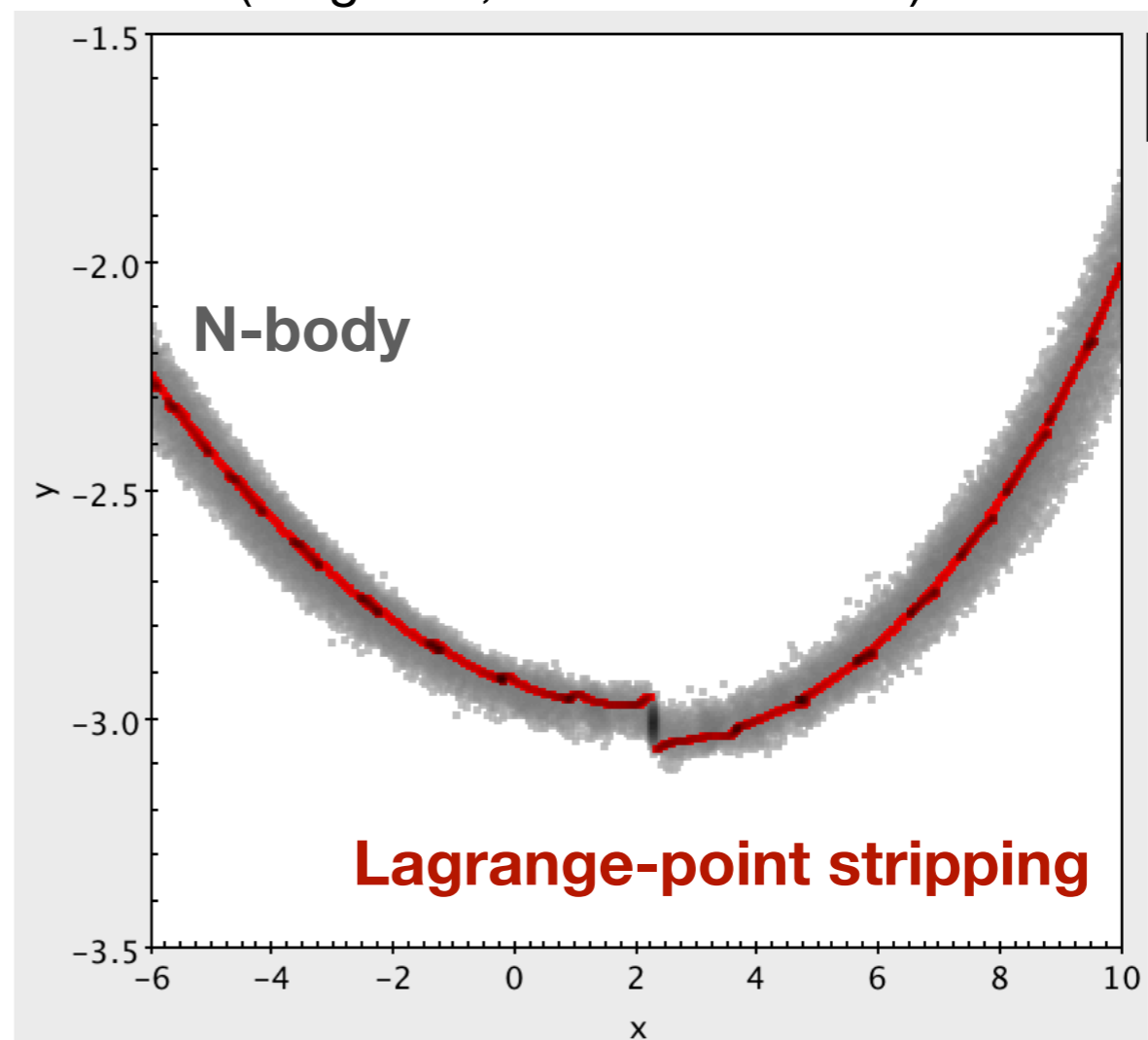
<https://github.com/Rodrigolbata/ActionFinder>

But we don't observe orbits, as  $T_{\text{human}} \ll T_{\text{orbit}}^{\dagger}$

$\dagger$  except in the Galactic center

Could we apply  
the ActionFinder  
to **real data** from  
**stellar streams**?

Stream model by  
Lagrange-point stripping  
(Varghese, RI & Lewis 2011)

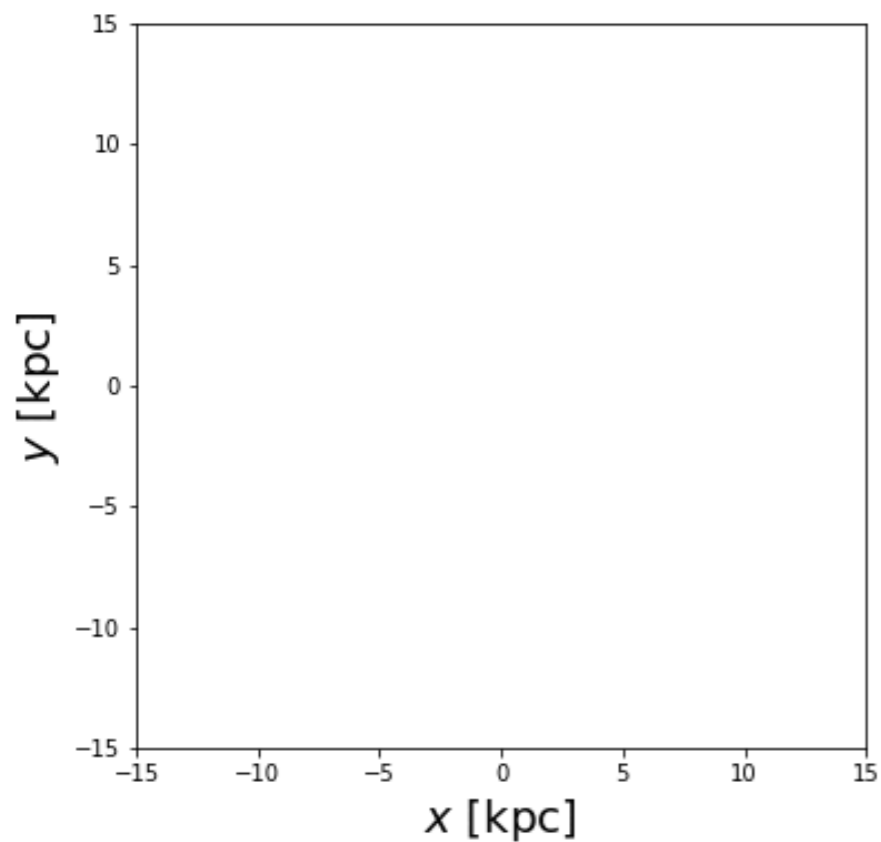


Palomar 5 -like model

# Simple stream by Lagrange-point stripping

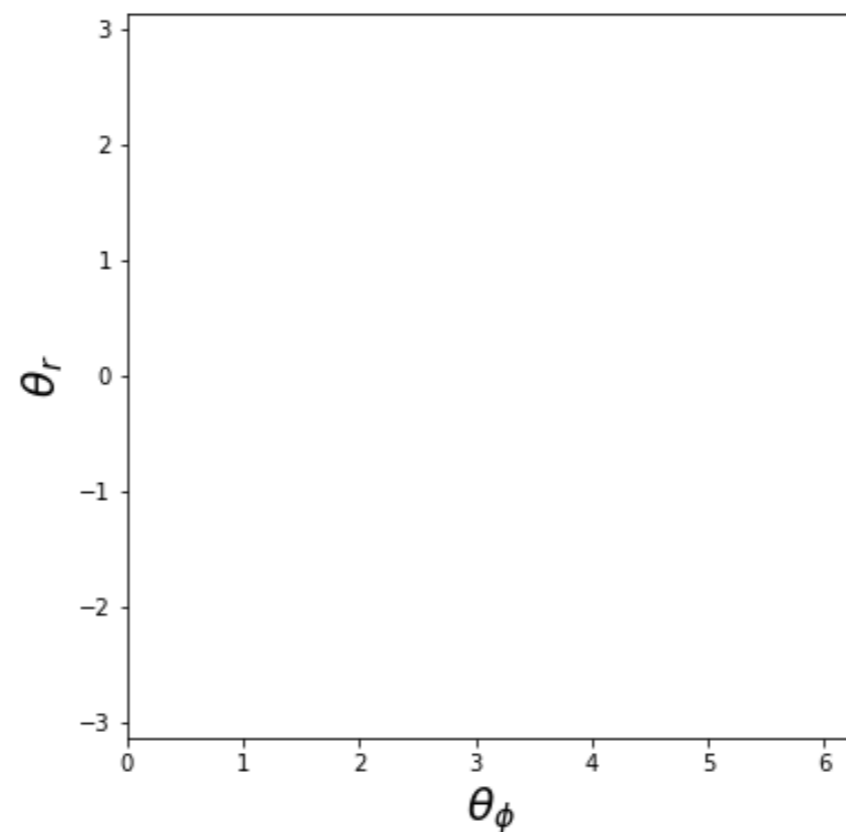
Blue: orbit

Red: evolving stream



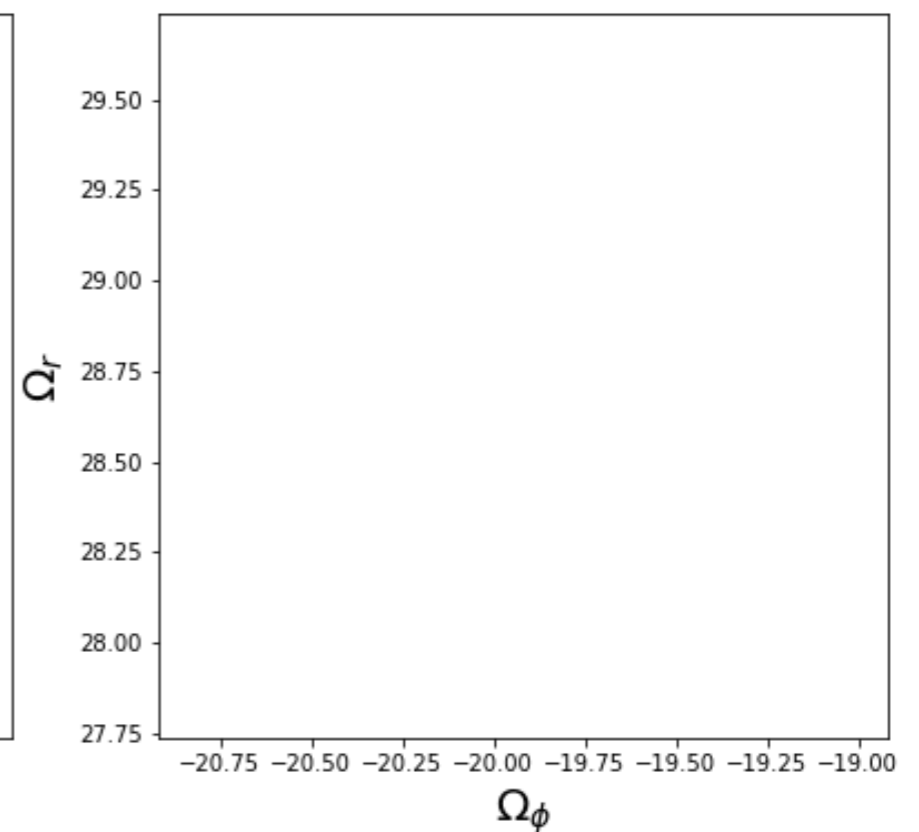
positions

Red: evolving stream



angles

Red: evolving stream



frequencies

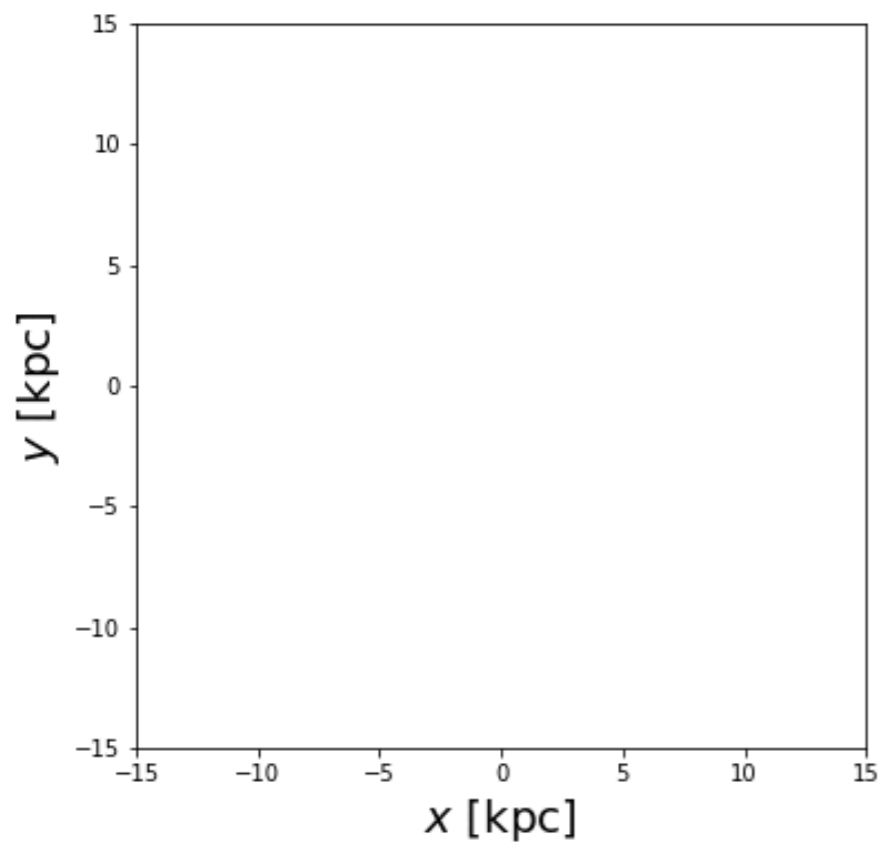
Palomar 5 -like model

2 Gyr integration in isochrone model with  $v_c(R = 8 \text{ kpc}) = 220 \text{ km s}^{-1}$

# Simple stream by Lagrange-point stripping

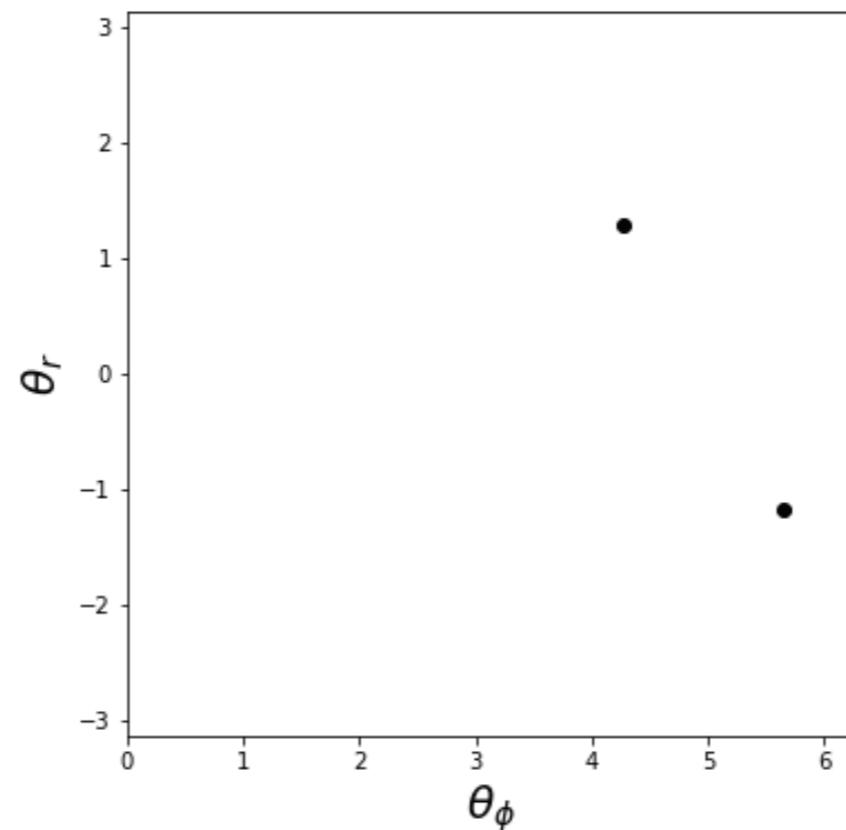
Blue: orbit

Red: final stream



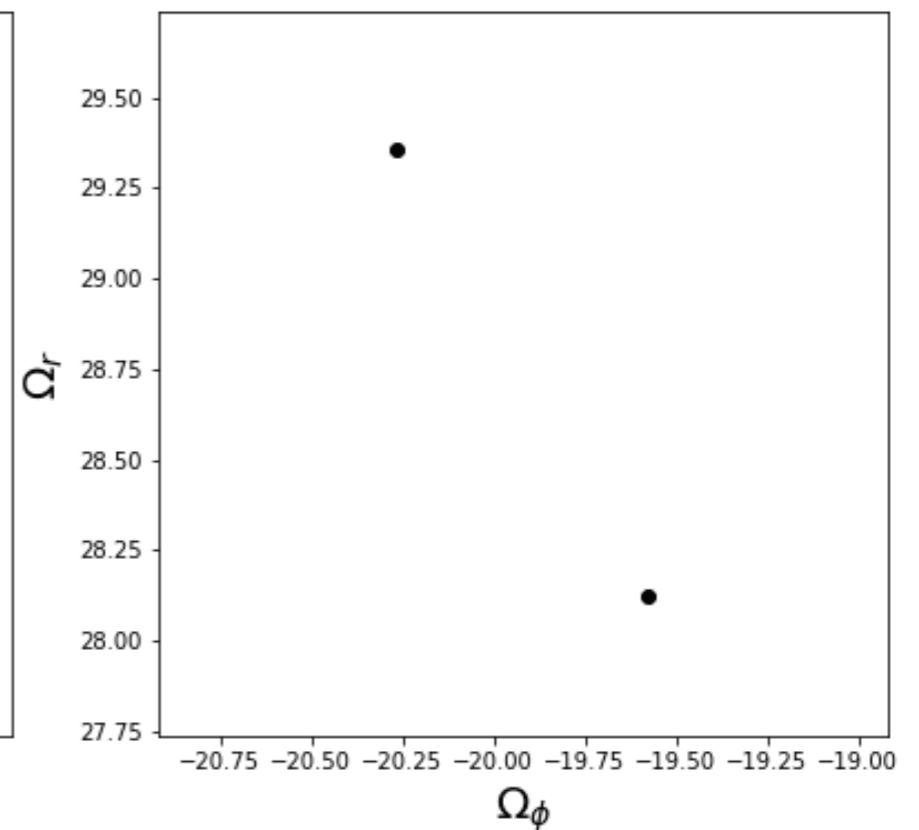
positions

Red: final stream



angles

Red: final stream



frequencies

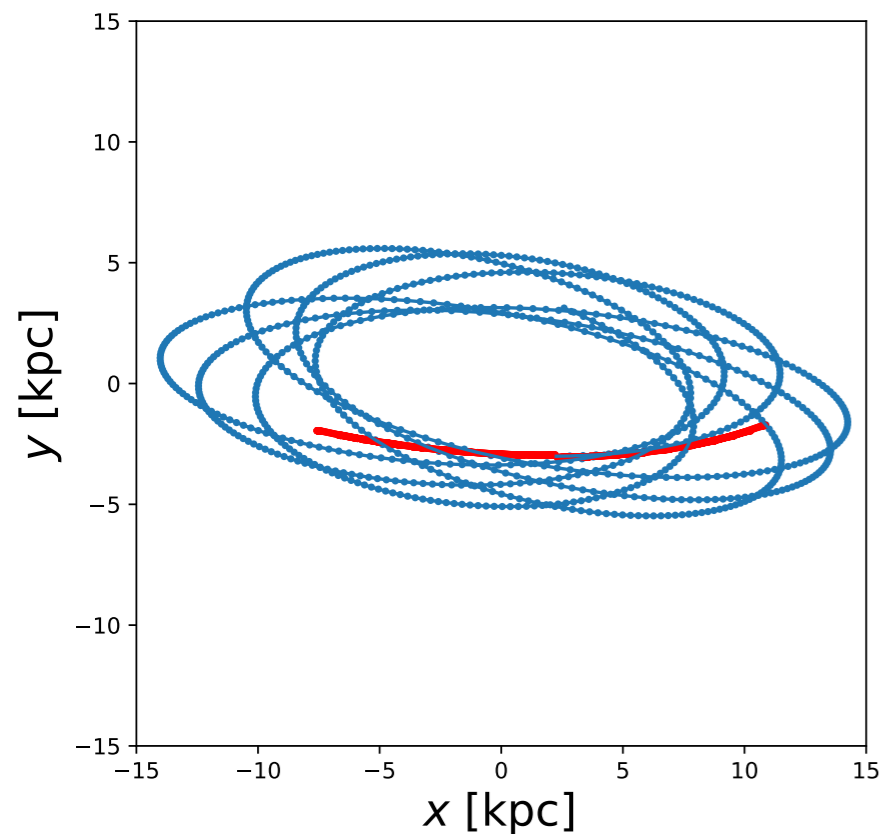
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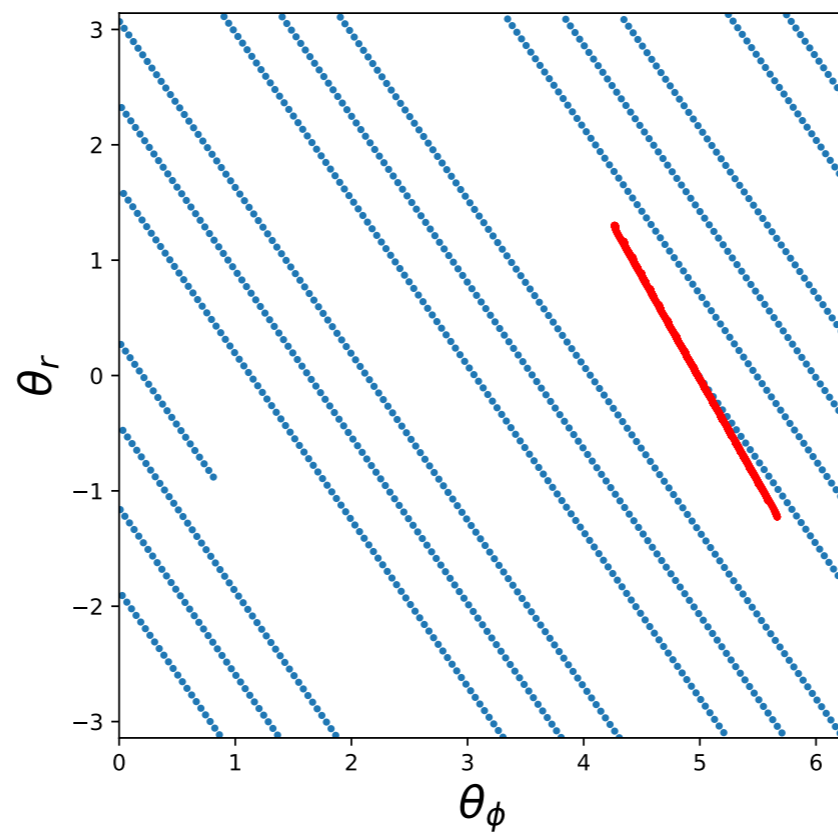
Blue: orbit

Red: final stream



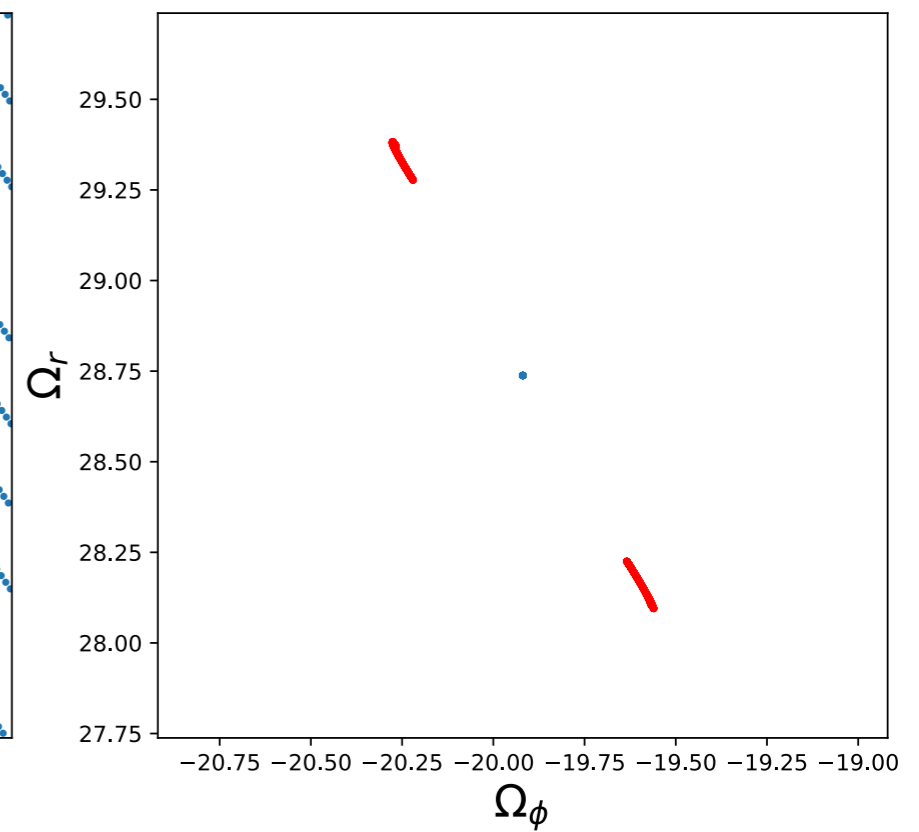
positions

Red: final stream



angles

Red: final stream



frequencies

Palomar 5 -like model

2 Gyr integration in isochrone model with  $v_c(R = 8 \text{ kpc}) = 220 \text{ km s}^{-1}$

# ActionFinder for **streams**

Start by treating stream as if it were an orbit. The objective (i.e. loss function) is to minimize:

$$\mathcal{L} = \left\langle \left\langle |\mathbf{J} - \langle \mathbf{J}_{\text{stream}} \rangle| \right\rangle_{\text{stream}} \right\rangle_{\text{set of streams}}$$

Then refine solution of neural net representation of  $S(\theta, J')$  using insight from Sanders & Binney (2013). As  $\Delta \vec{\theta} = t \vec{\Omega}$ , revised objective:

$$\mathcal{L} = 1 - \left\langle \left\langle \hat{\mathbf{e}}_{\theta} \right\rangle_{\text{stream}} \cdot \left\langle \hat{\mathbf{e}}_{\Omega} \right\rangle_{\text{stream}} \right\rangle_{\text{set of streams}}$$

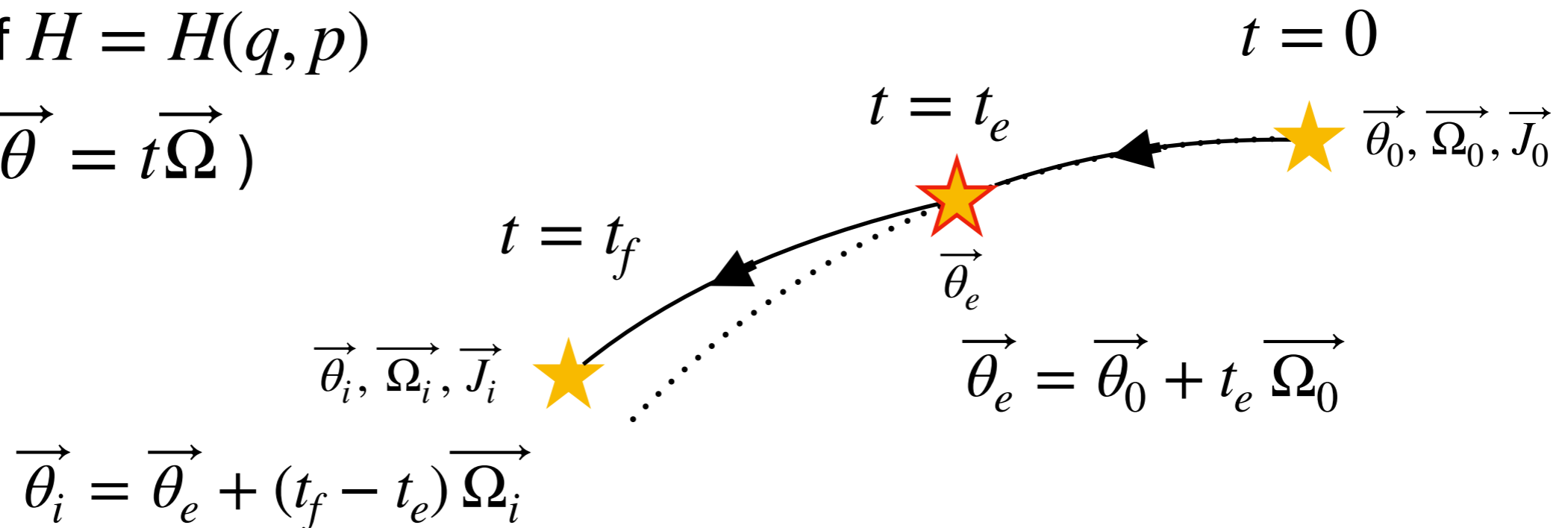
This gives <0.1% bias with simple potentials with 8,000 particles.

**We also only need full 6-D phase space information for small subsample of each stream** (with further loss function tweak)

# A Time Machine for Streams

Easy if  $H = H(q, p)$

(as  $\Delta \vec{\theta} = t \vec{\Omega}$ )



$$\vec{\theta}_i = \vec{\theta}_0 + t_e (\vec{\Omega}_0 - \vec{\Omega}_i) + t_f \vec{\Omega}_i$$

global parameter of stream  
parameter of star

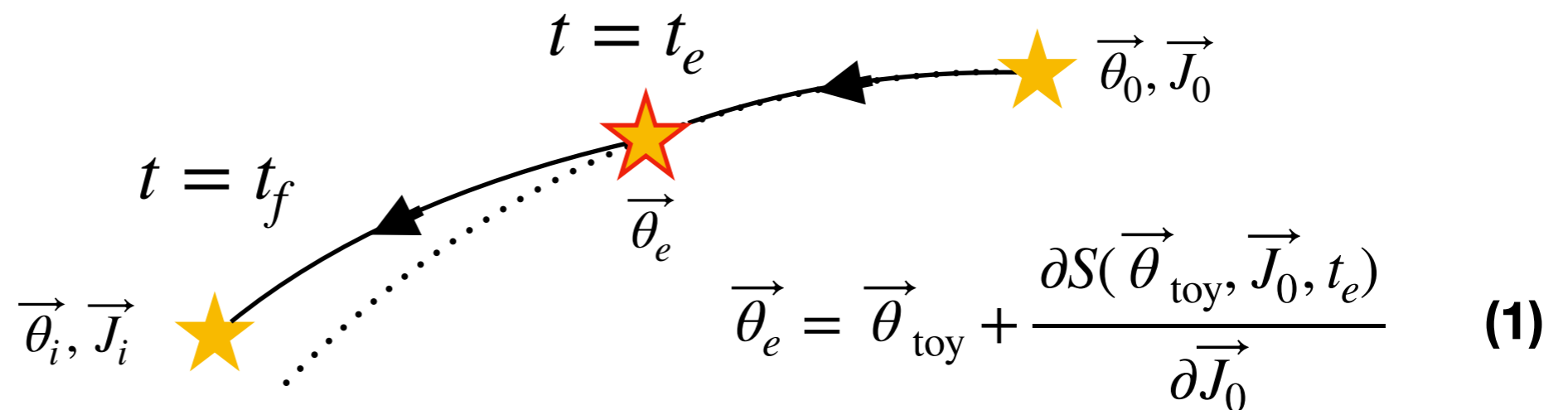
$$\vec{\theta}(\text{remnant}) = \vec{\theta}_0 + t_f \vec{\Omega}_0$$

This gives progenitor orbit and past path  $(\vec{x}, \vec{v}, t)$  of stars, and hence also when (and perhaps “why?”) stars were lost



# A Time Machine for Streams

If  $H = H(q, p, t)$ , i.e.,  $S = S(\vec{\theta}_{\text{toy}}, \vec{J}, t)$   $\vec{\theta}_0 = \vec{\theta}_{\text{toy}} + \frac{\partial S(\vec{\theta}_{\text{toy}}, \vec{J}_0, 0)}{\partial \vec{J}_0}$   
 (note  $\Delta \vec{\theta} \neq t \vec{\Omega}$ )  $t = 0$



$\vec{\theta}_i = \vec{\theta}_{\text{toy}} + \frac{\partial S(\vec{\theta}_{\text{toy}}, \vec{J}_i, t_f)}{\partial \vec{J}_i}$   $\vec{\theta}_e = \vec{\theta}_{\text{toy}} + \frac{\partial S(\vec{\theta}_{\text{toy}}, \vec{J}_i, t_e)}{\partial \vec{J}_i}$  **(2)**

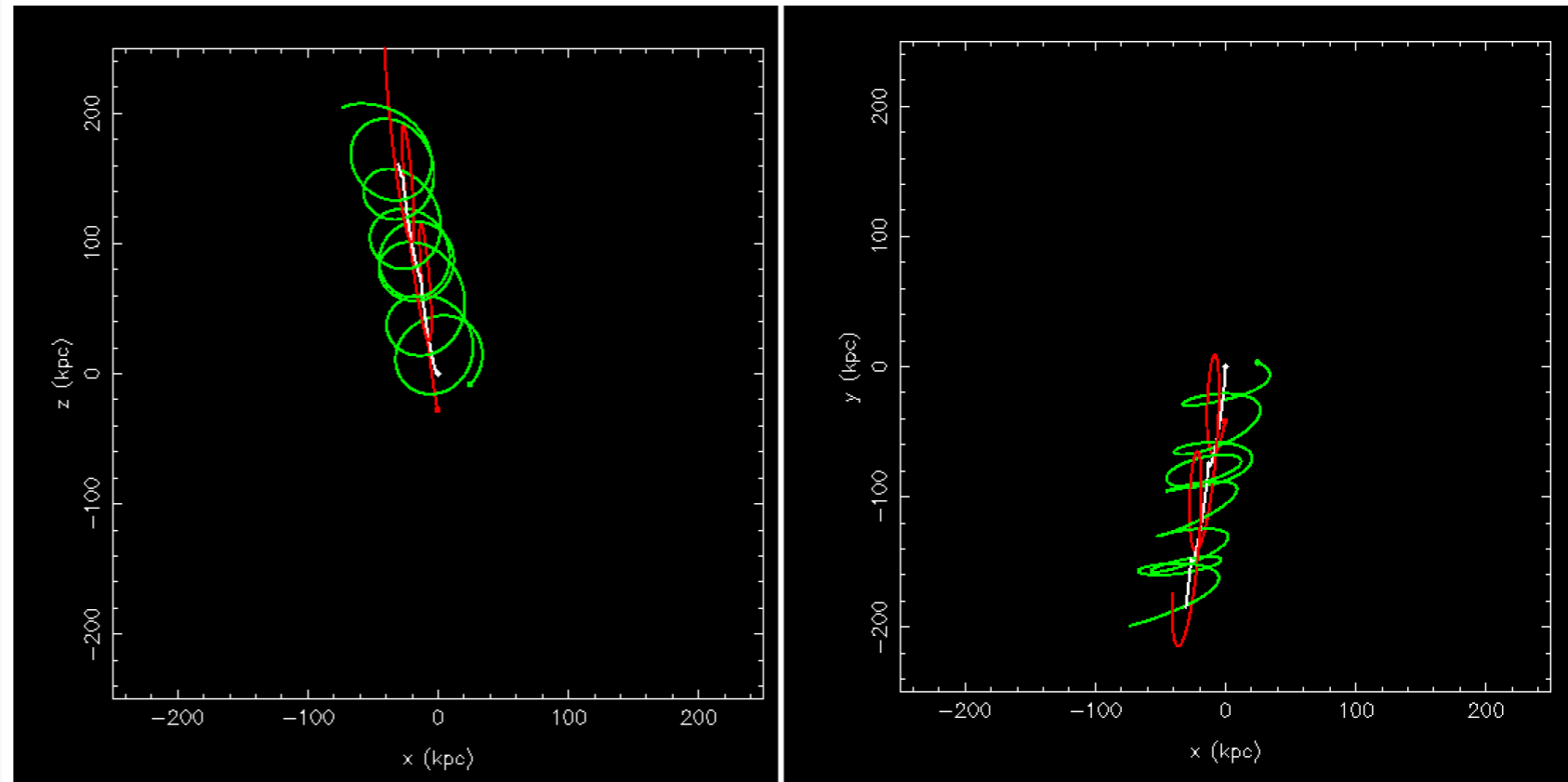
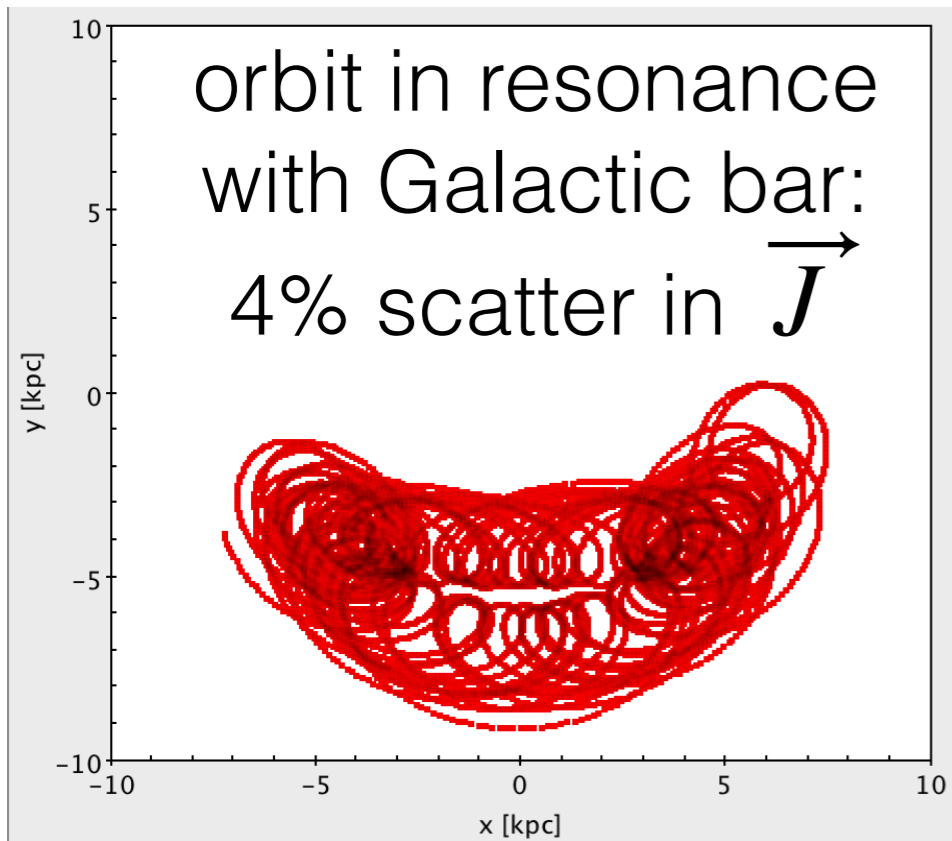
We haven't coded this up yet, but it *should* be straightforward.

Search for  $t_e$  values that obey **(1)** & **(2)**

This could reveal how the acceleration field changes in time.

# Time-dependence in orbits

We have already upgraded the ActionFinder to find time-dependent generating functions  $S(\vec{\theta}_{\text{toy}}, \vec{J}, t)$  when  $t$  is provided explicitly

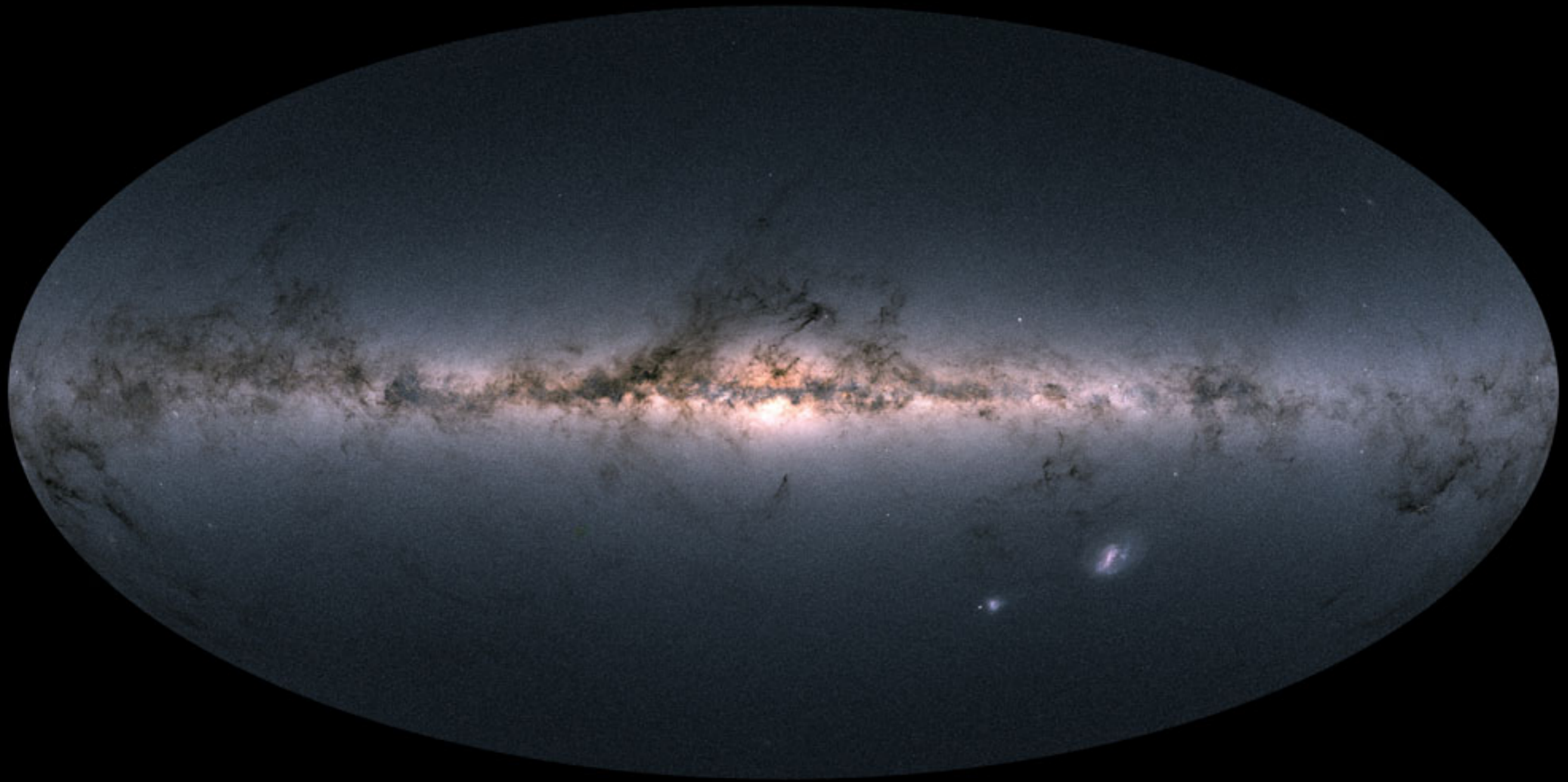


Toy MW+LMC+Sgr system, 1% scatter in  $\vec{J}$

# Outline

- Dynamics with Machine Learning (ActionFinder)
- **The STREAMFINDER search for stellar streams**
- Perturbations of the GD-1 stream

How do we best find streams with Gaia?

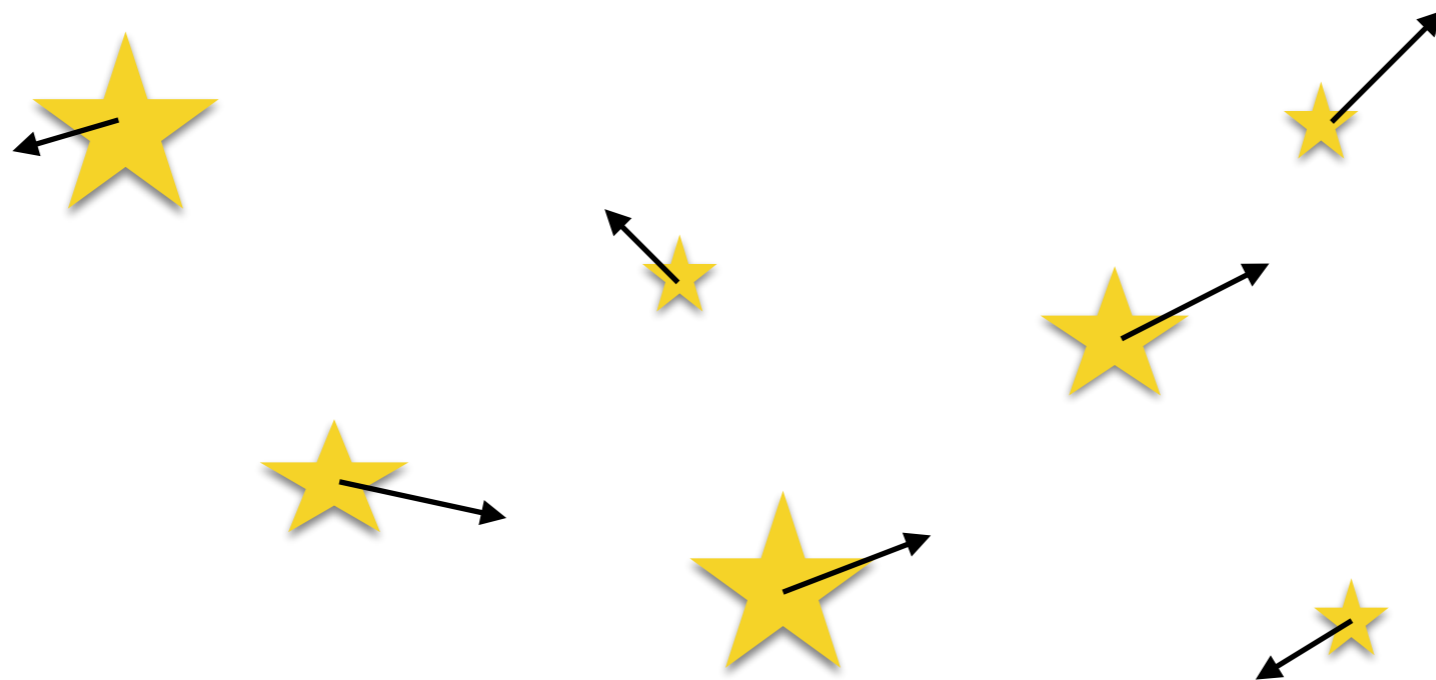


# Finding streams in heterogeneous datasets

Gaia:  $\alpha, \delta, \varpi, \mu_\alpha, \mu_\delta, G, G_{BP}, G_{RP}$  for  $> 10^9$  stars

heliocentric velocity for  $7.2 \times 10^6$  stars

Distance, chemistry & line of sight velocity information much poorer than proper motions and photometry



Malhan, RI (MNRAS 2018)

Malhan, RI, Martin (MNRAS 2018);

Malhan et al. (MNRAS 2019)

RI, Malhan, Martin (ApJ 2019)

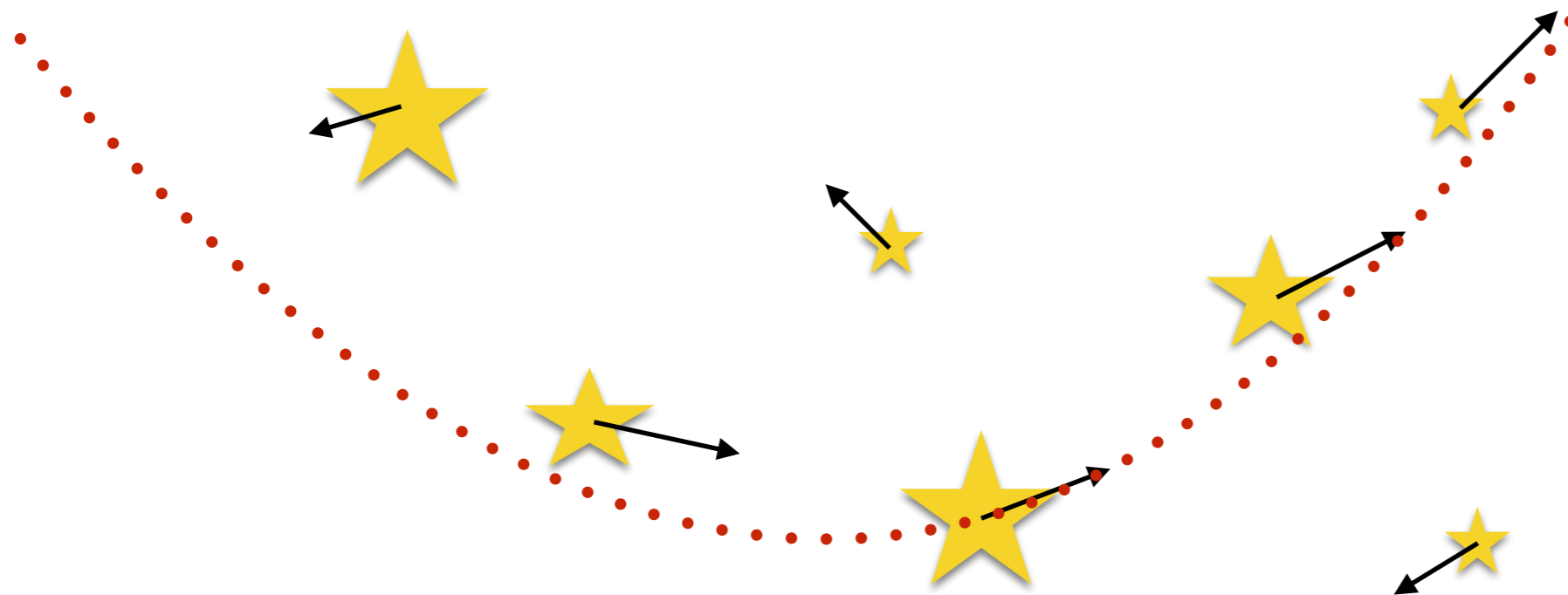
RI, Bellazzini, Malhan, Martin, Bianchini (Nature Ast. 2019)

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model in:

$\alpha, \delta, \varpi,$

$\mu_\alpha, \mu_\delta,$

$G, G_{BP} - G_{RP}$

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Malhan et al. (MNRAS 2019)

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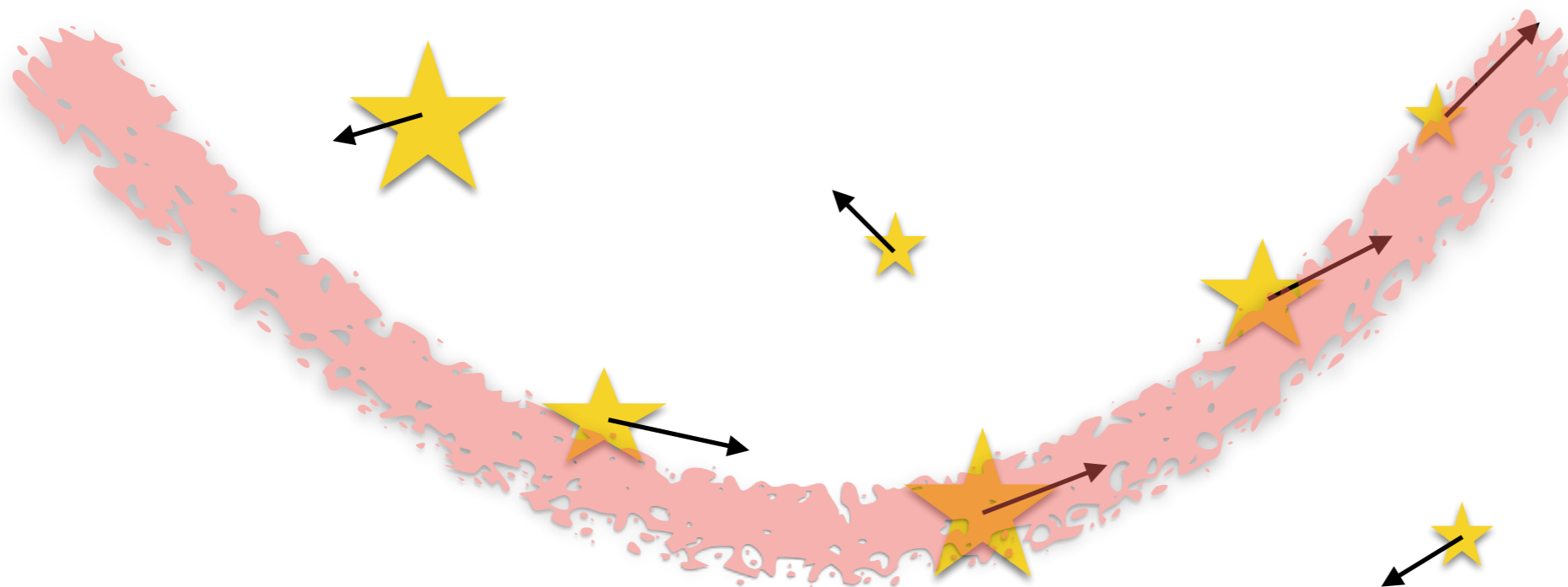
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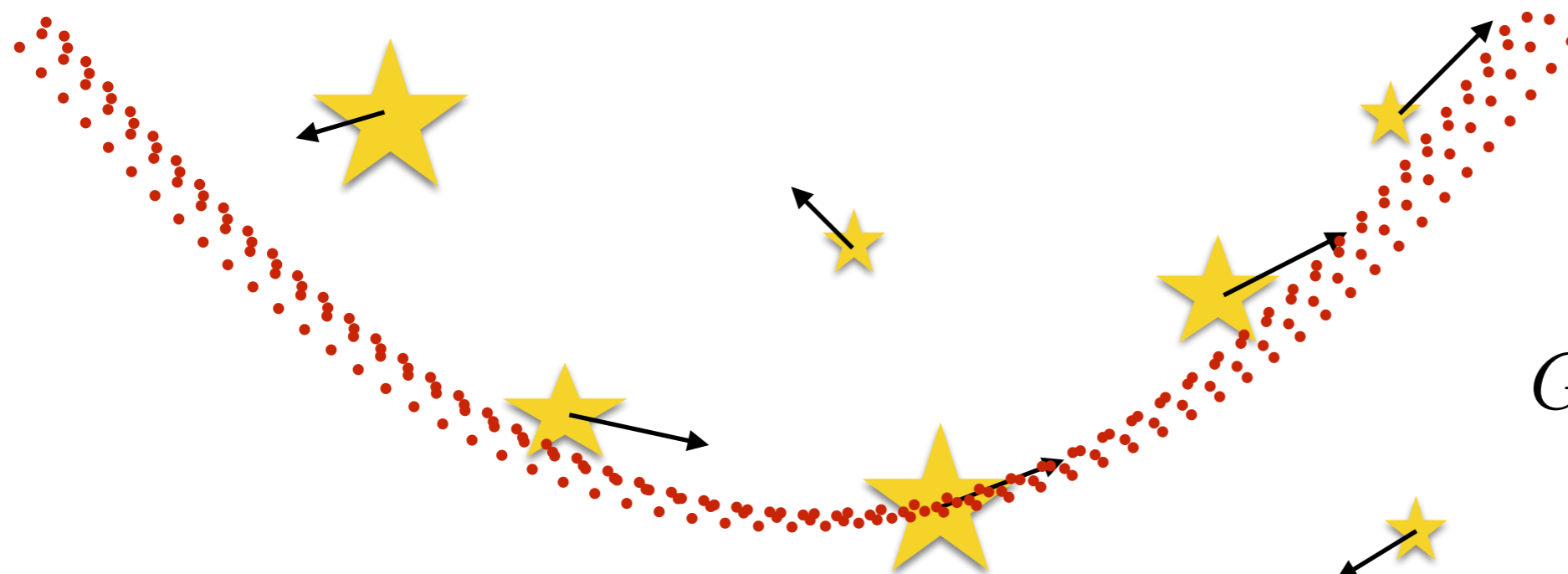
RI, Bellazzini, Malhan, Martin, Bianchini (Nature Ast. 2019)

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Distance, chemistry & line of sight velocity information much poorer than proper motions and photometry



model in:

$\alpha, \delta, \varpi,$

$\mu_\alpha, \mu_\delta,$

$G, G_{BP} - G_{RP}$

$$\ln \mathcal{L} = \sum_{\text{data}} \ln [\eta \mathcal{P}_{\text{stream}}(\theta) + (1 - \eta) \mathcal{P}_{\text{cont}}]$$

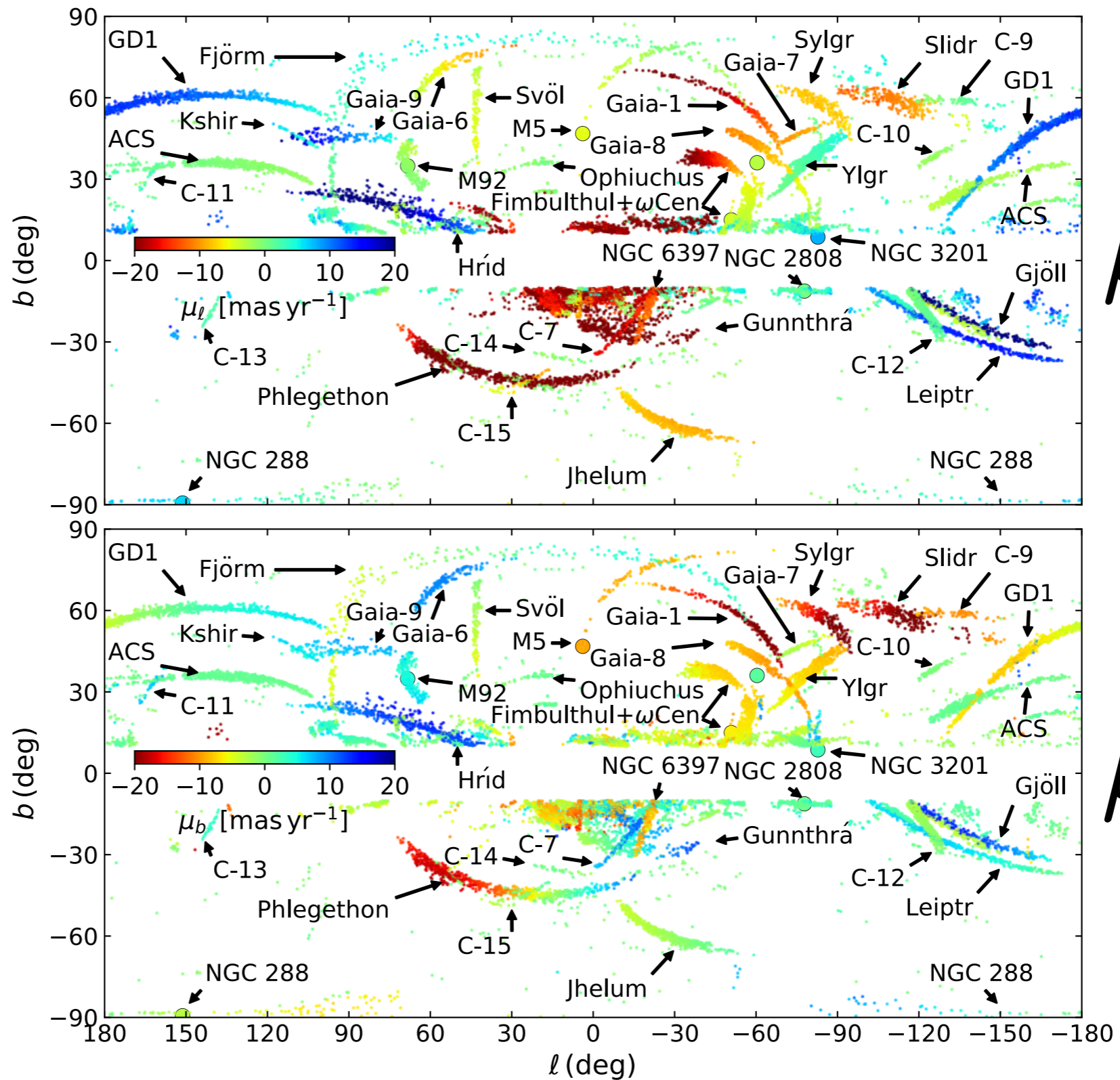
For each star: What is  $\ln \mathcal{L}$  of the most likely value of eta?

Hence maps of “streaminess”



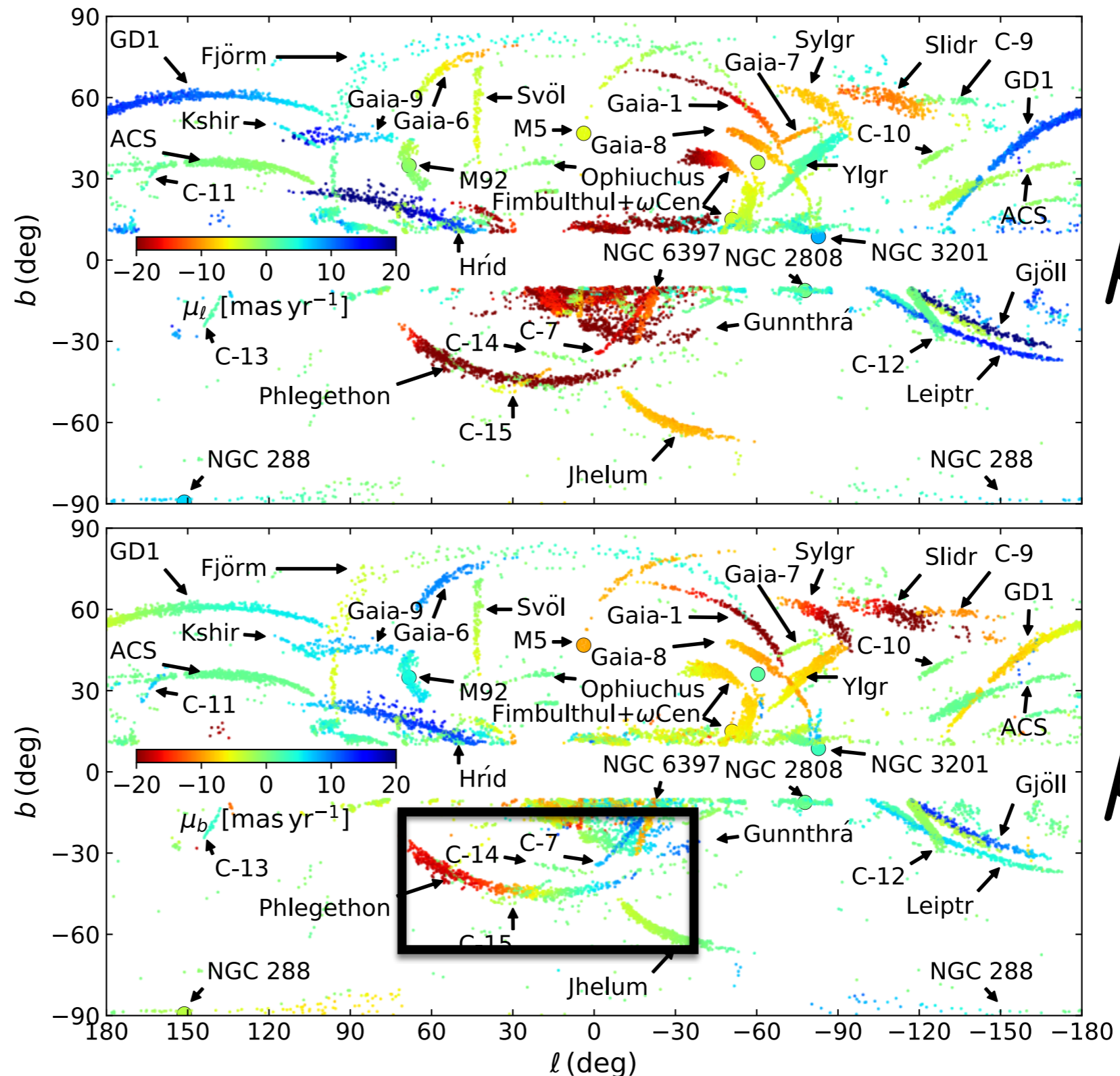
# New stream maps [3, 12] kpc

10 $\sigma$  detections,  
50 pc half-width



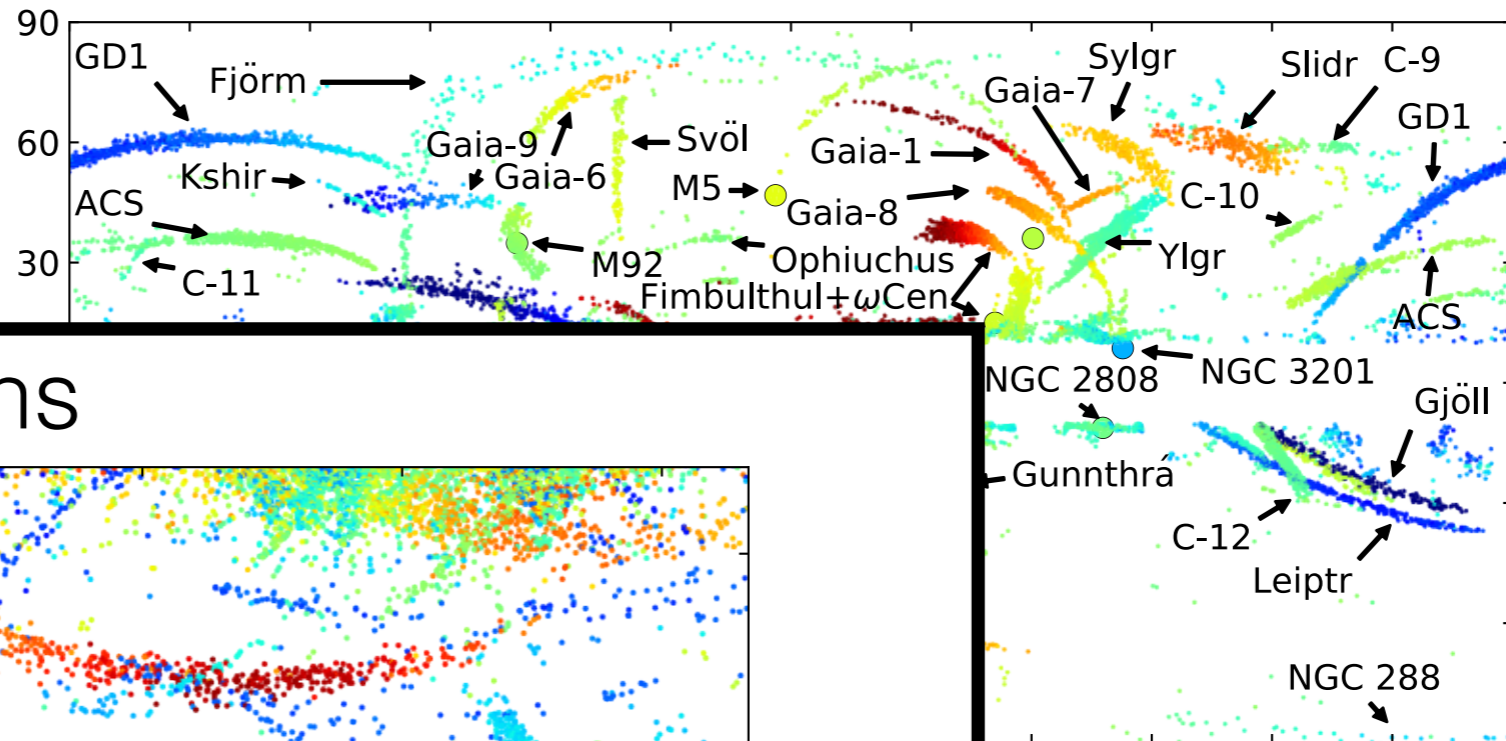
# New stream maps [3, 12] kpc

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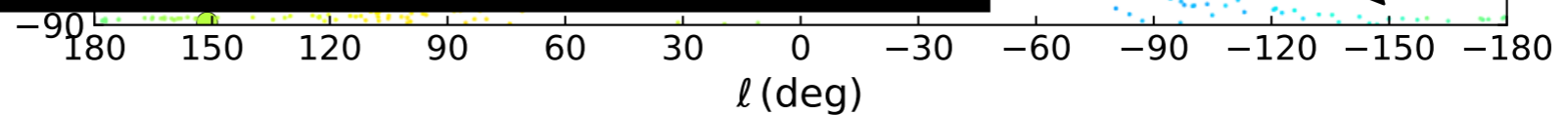
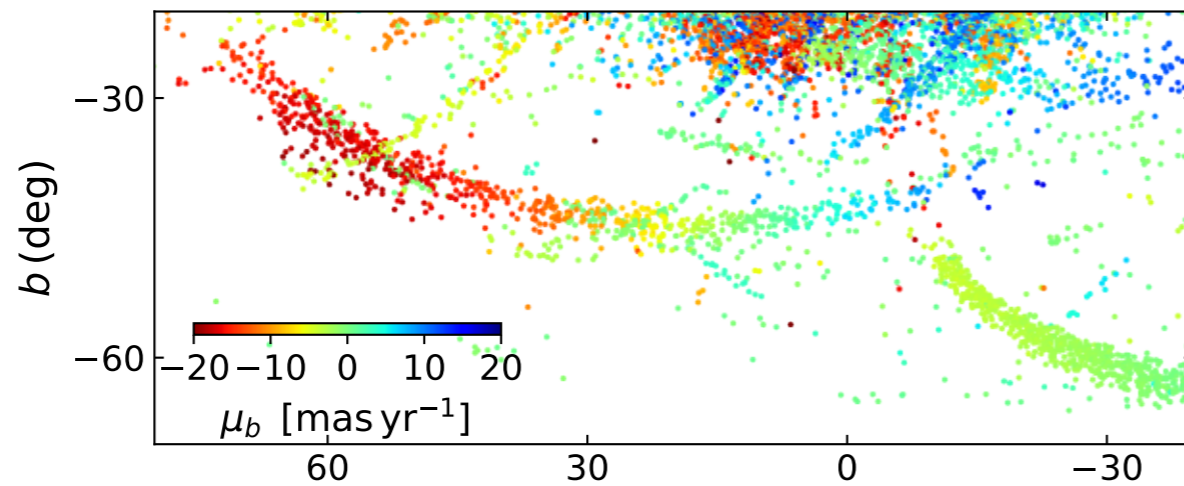
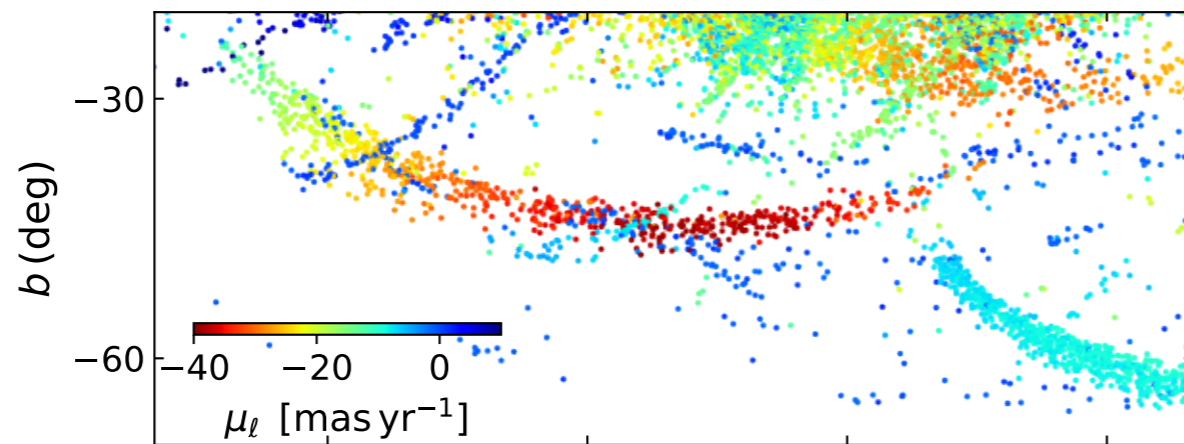


# New stream maps [3, 12] kpc

10 $\sigma$  detections,  
50 pc half-width



8 $\sigma$  detections

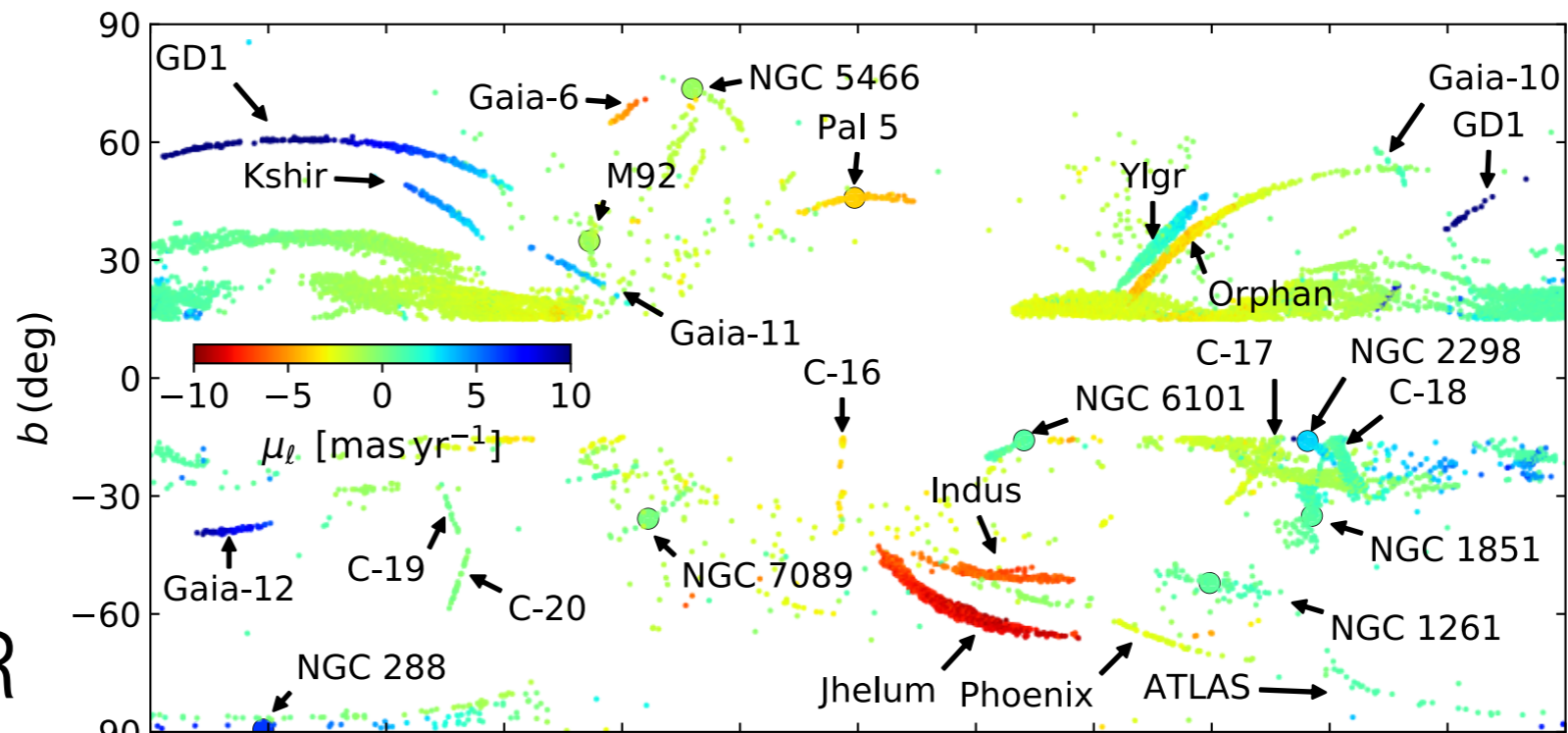


$\mu_l$

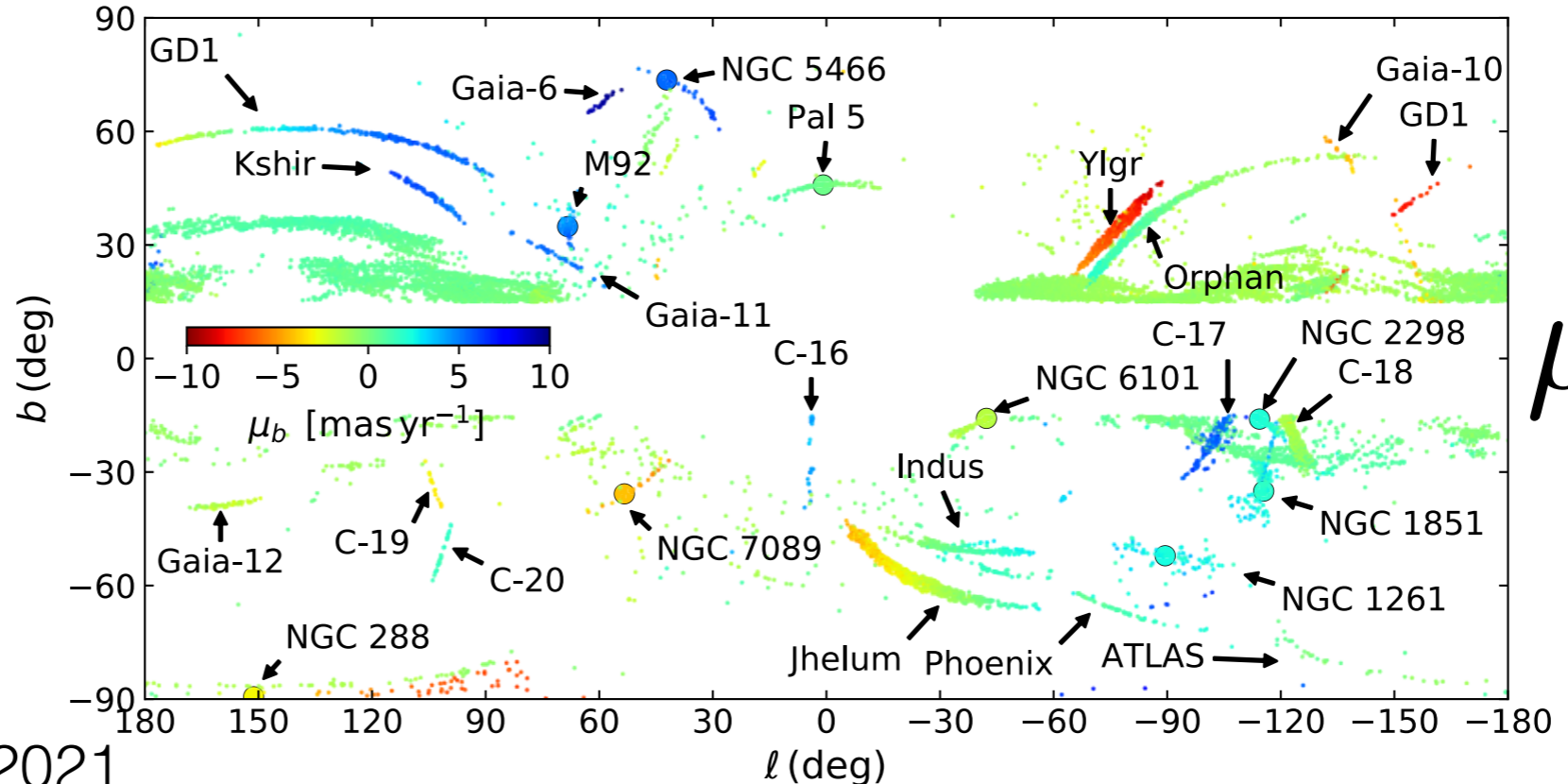
$\mu_b$

# New stream maps [10,30] kpc

10 $\sigma$  detections,  
50 pc half-width



$\mu_l$



$\mu_b$

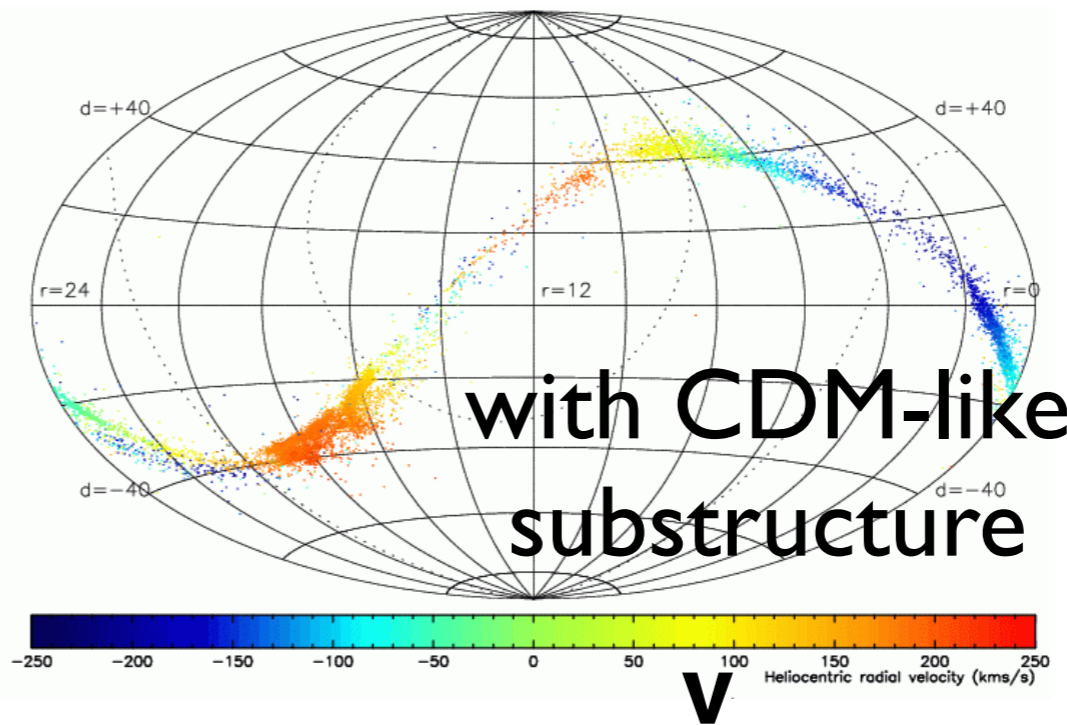
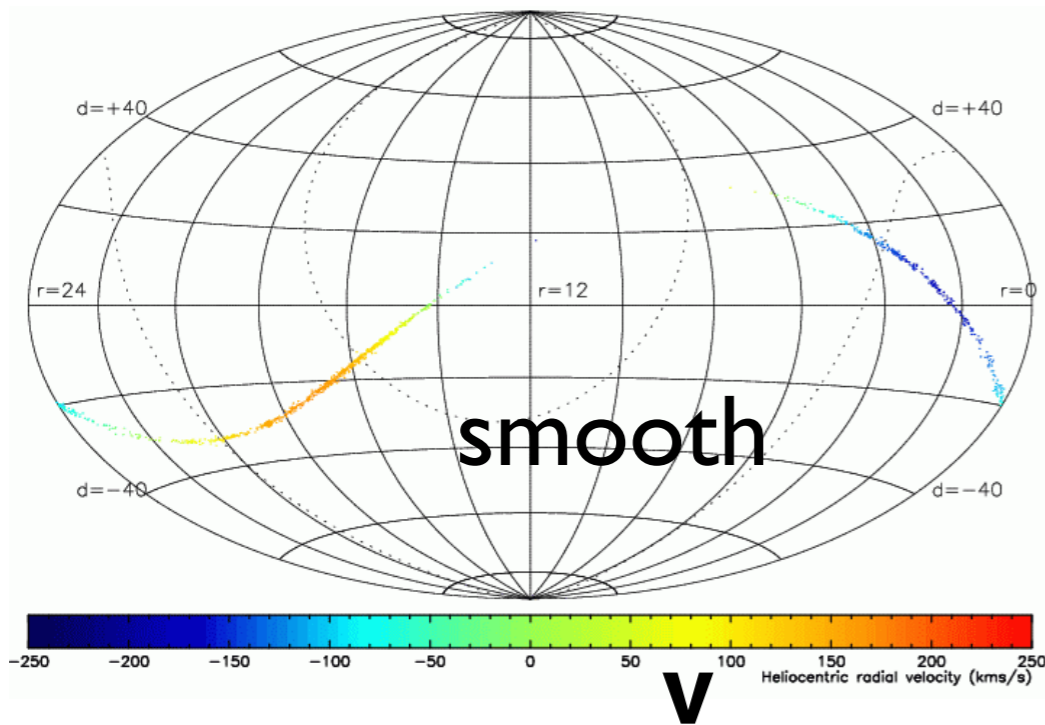
STREAMFINDER  
high-resolution  
spectroscopic  
survey

ESPaDOnS(CFHT)  
+ UVES(VLT)

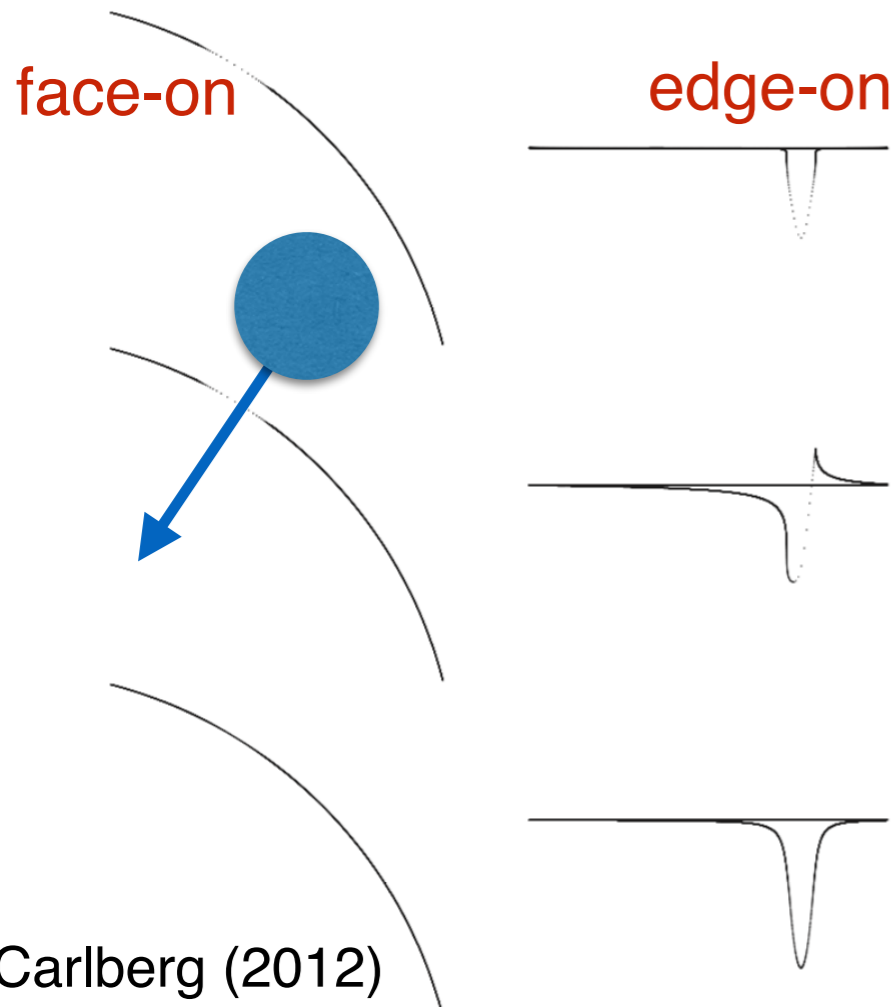
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# Stellar Streams as seismometers

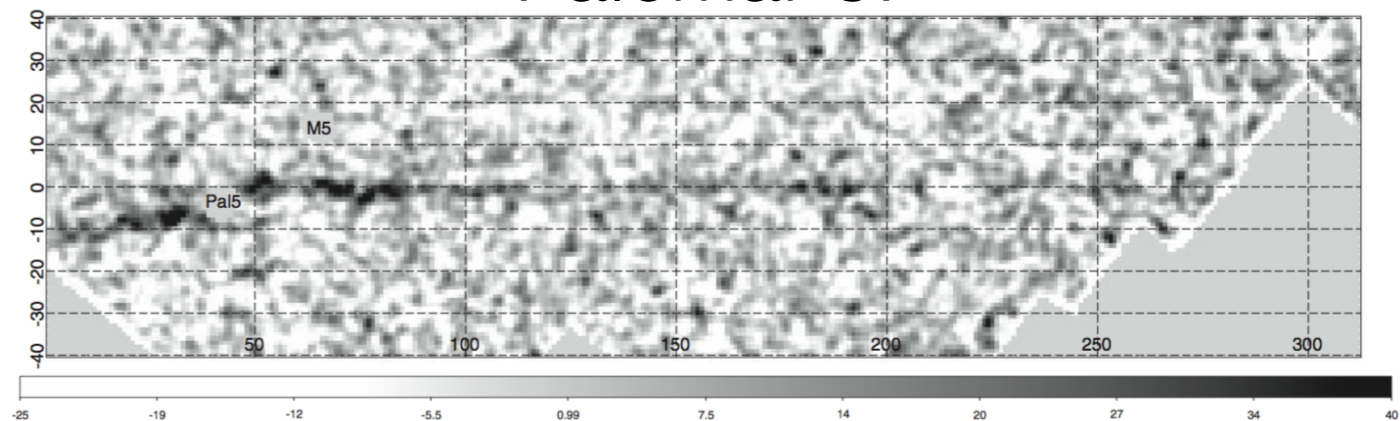


RI, Lewis, Irwin,  
Quinn (2002)  
Johnston et al.  
(2002)  
Dalal & Kochanek  
(2002)



## Gaps in streams from sub-halo encounters

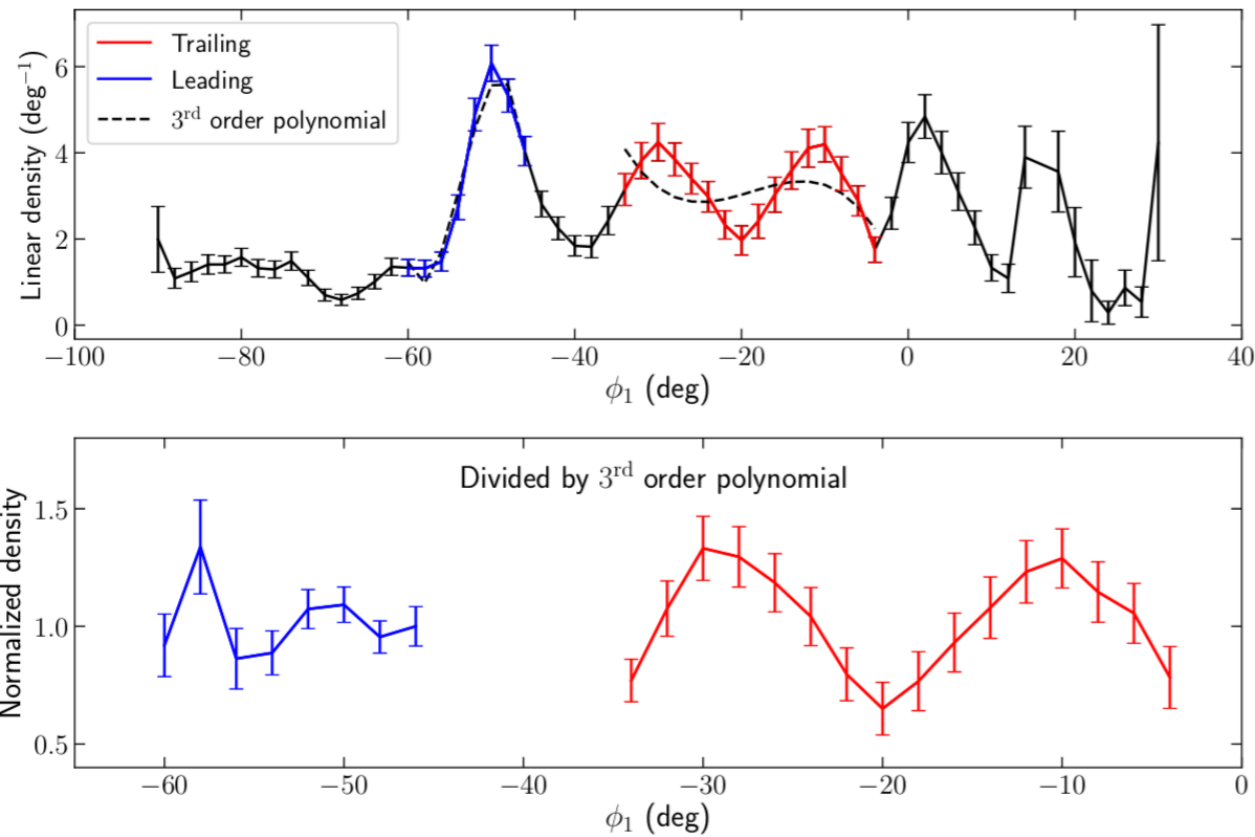
matched filter map around  
Palomar 5:



**Figure 2.** Matched filtered star map of the Pal 5 field, with Pal 5 and the foreground M5 cluster masked out. To remove the varying background, the masked image has been smoothed over  $4^\circ$  subtracted from the original image, and then smoothed with a 2 pixel, or,  $0.2$  Gaussian. The analysis is conducted on the original uncorrelated pixels. We have made no attempt to straighten the southern part of the stream, left of the cluster in this image.

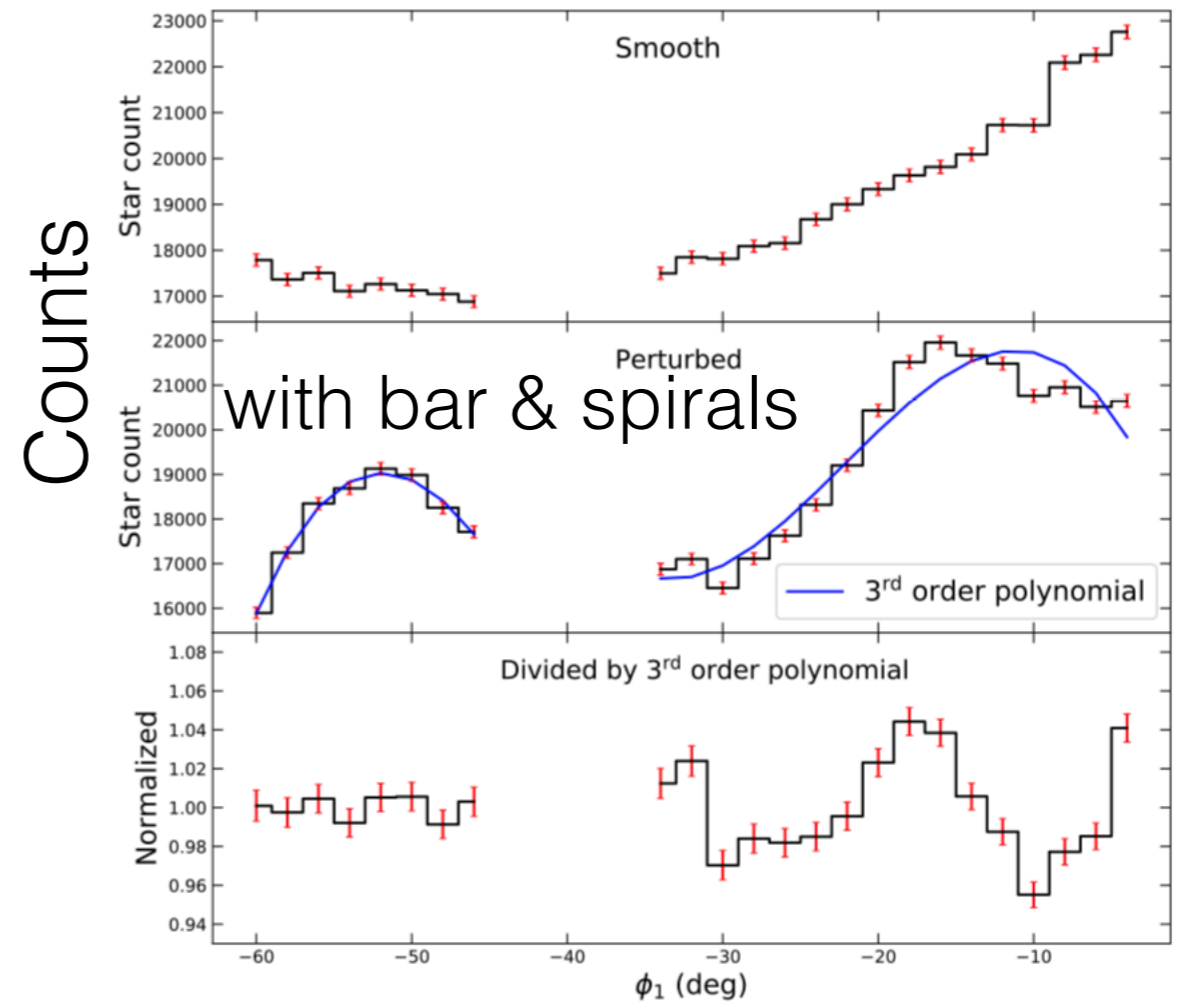
# Gaps in the GD1 stream

Data

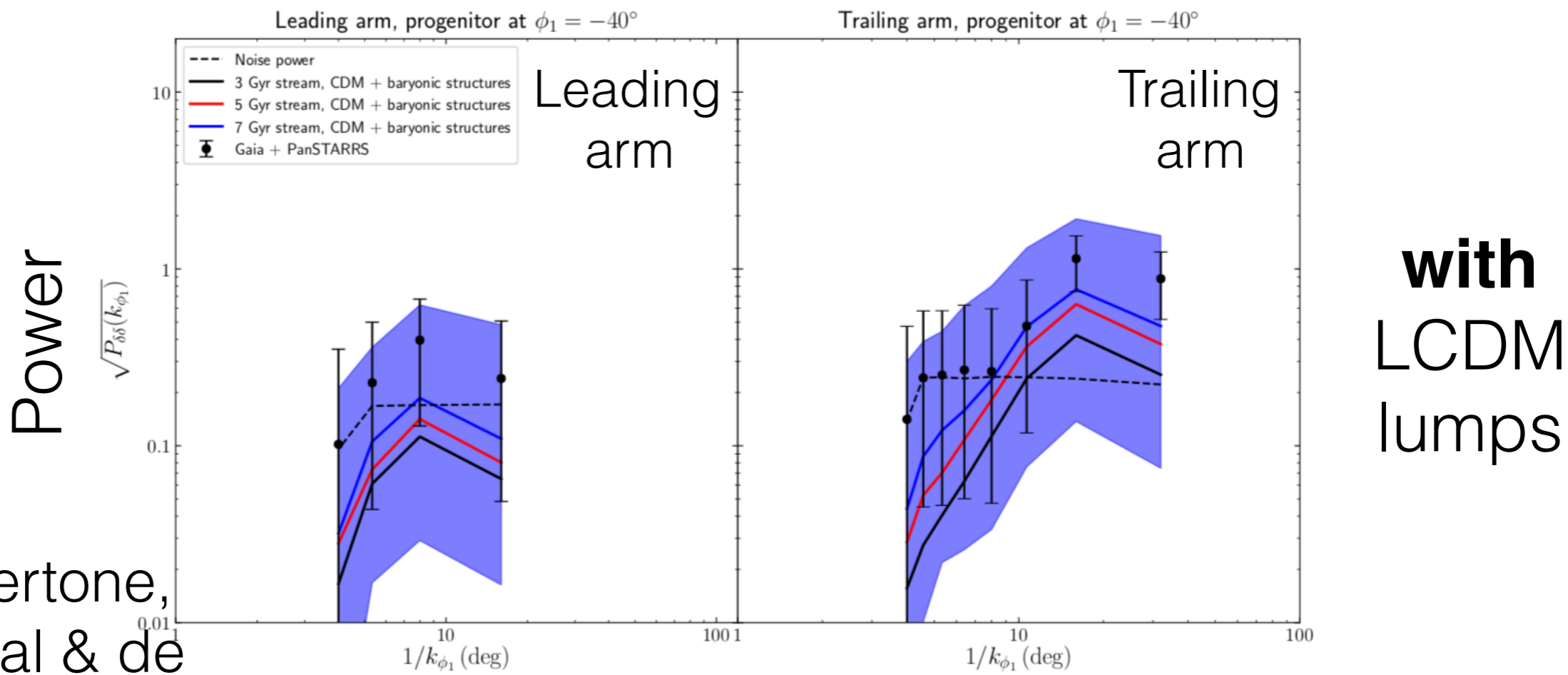
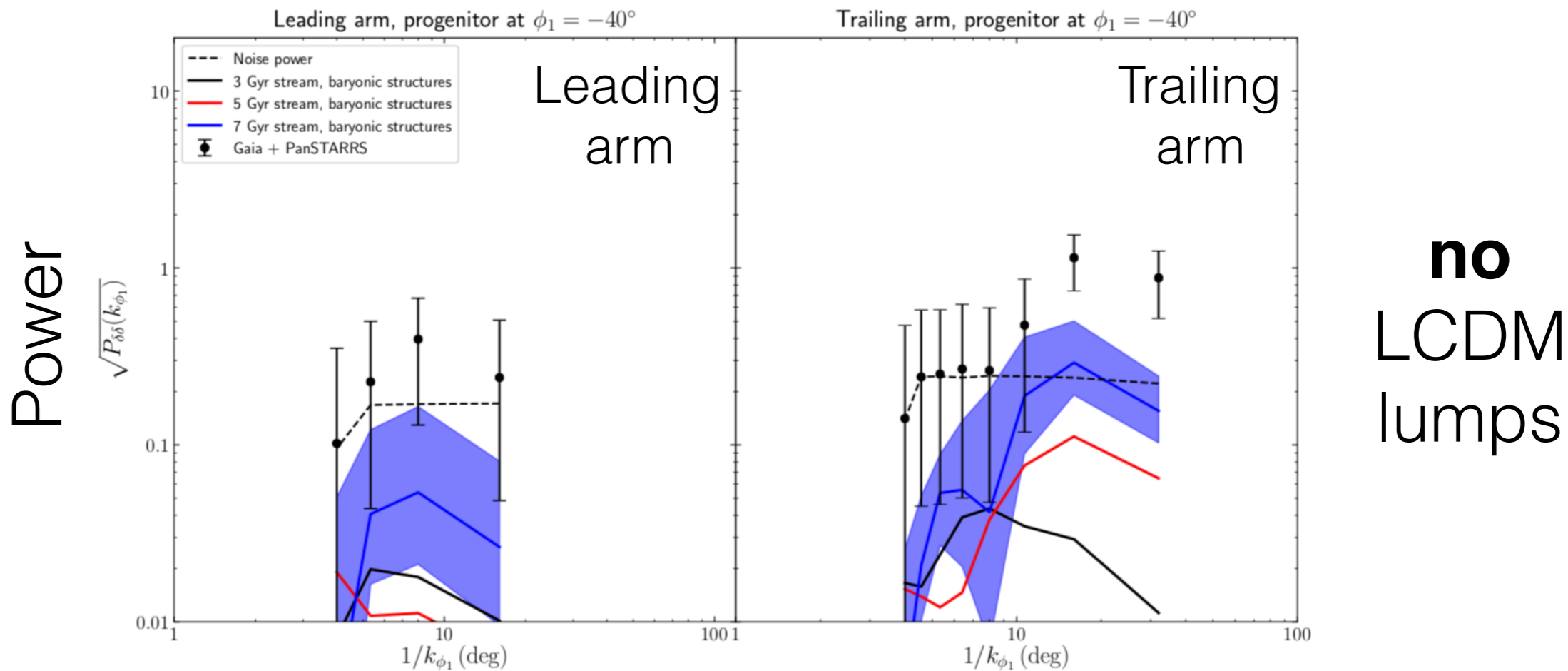


angle along stream

Simulations



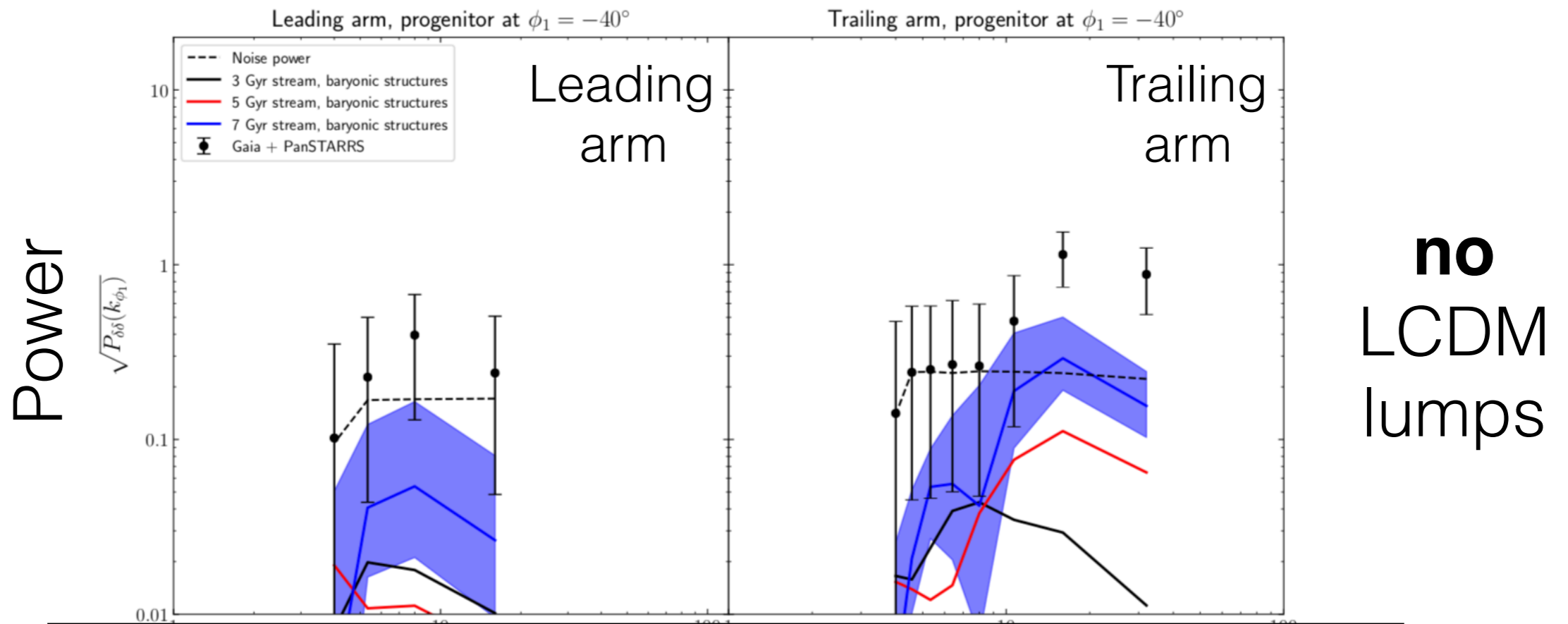
angle along stream



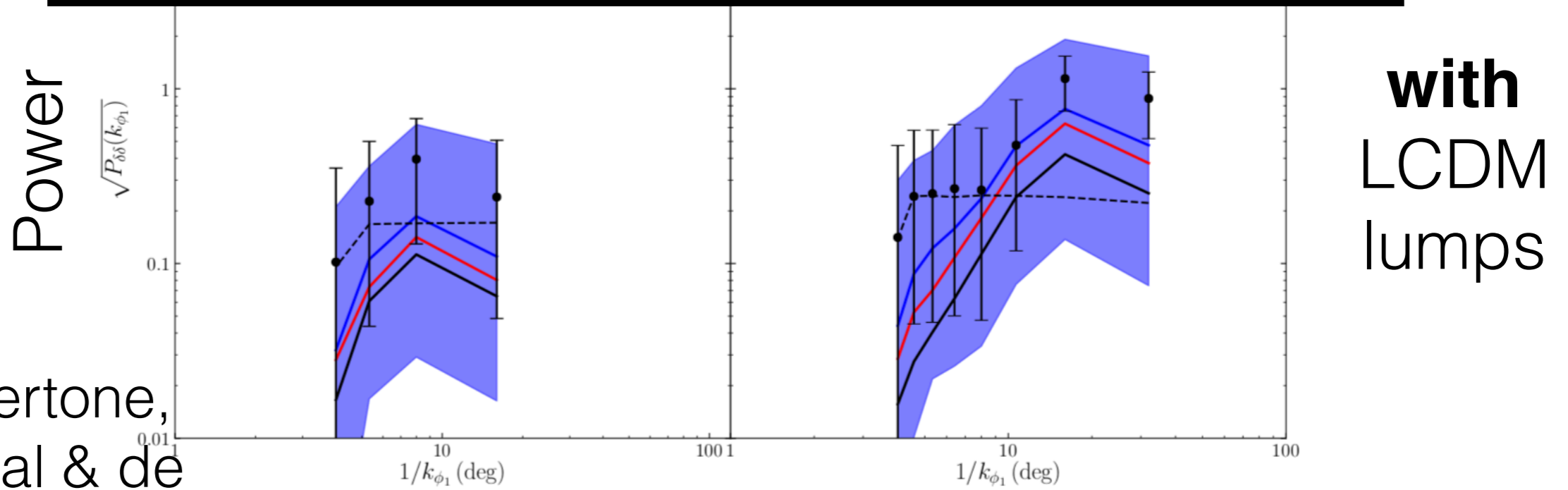
Banik, Bertone,  
Bovy, Erkal & de  
Boer (2021)

inverse angular wavenumber **[deg]**





Implies abundance of sub halos of  $0.4^{+0.3}_{-0.2}$  compared to CDM

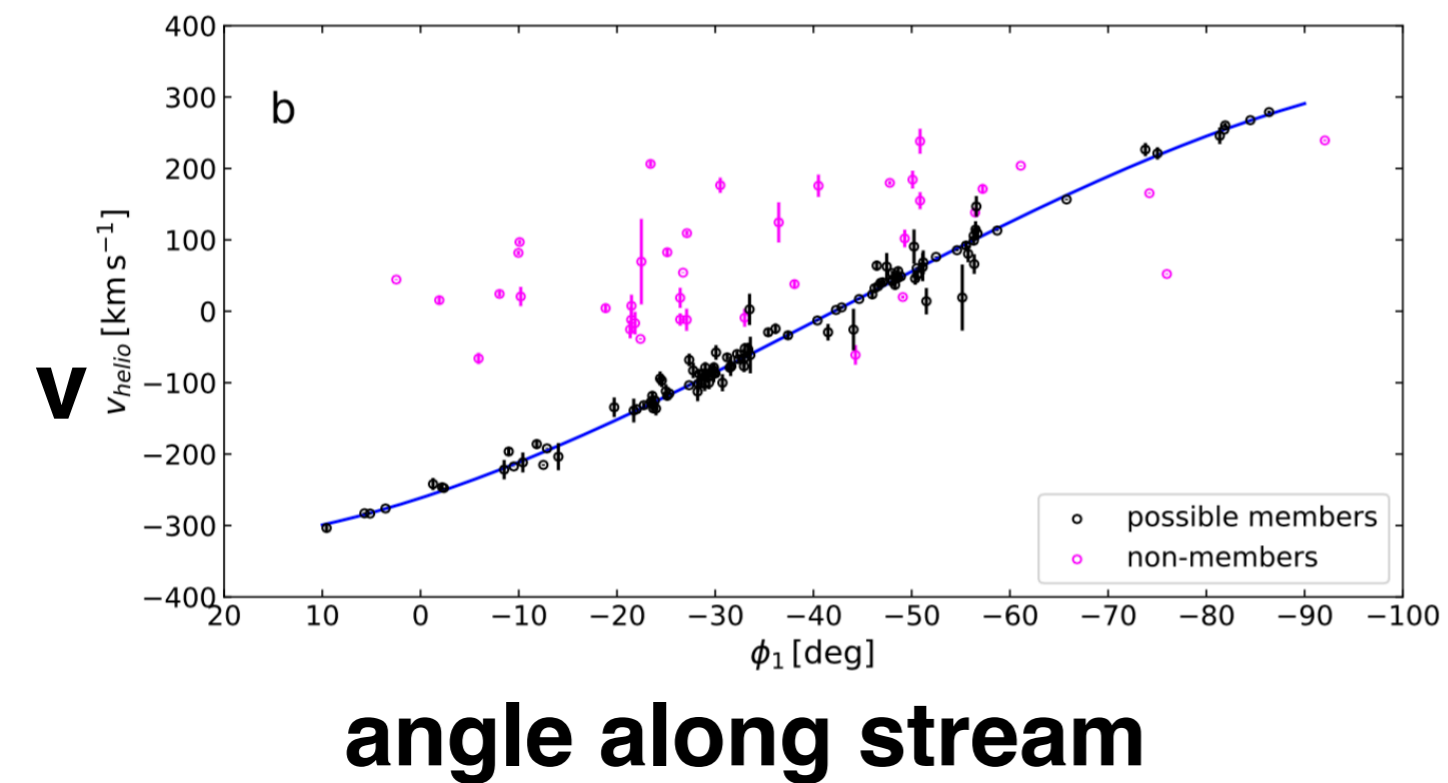
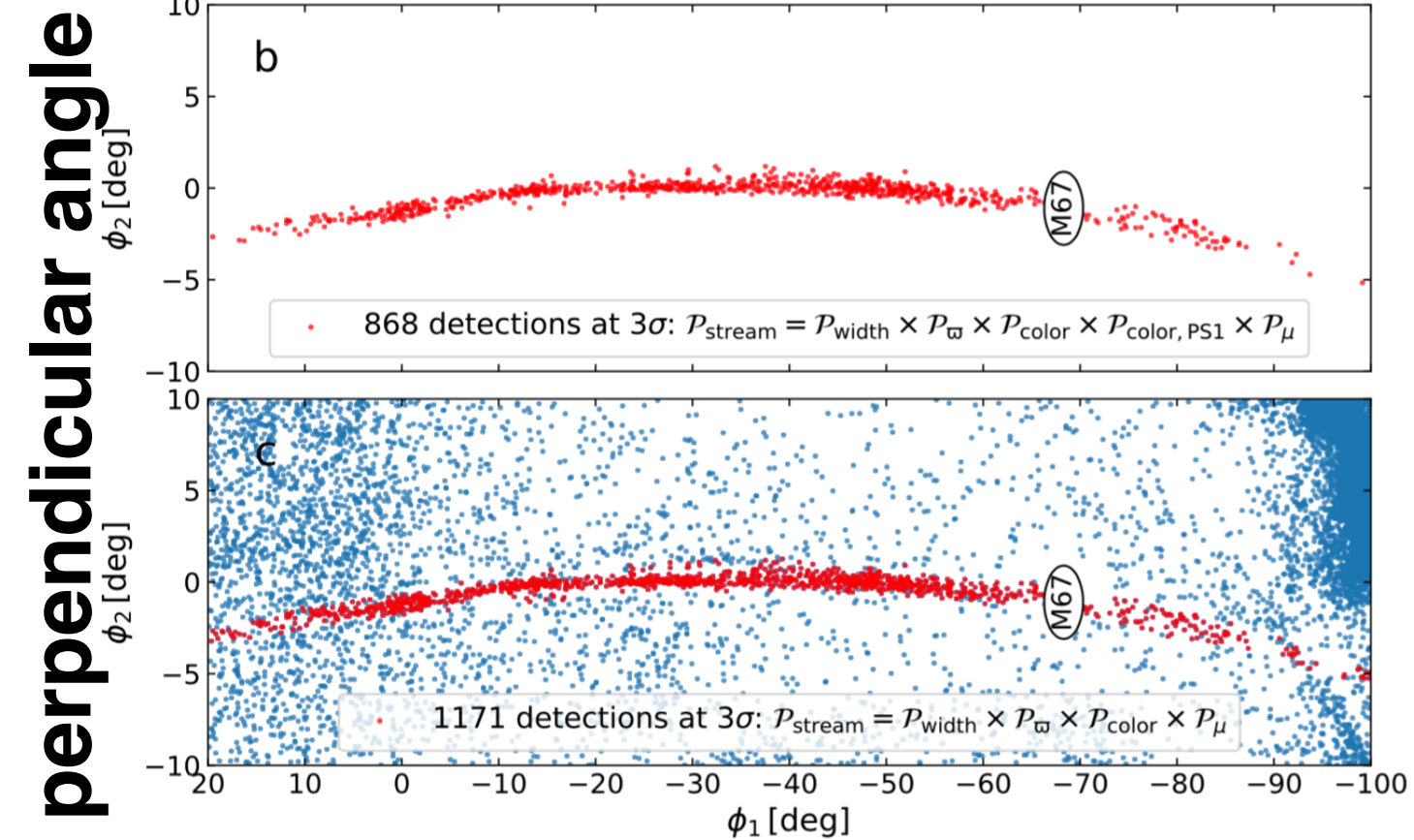


Banik, Bertone,  
Bovy, Erkal & de  
Boer (2021)

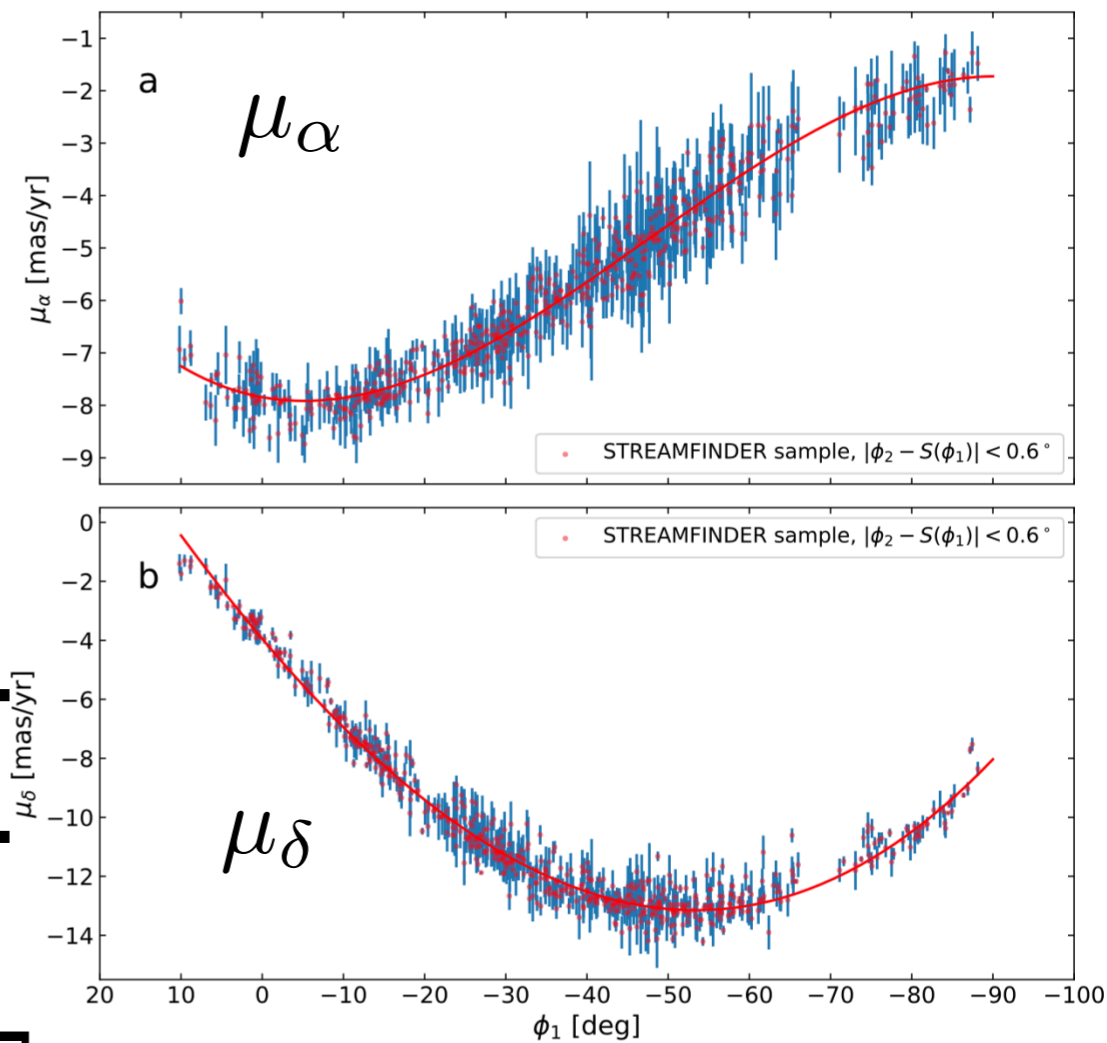
inverse angular wavenumber **[deg]**

# GD1 stream

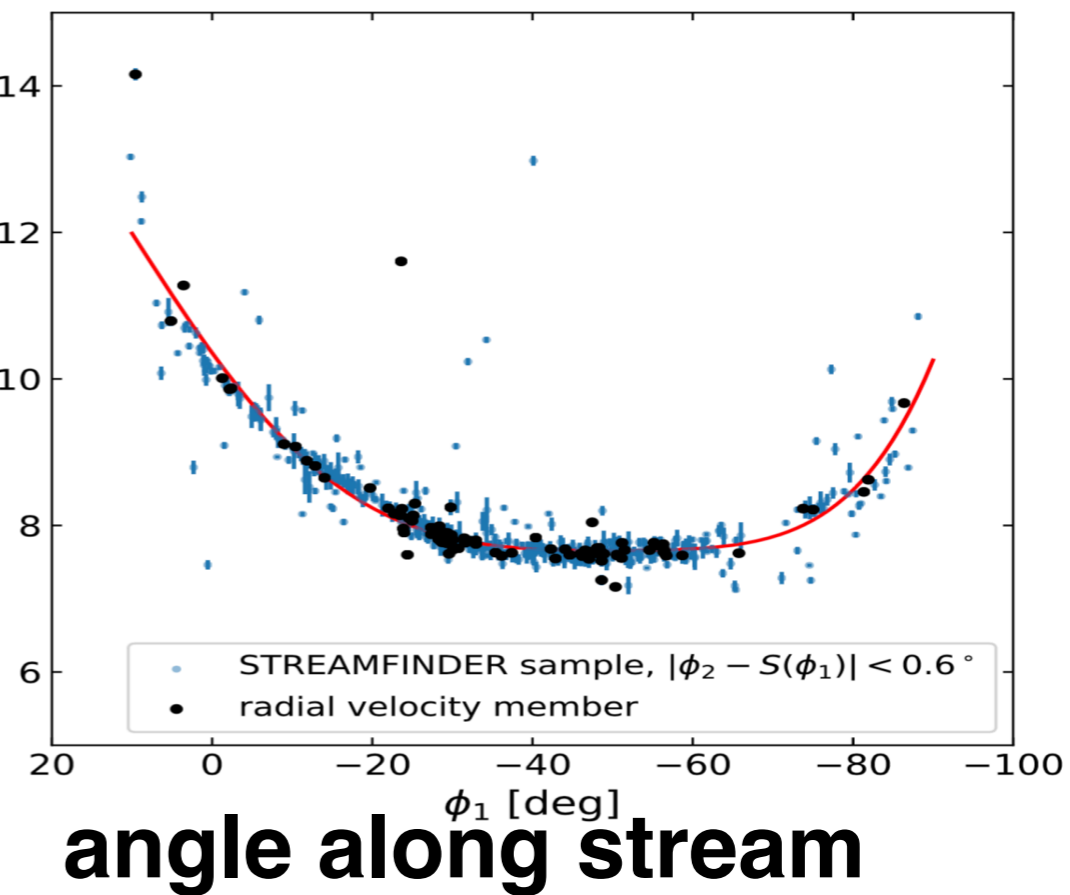
RI, Thomas, Famaey, Malhan, Monari (2020)



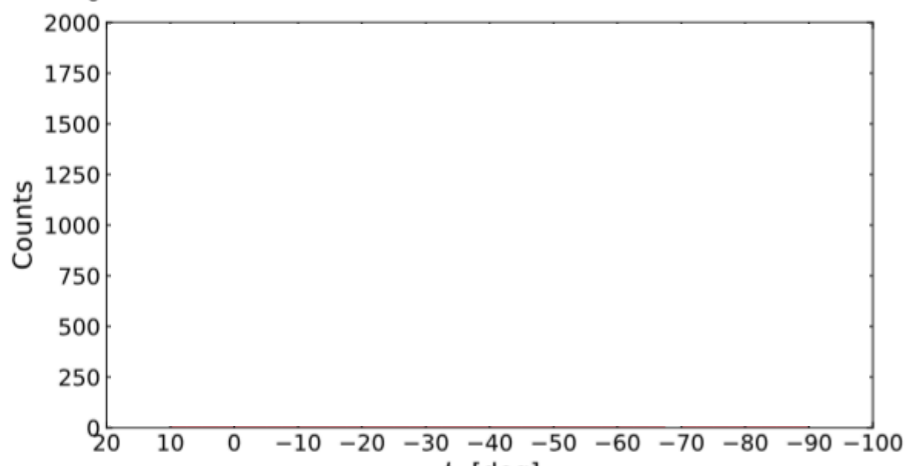
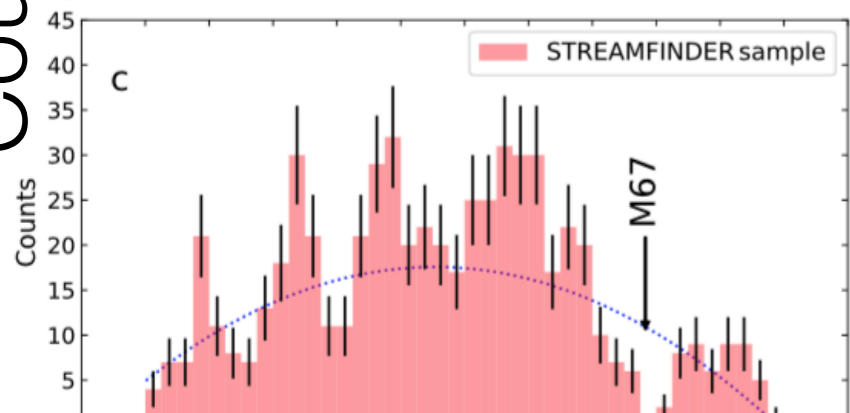
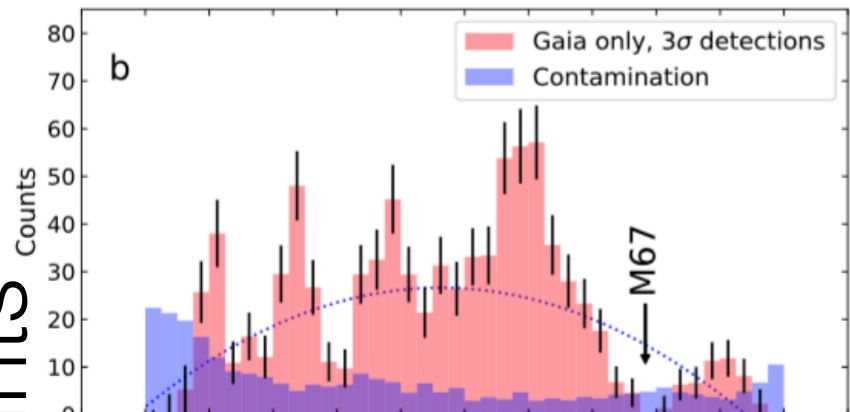
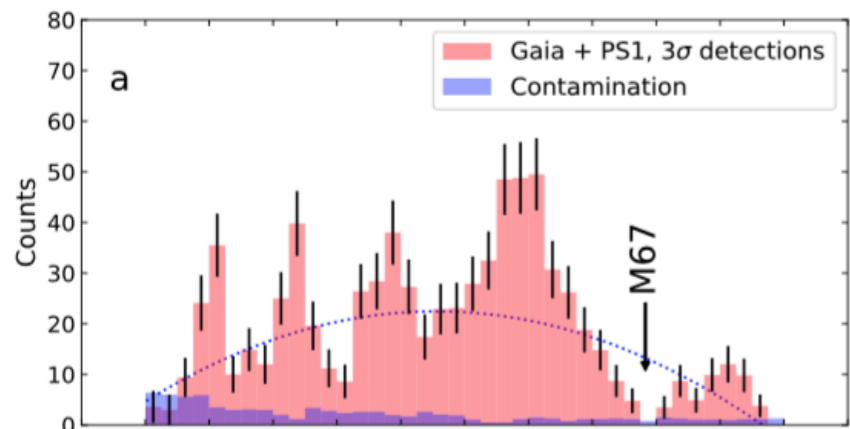
**proper motions**



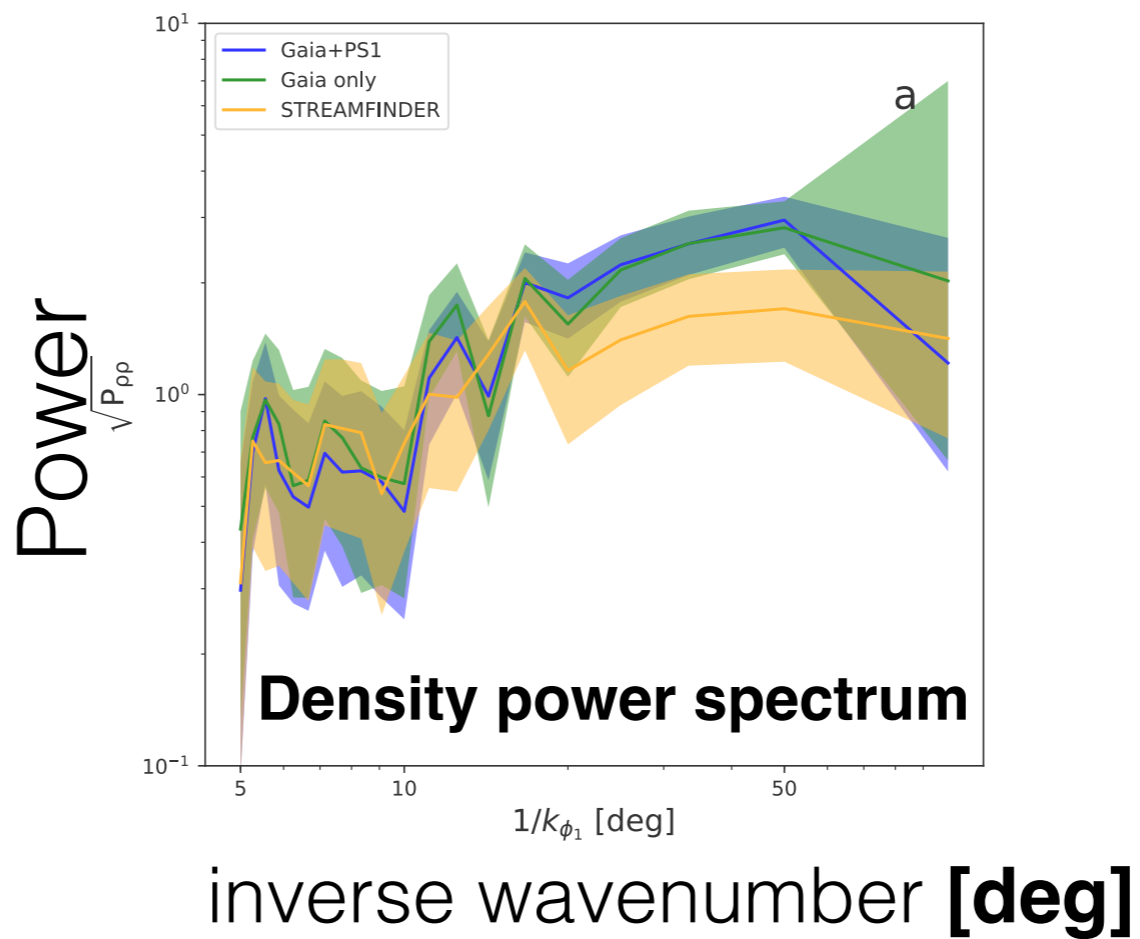
**photometric distance [kpc]**



RI, Thomas, Famaey,  
Malhan, Monari (2020)

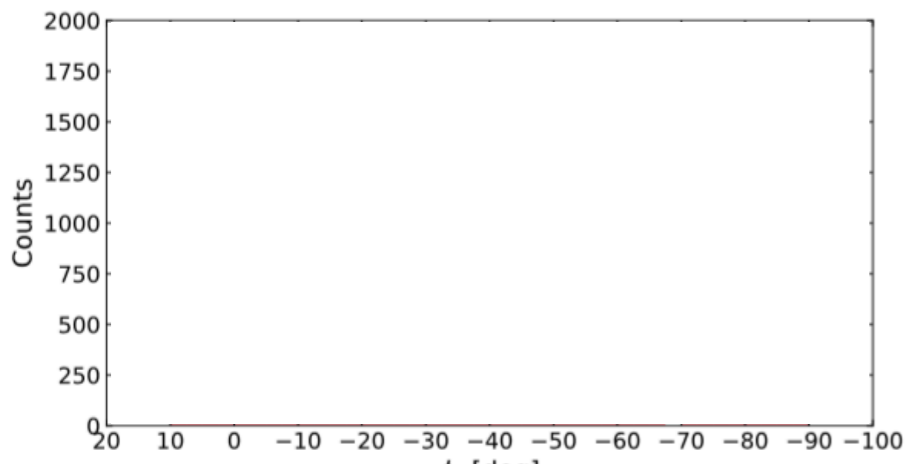
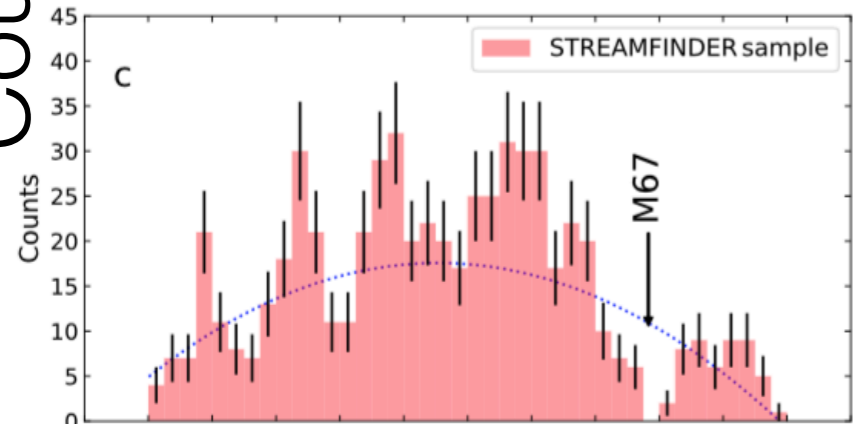
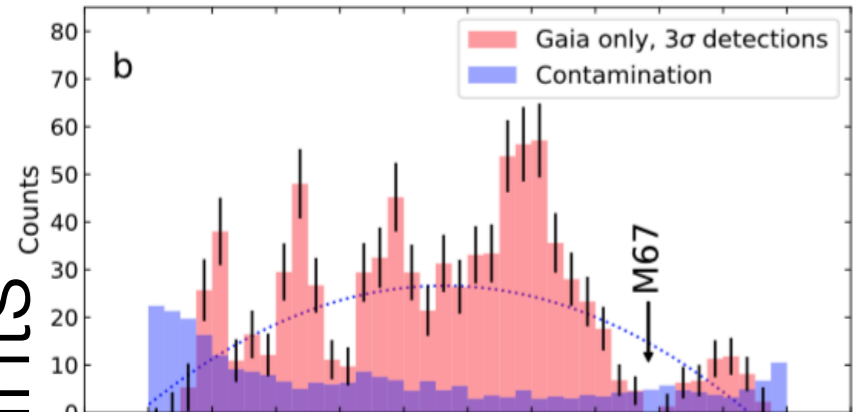
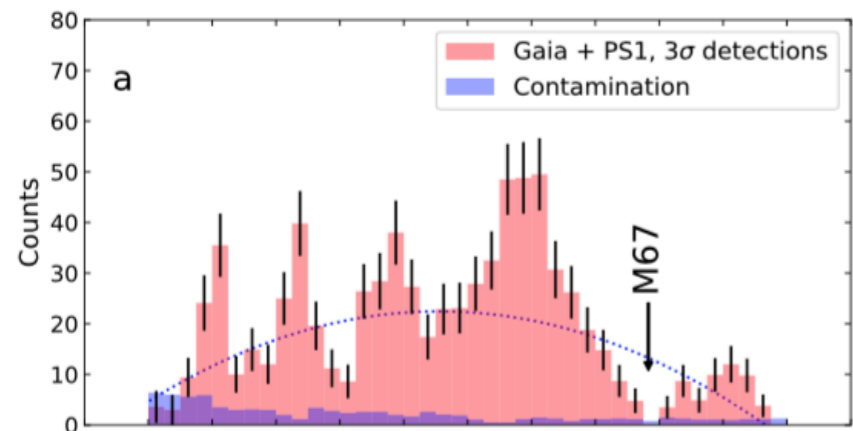
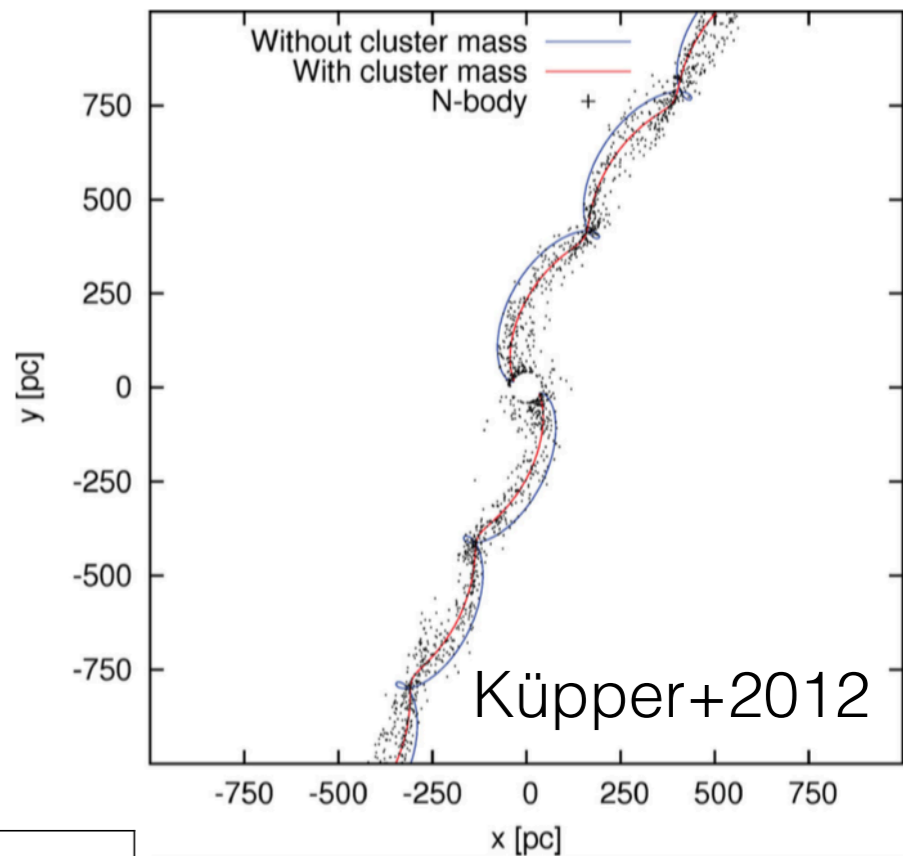


angle along stream



# Epicycles!

RI, Thomas, Famaey,  
Malhan, Monari (2020)

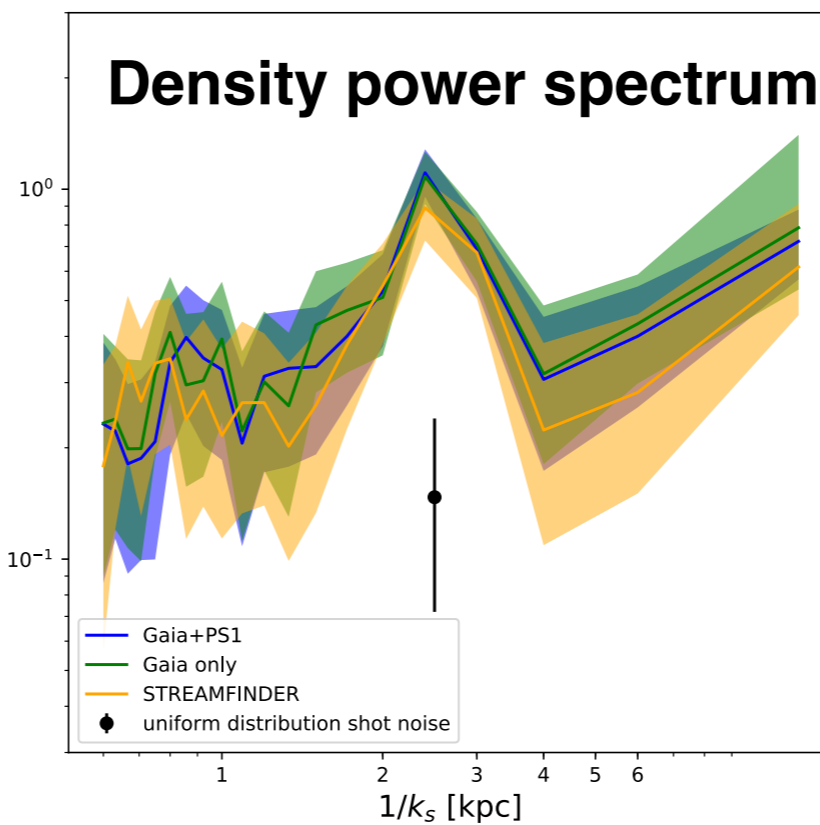


angle along stream

Power

$\sqrt{P_{pp}}$

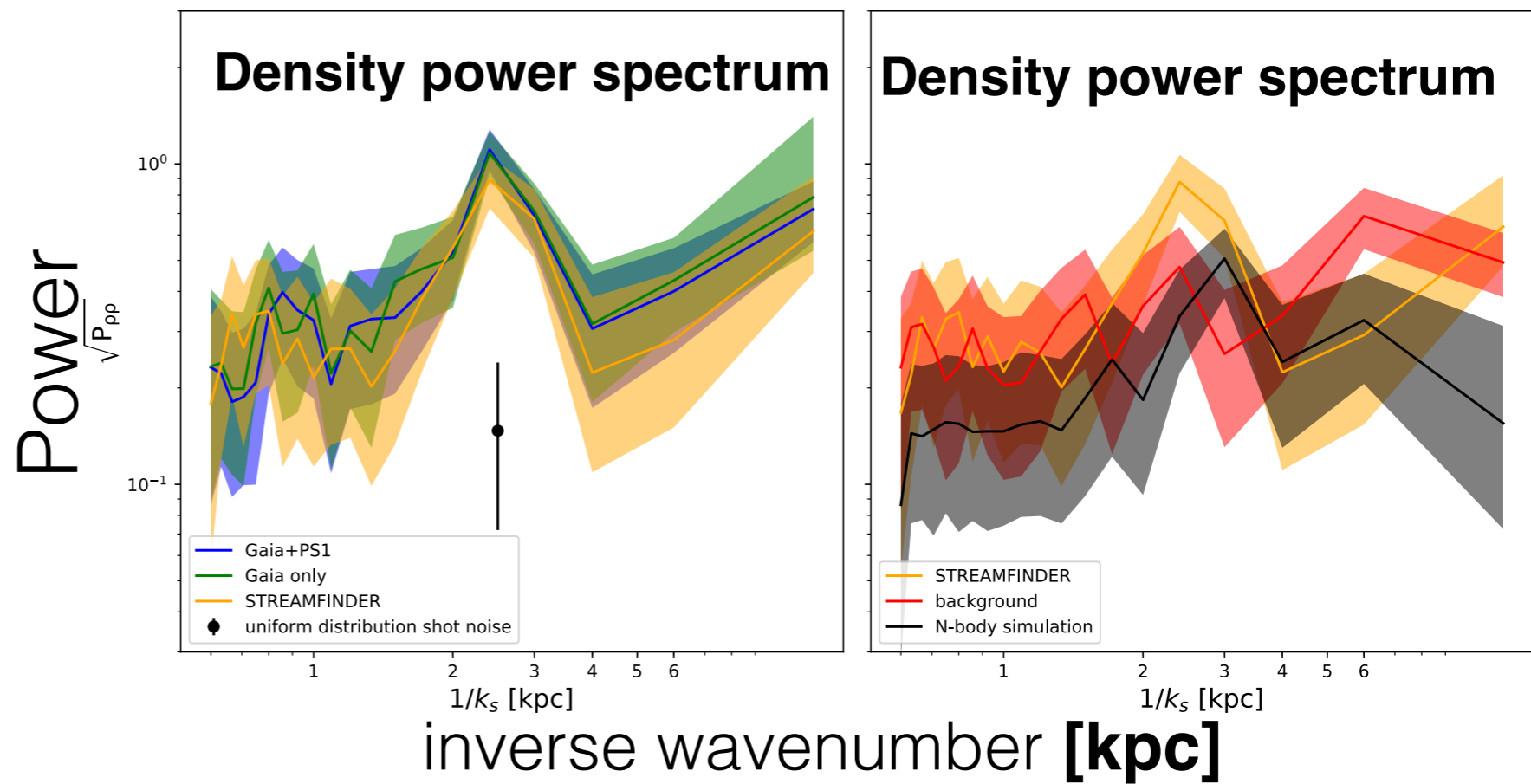
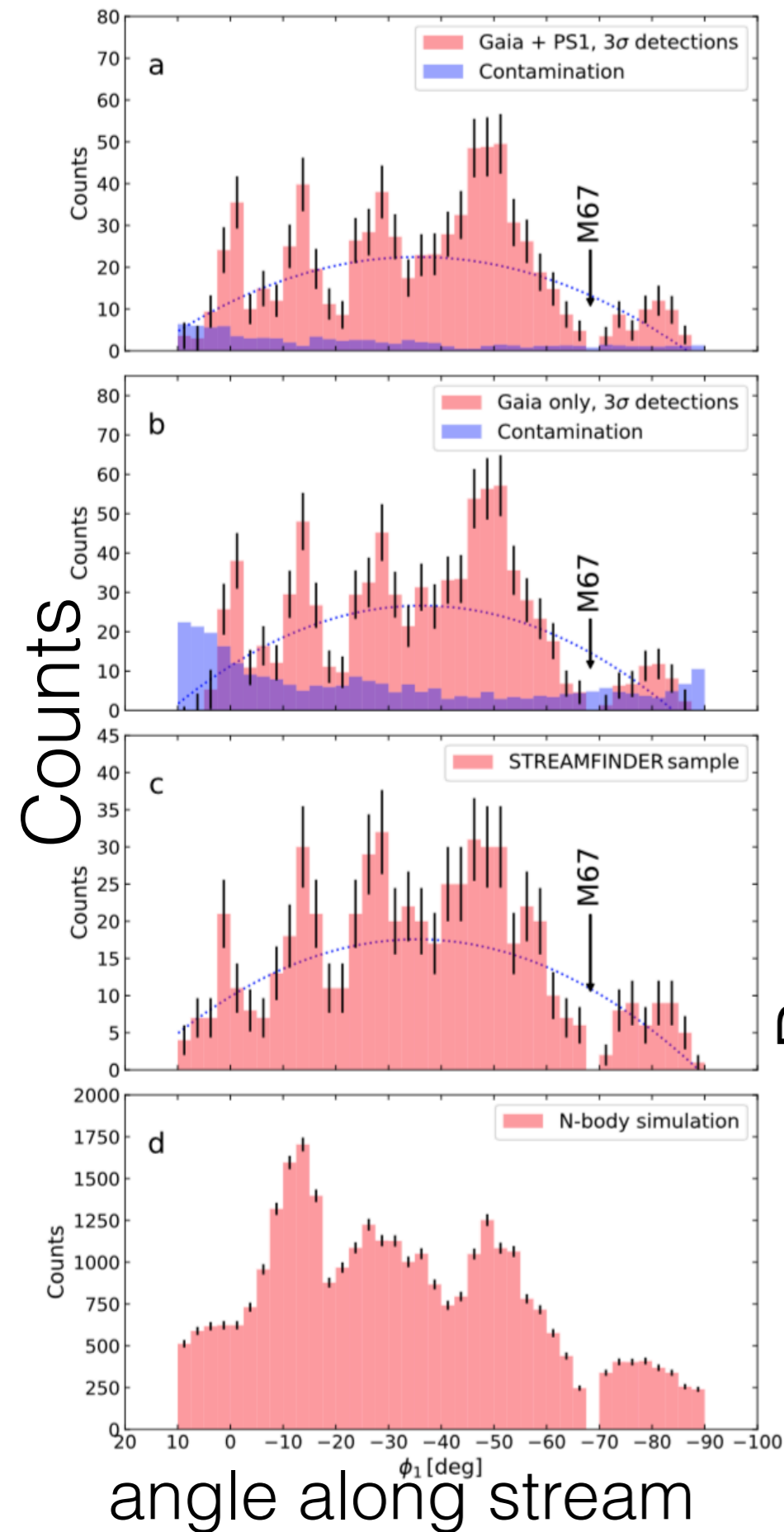
## Density power spectrum



inverse wavenumber [kpc]

# Epicycles!

RI, Thomas, Famaey,  
Malhan, Monari (2020)



Epicycles, not LCDM subhalo flybys....!

# Wrapping up...

- Stellar streams hold much promise as probes of the dark matter
- Have developed a Deep Learning algorithm to find canonical transformations  $(x, v) \rightarrow (\theta, J)$ , and recover  $\Phi$  and  $H(J)$ . Now works with realistic stream (6D+5D) astrometric observables.
- This is a fairly generic method that may work throughout much of physics!
- Very soon: Galactic acceleration field from streams
- TBD: Confront acceleration field with predictions of different theories of dark matter & gravity
- TBD: Confront stream properties with predictions of DM & gravity theories

